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TUNNELING OF QUANTUM SPINS

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A WKB formalism is presented whereby the tunneling rate of a quantum spin is obtained in the semiclassical limit when $\hbar \to 0$ and the spin quantum number $S \to \infty$ in such a way that $\hbar S$ remains constant. The main idea is to single out one of the anisotropy axes, say the z-axis, to work in a representation with S_z, the z-component of the spin, diagonal and to describe quantum tunneling as a hopping process *on the spectrum* of S_z. This formalism enables us to efficiently handle tunneling problems, to incorporate dissipation, and to prove that the tunneling rate is universal, i.e. independent of the particular form of the anisotropy.

1. Introduction

The penetration of a particle in a classically forbidden region, i.e. tunneling, is one of the most striking manifestations of quantum mechanics [1]. It comes about as a direct consequence of the Schrödinger equation and is described most easily in terms of the semiclassical treatment of Wentzel [2], Kramers [3], and Brillouin [4] (WKB). Nearly sixty years after its conception the subject has obtained a considerable maturity but, surprisingly, a comprehensive treatment of quantum spins has never been attempted. Quantum spins do present, however, a fascinating challenge and it is the aim of the present paper to meet this challenge.

Through the early work of Dirac [5] and the introduction of the path integral by Feynman [6] the semiclassical motion of a particle can be interpreted as a small deviation from its *classical* path, which is a stationary point of the action functional $S(q, \dot{q}, t)$ and, as such, the dominating contribution to the path integral [7]

$$\int \mathcal{D}[q(t)] \exp \frac{\mathrm{i}}{\hbar} \int_0^t \mathrm{d}s\, L(q, \dot{q}, s) = \int \mathcal{D}[q(t)] \exp \frac{\mathrm{i}}{\hbar} S(q, \dot{q}, t)\,, \tag{1.1}$$

in the limit $\hbar \to 0$. Here L denotes a Lagrangian. In the classically allowed region, $S(q, \dot{q}, t)$ is real and in the limit $\hbar \to 0$ the classical path appears via a stationary phase argument. In the classically forbidden region, however, where tunneling becomes important, the action is imaginary and the stationary phase argument does not apply directly. In spite of that, tunneling has found an appealing interpretation in the context of functional integration through the instanton technique [8].

In his epoch-making 1948 paper [6] Feynman already indicated that quantum spins do not allow a satisfying path-integral representation. Accordingly, a semiclassical treatment is hard to obtain. See Schulman [7] for an extensive discussion of this problem. The peculiar aspects of a quantum spin are already brought out by its commutation relations,

$$[\hat{S}_x, \hat{S}_y] = \mathrm{i}\hbar \hat{S}_z\,, \quad \text{and cycl.}\,, \tag{1.2}$$

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Let us consider the Hamiltonian

$$\hat{\mathcal{H}} = -F_0(\hat{S}_z) - \frac{1}{2}\sum_{n=1}^{N}[F_n(\hat{S}_z)\hat{S}_+^n + \hat{S}_-^n F_n(\hat{S}_z)]\,. \tag{7.2}$$

van Hemmen & Sütö, Europhys. Lett. **1** (1986) 481-490
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Motivated by spin-glass work [JLvH, PRL **49** (1092) 409] and, in particular, by the observation of John Mydosh (Leiden) that in generic spin glasses such as *Fe*Au the Fe ions often form ferromagnetic clusters of typical size 5-20 and hence dominate the relaxation of magnetization, we got interested in the problem of spin quantum tunneling. At low temperatures the magnetic moments constitute giant spins that only can tunnel. How could we explain this? The answer has been given by the above two papers through developing a full-blown WKB formalism for spins.

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WKB for quantum spins

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Abstract

A cluster of ferromagnetically coupled magnetic moments behaves at low temperatures as a single spin with large spin quantum number. Here we show the simplicity of a Wentzel–Kramers–Brillouin analysis of spin quantum tunneling in a magnetic anisotropy.

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Gamow [1,2] explained alpha decay as a consequence of tunneling particles in 1928, at the dawn of quantum mechanics. The first papers analytically solving the problem of spin quantum tunneling and the associated level splitting [3–5] appeared as late as 1986, one of them in Physica B. In retrospect, they proved to be the first publications in a new and prosperous field of research. Enz and Schilling [3] mapped a specific Hamiltonian onto a particle problem and applied instanton techniques whereas the present authors generalized the Wentzel–Kramers–Brillouin (WKB) method so as to be generally applicable to the spin case; see [4–8] for additional information. In passing we note that Gamow [1] also used WKB.

As it behooves a good scientific paper, our original work [5] contained a whole lot of details—description of the classical motion, approximate solutions, generalizations, four long appendices—that served to familiarize the reader with the in those days novel notion of spin tunneling. As we realized later, it could also hide the basic simplicity of the underlying ideas. Since 1986 there has been a kind of competition between instanton and WKB description in various applications. In addition, papers

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Indeterminacy and Image Improvement in Snake Infrared „Vision“

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Overview

1. Biological introduction
2. Optics of the “pithole camera”
3. The reconstruction algorithm

A.B. Sichert, P. Friedel, and J.L. van Hemmen
Phys. Rev. Lett. **97** (2006) 068105

B. Schwarzschild, Physics Today IX.06, pp. 18-20

Two types of organs

Groove organs (boid snakes)

Pit organs (pit vipers)

Organ size ≈ 1 mm.

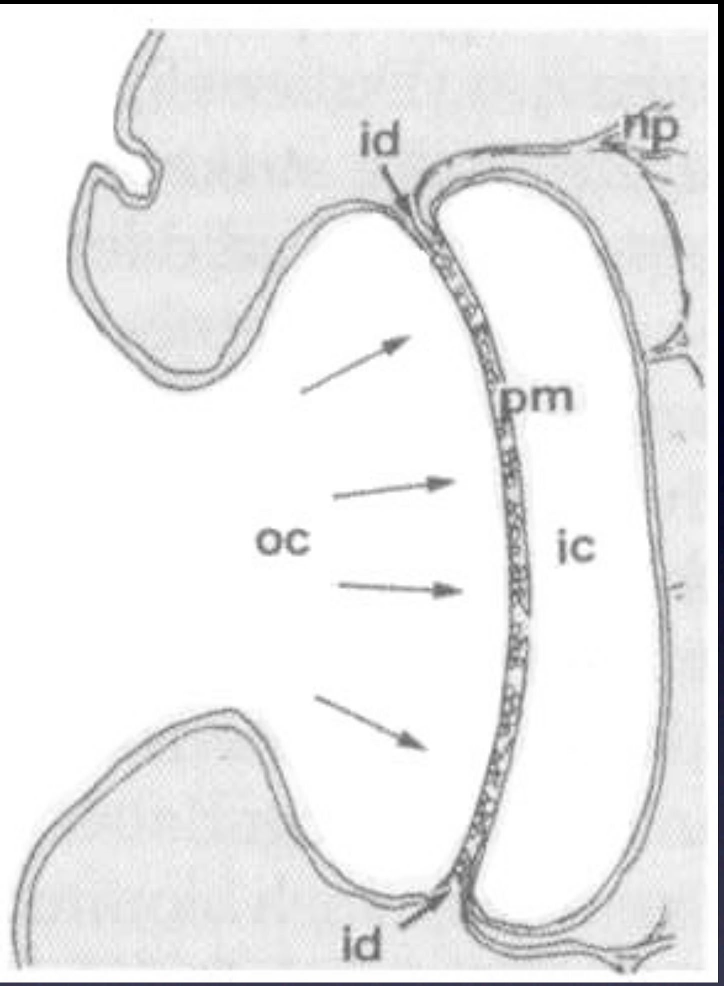
Sensitive to temperature variations down to 1 mK.

Neuronal map is built in *optic tectum*.

Film courtesy: Guido Westhoff (Bonn)

Using infrared vision snakes can make an *accurate* representation of their surroundings.

The pit organ



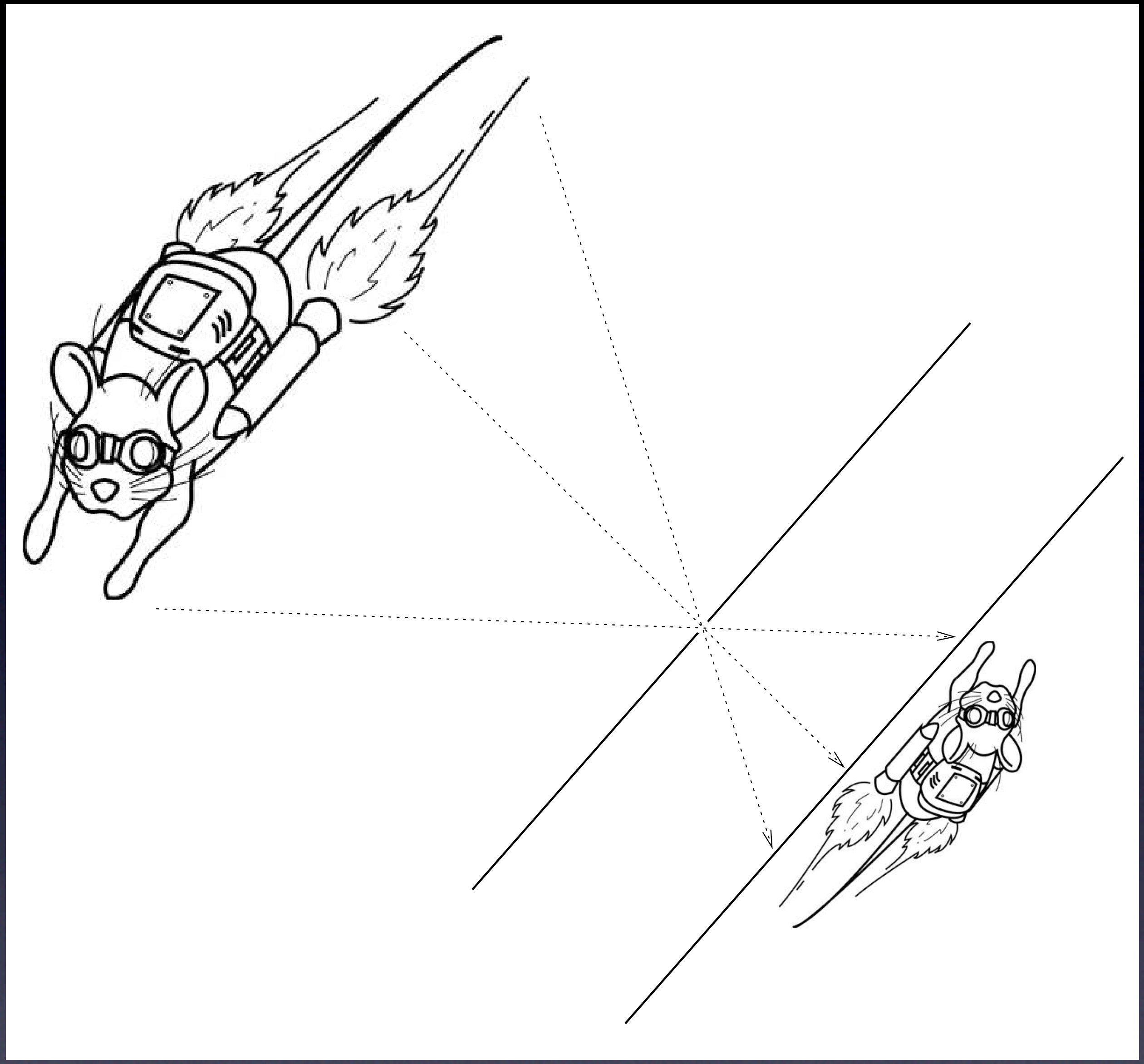
id = intradermal receptors
oc = outer cavity
ic = inner cavity
pm = pit membrane
np = plexuslike structure

Freely suspended membrane enables high sensitivity.

Organ aperture (pinhole) is wide → poor optical quality.

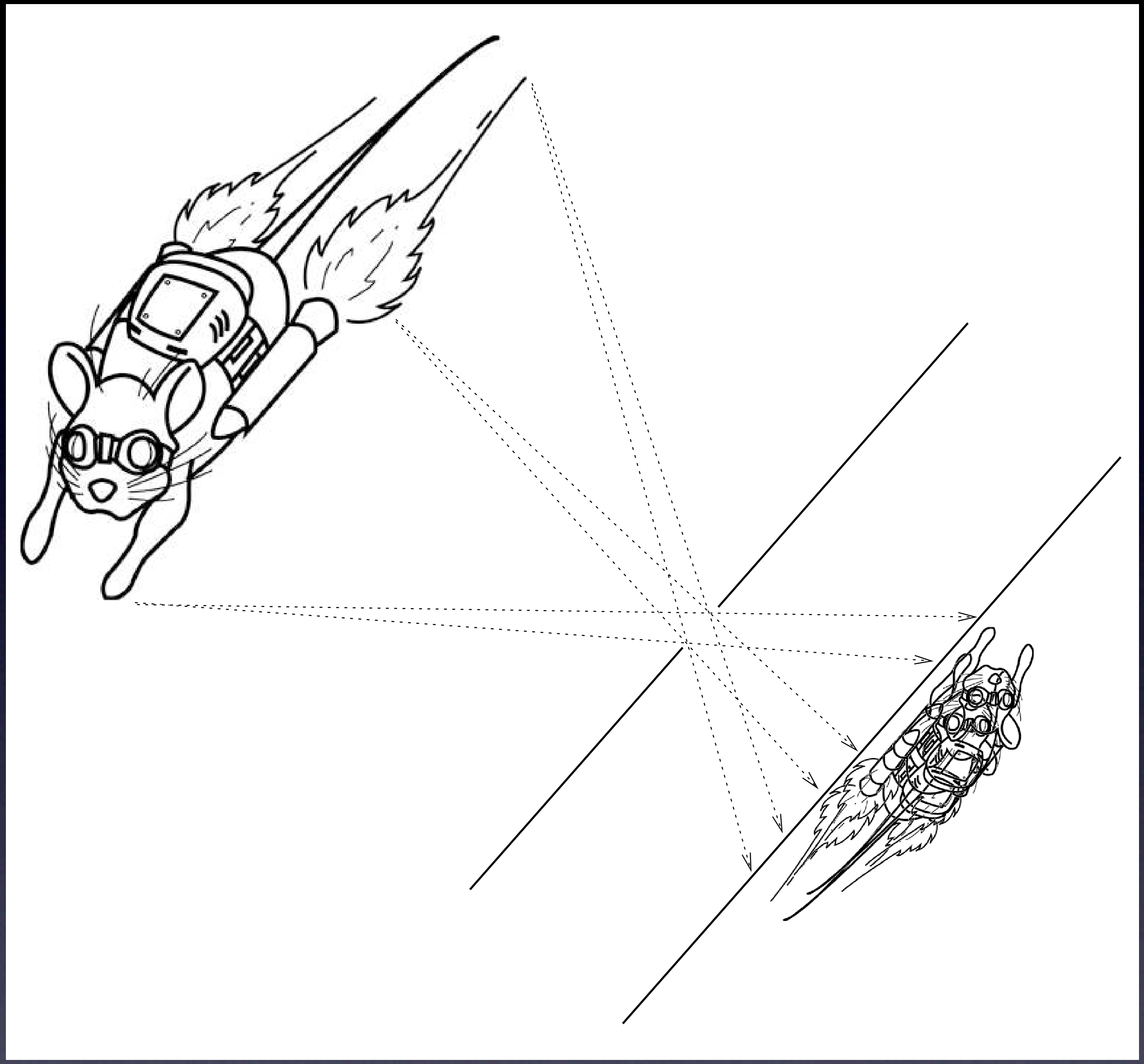
Typically 40x40 receptors on pit membrane.

How does a pinhole camera work?



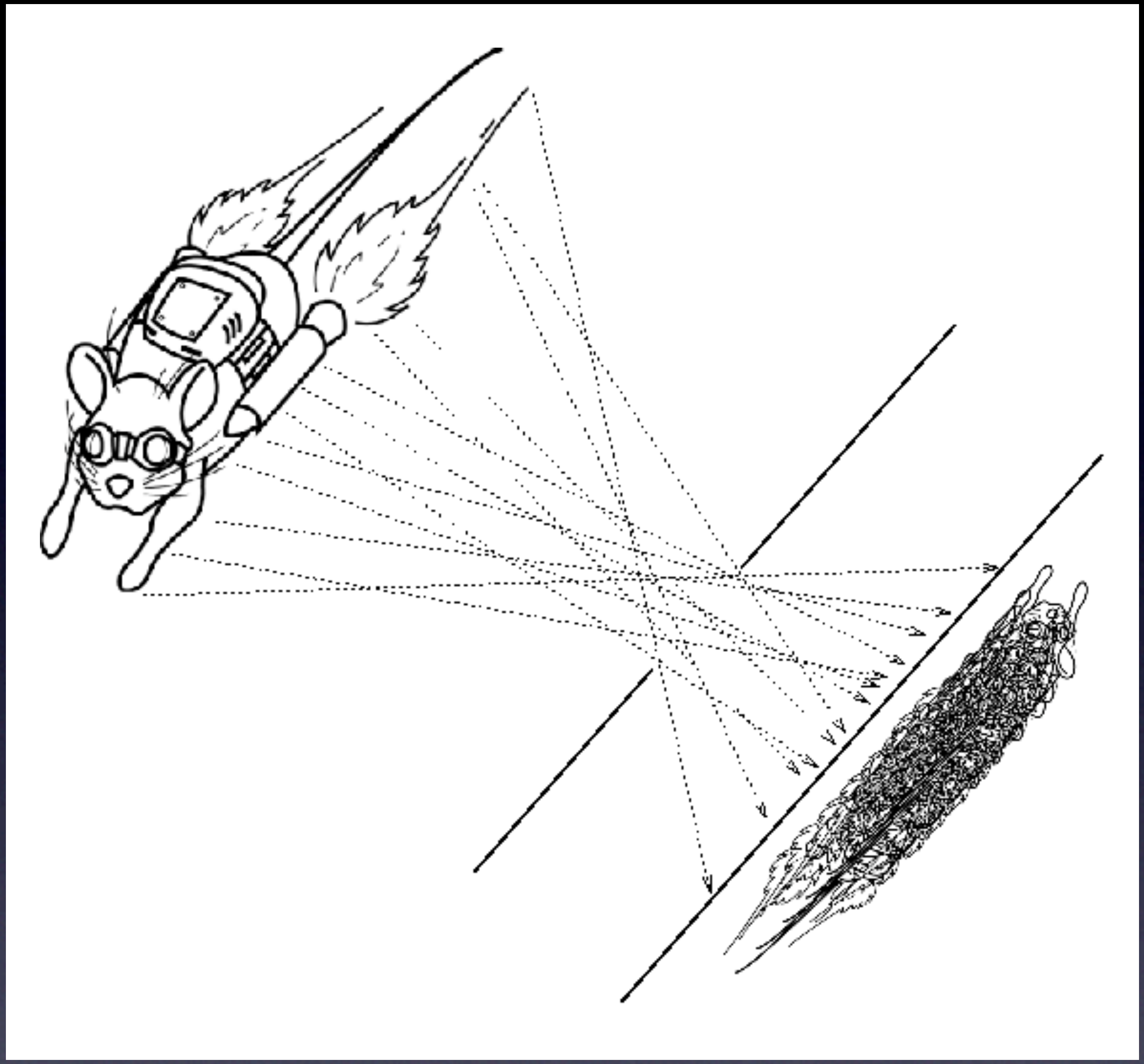
If we use a small pinhole, every point in space projects onto *one* membrane point.
We get a *sharp* image.

How does a pinhole camera work?



If the pinhole is slightly larger, the projection is not one-to-one anymore.
The image will be somewhat blurred.

How does a pinhole camera work?

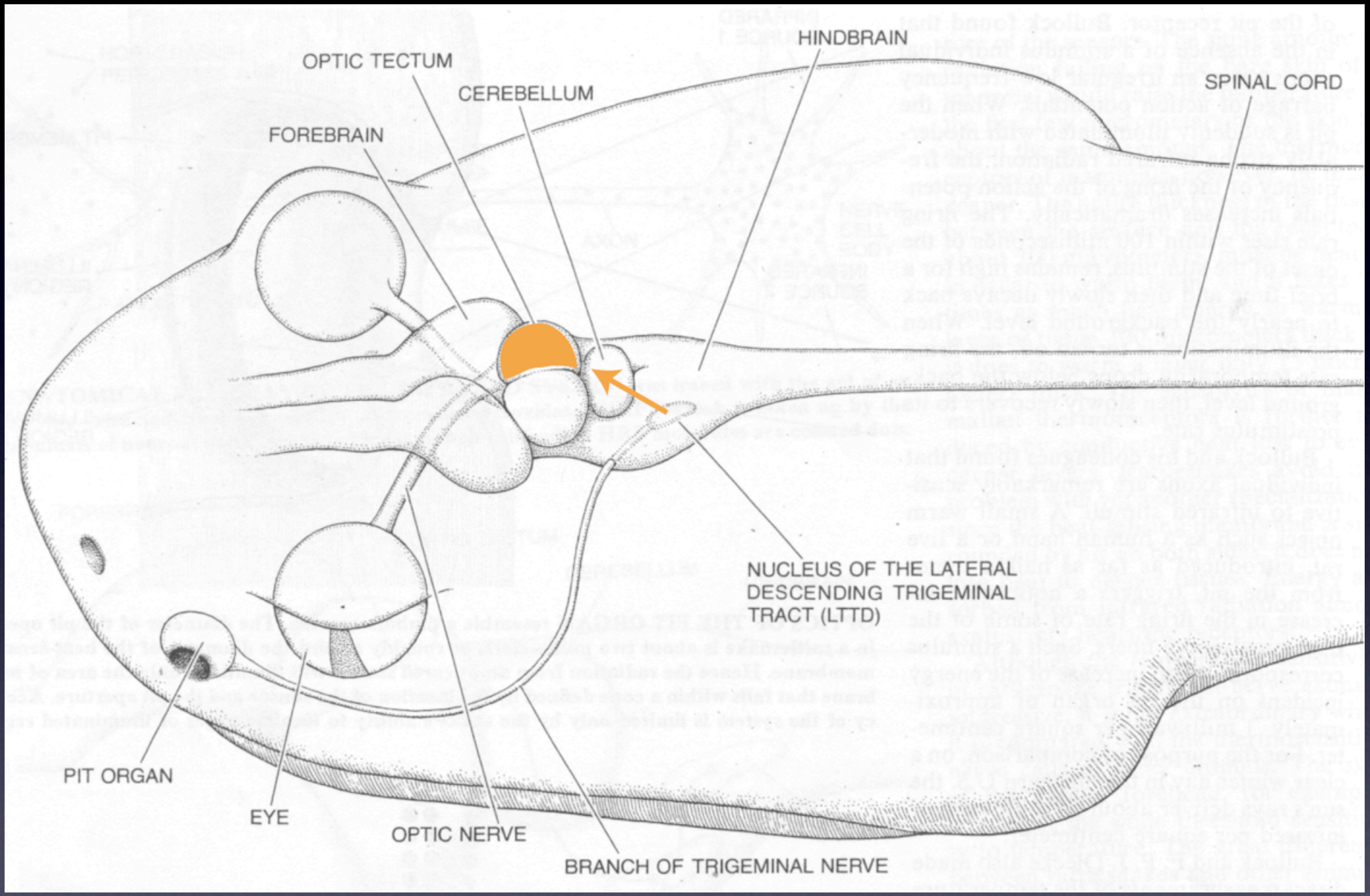


If the pinhole is very large, every membrane point receives input from many spatial locations.
The image is completely blurred.

Why is the organ aperture so large?

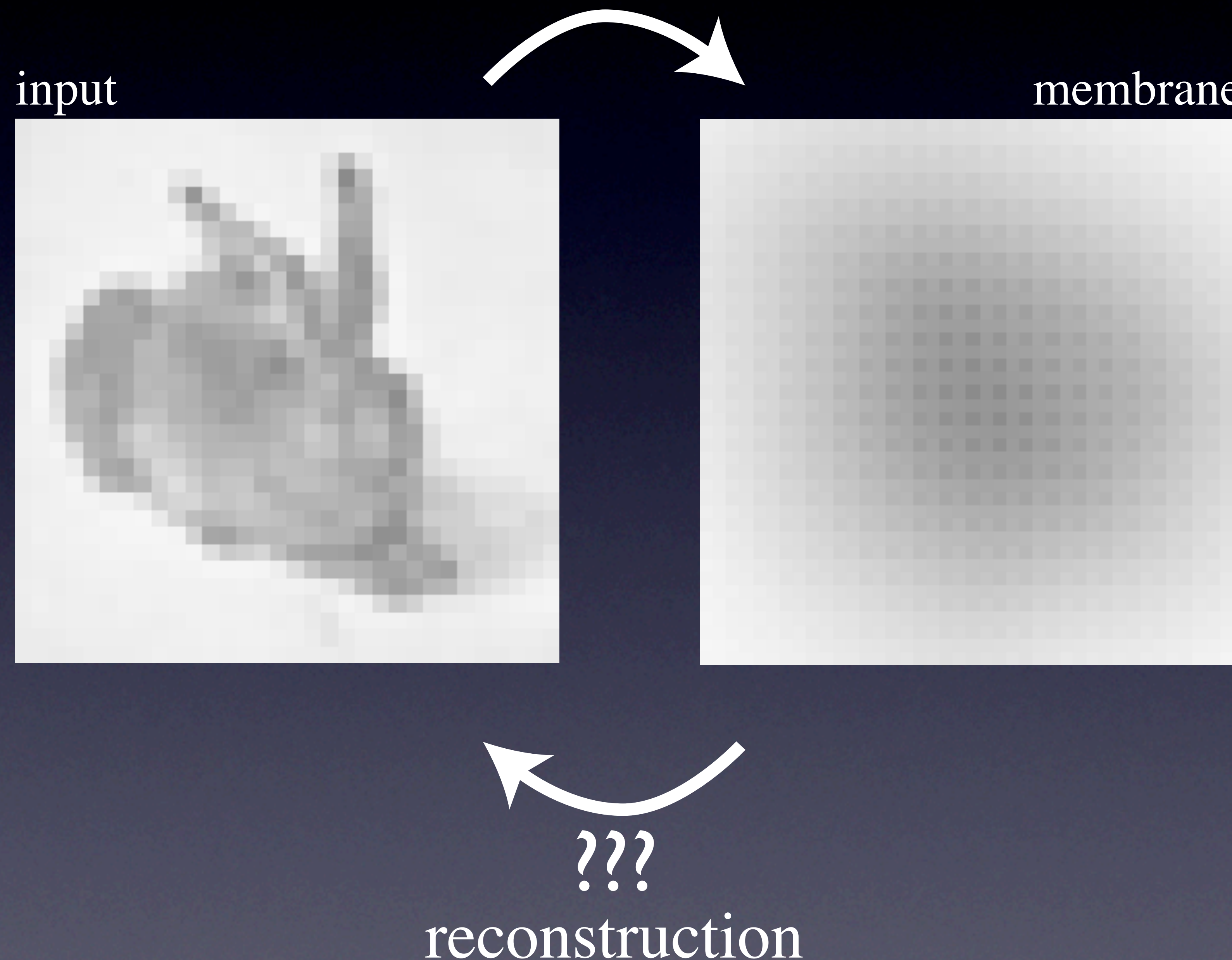
A very small pinhole can make very sharp images but...

The pit organ



Pit-membrane receptors project into the *optic tectum*, where the *visual* and infrared maps are “merged”.

Optical quality of the organ



Since the pit organ does not have a lens, the snake needs to *reconstruct* the input image *neuronally* so as to obtain a high quality image of its environment!

Reconstruction algorithm: the *Virtual Lens*

1. Mapping between input *I* and membrane-receptor response *S* is known and linear, but contains noise,

$$S_\alpha = \sum_\beta T_\alpha{}^\beta (I_\beta + \psi_\beta) + \chi_\alpha$$

2. Goal: finding an optimal reconstruction of the input using the sensory response at the pit membrane,

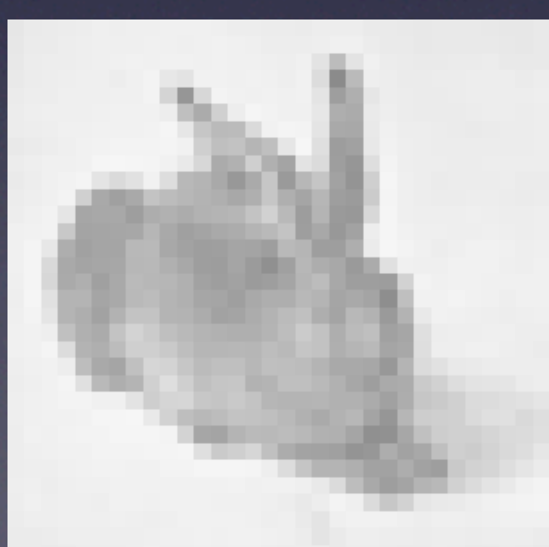
$$\hat{I}_\alpha = \sum_\beta R_\alpha{}^\beta S_\beta$$

The mapping

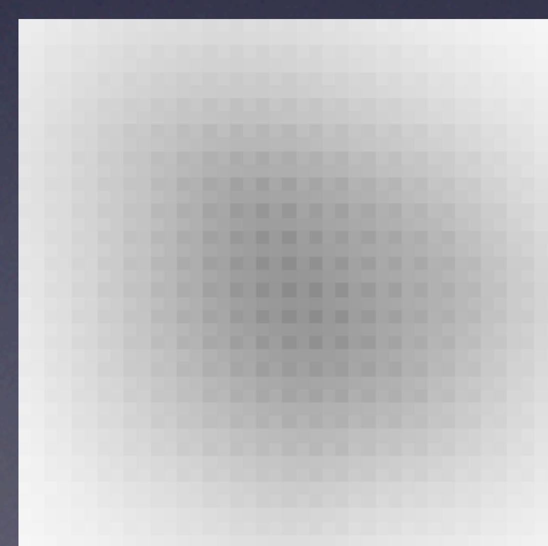
A 2-dimensional projection of the heat distribution in space is mapped onto the pit membrane.

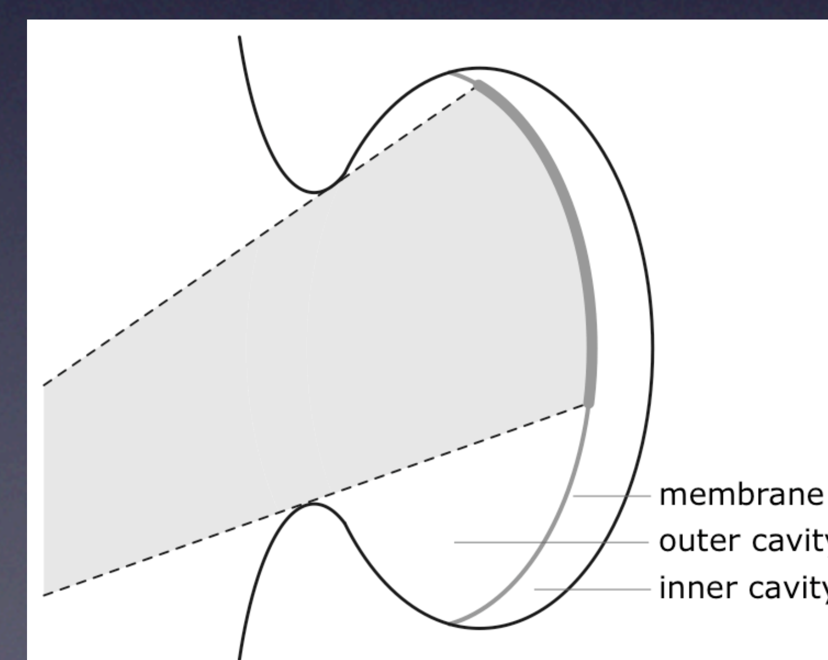
The transfer function between a point in space α and a membrane point β determines the intensity on the membrane,

$$
{T_\alpha}^{\beta} = \begin{cases} \mathbf{r}_{\alpha\beta}^{-2} \cos\phi_{\alpha\beta} & \text{if } \alpha \text{ is visible from } \beta, \\ 0 & \text{otherwise} \end{cases}
$$



$$\xrightarrow{{T_\alpha}^{\beta}}$$





The optimal reconstruction

1. Define the quadratic error as

$$E = \left\langle \left| \hat{I}_\alpha - I_\alpha \right|^2 \right\rangle$$

2. Minimizing the error leads to an optimal reconstruction in terms of the matrix equation

$$\sum_\beta R^\alpha{}_\beta \left[\sum_\delta T_\gamma{}^\delta T^\beta{}_\delta (1 + \tau^2) + \sigma^2 \delta^\gamma_\beta \right] = T_\gamma{}^\alpha$$

where σ and τ represent the noise level.

Biological implementation

The snake must "know" what type of image to expect

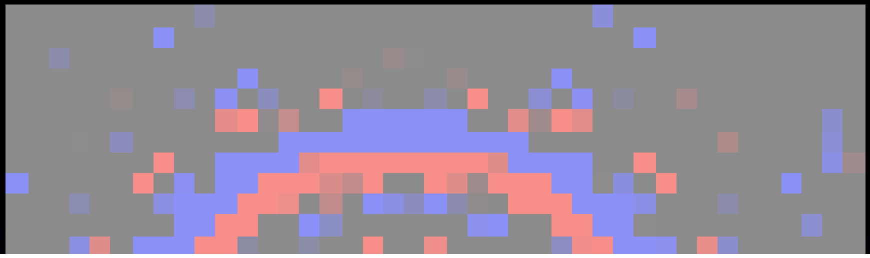
$\longrightarrow$ Learn σ and τ so as to match typical input

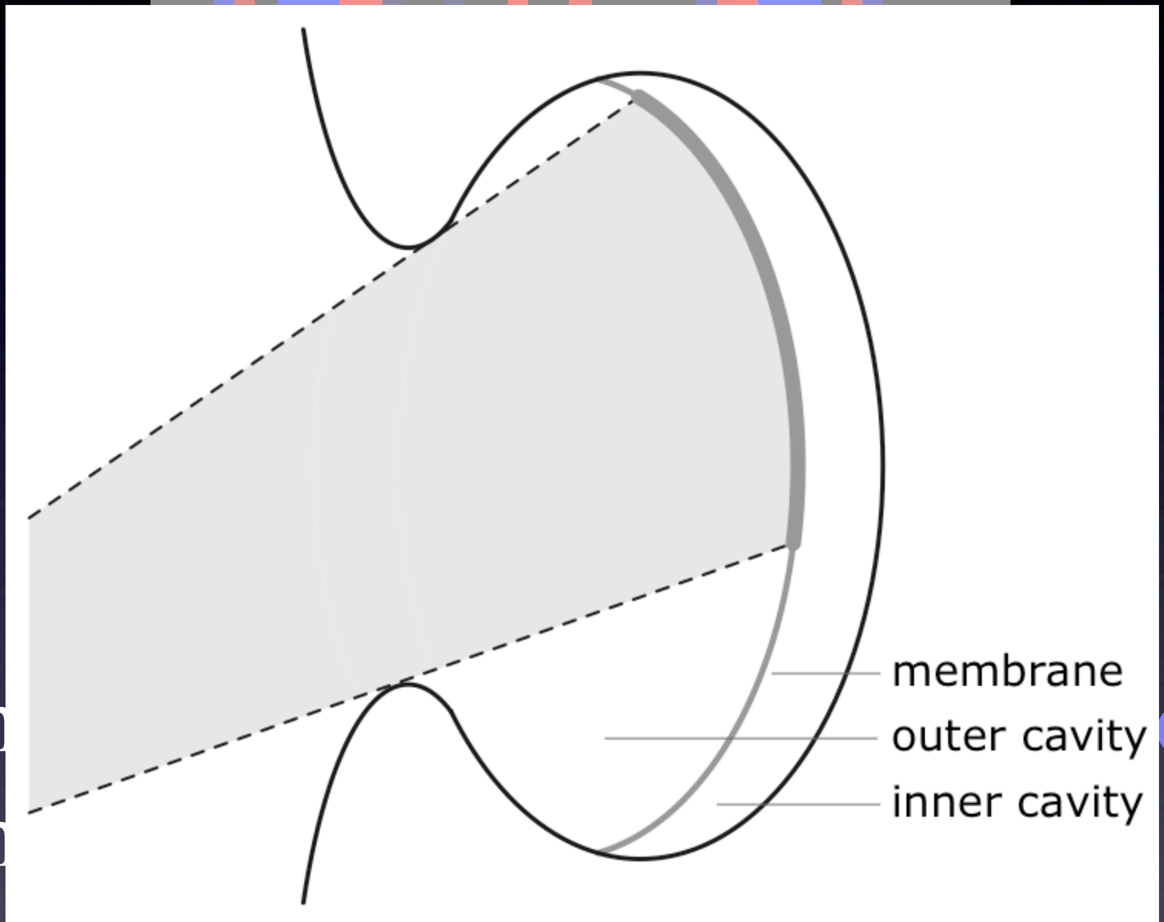
Once σ and τ have been learned,
reconstruction is just matrix multiplication,

$$\hat{I}_\alpha = \sum_\beta {R_\alpha}^\beta S_\beta$$

Advantage:
Calculations are easy and the model can be realized straightforwardly in a neuronal circuit.

Receptive field of the map neurons



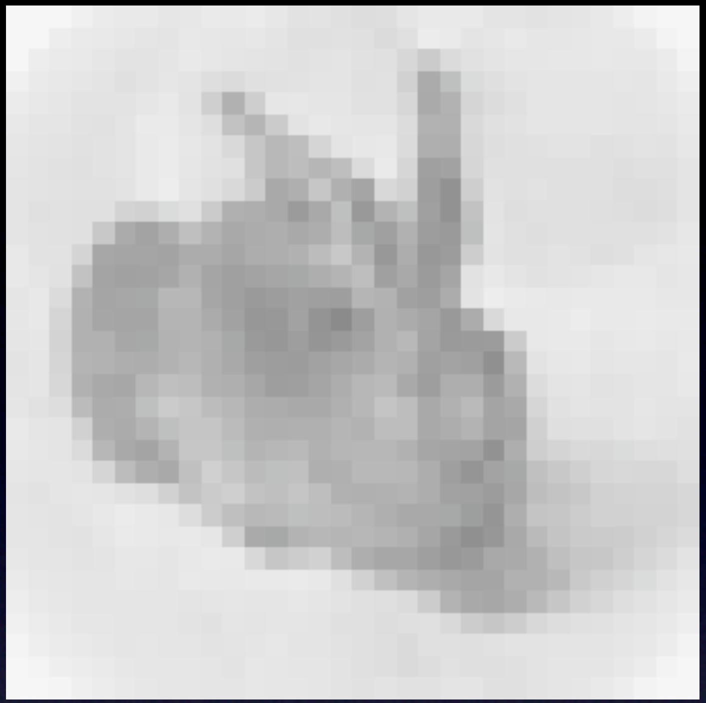


A map neuron e input
from the mem

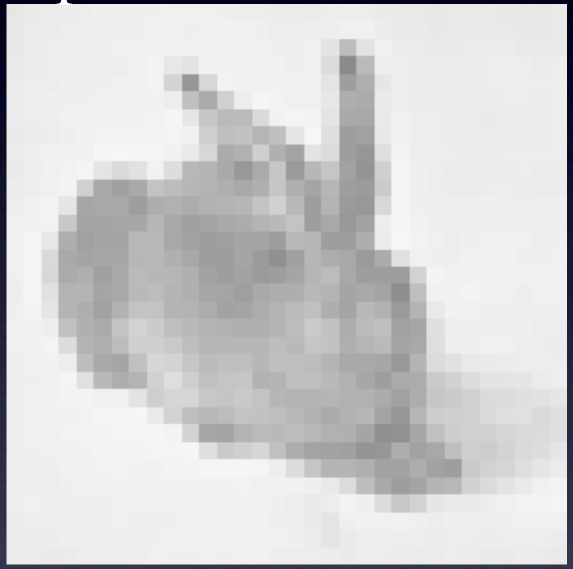
The model "recognizes" ring-like correlations, indicating a point source at the image pixel coded for by the map neuron.

Reconstruction performance

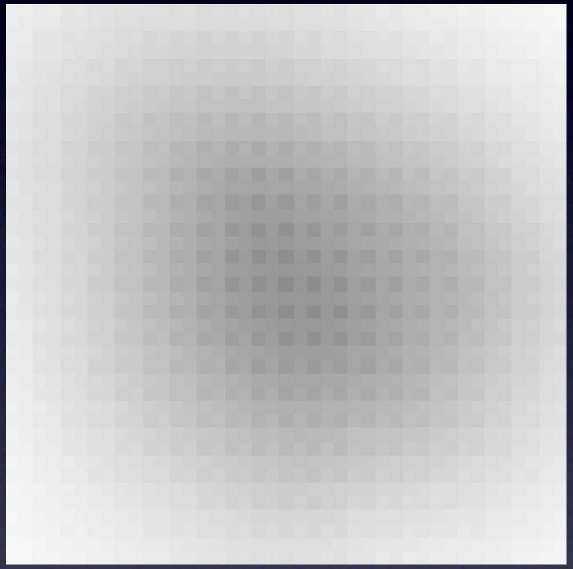
without noise



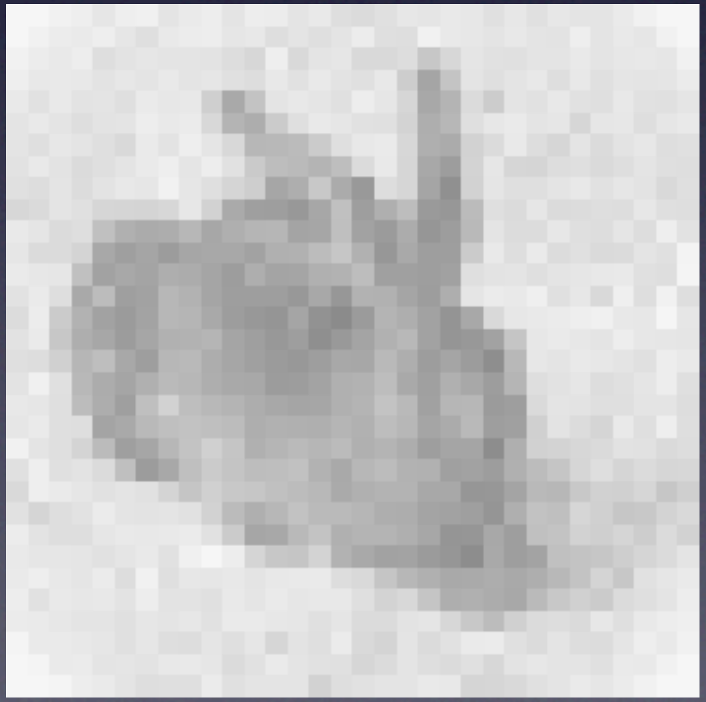
input



membrane



with noise



A collection of “edge detectors” for all positions *automatically* forms a complete reconstruction.

Influence of noise

Because of the wide organ aperture, sensitivity to detector noise is high.

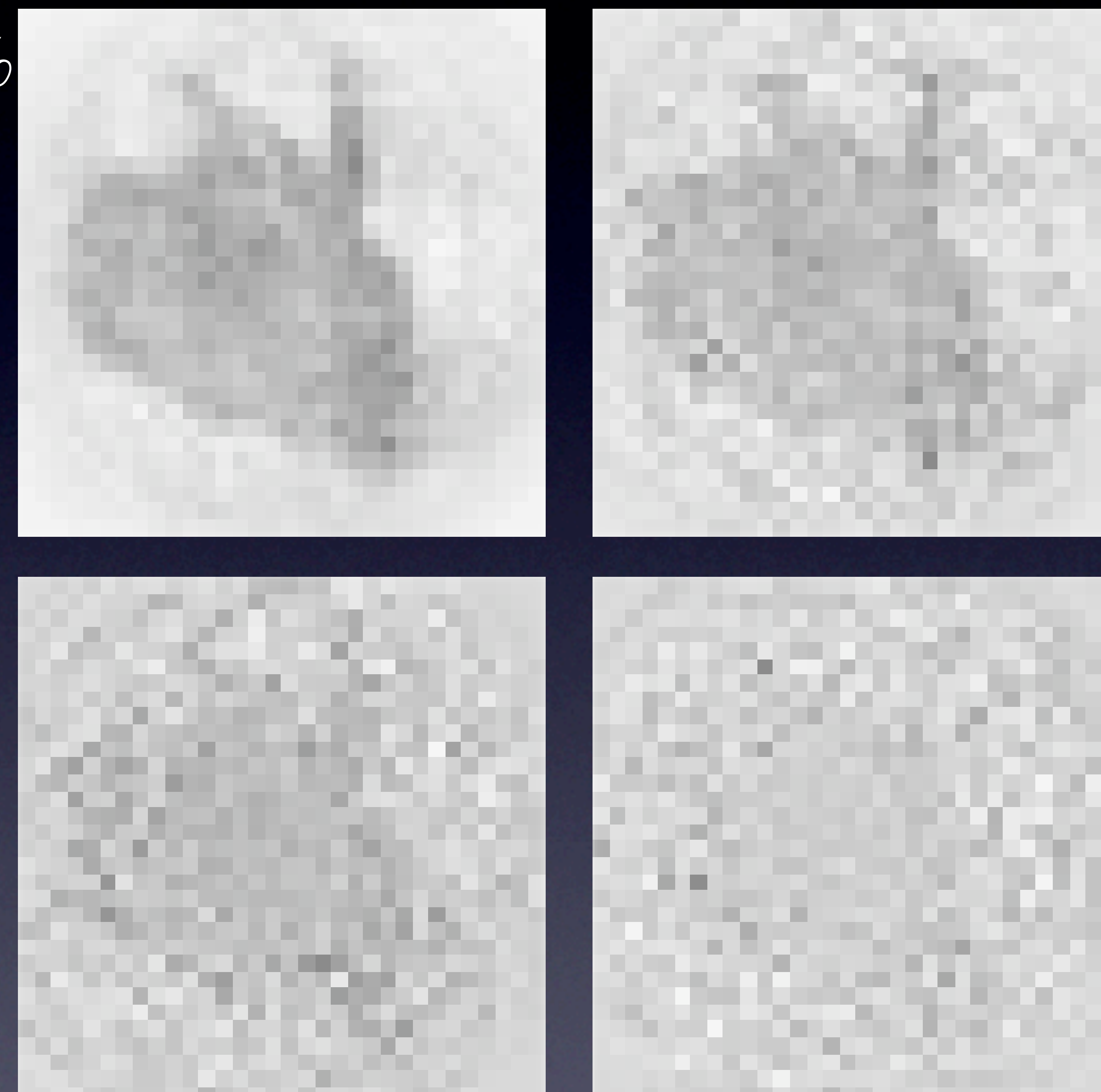
Maximum resolution ("one pixel") is about 3°, resulting from number of heat receptors.

0.25%

1.0%

2.0%

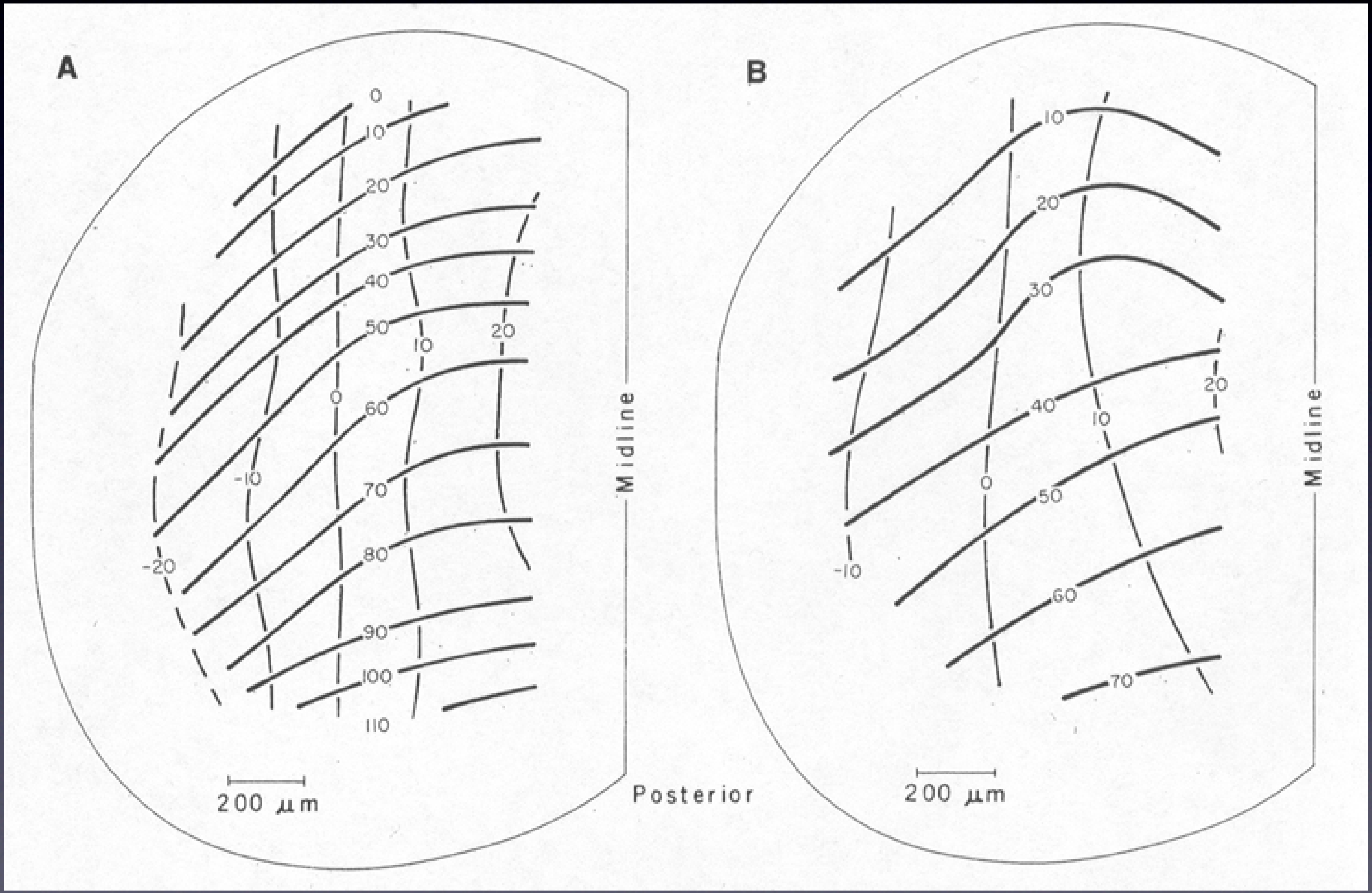
5.0%



Thresholding can still improve reconstruction quality.

Merging the senses

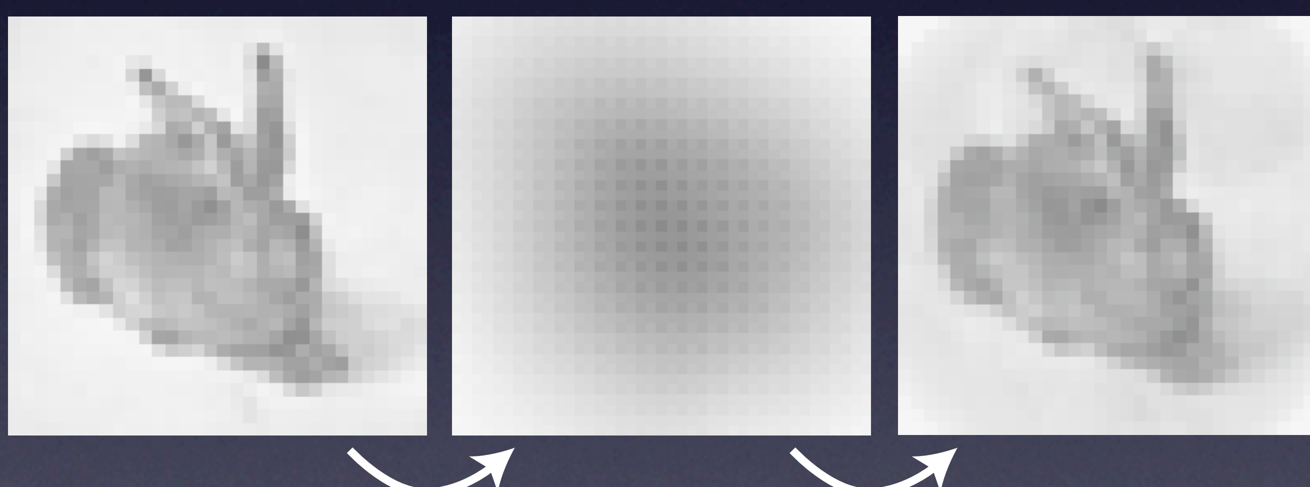
IR and visual map are aligned in the *optic tectum*.



vision IR

Merging the senses

input measurement reconstruction



Neuronal processing allows for good reconstruction despite bad measurement

Further processing: object recognition

Summary

High-quality image reconstruction from the blurred membrane heat distribution is neuronally feasible.

Resolution is limited to approximately 3° by the limited number of membrane heat receptors

The model explains the high thermal sensitivity (1 mK) of the snake pit organ.

Multimodal integration works in nature - but how?
How can we benefit from all this in technical applications?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