Dissipation in thin narrow superconducting films
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1. Dark counts in photon detectors (dissipation in thin films).
2. Dissipation in superconducting wires, ALMH theory.
3. Dissipation due to vortex motion in thin films.

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Motivation: dark counts in photon superconducting detectors

- Bias current close to $I_c$ for sensitivity.
- Counts without photons (dark counts), $V_{dc} > 0$.
- Origin: vortex motion across strip

NbN
$T_c = 15$ K

$\xi = 4$ nm
$W = 120$ nm
$d = 5$ nm

Kitaigorsky
Sobolevsky

Reduced bias current
Dissipation in thin wire, length $L$, ALMH theory.

\[ \Psi_n(x) = \Psi_0 e^{i\kappa_n x + \phi}, \quad \kappa_n = \frac{2\pi n}{L}, \]

\[ I = I_0 \kappa_n (1 - \kappa_n^2), \quad \kappa_{\text{max}} = \frac{1}{\sqrt{3}}. \]

- Change of current: $\kappa_n \rightarrow \kappa_{n-1}$.
- Phase changes locally inside normal region by $2\pi$.
- Saddle point solution:

\[ \Psi_{s,n \rightarrow n-1} = [\alpha \tanh(\alpha x / \xi \sqrt{2}) - i \kappa \sqrt{2}] e^{i\kappa x + \phi}, \quad \alpha = (1 - 3\kappa^2)^{1/2}. \]

- Barrier

\[ \Delta F_{n \rightarrow n-1} = \frac{\sigma H_c^2 \xi}{8\pi} f(\kappa) \tan^{-1}(\alpha / \kappa \sqrt{2}), \quad R = \Omega e^{-\Delta F / T}, \]

\[ \Omega = \frac{L}{\tau_{TDGL} \xi} \left( \frac{\sqrt{2\sigma H_c^2 \xi \alpha^{3/2}}}{24\pi^4 T} \right)^{1/2} \sqrt{\epsilon_1 s} \left[ \epsilon_{n1} \epsilon_{n2} \prod_{m=4}^{\infty} \epsilon_{nm} \epsilon_{sm} \right]^{1/2} \approx \frac{1}{\tau_{TDGL}} \left[ \frac{\Delta F_{n \rightarrow n-1}}{T} \right]^{1/2}. \]

[Fluctuation factor]
Thin film, length $L$, width $w$, $\xi \ll w$, $\Lambda = 2\lambda_L^2/d$

- Pearl vortex energy inside strip (kinetic energy of currents), $A \propto 1/\Lambda \to 0$.

$$I_0 = \frac{c\Phi_0}{8\pi\Lambda}$$

$U$ in units

$$\mathcal{E}_0 = \frac{\Phi_0^2}{8\pi^2\Lambda}$$

- Phase difference between $y$-edges for $L \gg w$, $\varphi(L/2) - \varphi(-L/2) = 2\pi x$.

- Vortex moving $x=0$ and $x=w$ changes phase difference by $2\pi$.

- Saddle point solution: vortex at maximum of the potential at given current.

- Barrier: $\Delta F_{n \to n+1} = \mathcal{E}_0 \ln \frac{2wI_0}{e\pi\xi I}$, $I \ll I_0$, $I_c = I_0 \frac{2w}{e\pi\xi}$. 

V. Kogan
Attempt frequency and fluctuation factor

- Rate of vortex crossings: \( R_{n\to n-1} = \Omega e^{-\Delta F/T} \),

- DC voltage given by Josephson relation: \( V = (\hbar / 2 e_{el}) R_{n\to n-1} \).

- Attempt frequency: \( \Omega \approx \frac{L}{\tau_{TDGL} \xi} \left[ \varepsilon_{n2} \varepsilon_{n3} \prod_{m=4}^{\varepsilon_{nm}} \varepsilon_{sm} \right]^{1/2} \cdot \varepsilon_{sm} > \varepsilon_{nm} > 0 \).

- Exponential factor: \( e^{-\Delta F/T} = (2\nu / \pi)^{1/2} (I / I_c)^\nu \), \( \nu = \varepsilon_0 / T \),

- Near critical current exponential dependence: \( (I / I_c)^\nu \approx \exp[\nu(I / I_c - 1)] \).

- Estimate for fluctuation factor (numerical calculations are in progress):

\[
\left[ \varepsilon_{n2} \varepsilon_{n3} \prod_{m=4}^{\varepsilon_{nm}} \varepsilon_{sm} \right]^{1/2} \propto \exp \left( - \frac{r w^2}{\xi^2} \right), \quad r(I) = ?
\]

- 1D: fluctuation factor is of order unity \( w \approx \xi \).

- Strip: fluctuation factor diminishes strongly dissipation rate.