Conceptual Physics

Luis A. Anchordoqui

Department of Physics and Astronomy
Lehman College, City University of New York

Lesson VIII
October 24, 2017

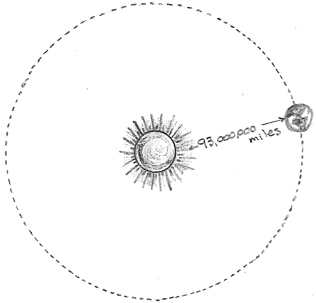
https://arxiv.org/abs/1711.07445

Table of Contents

- Rate at which solar energy reaches Earth's upper atmosphere is

$$S_0 = 1.5 \text{ kilowatts/square meter} (= 1.5 \text{ kW/m}^2)$$

- Since $1\text{kW} \approx 1 \text{ Btu/s}$ ☞ this means energy flux of 1.5 Btu/(m^2 s)
- We can use this number
  to obtain total rate at which energy is being radiated by Sun
- Imagine sphere drawn with Sun at center and Earth at surface



- Since we know rate at which radiation arrives at 1 m^2 of sphere total radiation emitted by Sun $\sim$ 1.5 Btu/(m^2 s) $\times$ area of sphere

- Distance from sun to earth is 93×10^6 miles
- 1 mile = 1.6 km = 1600 m ☞ so

$$\text{the earth} - \text{sun distance} = 93 \times 10^6 \times 1600 \text{ m} = 150 \times 10^9 \text{ m}$$

- Area of whole sphere is

$$A = 4\pi r^2 = 4\pi(150 \times 10^9)^2$$

- Now ☞ $\pi \approx 3$ so $4\pi \approx 12$ and $(150)^2 = 22500 = 2.25 \times 10^4$

$$A \approx 12 \times 2.25 \times 10^4 \times 10^{18} \approx 27 \times 10^{22} \text{ square meters}$$

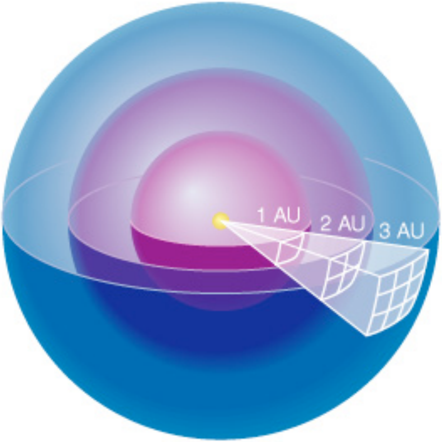
- Total radiation arriving at sphere is

$$\begin{aligned} L_\odot &\approx 1.5 \text{ Btu/s/m}^2 \times 27 \times 10^{22} \text{ m}^2 \\ &\approx 40 \times 10^{22} \text{ Btu/s} \\ &\approx 4 \times 10^{23} \text{ Btu/s} \end{aligned}$$

- This must be rate at which energy is being radiated by Sun
- We know that Sun is not heating up ☞ so 4×10^{23} Btu/s must also be rate at which energy is being created at Sun

- Suppose Sun were burning oil
  or some other chemical with energy content $\sim$ 20,000 Btu/lb
- If entire mass $M_\odot = 4 \times 10^{30}$ lb were composed of such fuel and it were all to burn ☞ then *total* energy release would be
  $$4 \times 10^{30}\ \text{lb} \times 2 \times 10^4\ \text{Btu/lb} = 8 \times 10^{34}\ \text{Btu}$$
- Since rate of energy release is 4×10^{23} Btu/s
  $$\text{amount} = \text{rate} \times \text{time} \Rightarrow 8 \times 10^{34}\ \text{Btu} = 4 \times 10^{23}\ \text{Btu/s} \times \text{time}$$
- Time available for burning
  $$\text{time} = \frac{8 \times 10^{34}}{4 \times 10^{23}}\ \text{s} = 2 \times 10^{11}\ \text{s}$$
- Since a year has 3×10^7 seconds ☞ time is equivalent to
  $$\frac{2 \times 10^{11}}{3 \times 10^7}\ \text{yr} = \frac{2}{3} \times 10^4\ \text{yr} = \frac{2}{3} \times 10{,}000\ \text{yr} = 6{,}666\ \text{yr}$$
- Way too short
- To last 6×10^9 yr (or so)
  fuel must have energy content of 10^6 times greater than oil

Inverse square law



Luminosity passing through each sphere is the same

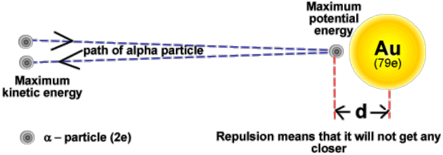
Area of sphere:

$$4\pi\,(\text{radius})^2$$

Divide luminosity by area to get flux desnity

- Size and structure of nuclei
  were 1st investigated in Rutherford's scattering experiments
- Rutherford directed positively charged nuclei of helium atoms
  (alpha particles)
  at a thin piece of metal foil
- As particles moved through foil
  they often passed near a metal nucleus
- Because of positive charge on both incident particles and nuclei
  particles were deflected from their straight-line paths
  by Coulomb repulsive force
- In fact ☞ some particles were even deflected backward
  through an angle of 180° from incident direction
- Those particles were apparently moving directly toward a nucleus
  in a head-on collision course

- Using conservation of energy we find distance d at which particle is turned around by Coulomb repulsion
- In head-on collision ☞ kinetic energy of incoming alpha particle must be converted completely to electrical potential energy when particle stops at point of closest approach and turns around



- Equate initial kinetic energy of alpha particle to electrical potential energy of system (α particle + nucleus)

$$\frac{1}{2} m_\alpha v_\alpha^2 = k_e \frac{q_1 q_2}{r} = k_e \frac{(2e)(Ze)}{d}$$

- Solving for d ☞ we get distance of closest approach

$$d = \frac{4 k_e Z e^2}{m_\alpha v_\alpha^2}$$

- For α particles with $K_\alpha = 1.12 \times 10^{-12}$ J ☞ $d \sim 3.2 \times 10^{-14}$ m when foil is made of gold
- Thus ☞ radius of gold nucleus must be less than 3.2×10^{-14} m
- For silver atoms ☞ distance of closest approach $= 2 \times 10^{-14}$ m
- From these results ☞ Rutherford concluded that:
  positive charge in atom is concentrated in small sphere (nucleus) with radius of approximately 10^{-14} m
- Modern particle colliders are a repeat of Rutherford experiment but at a much higher energy
- We now know ☞ nucleons are all made up of *fundamental* quarks which glue together via strong interaction

- World's largest microscope ☞ CERN Large Hadron Collider
- LHC has directly probed distance scales well inside proton as short as 2×10^{-20} m
- LHC consists of a 27 km ring of superconducting magnets with a number of accelerating structures to boost particle's energy along the way
- Inside accelerator ☞ two high energy particle beams travel at close to speed of light before they are made to collide

Answer after 2017

Answer before 2017

- Some nuclei are unstable
- An unstable nucleus tries to achieve balanced state
  by given off neutrons or protons via radioactive decay
- When ^{238}U (we omit the "92") undergoes radioactive decay
  it emits α-particle which consists of 2 protons and 2 neutrons
- This leaves atom with 90 p (92 minus 2) and 234 N (238 minus 4)
  which is called thorium-234
- ^{234}Th decays and we have decay chain ending with lead ($^{206}_{82}$Pb)
- Beta decay takes place by the emission of e (or β-ray) + ν
  from nucleus at same time that one of n changes to p
- So when a carbon-14 (6 protons, 8 neutrons) β-decays
  it changes to ^{14}N (7 protons, 7 neutrons – ordinary nitrogen!)

- There is law describing substance undergoing radioactive decay: constant *fraction* of atoms in sample disintegrate per second

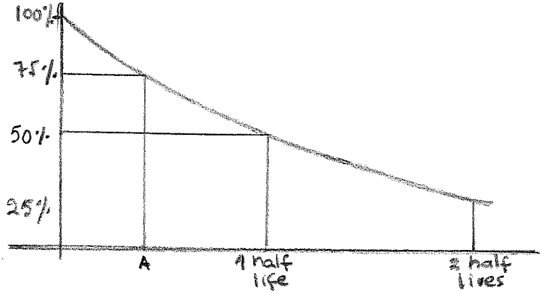
- Time it takes for half of sample to decay ☞ *half-life*

- E.g. ☞ if 100 atoms are initially present and half life is 10 years

  then { after ten years ☞ 50 atoms will remain undecayed
  after another 10 years ☞ 25 will be left
  after 30 years ☞ 12 will be left, etc.

- Since radioactivity level
  depends directly on number of undecayed atoms remaining
  we can say that after each half-life time radioactivity is cut in half

- Knowing half-life ☞ we know how to draw graph
  which will allow us to find radioactivity level
  after any amount of time



- E.g. ☞ to find out how much long substance must decay
  so that its radioactivity level drop by 25%
  we look at graph for time corresponding to 75% activity
- This is point A on time axis ☞ and is about 0.4 of a half-life

- Due to cosmic ray bombardment ☞ tiny fraction (1 millionth of 1%) of carbon in atmosphere (in CO_2) is carbon-14
- CO_2 is breath by plants
- Hence ☞ all plants have a tiny bit of radioactive ^{14}C in them
- Via radioactive decay ☞ ^{14}C in living things changes to stable ^{14}N but because living plants breathe ☞ decayed ^{14}C is replenished and there is a constant ratio of ^{14}C to ^{12}C (ordinary carbon)
- Equilibrium ☞ radioactive level of 15 disintegrations per second for every gram of the carbon mixture
- When a plant dies ☞ replenishment stops

- Percentage of ^{14}C steadily decreases with half-life of $5,730$ yr
- Since we know radioactivity of plants today
  we are able to determine the ages of ancient objects
  by measuring their radioactivity
- E.g. ☞ extract small quantity of carbon from ancient papyrus scroll find it has 1/2 as much radioactivity as same amount of carbon extracted from living tree ☞ papyrus must be $5,730$ yr old
- If it is 75% as radioactive ☞ 0.4 of one half-life
  or $2,292$ years have elapsed since papyrus was alive

1. ^{238}U:
   - Half-life of ^{238}U is 4.5×10^9 years
   - This isotope isn't less abundant than other heavy elements (bismuth, mercury, gold, etc.) we conclude that these elements were formed not much longer than 4.5×10^9 or 6×10^9 yr ago

2. ^{235}U:
   - ^{235}U is $\frac{1}{140}$ as abundant as ^{238}U and has half-life of 0.9×10^9 years
   - If ^{238}U and ^{235}U were formed in roughly equal amounts it must have taken about 7 half-lives to get them to the present ratio (since $(\frac{1}{2})^7 = \frac{1}{128}$ close to $\frac{1}{140}$)
   - We estimate that both these elements were formed $7 \times 0.9 \times 10^9 = 6.3 \times 10^9$ years ago (6.3 billion)

Age of the rocks

- When ^{238}U undergoes radioactive disintegration
  final products of sequence of decays are:
  isotope of lead (^{206}Pb) + 8 ^{4}He + e's + ν's
- When ^{238}U became encased in rock
  lead and helium were locked into close proximity to ^{238}U
- As time passes ratio ^{206}Pb/^{238}U increases
- By knowing this ratio and half-life of ^{238}U (4.5×10^9 years)
  we estimate time that has passed since ^{238}U was encased in rock
- Same procedure can be used with other "mother-daughter" pairs
  (^{232}Th $\to$ ^{208}Pb and ^{235}U $\to$ ^{207}Pb)
- Using this method ☞ oldest rocks found on Earth
  have been dated 4 billion years
- More sophisticated methods
  date formation of Earth's crust at 4.5×10^9 years

Age of the oceans

- Oceans have become salty
  as a result of minerals being washed into them
  by rivers flowing to sea
- Evaporation of water from ocean (leaving behind the brine)
  and its subsequent return to rivers as fresh-water rain
  leads to increase in salinity from year to year
- From knowledge of { total volume of oceans; rate of fresh water flow to sea; and mineral content of river water }
  we estimate salinity of oceans has been increasing at rate
  of one billionth of a percent per year (10^{-9}% per year)
- Thus ☞ to reach a concentration of 3%
  it must have taken about 3 billion years