Research Topics in Theoretical High Energy Physics

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NONPERTURBATIVE ASPECTS OF GAUGE THEORIES

All fundamental interactions are described by gauge theories. Quantum Chromodynamics (QCD) is the gauge theory describing the strong forces, the interactions among quarks and gluons.

Remarkable properties of QCD: effective strength of interaction increases with distance (asymptotic freedom); there are no free quarks or gluons (confinement).

Key theoretical challenge: analytic understanding of confinement.

OUR APPROACH

- Analyze lower dimensional gauge theory with only dynamical gluons = $3d$ Yang-Mills theory.
- Easier to analyze, nontrivial, guide to 4d case
- Approximation to high-temperature 4d YM, physically relevant (mass gap in $YM_3 \approx$ magnetic mass in high-temperature YM_4)

METHOD AND RESULTS

- Hamiltonian analysis in terms of gauge invariant variables
- Use conformal field theory to study properties of gauge invariant configuration space
- Analytical calculation of vacuum wavefunction and string tension (excellent agreement with lattice calculations)
- Extend analysis to YM + adjoint scalars in order to study adjoint screening/string-breaking effects

SOME FUTURE PROJECTS

- Calculate higher order corrections to string tension
- Glueball and meson masses
- Inclusion of dynamical quarks (string/gauge theory duality)

MORE DETAILS

The Hamiltonian for 3d YM is given by ($A_0 = 0$ gauge)

$$\mathcal{H} = \frac{e^2}{2}\int E_i^a E_i^a + \frac{1}{2e^2}\int B^a B^a$$

where $e^2 \sim$ [mass]

- Gauge invariant parametrization

$$A = \tfrac{1}{2}(A_1 + iA_2) = -\partial M\, M^{-1}$$
$$\bar{A} = \tfrac{1}{2}(A_1 - iA_2) = M^{\dagger -1}\bar{\partial} M^\dagger$$

M: $(N\times N)$ complex matrix ($M \in SL(N, \mathbf{C}) = SU(N)^{\mathbf{C}}$)
Under gauge transformations, $g \in SU(N)$:

$$A \to A_i^g = gA_i g^{-1} - \partial_i g g^{-1}$$

$$M \to g\,M \qquad M^\dagger \to M^\dagger g^{-1}$$

$H = M^\dagger M$ is gauge invariant hermitian field

- Measure of gauge invariant configuration space $\mathcal{C}$

$$\mathcal{C} = \frac{\text{gauge potentials}}{\text{gauge transformations}} = \frac{\mathcal{A}}{\mathcal{G}}$$

$$d\mu(\mathcal{C}) = d\mu(H)\, \exp\left[2N\, S_{wzw}(H)\right]$$

where $S_{wzw}(H)$ is the Wess-Zumino-Witten action,

$$S_{wzw}(H) = \frac{1}{2\pi}\int \mathrm{Tr}(\partial H \bar{\partial} H^{-1}) + \frac{i}{12\pi}\int \mathrm{Tr}(H^{-1}dH)^3$$

- Inner product for physical states

$$\langle 1|2\rangle = \int d\mu(H)\, \exp\left[2N\, S_{wzw}(H)\right]\; \Psi_1^*\Psi_2$$

$$\left.\begin{matrix}\text{Matrix elements in}\\ 3d\ \text{YM}\end{matrix}\right\} = \left\{\begin{matrix}\text{Correlators of a hermitian}\\ WZW\ \text{model (CFT)}\end{matrix}\right.$$

Finite norm wavefunctions Ψ are functions of $J = \frac{N}{\pi}\partial H H^{-1}$

- Hamiltonian in terms of J

Solve Shrödinger equation $\mathcal{H}\Psi = E\Psi$, where

$$\mathcal{H} = m\left[\int J^a \frac{\delta}{\delta J^a} + \int \Omega_{ab}(x,y)\frac{\delta^2}{\delta J^a(x)\delta J^b(y)}\right] + \frac{\pi}{mN}\int :\bar{\partial}J^a\,\bar{\partial}J^a:$$

$$\Omega_{ab}(x,y) = \left[\frac{N}{\pi}\delta_{ab}\partial_y - i f_{abc}J^c(y)\right]\frac{1}{\pi(x-y)} + \mathcal{O}(\epsilon)$$

- Vacuum wavefunction Ψ_0

$$\Psi_0 = \exp\left\{-\frac{2}{e^2}\frac{\pi^2}{N^2}\int \bar{\partial}J^a(x)\left[\frac{1}{m+\sqrt{m^2-\nabla^2}}\right]_{x,y}\bar{\partial}J^a(y) + \text{higher } J - \text{terms}\right\}$$

Smooth interpolation between short and long distance regimes

$$\Psi_0 \approx \exp\left[-\frac{1}{2e^2}\int B\frac{1}{\sqrt{-\nabla^2}}B\right] \qquad \frac{k}{m} \gg 1$$

(perturbative limit)

$$\approx \exp\left[-\frac{1}{4e^2 m}\int B^2\right] \qquad \frac{k}{m} \ll 1$$

- String tension calculation

Using this wavefunction we find that the potential between external quarks is confining ($\sim \sigma r$) and derive an analytic expression for the string tension, in excellent agreement with lattice results.

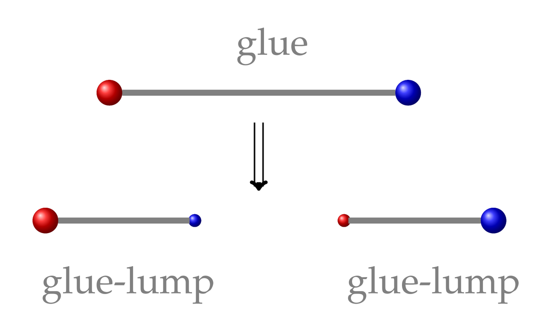
Compare $\sqrt{\sigma}/e^2$ with numerical (lattice) estimates by Teper et al. Our predictions are in black, lattice values are in red.

Group	Representations					
	k=1 Fund.	k=2 antisym	k=3 antisym	k=2 sym	k=3 sym	k=3 mixed
$SU(2)$	0.345 0.335					
$SU(3)$	0.564 0.553					
$SU(4)$	0.772 0.759	0.891 0.883		1.196 1.110		
$SU(5)$	0.977 0.966					
$SU(6)$	1.180 1.167	1.493 1.484	1.583 1.569	1.784 1.727	2.318 2.251	1.985 1.921
$SU(N)$ $N\to\infty$	0.1995 N 0.1976 N					

The difference between predictions and lattice values is $\leq 3\%$, $\leq 0.88\%$ as $N \to \infty$.

- Adjoint screening/string breaking effects

Gauge invariant Hamiltonian approach has been extended to account for screening, when external particles are in the adjoint representation/adjoint string breaking



Results for string energy breaking : our analytic result $V = 7.9\, m$, lattice result $V = 8.7\, m$ (9% agreement)

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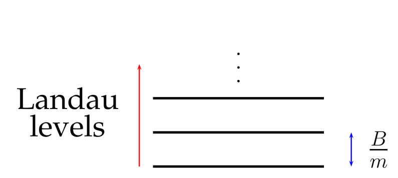
QUANTUM HALL EFFECT, NONCOMMUTATIVE FIELD THEORIES AND BOSONIZATION IN HIGHER DIMENSIONS

Quantum Hall effect in higher dimensions, similarly to the classic two-dimensional case, provides a rich framework for new ideas on fuzzy spaces, noncommutative field theories, matrix models and bosonization in higher dimensions.

- Connection between QHE and noncommutative plane

Spectrum of charged particle in magnetic field B



Projection to lowest Landau level (LLL)

$$[x_i,\, x_j] = \frac{i}{B}\,\epsilon_{ij}$$

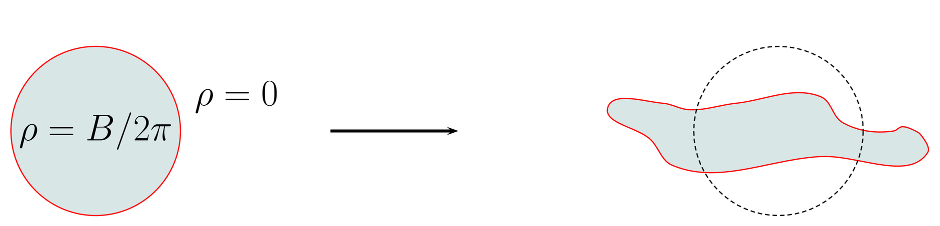
Coordinate space $\Rightarrow$ Phase space

LLL provides physical realization of NC space

- QH droplets, chiral bosons, Chern-Simons action

N-body LLL ground state $\to$ incompressible droplet

Edge excitations : area preserving boundary fluctuations



Edge excitations collectively described by chiral boson Φ

$$S_{\rm edge} = \int_{\partial D}\left(\partial_t\Phi + \omega\partial_\theta\Phi\right)\partial_\theta\Phi$$

In the presence of electromagnetic interactions

$$S_{\rm bulk} \sim \int_D \epsilon_{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda \quad \text{(Chern}-\text{Simons action)}$$
$$S_{\rm edge} \sim \text{gauged chiral action}$$

Anomaly cancellation : $\delta S_{\rm bulk} + \delta S_{\rm edge} = 0$ (total action is gauge invariant)

Main results:

- Generalization of QHE to arbitrary even dimensions (on $\mathbf{CP}^k$, $2k$-dimensional space) with abelian ($U(1)$) and nonabelian ($SU(k)$) background gauge fields.
- Universal matrix action description of LLL dynamics

$$S_0 = \int dt\ \mathrm{Tr}\left[i\hat{\rho}_0\hat{U}^\dagger\partial_t\hat{U} \;-\; \hat{\rho}_0\hat{U}^\dagger\hat{V}\hat{U}\right]$$

which leads to the evolution equation for density matrix

$$i\frac{d\hat{\rho}}{dt} = [\hat{V}, \hat{\rho}]$$

$\hat{U}$ = $N\times N$ unitary matrix; N is the dimension of the LLL space.
(No explicit dependence on properties of space on which QHE is defined, abelian or nonabelian nature of fermions etc.)

- Bosonization, noncommutative field theory description of LLL dynamics

LLL interactions of original nonrelativistic fermions can be also described by a bosonic noncommutative field theory

$$S_0 = N\int d\mu dt\ \left[i(\rho_0 * U^\dagger * \partial_t U) \;-\; (\rho_0 * U^\dagger * V * U)\right]$$

(Actual definition of $*$ and transition rule from matrices to fields is omitted here.)

- Dynamics of higher-dimensional non-Abelian QH droplets

Dynamics of LLL ground states are confined on the droplet boundary and are described by novel bosonic chiral actions which are higher dimensional generalizations of the Wess-Zumino-Witten action.

In the presence of gauge interactions, there is also bulk dynamics, described by a higher dimensional Chern-Simons action.

Anomaly cancellation between the edge and bulk actions ensures the gauge invariance of the theory.

FUTURE PROJECTS

- Extension of previous work to the droplet dynamics of different filling fractions
- Investigate the mathematical structure underlying the incompressibilty of the higher dimensional droplets
- Use the QHE scheme as an alternative way to introduce gauge fields and gravity on fuzzy spaces

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