## Units

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Unit</th>
<th>Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>Meter</td>
<td>Length traveled by light in a specific fraction (1/299,792,458) of a second</td>
</tr>
<tr>
<td>Time</td>
<td>Second</td>
<td>Duration of 9,192,631,770 periods of radiation emitted during transition between two hyperfine levels of the ground state of cesium 133 atom</td>
</tr>
<tr>
<td>Mass</td>
<td>Kilogram</td>
<td>Platinum cylinder in International Bureau of Weights and Measures (Paris)</td>
</tr>
</tbody>
</table>
Measurement & Uncertainty

No measurement is exact, there is always some uncertainty due to limited instrument accuracy and difficulty reading results.

For example, it would be difficult to measure the width of this table to better than a millimeter.
Measurement & Uncertainty

Estimated uncertainty is written with a ± sign

\[ 8.8 \pm 0.1 \text{cm} \]

Percent uncertainty

ratio of uncertainty to measured value multiplied by 100

\[ \frac{0.1}{8.8} \times 100\% \approx 1\% \]
A friend asks to borrow your precious diamond for a day to show her family

You are a bit worried, so you carefully have your diamond weighed on a scale which reads 8.17 g

Scale accuracy is claim to be ± 0.05 g.

Next day you weigh returned diamond again getting 8.09 g.

Is this your diamond?
Scale readings are measurements

Each measurement could have been high or low by up to 0.05 g

Actual mass of your diamond lies most likely between 8.12 g and 8.22 g

Actual mass of return diamond lies most likely between 8.04 and 8.14 g

These two ranges overlap so there is not a strong reason to doubt that return diamond is yours
**Significant Figures**

Number of significant figures: number of reliably known digits in a number

It is usually possible to tell the number of significant figures by the way the number is written:

- **23.21 cm** has 4 significant figures
- **0.062 cm** has 2 significant figures (the initial zeroes don’t count)
- **80 km** is ambiguous - it could have 1 or 2 significant figures; if it has 2 it should be written **80. km**
**Significant Figures**

When multiplying or dividing numbers, result has as many significant figures as number used in calculation with fewest significant figures.

Example: \(11.3 \text{ cm} \times 6.8 \text{ cm} = 77\text{ cm}\)

When adding or subtracting answer is no more accurate than the least accurate number used.
Global positioning satellites (GPS) can be used to determine positions with great accuracy. System works by determining distance between observer and each of several satellites orbiting Earth.

If one of satellites is at a distance of 20,000 km from you, what percent accuracy in distance is required if we desired a 2 m uncertainty? How many significant figures do we need to have in that distance?
The percentage accuracy is

\[ \frac{2 \text{ m}}{2 \times 10^7 \text{ m}} \times 100\% = 10^{-5}\% \]

The distance of 20,000,000 m needs to be distinguishable from 20,000,002 m which means that 8 significant figures are needed in distance measurements.
Point particle

• In a horse race the winner is the horse whose nose first crosses the finish line.

• One could argue that what really matters during the race is the motion of that single point of the horse.

• In physics this type of simplification turns out to be useful for examining the motion of idealized objects called point particles.
To describe motion of a particle we need to be able to describe position of particle and how that position changes as it moves.

Change of bicycle’s position is called a displacement:

$$\Delta x = x_f - x_i$$
You are playing a game of catch with a dog!

Dog is initially standing near your feet.

Then he jogs 20 feet in a straight line to retrieve a stick and carries stick 15 feet back towards you to chew stick.

(a) What is total distance dog travels?
(b) What is displacement of dog?
(c) Show that net displacement for trip is sum of sequential displacements.
You are playing a game of catch with a dog.

Dog is initially standing near your feet.

Then he jogs 20 feet in a straight line to retrieve a stick and carries stick 15 feet back towards you to chew stick.

(a) What is total distance dog travels? \(35 \text{ ft}\)

(b) What is displacement of dog?

(c) Show that net displacement for trip is sum of sequential displacements.
You are playing a game of catch with a dog
Dog is initially standing near your feet
Then he jogs 20 feet in a straight line to retrieve a stick
and carries stick 15 feet back towards you to chew stick

(a) What is total distance dog travels? 35 ft
(b) What is displacement of dog? 5 ft
(c) Show that net displacement for trip is sum of sequential displacements
Average Velocity & Speed

Average speed = \frac{\text{Total distance traveled by particle}}{\text{Total time from start to finish}}

\[ \Delta x \] \quad \Delta t = \frac{\Delta x}{\Delta t} = \text{slope} = v_{x, av} \quad \text{Average Velocity} \]
Dog that you were playing catch with jogged 20 ft away from you in 1 s to retrieve stick and ambled back 15 ft in 1.5 s.

Calculate

(a) Dog's average speed

(b) Dog's average velocity for total trip
Dog that you were playing catch with jogged 20 ft away from you in 1s to retrieve stick and ambled back 15 ft in 1.5 s.

Calculate
(a) Dog's average speed  \(14 \text{ ft/s}\)
(b) Dog's average velocity for total trip
Dog that you were playing catch with jogged 20 ft away from you in 1s to retrieve stick and ambled back 15 ft in 1.5 s.

Calculate
(a) Dog's average speed \(14 \text{ ft/s}\)
(b) Dog's average velocity for total trip \(2 \text{ ft/s}\)
The instantaneous velocity is the limit of the ratio $\Delta x / \Delta t$ as $\Delta t$ approaches zero. It is the slope of the line tangent to the $x$-versus-$t$ curve.

$$v_x(t) = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$
Instantaneous Velocity

In calculus, limit that defines instantaneous velocity is called derivative of x with respect to time (t).

A line's slope may be positive, negative, or zero → instantaneous velocity in 1 dimension may be positive (x increasing), negative (x decreasing), or zero (no motion).

For an object moving with constant velocity →

- the object's instantaneous velocity is equal to its average velocity over any time interval.
- Position versus time of this motion will be a straight line.

Instantaneous velocity is a vector and magnitude of instantaneous velocity is instantaneous speed.

From now on: velocity → denotes instantaneous velocity and speed → denotes instantaneous speed.
Position of a particle as a function of time

Figure shows position of a particle as a function of time
Find instantaneous velocity at t=1.8 s?

When velocity is greatest?

When is it zero?

Is it ever negative?
Position of a particle as a function of time

Figure shows position of a particle as a function of time

Find instantaneous velocity at t= 1.8 s?

\[ v_x = \text{slope} \sim \frac{(8.5 \text{ m} - 4.0 \text{ m})}{(5.0 \text{ s} - 2.0 \text{ s})} = 1.5 \text{ m/s} \]

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When velocity is greatest?

Tangent line is steepest and hence velocity is greatest at \( t \sim 4 \text{ s} \)

When is it zero?

Is it ever negative?
Position of a particle as a function of time

Figure shows position of a particle as a function of time

Find instantaneous velocity at \( t=1.8 \) s?

\[ v_x = \text{slope} \sim \frac{(8.5 \, \text{m} - 4.0 \, \text{m})}{(5.0 \, \text{s} - 2.0 \, \text{s})} = 1.5 \, \text{m/s} \]

When velocity is greatest?
Tangent line is steepest and hence velocity is greatest at \( t\sim 4 \) s

When is it zero?
Velocity is zero at \( t=0 \) s and \( t=6 \) s

Is it ever negative?
Figure shows position of a particle as a function of time

Find instantaneous velocity at \( t = 1.8 \) s?

\[
x_v = \text{slope} \sim \frac{(8.5 \text{ m} - 4.0 \text{ m})}{(5.0 \text{ s} - 2.0 \text{ s})} = 1.5 \text{ m/s}
\]

When velocity is greatest?

Tangent line is steepest and hence velocity is greatest at \( t \sim 4 \) s

When is it zero?

Velocity is zero at \( t = 0 \) s and \( t = 6 \) s

Is it ever negative?

Velocity is negative for \( t < 0 \) s and \( t > 6 \) s
Acceleration

Acceleration is rate of change of velocity with respect to time

**Average acceleration**

\[ a_{x,av} = \frac{\Delta v_x}{\Delta t} = \frac{v_{x,f} - v_{x,i}}{t_f - t_i} \]

**Instantaneous acceleration**

\[ a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d(dx/dt)}{dt} = \frac{d^2x}{dt^2} \]
Motion with constant acceleration

\[ x(t) = x_0 + v_0 t + \frac{1}{2} a t^2 \]

\[ v(t) = v_0 + a t \]

Motion diagrams: moving object is drawn at equally space time intervals

(a) Velocity is increasing so acceleration is in direction of velocity vector

(b) Velocity vector is decreasing so acceleration is in direction opposite to that of velocity vector
Upon graduation, a joyful physics student throws her cap straight upward with an initial speed of 14 m/s.
Flying cup

(a) How long does it take for the cap to reach its highest point?

(b) What is the distance to the highest point?

(c) Assuming the cap is caught at the same height from which it was released, what is the total time the cap is in flight?
Start with

\[ v_y(t) = v_{0y} - gt \]

When the cap is at the top - the instantaneous velocity is zero

\[ t_{max} = \frac{v_{0y}}{g} = 1.5 \text{ s} \]

\[ y_{max} = v_{0y} \cdot t_{max} - \frac{1}{2} g \cdot t_{max}^2 = 10 \text{ m} \]

By symmetry

\[ t_{tot} = 3 \text{ s} \]
Flying cup

Plot position as a function of time and velocity as a function of time

Note that slope is equal to instantaneous acceleration \(= 9.8 \text{ m/s}^2\)
Flying cup on Moon

Acceleration due to gravity on the Moon is about one sixth what it is on the Earth. If the cap is thrown vertically upward on the Moon, how many times higher will it go than it would on Earth, assuming the same initial velocity.
Choose the upward direction to be positive

and \( y_0 = 0 \) to be the level from which the object is thrown

The initial velocity is \( v_0 \) and the velocity at the top is zero

\[
 t_{\text{max}} = \frac{v_0}{a} \Rightarrow y = \frac{v_0^2}{a} - \frac{1}{2} \frac{v_0^2}{a} = \frac{1}{2} \frac{v_0^2}{a}
\]

From this we see that the displacement is inversely proportional to the acceleration and so if the acceleration is reduced by a factor of 6 by going to the Moon and the initial velocity is unchanged the displacement increases by a factor of 6
Catching a speeding car

A car is speeding at constant 56 mi/h in a school zone. A police car starts from rest just as speeder passes by it and accelerates at constant rate of 5 m/s².

(a) When does police car catch speeding car?

(b) How fast is police car traveling when it catches up with speeder?
Equations for the speeding car and police car are

\[ x_1 = v_0 t \quad \text{and} \quad x_2 = \frac{1}{2} at^2 \]

Catching equation

\[ x_1 = x_2 \Rightarrow v_0 t = \frac{1}{2} at^2 \]

\[ t = \frac{2v_0}{a} = 10 \text{ s} \]

Velocity when catching the speeder

\[ v = at = 50 \text{ m/s} \]
Catching a speeding car

How fast is police traveling when is 25 m behind speeding car

\[ v_{S0x} = 90 \text{ km/h} \]
\[ v_{P0x} = 0 \]

\[ a_{Sx} = 0 \]
\[ a_{Px} = 5.0 \text{ m/s}^2 \]

\[ x_0 = 0 \]
\[ D \]
\[ t_1 \]

\[ x_s = v_{S0x}t \]
\[ x_p = \frac{1}{2} a_{Px}t^2 \]

\[ D = 25 \text{ m} \]
$x_1 = x_2 + 25 \text{ m}$

$v_0 t = \frac{1}{2} a t^2 + 25 \text{ m}$

$\frac{1}{2} a t^2 - v_0 t + 25 \text{ m} = 0$

$t = \frac{-v_0 \pm \sqrt{v_0^2 - 50 a \text{ m}}}{a} = 5 \pm \sqrt{15} \text{ s}$

$v_{pol} = 5.64 \text{ m/s}$ and $v_{pol} = 44.4 \text{ m/s}$
Homework 1

Which of position-versus-time curves in figure best shows motion of an object

(a) with positive acceleration
(b) with constant positive velocity
(c) that is always at rest
(d) with positive velocity and negative acceleration
Homework 1

Which of position-versus-time curves in figure best shows motion of an object

(a) with positive acceleration ➡️ curve d
(b) with constant positive velocity
(c) that is always at rest
(d) with positive velocity and negative acceleration
Which of position-versus-time curves in figure best shows motion of an object

- (a) with positive acceleration ➡ curve d
- (b) with constant positive velocity ➡ curve b
- (c) that is always at rest
- (d) with positive velocity and negative acceleration
Homework 1

Which of position-versus-time curves in figure best shows motion of an object

(a) with positive acceleration ➔ curve d
(b) with constant positive velocity ➔ curve b
(c) that is always at rest ➔ curve e
(d) with positive velocity and negative acceleration
Which of position-versus-time curves in figure best shows motion of an object

(a) with positive acceleration ➡️ curve d
(b) with constant positive velocity ➡️ curve b
(c) that is always at rest ➡️ curve e
(d) with positive velocity and negative acceleration ➡️ curve c
Homework 2

Which of velocity-versus-time curves in figure best describes motion of an object

(a) with constant positive acceleration
(b) with positive acceleration that is decreasing with time
(c) with positive acceleration that is increasing with time
(d) with no acceleration
Homework 2

Which of velocity-versus-time curves in figure best describes motion of an object

(a) with constant positive acceleration ➫ curve b
(b) with positive acceleration that is decreasing with time
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Thanks & questions