Prof. Anchordoqui

Problems set # 10 **Physics 131** December 6, 2018

1. The escape velocity from Earth is 4×10^4 km/h. What would be the percent decrease in length of a 95.2 m long spacecraft traveling at that speed?

2. At what speed do the relativistic formulas for length and time intervals differ from the classical values by 1%?

3. Space explorer Harry sets off at a steady $0.95c$ to a distant star. After exploring the star for a short time, he returns at the same speed and gets home after a total absence of 80 yr (as measured by earth-bound observers). How long do Harry's clocks say he was gone, and by how much has he aged as compared to his twin Sally who stayed behind on Earth. [Note: This is the famous "twin paradox." It is fairly easy to get the right answer by judicious insertion of a factor of γ in the right place, but to understand it, you need to recognize that it involves three uniformly moving reference frames: the earth-bound frame S, the frame S' of the outbound rocket, and the frame S'' of the returning rocket. Write down the time dilation formula for the two halves of the journey and then add. Noticed that the experiment is not symmetrical between the two twins: Sally stays at rest in the single uniformly moving frame S, but Harry occupies at least two different frames. This is what allows the result to be unsymmetrical.]

4. A distant galaxy is observed to have a redshift $V/c = 0.1$, where V is the recession velocity of the galaxy, and c is the speed of light. *(i)* What is the recession velocity of the galaxy in units of km/s? *(ii)* Using the Hubble expansion formula calculate the distance to the galaxy in units of ly? *(iii)* How long ago was the light we are now seeing from the galaxy emitted?

5. The electromagnetic spectrum includes a wide range of light waves, some that we cannot see. Some of the non-visible types of waves are radio waves, microwaves, infrared rays, and X-rays. In the visible spectrum of light, the color of the light depends on the frequency. We have seen that the visible spectrum is always the same for a rainbow, or the separated light from a prism. The order of colors is red, orange, yellow, green, blue, indigo, and violet. Wien's Law tells us that objects (such as stars) of different temperature emit spectra that peak at different wavelengths. Hotter objects emit most of their radiation at shorter wavelengths; hence they will appear to be *bluer*. Cooler objects emit most of their radiation at longer wavelengths; hence they will appear to be *redder*. The wavelength of the peak of the spectrum emitted by a star gives a measure of the temperature,

$$\text{wavelength of maximum intensity} = \frac{0.29}{T}\ \text{cm}\,{}^\circ\text{K}\,. \qquad (1)$$

(i) Use this information to determine the temperature of the sun, which has a radius $R_\odot = 432,288$ miles. *(ii)* Using (1) convince yourself that while the sun does emit ultraviolet radiation, the majority of solar energy comes in the form of *light* in the visible regions of the electromagnetic

spectrum, $390 < \lambda/\text{nm} < 700$, where one nanometer (nm) equals 10^{-9} m.

6. We have seen that the average translational kinetic energy of molecules in a gas is directly proportional to the temperature of the gas. We can invert this relation to find the average speed of molecules in a gas as a function of the temperature,

$$\text{average speed} = \sqrt{\frac{3 \times k_B \times \text{temperature}}{\text{average mass}}}\,. \tag{2}$$

If the average speed of a gas is greater than about 15% to 20% of the escape speed of a planet, virtually all the molecules of that gas will escape the atmosphere of the planet. *(i)* At what temperature is average speed for O_2 equal to 15% of the escape speed of the Earth? *(ii)* At what temperature is the average velocity for H_2 equal to 15% of the escape speed for Earth? *(iii)* Temperatures in the upper atmosphere reach $1,000°$K. How does this help account for the low abundance of hydrogen in Earth's atmosphere? *(iv)* Compute the temperatures for which the average speeds of O_2 and H_2 are equal to 15% of the escape speed at the surface of the moon, where the mass of the moon is 1.6×10^{23} lb and its radius $1,080$ miles. How this account for the absence of an atmosphere on the moon? The average mass of a hydrogen atom is 1.674×10^{-24} g and that of a oxygen atom is 2.657×10^{-23} g.