CUNY Common Core
Course Submission Form

Instructions: All courses submitted for the Common Core must be liberal arts courses. Courses may be submitted for only one area of the Common Core. All courses must be 3 credits/3 contact hours unless the college is seeking a waiver for another type of Math or Science course that meets major requirements. Colleges may submit courses to the Course Review Committee at any time. Courses must also receive local campus governance approval for inclusion in the Common Core.

College	Lehman College
Course Prefix and Number (e.g., ANTH 101, if number not assigned, enter XXX)	MAT 123
Course Title	Number Systems and Number Theory For Educators
Department(s)	Mathematics
Discipline	Mathematics
Credits	3
Contact Hours	3
Pre-requisites (if none, enter N/A)	Departmental permission
Co-requisites (if none, enter N/A)	n/a
Catalogue Description	Properties of counting numbers, integers, rationals and reals; elementary number theory. Operations, computations, and historical developments of these ideas also included. Note. Intended for pre-service elementary and middle school teachers.
Special Features (e.g., linked courses)	
Sample Syllabus	Syllabus must be included with submission, 5 pages max recommended

Indicate the status of this course being nominated:

☐ current course ☐ revision of current course ☒ a new course being proposed

CUNY COMMON CORE Location

Please check below the area of the Common Core for which the course is being submitted. (Select only one.)

Required	Flexible	
☐ English Composition	☐ World Cultures and Global Issues	☐ Individual and Society
☒ Mathematical and Quantitative Reasoning	☐ US Experience in its Diversity	☐ Scientific World
☐ Life and Physical Sciences	☐ Creative Expression	

Waivers for Math and Science Courses with more than 3 credits and 3 contact hours

Waivers for courses with more than 3 credits and 3 contact hours will only be accepted in the required areas of "Mathematical and Quantitative Reasoning" and "Life and Physical Sciences." Three credit/3-contact hour courses must also be available in these areas.

If you would like to request a waiver please check here:	☐ Waiver requested
If waiver requested: Please provide a brief explanation for why the course will not be 3 credits and 3 contact hours.	
If waiver requested: Please indicate whether this course will satisfy a major requirement, and if so, which major requirement(s) the course will fulfill.	

Learning Outcomes **In the left column explain the course assignments and activities that will address the learning outcomes in the right column.**	
B. Mathematical and Quantitative Reasoning: Three credits A course in this area must meet all the learning outcomes in the right column. A student will:	

This SLO is assessed by written assignments, quizzes, and exams throughout the semester. Throughout the course, students will be required to demonstrate that they know, can describe, and can interpret various types of real numbers (whole, integer, rational, irrational, and decimal) using multiple representations including graphs and tables.
 	
• Examples: Below are several example questions that students will be asked to solve in class, on written assignments, and on exams:
  • Represent the number 1.234 as a length; then using the base-ten structure represent this decimal as with bundled objects.
  • Ken ordered ¾ of a ton of gravel. He wants 25% of his order of gravel delivered now 75% delivered later. What fraction of a ton of gravel should Ken get delivered now? (Make drawings to describe the situation and to explain your answer).
  • Use the decimal representation of 1.777… to show that the square root of this number is rational. Then, sketch a picture showing the original number and its square root.
	Interpret and draw appropriate inferences from quantitative representations, such as formulas, graphs, or tables.

This SLO is assessed by written assignments, quizzes, exams, and in-class group work. Students will not only be expected to develop their algebraic thinking using variables and formulas, but they will also be expected to use models with different number system constraints. Additionally, students will need to understand, use, and describe the algebraic operations of addition, subtraction, multiplication, and division to solve problems and model situations,
 	
• Examples: Below are several examples of questions that will be used to assess students.
  • A lot of gumballs are in a glass container. The container is shaped like a box with a square base. When you look down on the top of the container, you see about 50 gumballs at the surface. When you look at one side of the container, you see about 60 gumballs. You also notice that there are about 9 gumballs against each vertical edge of the container. Given this information, estimate the total number of gumballs in the container. What assumptions are you making? How does the problem change if some of the gumballs have broken into pieces?
  • Suppose that 4 painters take 20 hours to paint a house. (Assume that all house painters work at the same steady rate.) Make a table to show the relationship between the number of house painters
	• Use algebraic, numerical, graphical, or statistical methods to draw accurate conclusions and solve mathematical problems.

and the number of hours it takes to paint the house. Include the case of 3 house painters in your table. • Show and describe how to write the fraction 5/8 as a decimal. Then, rewrite the fraction as the sum of fractions whose denominators are powers of 10.	

This SLO is assessed by in-class activities, homework assignments, quizzes, and exams. In the course, students will have to use various representations (e.g. diagrams, drawings, graphs, equations, and tables), conceptual models, and appropriate tools to solve problems.
 	
• Examples: Several examples of questions that students will be asked to solve in class, for homework, and on other assessments are below.
  • A restaurant server received a $7.00 tip on a meal he served. If this tip represents 20% of the cost of the meal, then how much did the meal cost? Solve this problem with the aid of a drawing and using a table.
  • Which of the following mixtures will be saltier: 3 tablespoons of salt mixed in 4 cups of water or 4 tablespoons of salt mixed in 5 cups of water? Explain your answer in at al east 2 different ways.
	• Represent quantitative problems expressed in natural language in a suitable mathematical format.

This SLO is assessed by classroom discussion activities, group work, quizzes, and exams. In the course, students will have to explain, describe, and effectively communicate similarities and differences between the various number systems to audiences of varied mathematical maturity: K-8 learners, 9-12 learners, and college-level peers. Students will also have to familiarize themselves with and interpret common mathematical errors made by elementary and middle school students.
 	
• Examples: Some problems and questions that students will be expected to address are as follows:
  • Use the scaffold method to calculate 72,125 divided by 31. Be sure to explain how other operations like addition, subtraction, and multiplication are used to solve this problem using this method.
  • Write equations with numbers in expanded form showing how to regroup the number 104 so that 69 can be subtracted from it. Explain your steps and reasoning.
  • Sam has a method for comparing fractions: He just looks at the denominator. Sam says the fraction with the larger denominator is smaller because, if there are more pieces, then each piece is smaller. Discuss Sam's ideas.
	• Effectively communicate quantitative analysis or solutions to mathematical problems in written or oral form.

This SLO is assessed by in-class and take-home assignments, including homework, quizzes, and exams. Throughout the course, students will have to explain why standard numerical algorithms for arithmetic work, recognize when they do not, and determine an appropriate alternative when feasible. • Examples: Below are examples of problems that students will be expected to solve.	• Evaluate solutions to problems for reasonableness using a variety of means, including informed estimation.

• In your own words, explain in detail why we can determine which of two fractions is greater by giving the two fractions common denominators. What is the rationale behind this method? What are we really doing when we give fractions the same denominator? • Leah is working on the multiplication problem 2.43 x 0.148. Ignoring the decimal places, Leah multiplies 243 x 148 and gets the answer 35964. But Leah cannot remember the rule about where to put the decimal point in this answer to get the correct answer. Explain how Leah can use reasoning about the sizes of the numbers to determine where to put the decimal place.	

This SLO is assessed on homework assignments, quizzes, and exams. Students will be expected to apply properties of the various number systems to solve problems in basic number theory, financial math, and everyday life.
 	
• Examples: Below are several examples of problems that students will be expected to solve.
  • Keiko has a rectangular piece of fabric that is 48 inches wide and 72 inches long. She wants to cut her fabric into identical square pieces leaving no fabric remaining. She wants all side lengths to be whole numbers. What are her options?
  • If 10 workers take 8 hours to sew a store's order of pants, then how long would it take 15 workers to sew the store's order of pants?
  • Last year's profits were $16 million, but this year's profits are only $6 million. By what present did profits decrease from last year to this year?
	• Apply mathematical methods to problems in other fields of study.

MAT 123 Syllabus

General Information

MAT 123: Number Systems and Number Theory for Educators *(3hr, 3cr)*

Course Description: This course studies number systems, their representations, their development, their properties, and their relationship to one another. An in-depth development of number system operations, computations within these systems as a foundation for algebra, and the historical development of these ideas is included.

Prerequisites: Departmental Permission

Note: Material covered in this class will help teachers/teacher candidates prepare for a leadership position as elementary mathematics specialist.

Instructor: Your instructor will provide contact information, office hours and meeting times for your section.

Course Format and Grading

Expectations: This course studies number systems, their representations, their historical development, their properties, and their relationship to one another. This course uses the problem-solving approach to teaching and learning mathematics concepts. Students are encouraged to ask questions. Class participation is essential. You are strongly encouraged to take good notes and do not miss class. Bring your concerns and challenges to the instructor's attention early on in the course so that they can address them effectively.

Homework: Homework will be assigned in class. Solutions to most problems from the previous session will be reviewed and discussed in class. In order to be successful in this course it is essential that you devote a lot of time to your homework.

Grades: Your grade will be made up of 70% exams and 30% assignments that include homework.

Text, Materials, and Accommodating Disabilities

References:

- Beckmann, S. (2018). Mathematics for elementary teachers (5^{th} ed). Pearson.
- Billstein, R., Libeskind, S., & Lott, J. W. (2016). A problem solving approach to Mathematics for elementary school teachers (12^{th} ed). Pearson.
- Sonnabend, T. (2010). Mathematics for teachers: an interactive approach for grades k-8 (4^{th} ed). Brooks/Cole Cengage Learning.

Materials: Physical and Virtual Manipulatives; Learning Tools

Calculator: Texas Instruments and Scientific Calculators

Accommodating Disabilities: Lehman College is committed to providing access to all programs and curricula to all students. Students with disabilities who may need classroom accommodations are encouraged to register with the Office of Student Disability Services. For more info, contact the Office of Student Disability Services, Shuster Hall, Room 238, 718-960-8441.

Course Objectives and Content

Course Objectives: This course meets all of the overall objectives for a CUNY common core Quantitative Reasoning course; these objectives and how they are met in this course are detailed below.

At the end of this course, students will be able to:

1. Interpret and draw appropriate inferences from quantitative representations, such as formulas, graphs, or tables.
    - Know, describe, use, and interpret various types of real numbers (whole, integer, rational, irrational, and decimal) using multiple representations.

2. Represent quantitative problems expressed in natural language in a suitable mathematical format.
    - Use various representations (e.g. diagrams, math drawings, tables), conceptual models, and appropriate tools to solve problems.

3. Use algebraic, numerical, graphical, or statistical methods to draw accurate conclusions and solve mathematical problems.
    - Understand, describe, and use algebraic properties/operations of addition, subtraction, multiplication, and division to solve problems and model situations.
    - Develop algebraic thinking by using variables, formulas, and models with different number theory constraints.

4. Effectively communicate quantitative analysis or solutions to mathematical problems in written or oral form.
    - Explain, describe, and effectively communicate similarities and differences between the various number systems to audiences of varied mathematical maturity: K-8 learners, 9-12 students, and college-level peers.
    - Familiarize self with and interpret common mathematics errors made by elementary and middle school students.

5. Evaluate solutions to problems for reasonableness using a variety of means, including informed estimation.
    - Explain why standard numerical algorithms for arithmetic work, recognize when they do not, and determine an appropriate alternative when feasible.

6. Apply mathematical methods to problems in other fields of study.
    - Apply properties of the various number systems to solve problems in basic number theory, financial math, and everyday life.

Course Topics
There is flexibility in the order and time allotted to each of the topics below, but all topics must be covered by the instructor and understood by the student. Historical development and perspective will be embedded within the topics where appropriate.

1. Numeration Systems
    - Ancient numeration systems
    - Number-base systems and Place Values

2. Whole Number Operations
    - Addition and Subtraction of Whole Numbers
    - Multiplication and Division of Whole Numbers
    - Properties of Whole Number Operations
    - Algorithms for Whole Number Operations
    - Mental Computations and Estimations

3. Number Theory
    - Factors and Divisibility
    - Prime and Composite Numbers
    - Common Factors and Common Multiples
    - Division and Euclidean Algorithm

4. Operations with Integers
    - Addition and Subtraction of Integers
    - Multiplication and Division of Integers

5. Rational Numbers and Proportional Reasoning
    - The Set of Rational Numbers
    - Addition and Subtraction of Rational Numbers
    - Multiplication and Division of Rational Numbers
    - Properties of, Estimations and Error Patterns with Rational Numbers
    - Quantitative and Proportional Reasoning

6. Decimals, Percents, and Real Numbers
    - Terminating and Repeating Decimals (Rationals)
    - Non-terminating and Non-Repeating Decimals (Irrationals)
    - Operations on Decimals
    - Percents
    - Real Numbers

Professional Standards

(Specific content and objectives will include the following standards from **NCTM CAEP Mathematics Content for Elementary Mathematics Specialist (Addendum to the NCTM CAEP Standards 2012)**

Upon completion of this course, students will have met the following professional standards:

C.1. Number and Operations -To be prepared to support the development of student mathematical proficiency, all elementary mathematics specialists should know the following topics related to number and operations with their content understanding and mathematical practices supported by appropriate technology and varied representational tools, including concrete models:

C.1.1 Counting and cardinality, comparing and ordering, understanding the structure of the base ten number system with particular attention to place value, order of magnitude, one-to-one correspondence, properties, and relationships in numbers and number systems – whole numbers, integers, rationals, irrationals, and reals

C.1.2 Arithmetic operations (addition, subtraction, multiplication, and division) including mental mathematics and standard and non-standard algorithms, interpretations, and representations of numbers – whole numbers, fractions, decimals, integers, rationals, irrationals, and reals

C.1.3 Fundamental ideas of number theory – divisors, factors and factorization, multiples, primes, composite numbers, greatest common factor, and least common multiple

C.1.4 Quantitative reasoning and relationships that include ratio, rate, proportion, and the use of units in problem situations

C.1.5 Historical development and perspectives of number, operations, number systems, and quantity including contributions of significant figures and diverse cultures