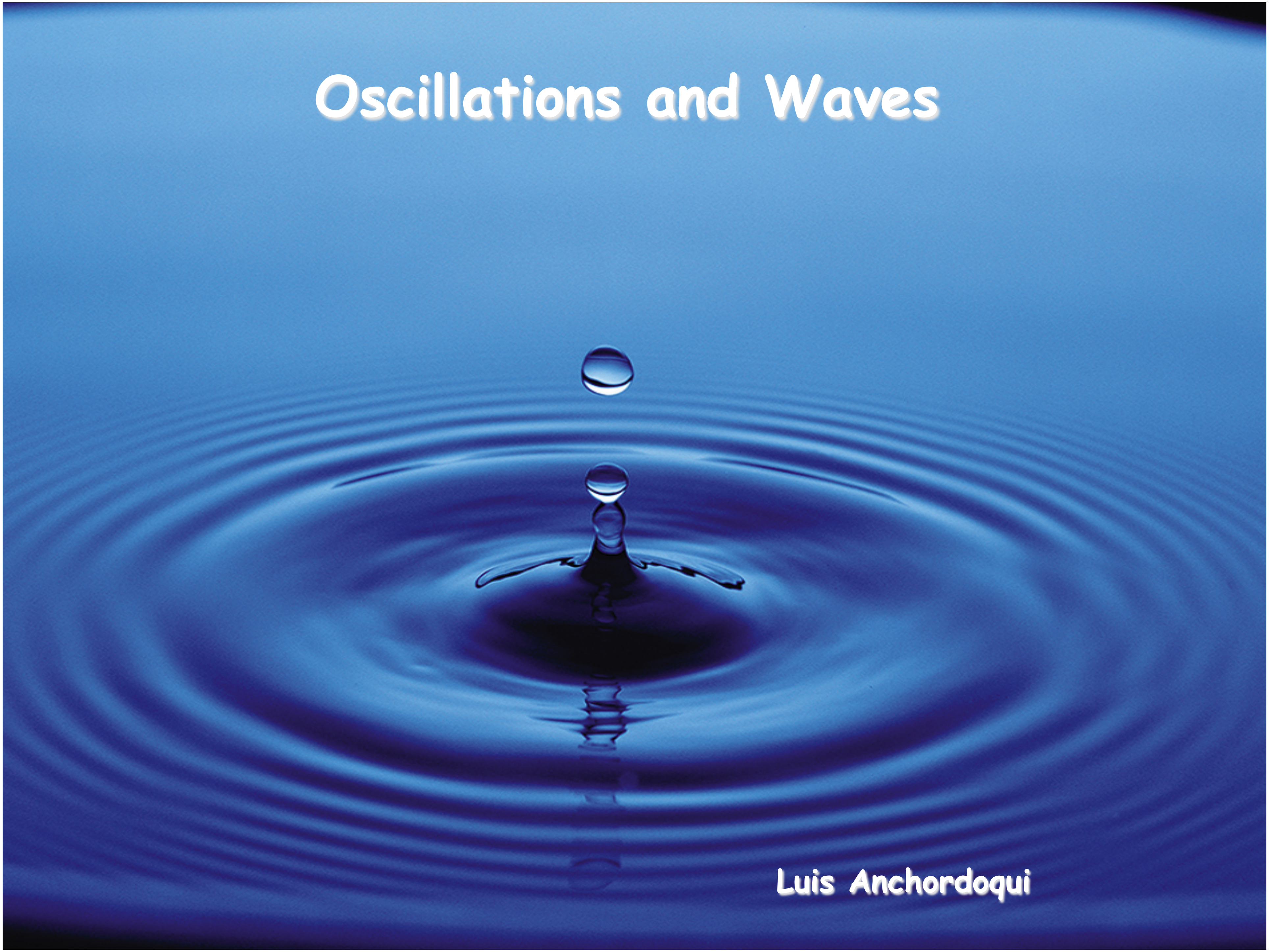
Oscillations and Waves



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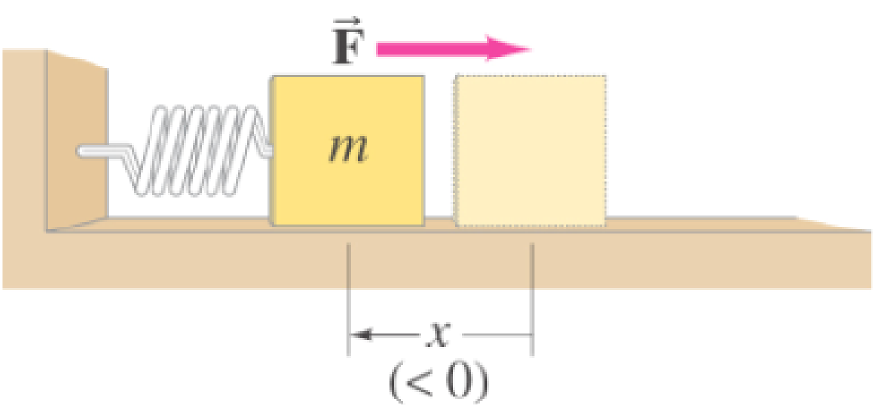
Simple harmonic motion

If an object vibrates or oscillates back and forth over the same path, each cycle taking the same amount of time, the motion is called periodic



The mass and spring system is useful model for a periodic system

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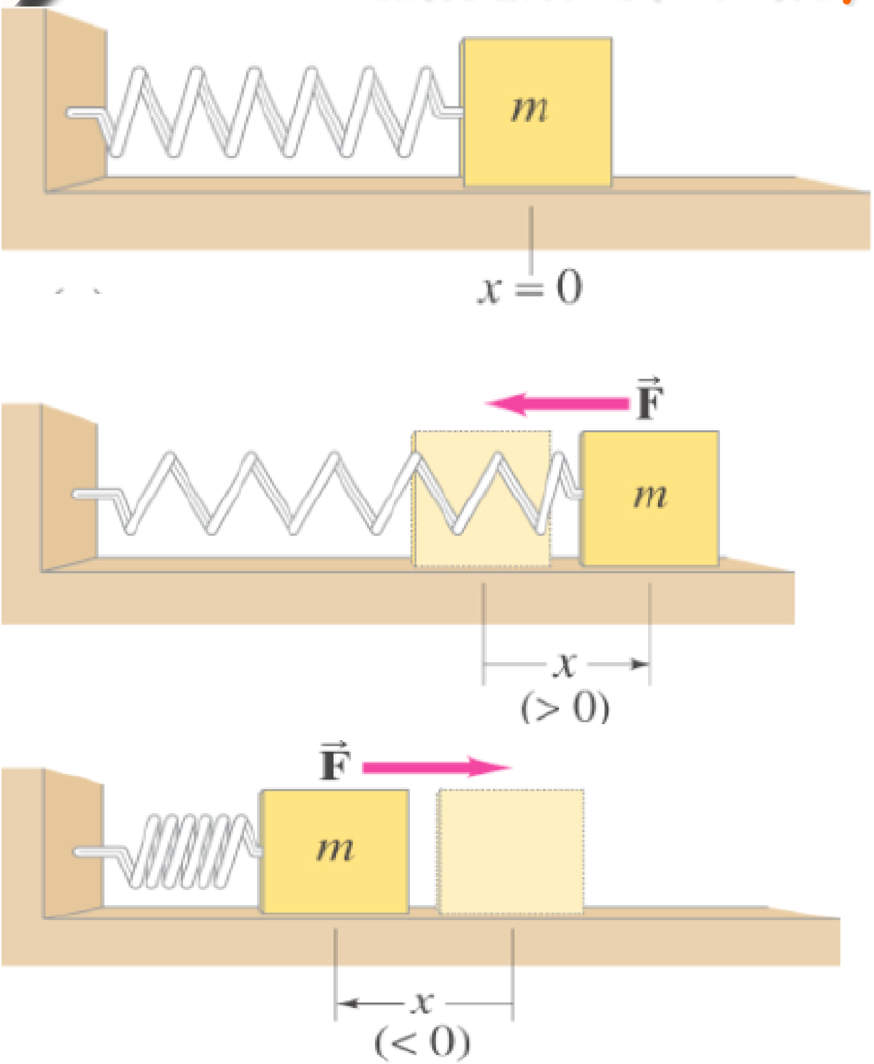
Simple harmonic motion (cont'd)

If an object vibrates or oscillates back and forth over the same path, each cycle taking the same amount of time, the motion is called periodic



The mass and spring system is useful model for a periodic system

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Simple harmonic motion (cont'd)

We assume that the surface is frictionless.

There is a point where the spring is neither streched nor compressed

⇩

The equilibrium position

We meassure displacement from that point (x = 0 on the previous figure)

The force exerted by the spring depends on the displacement

$$F = -kx$$

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Simple harmonic motion (cont'd)

The minus sign on the force indicates that it is a restoring force

↓

it is directed to restore the mass to its equilibrium position

k is the spring constant

The force is not constant, so the acceleration is not constant either

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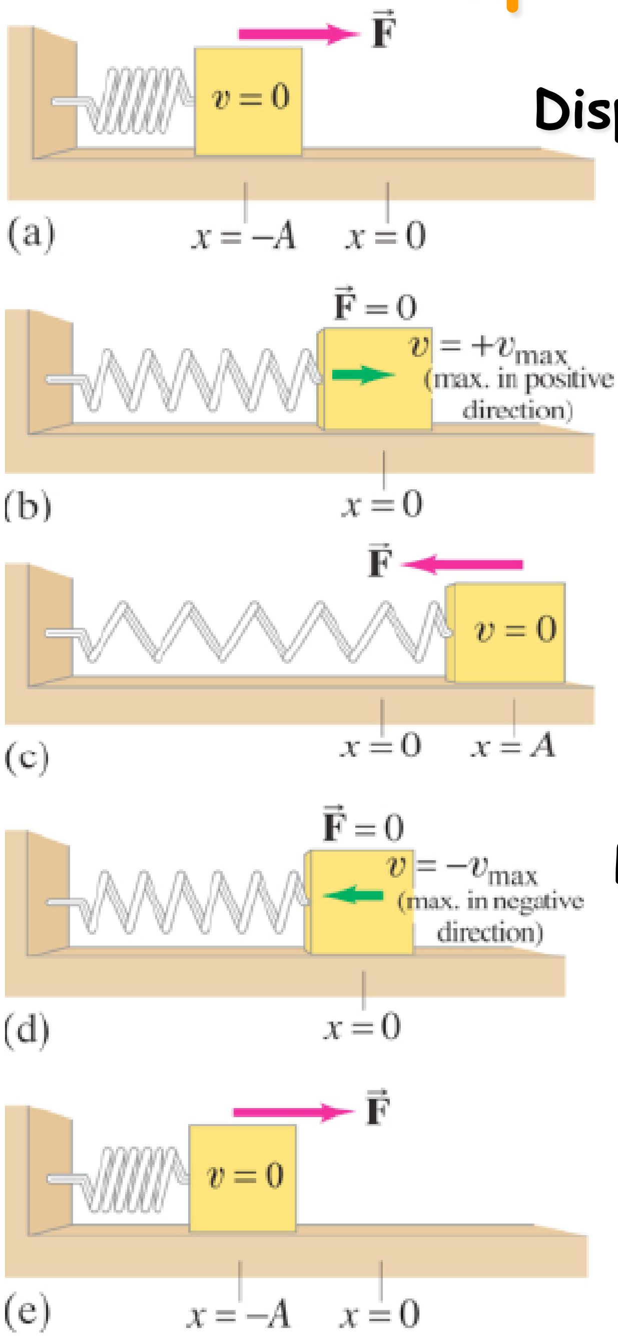
Simple harmonic motion (cont'd)



Displacement is measured from the equilibrium point

Amplitude is the maximum displacement

A cycle is a full to-and-fro motion.
This figure shows half a cycle

Period is the time required to complete one cycle

Frequency is the number of cycles completed per second

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Simple harmonic motion (cont'd)

Any vibrating system where the restoring force
is proportional to the negative of the displacement
is in simple harmonic motion (SHM)
and is often called a simple harmonic oscillator

We know that the potential energy of a spring is given by

$$PE = \tfrac{1}{2} kx^2$$

The total mechanical energy is then:

$$E = \tfrac{1}{2} mv^2 + \tfrac{1}{2} kx^2$$

The total mechanical energy will be conserved,
as we are assuming the system is frictionless

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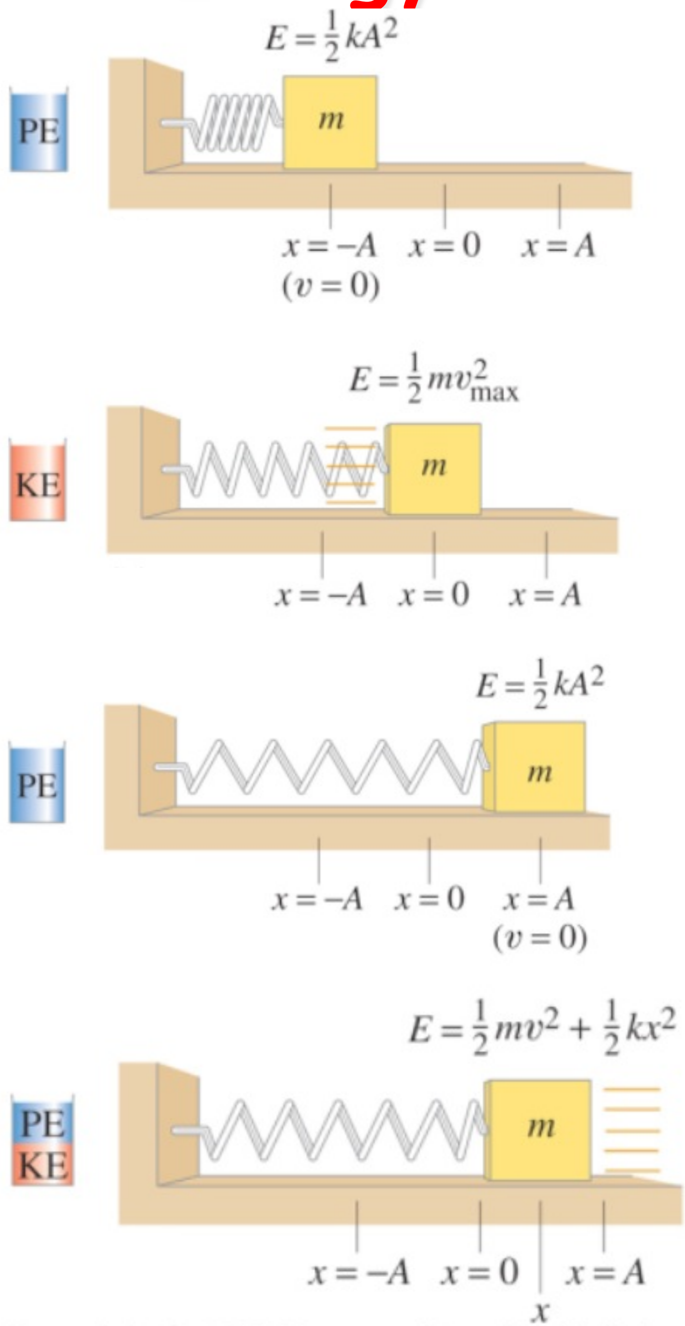
Energy in the simple harmonic oscillator



If the mass is at the limits of its motion, the energy is all potential

If the mass is at the equilibrium point, the enrgy is all kinetic

We know what the potential energy is at the turning points

$$E = \frac{1}{2} k A^2$$

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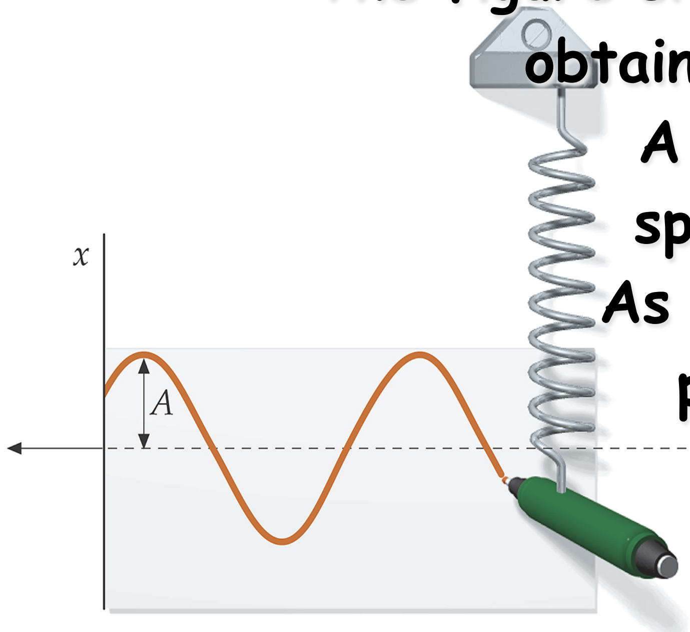
The period and sinusoidal nature of SHM

The figure shows how we can experimentally obtain x versus t for a mass on a spring



A marking pen is attached to a mass on a spring and the paper is pulled to the left. As the paper moves with constant speed the pen traces out the displacement x as a function of time.

The general equation for such curve is

$$x = A \cos(\omega t + \delta)$$

ω = $2\pi f$

δ = Phase constant

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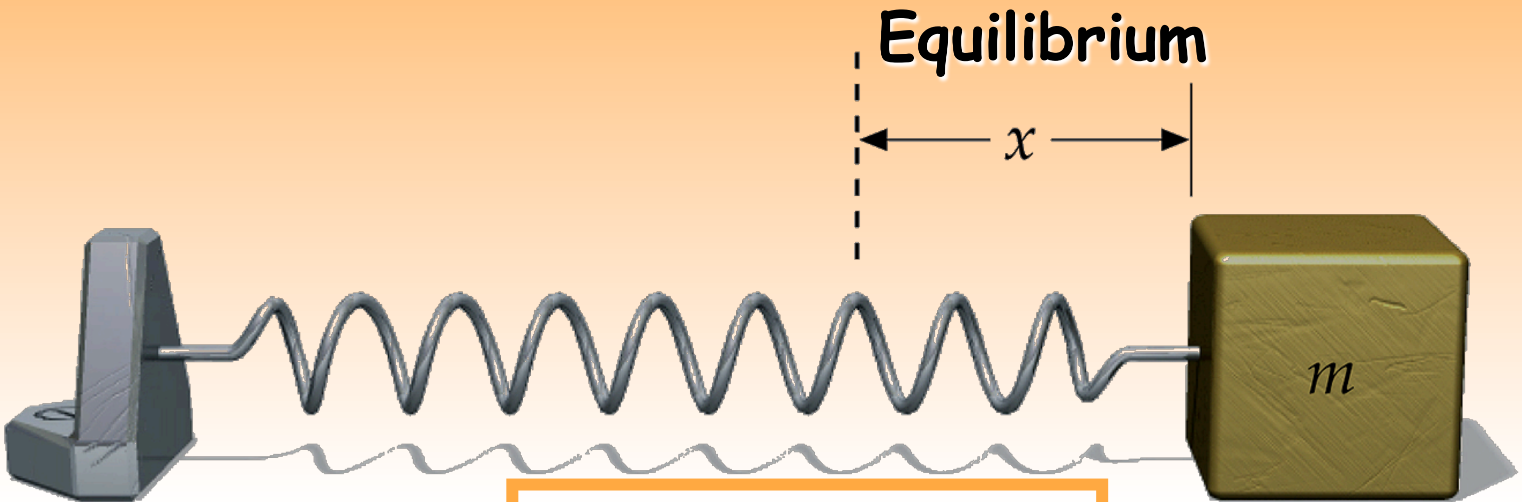
Sinusoidal Nature of SHM

Consider an object on a spring on a frictionless surface



$$F_x = -kx$$

Using Newton's second law

$$m \frac{d^2x}{dt^2} = -kx$$

The general solution is

$$x = A \cos(\omega t + \delta)$$

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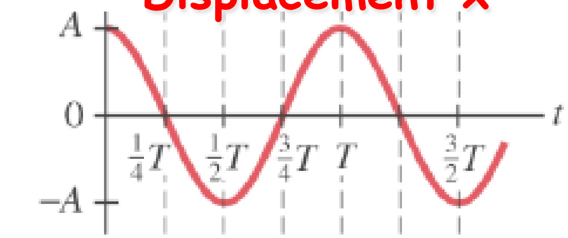
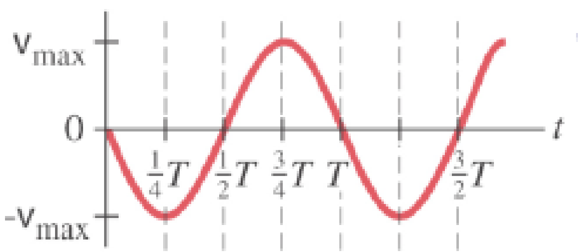
Sinusoidal Nature of SHM (cont'd)

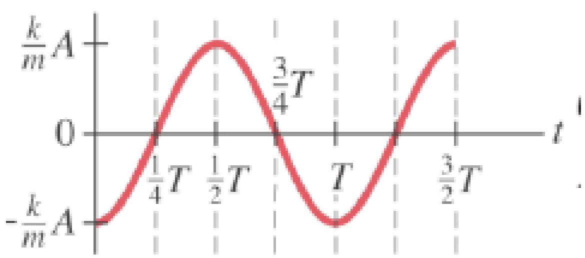
The velocity and acceleration can be calculated as function of time

Displacement x





Velocity v



Acceleration a

$$v = -v_{max} \sin \omega t$$

$$v_{max} = A\,(k/m)^{\frac{1}{2}}$$

$$a = -a_{max} \cos(2\pi t/T)$$

$$a_{max} = k\,A/m$$

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Spider Web

A spider of mass 0.3 g waits in its web of negligible mass.
A slight movement causes the web to vibrate
with a frequency of about 15 Hz.

a) Estimate the value of the spring stifness constant k for the web.

(b) At what frequency would you expect the web to vibrate if an insect mass 0.1 g were trapped in addition to the spider?

$k = 2.7$ N/m

$f = 13$ Hz

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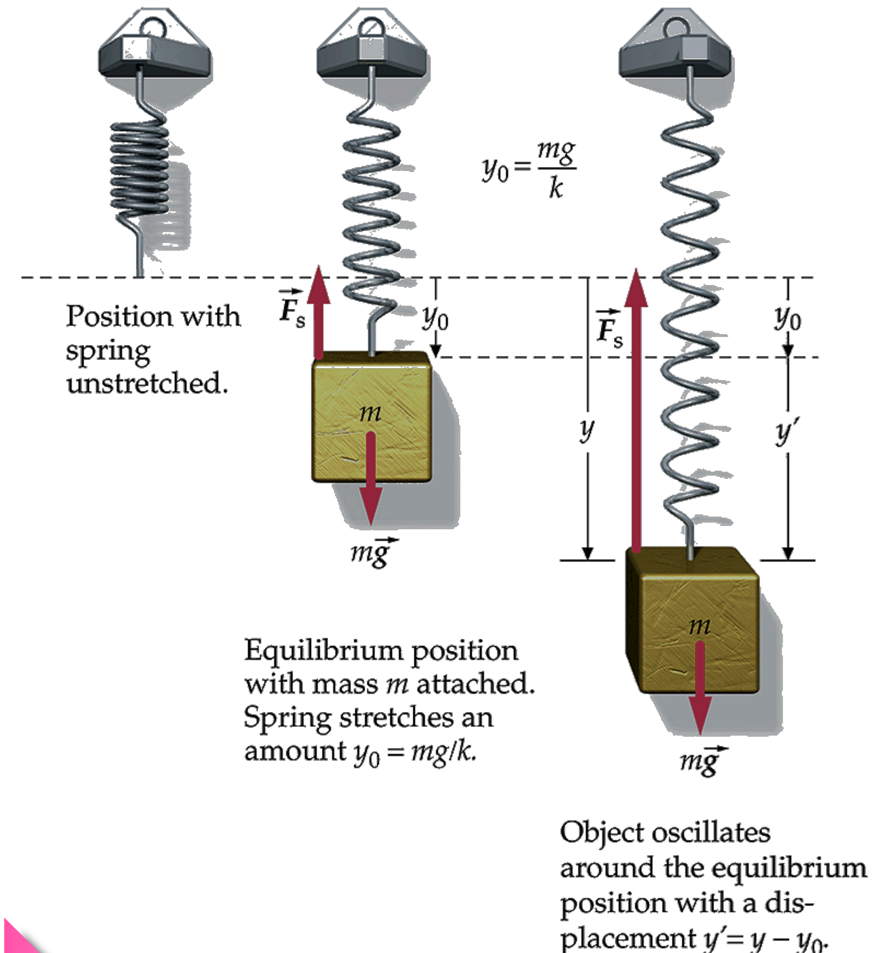
Oscillating systems: object on a vertical spring



$$\Sigma F_y = -ky + mg$$

Changing variables $y' = y - y_0$

$$\Sigma F_y = -k(y' + y_0) + mg$$

But $ky_0 = mg \rightarrow \Sigma F_y = -ky'$

From Newton's second law

$$\frac{-ky'}{m} = \frac{d^2y}{dt^2}$$

$$y = y' + y_0 \rightarrow \frac{d^2y}{dt^2} = \frac{d^2y'}{dt^2}$$

$$\downarrow$$

$$\frac{d^2y'}{dt^2} = -\frac{k}{m}y'$$

The solution is $y' = A\cos(\omega t + \delta)$

$$\omega = (k/m)^{\frac{1}{2}}$$

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You are teaching your sister how to make paper party decorations using paper springs.

She makes a paper string.

The spring is stretched 8 cm and has a single sheet of colored paper suspended from it.

You want decorations to bounce at approximately 1 cy/s.

How many sheets of colored papers should be used for the decoration on that spring?

Three sheets are needed

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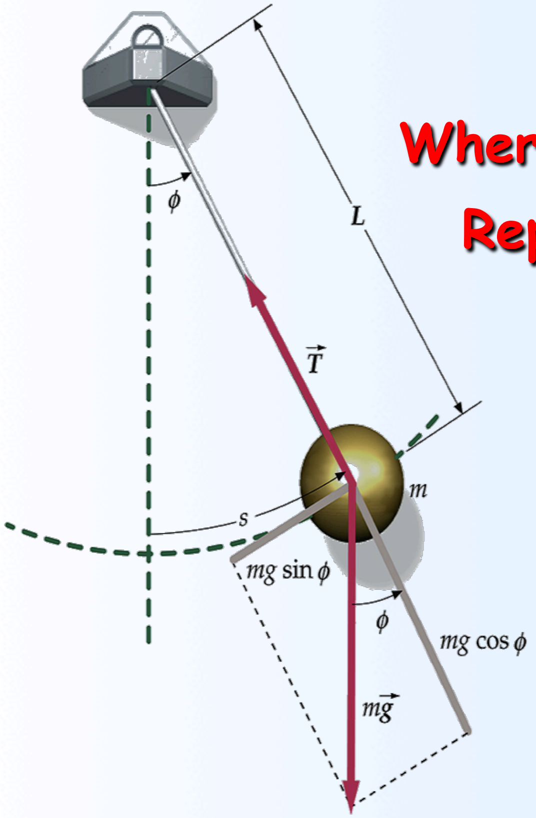
Oscillating systems: the simple pendulum



$$-mg \sin \Phi = m \frac{d^2 s}{dt^2}$$

Where the arc length $s = L\Phi$

Repeatedly differentiating on both sides of s gives

$$\frac{d^2 s}{dt^2} = L \frac{d^2 \Phi}{dt^2}$$

Substituting and re-arranging gives

$$\frac{d^2 \Phi}{dt^2} = -\frac{g}{L} \sin \Phi$$

Note that the motion of the pendulum does not depend on its mass

For small $\Phi \rightarrow \sin \Phi \approx \Phi$

$$\frac{d^2 \Phi}{dt^2} \approx -\frac{g}{L} \Phi \qquad \Phi \ll 1$$

General solution for small oscillation

$$\Phi = \Phi_0 \cos (\omega t + \delta)$$

Where

$$\omega^2 = \frac{g}{L} \text{ and } T = \frac{2\pi}{\omega} = 2\pi \, (L/g)^{\frac{1}{2}}$$

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The simple pendulum

As long as the cord can be considered massless and te amplitude is small, the period does not depend on the mass

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Sin θ at small angles

θ (degrees)	θ (radians)	sin θ	% Differen
0	0	0	0
1°	0.01745	0.01745	0.005%
5°	0.08727	0.08716	0.1%
10°	0.17453	0.17365	0.5%
15°	0.26180	0.25882	1.1%
20°	0.34907	0.34202	2.0%
30°	0.52360	0.50000	4.7%

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The pendulum in an old clock is made of brass and keeps perfect time at 17° C.

How much time is gained or lost in an year if the clock is kept at 25° C?

$\Delta T = 40$ min

(Assume the frequency dependence on length for a simple pendulum applies.)

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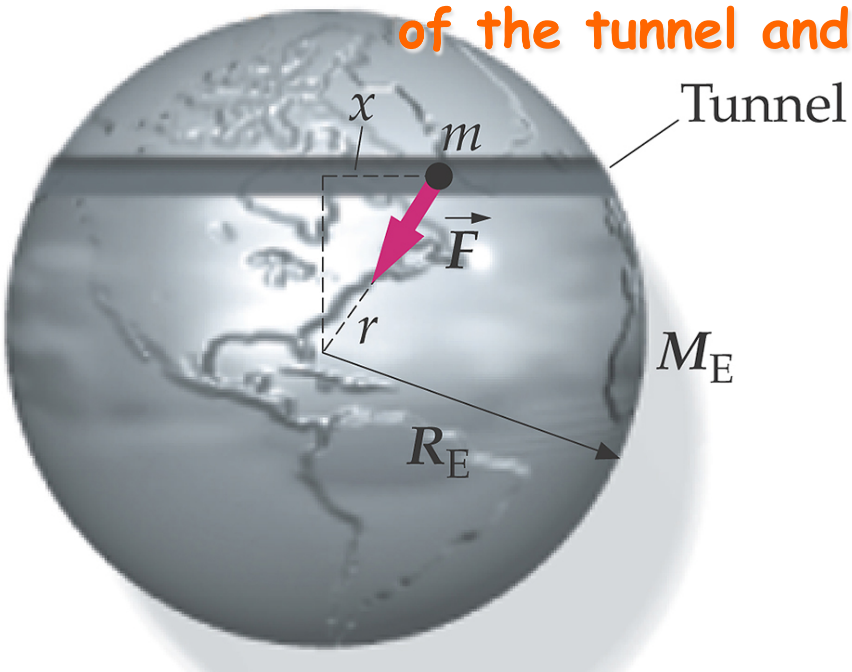
A straight tunnel is dug through Earth as shown in the figure. Assume that the walls of the tunnel are frictionless.

(a) The gravitational force exerted by Earth on a particle of mass m at a distance r from the center of Earth when $r < R_\oplus$ is $F_r = (G\, m\, M_\oplus \,/\, R_\oplus^3)\, r$.

Show that the net force on a particle of mass m at a distance x from the middle of the tunnel is given by $F_x = -(G\, m\, M_\oplus / R_\oplus^3)\, x$ and that the motion of the particle is therefore simple harmonic motion.

Show that the period of the motion is independent of the length of the tunnel and is given by $T = 2\,\pi\,(R_\oplus/g)^{\frac{1}{2}}$

Find its numerical value in minutes.



$T = 84.4$ min

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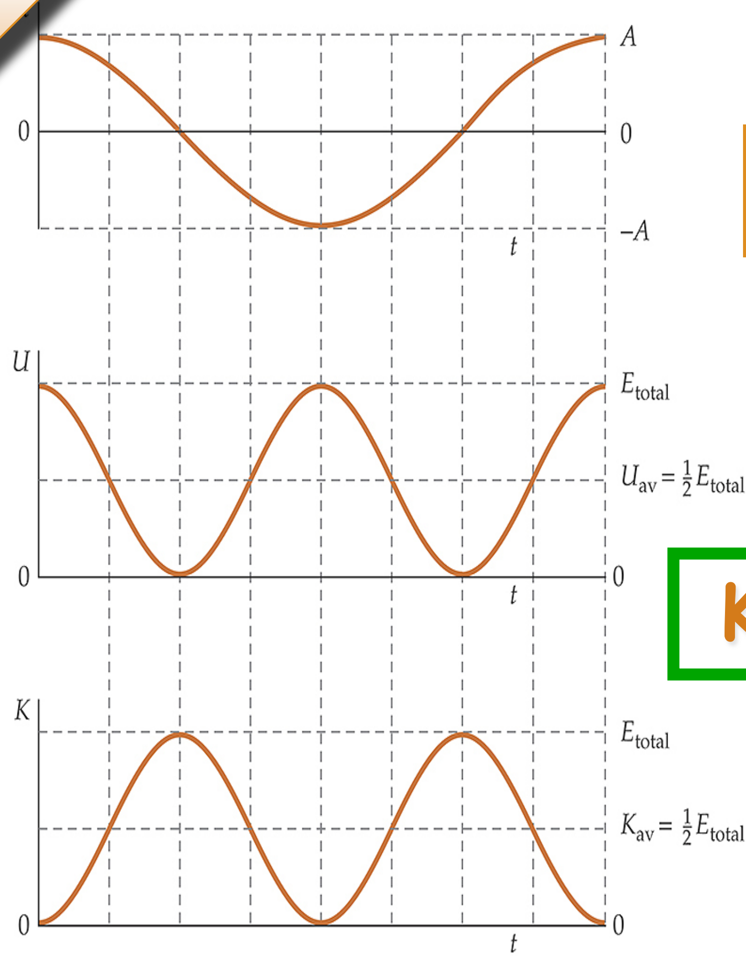
Energy in a simple harmonic motion



$$U = \frac{1}{2} kx^2$$

$$U = \frac{1}{2} kA^2 \cos^2 (\omega t+\delta)$$

$$K = \frac{1}{2} mv^2$$

$$K = \frac{1}{2} m\omega^2 A^2 \sin^2 (\omega t+\delta)$$

Using $\omega^2 = k/m$

$$E = U + K = \frac{1}{2} kA^2[\cos^2(\omega t+\delta) + \sin^2(\omega t+\delta)] = \frac{1}{2} kA^2$$

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Energy in a simple harmonic motion (cont'd)

The total energy is, therefore $\frac{1}{2} k A^2$

And we can write

$$\frac{1}{2} mv^2 + \frac{1}{2} kx^2 = \frac{1}{2} kA^2$$

This can be solved for the velocity as a function of position:

$$v = \pm v_{max}\sqrt{1 - \frac{x^2}{A^2}}$$

where

$$v^2_{max} = (k/m) A^2$$

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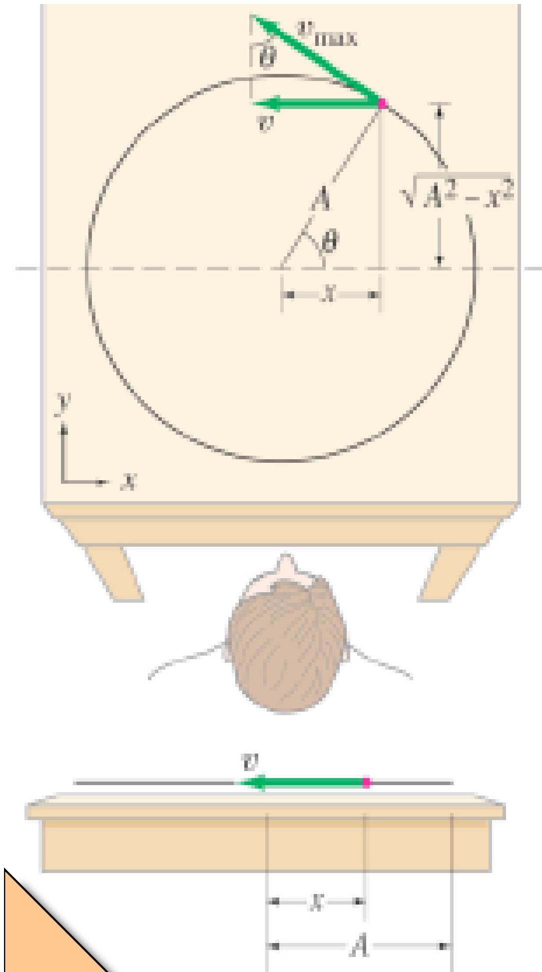
Simple Harmonic Motion and circular Motion



If we look at the projection onto the x axis of an object moving in a circle of radius A at a constant speed v_{max}, we find that the x component of its velocity varies as :

$$v = v_{max}\sqrt{1 - \frac{x^2}{A^2}}$$

This is identical to SHM

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SHM and Circular Motion (cont'd)

Therefore, we can use the period and frequency of a particle moving in a circle to find the period and frequency

$$T = \frac{2\pi r}{v}$$

$$r = A$$

$$v = (k/m)^{\frac{1}{2}} A$$

$$T = \frac{2\pi A}{(k/m)^{\frac{1}{2}} A}$$

Simplifying gives

$$T = 2\,\pi\,(m/k)^{\frac{1}{2}}$$

$$f = \frac{1}{T} = \frac{1}{2\pi}(k/m)^{\frac{1}{2}}$$

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Damped oscillations

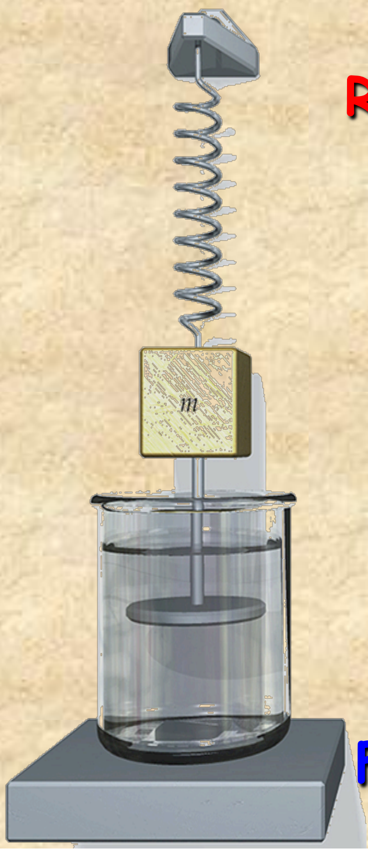
Left to itself a spring or pendulum eventually stops oscillations because the mechanical energy is dissipated by frictional forces

The damped force exerted on an oscillator can be represented by the empirical expression

$$\vec{F}_d = -b\vec{v}$$

Such a system is said to be linearly damped

The motion of a damped system can be obtain from Newton's second law



$$-kx - b\frac{dx}{dt} = m\frac{d^2x}{dt}$$

Rearranging

$$m\frac{d^2x}{dt} + b\frac{dx}{dt} + kx = 0$$

The solution to this equation is

$$x = A_0 e^{(-b/2m)\,t} \cos(\omega' t + \delta)$$

$$\omega' = \omega_0 \left[1 - \left(\frac{b}{2m\omega_0}\right)^2\right]^{\frac{1}{2}}$$

$\omega_0 = (k/m)^{\frac{1}{2}} \longrightarrow$ frequency with no damping

For weak damping $b/(2m\omega_0) \ll 1$ and ω' is nearly equal ω_0

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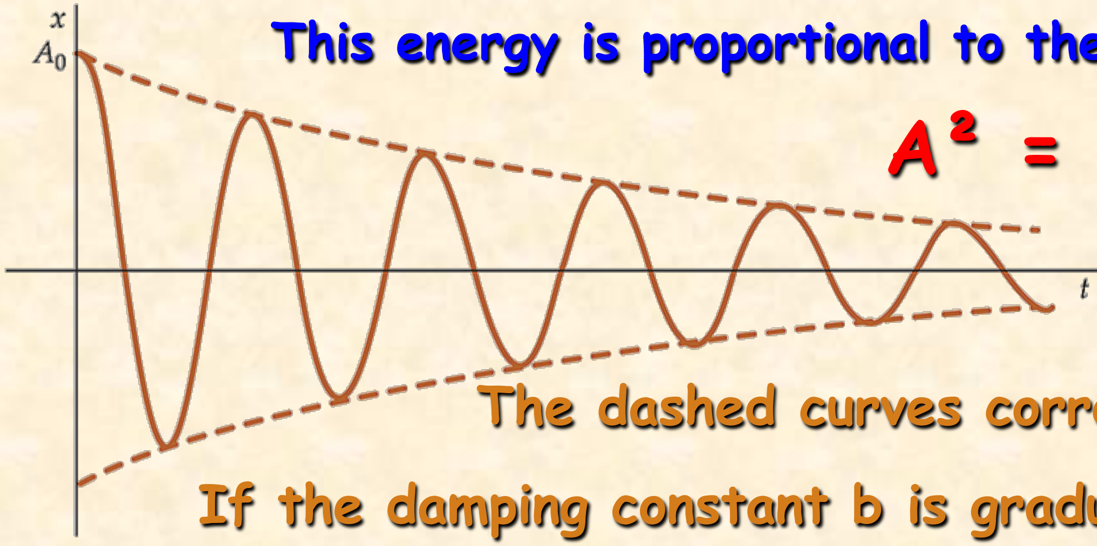
Damped oscillations (cont'd)

Because the damping force is opposite to the direction of motion it does negative work and causes the mechanical energy of the system to decrease

This energy is proportional to the square of the amplitude

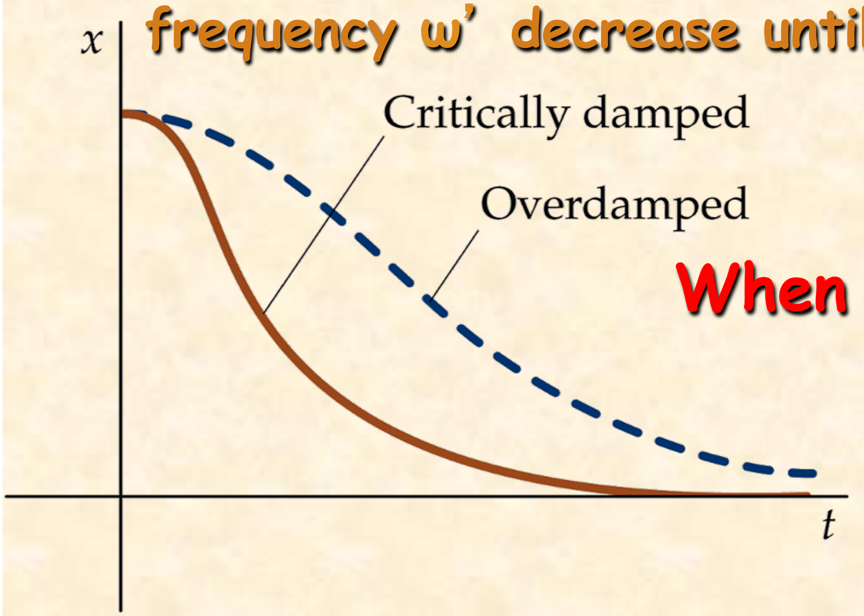


$$A^2 = A_0^2\, e^{-t/\tau} \longrightarrow \tau = m/b$$

The dashed curves correspond to $x = A$ and $x = -A$

If the damping constant b is gradually increased the angular frequency ω' decrease until it becomes zero at the critical value

$$b_c = 2m\omega_0$$



When $b \geq b_c \longrightarrow$ system is overdamped (does not oscillate)

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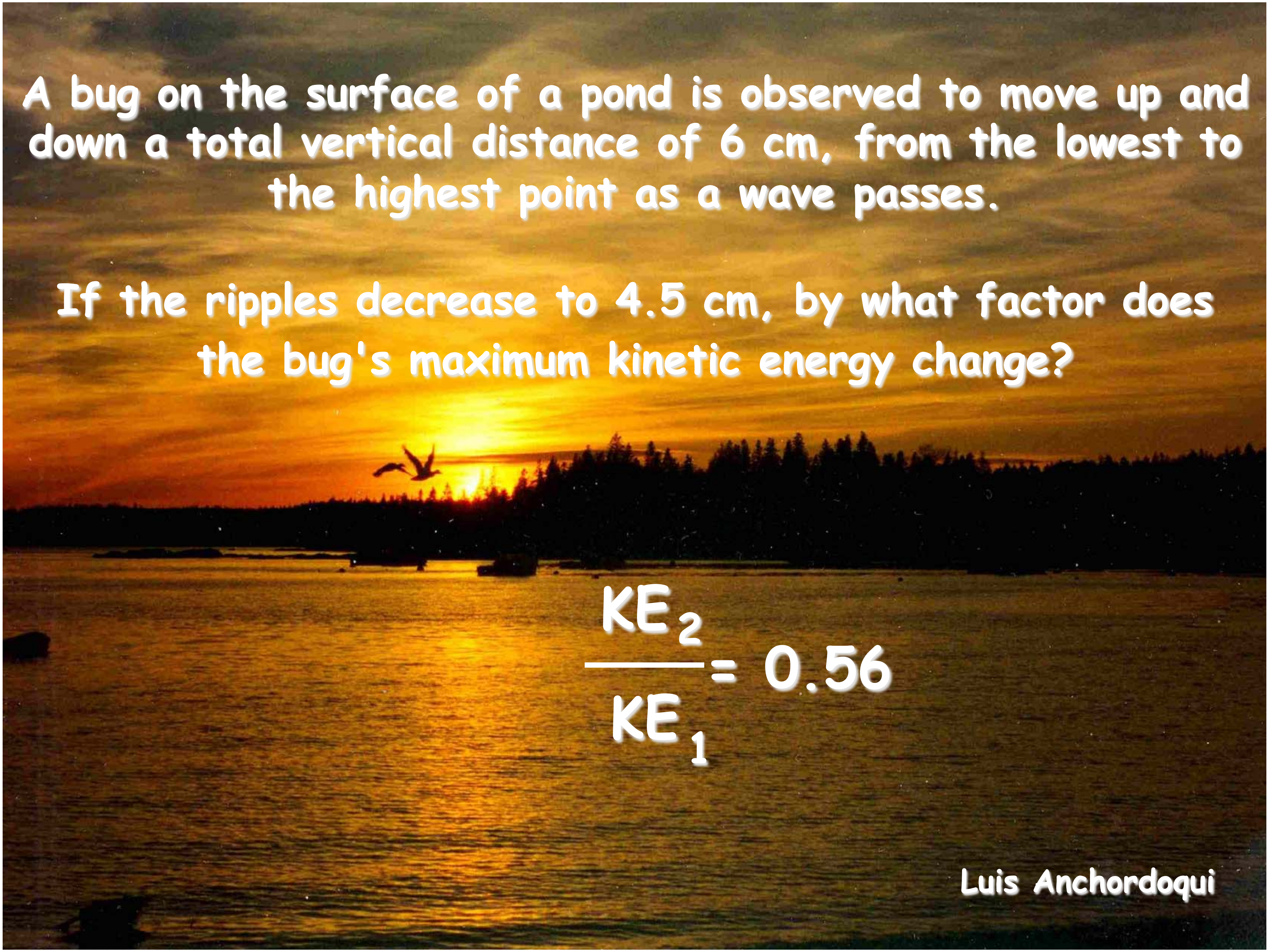

A bug on the surface of a pond is observed to move up and down a total vertical distance of 6 cm, from the lowest to the highest point as a wave passes.

If the ripples decrease to 4.5 cm, by what factor does the bug's maximum kinetic energy change?

$$\frac{KE_2}{KE_1} = 0.56$$

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A bungee jumper (with mass 65 kg) jumps from a high platform in Kuta Beach (Bali, Indonesia).

After reaching his lowest point, he oscillates up and down, hitting a low point eight more times in 38 s

He finally comes to rest 25 m below the level of the bridge. Calculate the spring stiffness constant and the unstretched length L of the bungee cord.

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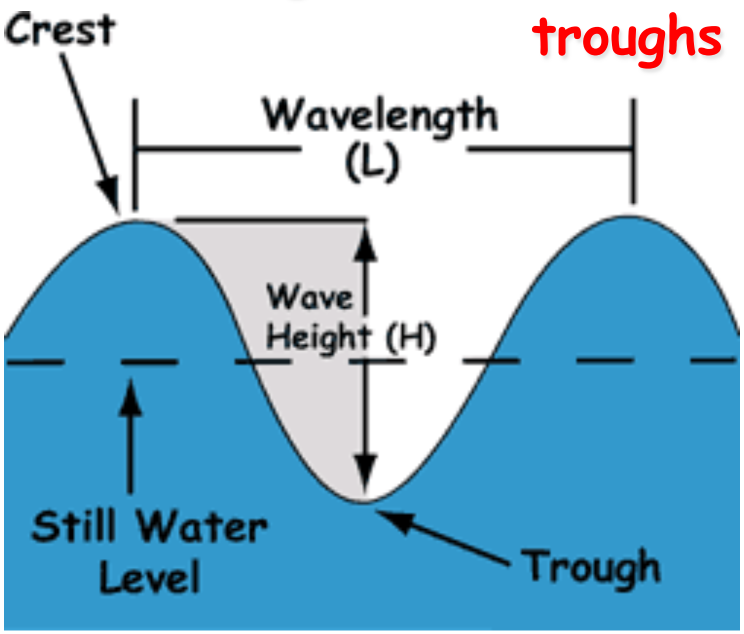
Wave Motion

A mechanical wave is caused by a disturbance in a medium

As wind passes over the water's surface friction forces it to ripple
The strength of the wind, the distance the wind blows
and the duration determine how big the ripples will become

The crest is the highest point on a wave
the trough between two waves is the lowest point
Wavelength is the horizontal distance, either between the crests or troughs of two consecutive waves



Wave height is a vertical distance between a wave's crest and the next trough

Wave period measures the size of the wave in time

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Wave Motion (cont'd)

If you have ever watched the ocean waves moving toward shore before they break you may have wondered if the waves were carrying water from far out at the sea into the beach

THEY DON'T

Water moves with a recognizable velocity but each of the molecule of water itself merely oscillates about an equilibrium point

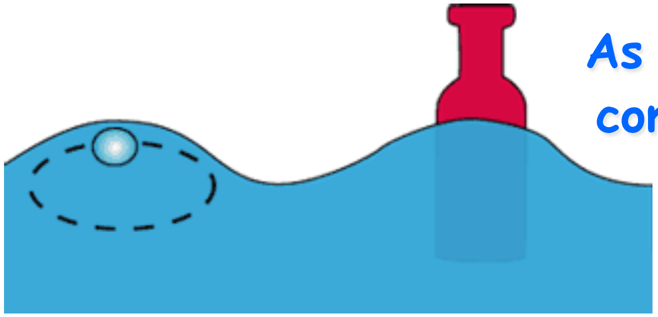
This is clearly demostrated by observing a bottle on a pond as waves move by

The bottle is not carried forward by the waves, but simply oscillate about an equilibrium point because this is the motion of water itself

Watch the water droplet move in a vertical circle as the wave passes

The droplet moves forward with the wave's crest and backward with the trough

These vertical circles are more obvious at the surface



As depth increases, their effects slowly decrease until completely disappearing about half a wavelength below the surface

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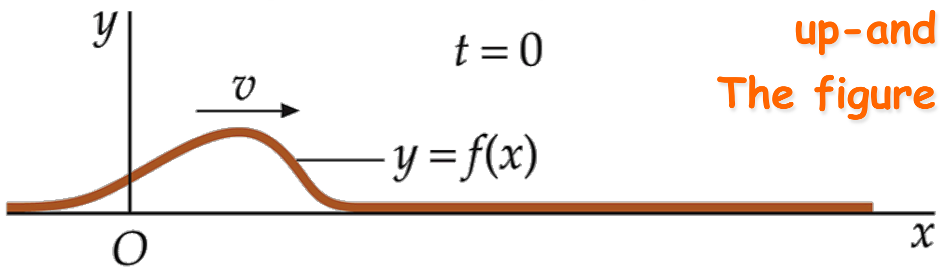
Looking a little more closely at how a wave its forme and how it comes to travel:

Pulse

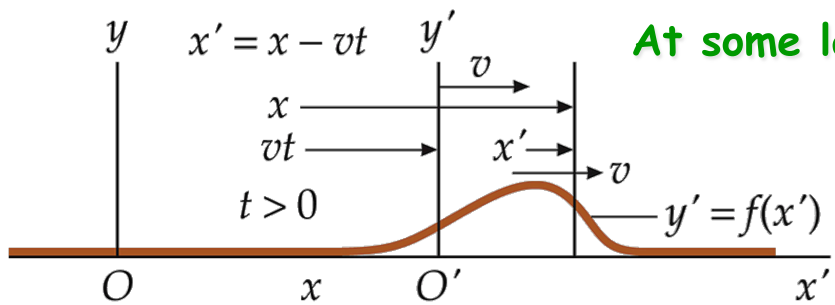
A single wave bump or pulse can be formed on a rope by a quick up-and- down motion of the hand

The figure shows a pulse on a string at time t =0



The shape of the string at this instant can be represented by some function y = f(x)

At some later time the pulse is farther down the string



In a new coordinate system with origin O' that moves to the right with the same speed as the pulse $\longrightarrow$ the pulse is stationary

The string is described in this frame by f(x') for all times

The x-coordinates of the two reference frames are related by

$$x' = x - vt \longrightarrow f(x') = f(x - vt)$$

The shape of the string in the original reference frame is y = f(x - vt)

wave moving in the +x direction

The same line of reasoning for a pulse moving to the left leads to y = f (x + vt)

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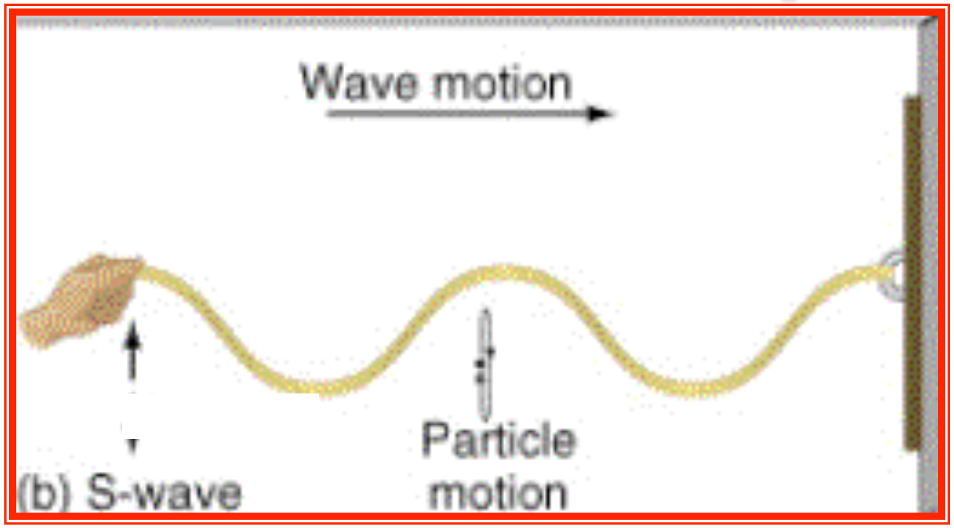
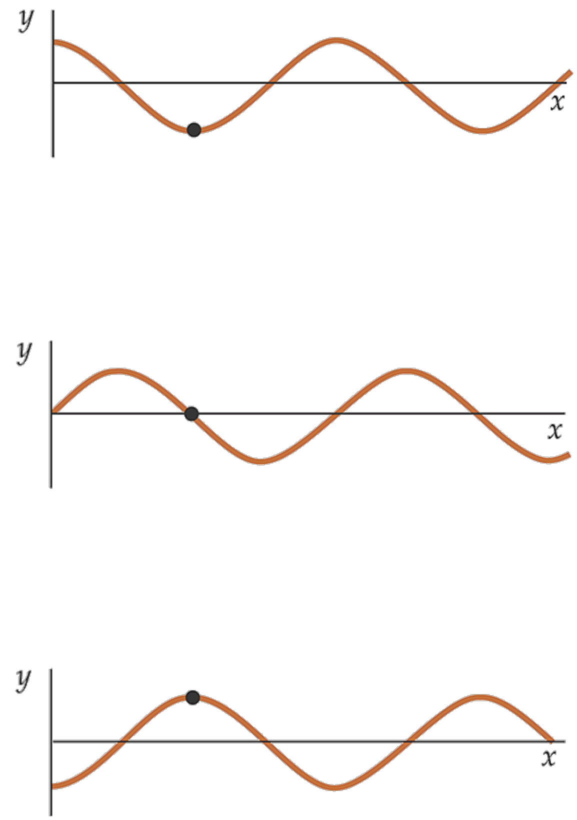
Looking a little more closely at how a wave its formed and how it comes to travel:

Periodic wave

A continuous or periodic wave has as its source a disturbance that is continuous and oscillating





When a taut string is plucked the disturbance in this case is the change in shape of the string from its equilibrium shape

Its propagation arises from the interaction of each string segment with the adjacent segments

The segments of the string move in the direction perpendicular to the string as the pulses propagate back and forth along the string

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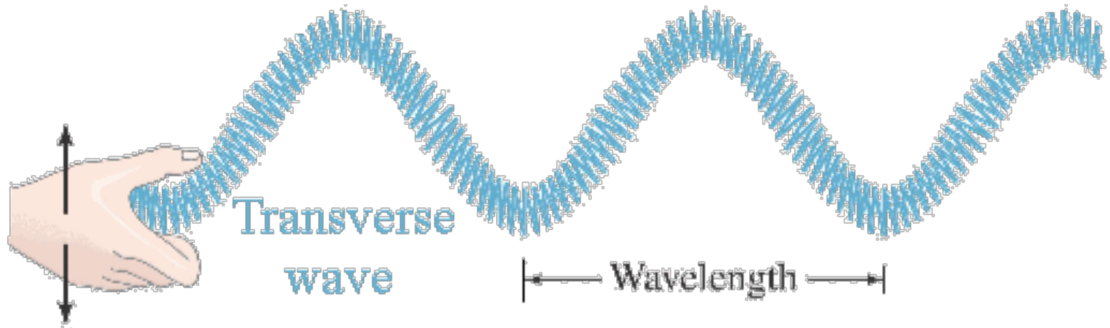
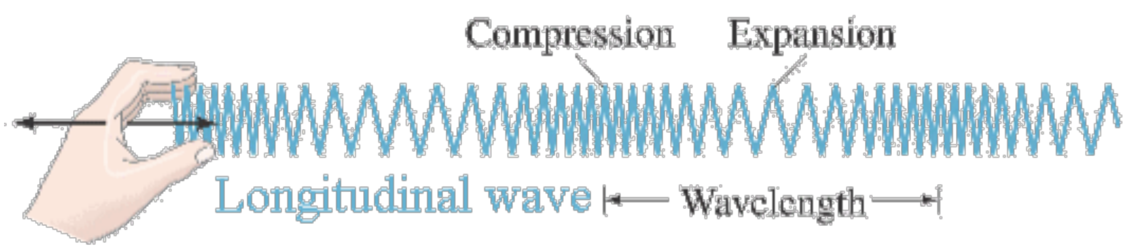
Types of waves



Waves in which the motion of the medium
(molecules of water, particles on the string)
is perpendicular to the direction of propagation are called transverse waves



Waves in which the motion of the medium is along (parallel to) the direction of propagation of the disturbance are called longitudinal waves. (Sound waves are examples of longitudinal waves)

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Speed of waves

The speed of the waves relative to the medium depends on elastic and inertial properties of the medium but is independent of the motion of the source of the waves

For a pulse on a rope

$$v = \left(\frac{F_T}{\mu} \right)^{\frac{1}{2}}$$

String tension (F_T)

Linear mass density (μ)

For sound waves

$$v = \left(\frac{B}{\rho} \right)^{\frac{1}{2}}$$

Bulk modulus (B)

Volume mass density (ρ)

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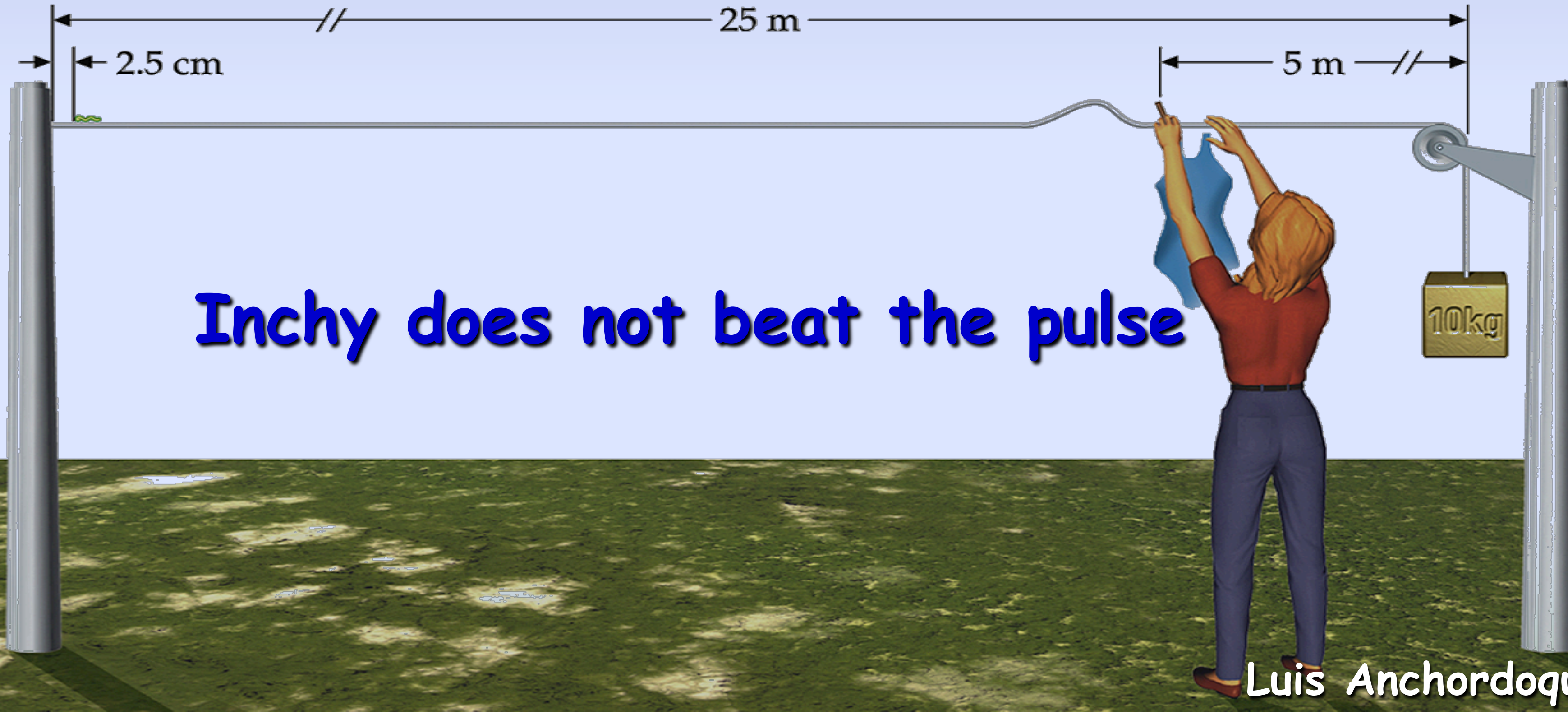
Inchy runs for his life

Inchy, an inchworm, is inching along a cotton clothesline.
The 25-m-long clothesline has a mass of 1 kg and is kept taut by a hanging object of mass 10 kg as shown in the figure.
Gaby is hanging up her swimsuit 5 m from one end when she sees Inchy 2.5 cm from the opposite end. She plucks the line sending a terrifying 3-cm-high pulse towards Inchy. If Inchy crawls at 1 in/s, will he get to the end of the clothesline before the pulse reaches him?



Inchy does not beat the pulse

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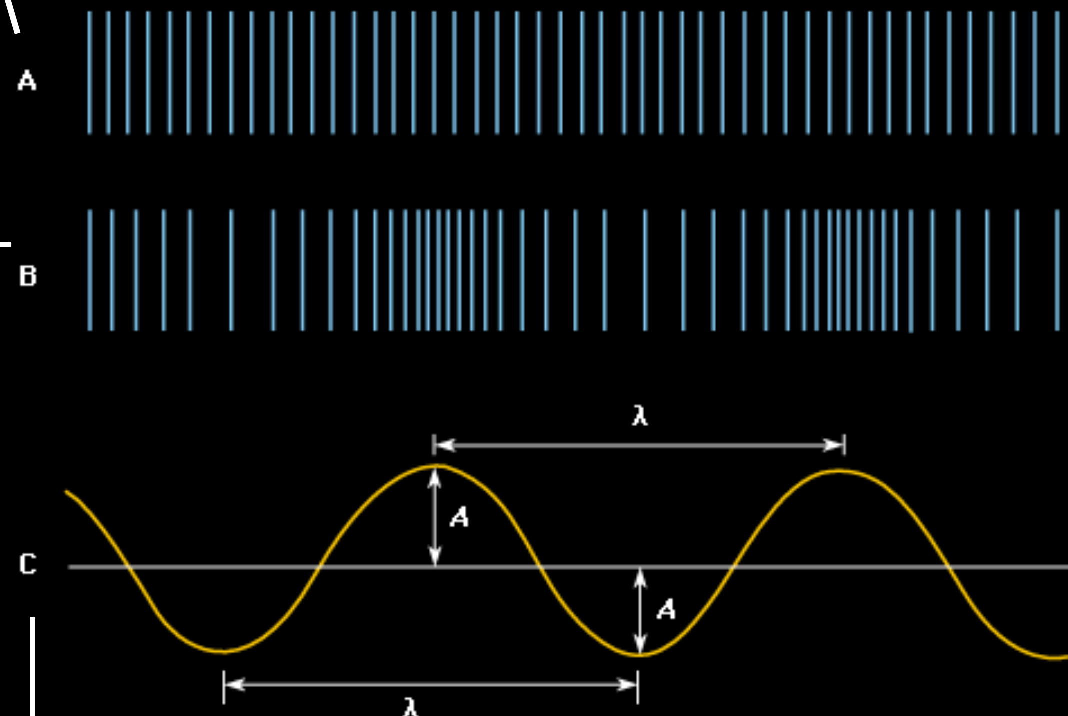
Graphic representations of a sound wave

(A) Air at equilibrium, in the absence of a sound wave

(B) Compressions and rarefactions that constitute a sound wave



(C) Transverse representation of the wave, showing amplitude (A) and wavelength (λ)

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Speed of sound in air

For sound waves in a gas the bulk modulus is proportional to the pressure wich in turn is proportional to the density and to the absolute temperature of the gas

The ratio B/ρ is independent of density and is merely proportional to the absolute temperature

$$v = (\gamma RT/M)^{\frac{1}{2}}$$

The dimensionless constant γ depends on the kind of gas

For diatomic molecules such as O_2 and N_2 $\gamma = 7/5$

Because O_2 and N_2 comprise 98% of the atmosphere for air $\gamma = 7/5$

For gases composed of monoatomic molecules such as He $\gamma = 5/3$

$$T = t_C + 273$$

$$R = 8.3145 \text{ J/(mol K)}$$

The molar mass from air is

$$M = 29 \times 10^{-3} \text{ kg/mol}$$

The speed of sound at 20°C is the about 343 m/s

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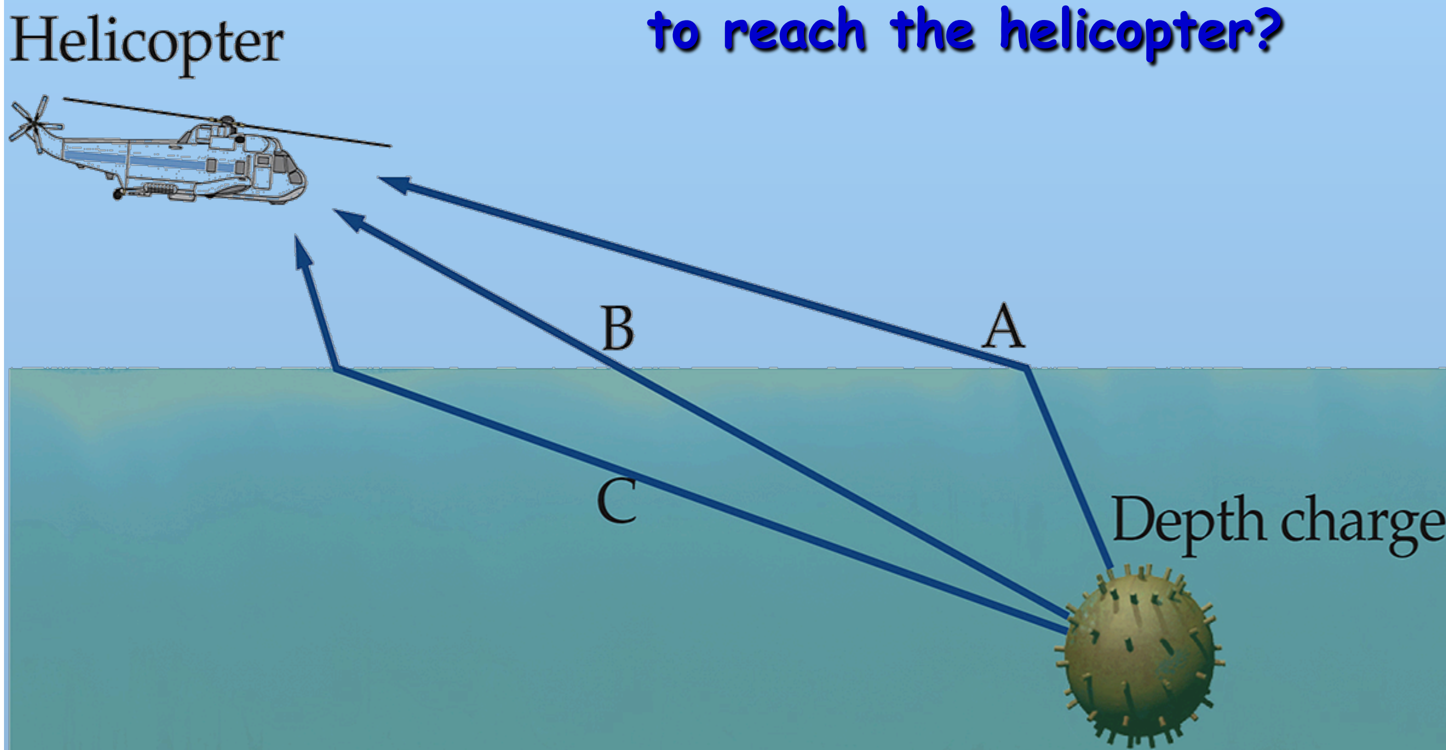
The explosion of a depth charge beneath the surface of a body of water is recorded by an helicopter hovering above the water's surface as shown in the figure.

Along which path (A, B, or, C) will the sound wave take the least time to reach the helicopter?



The speed of sound in water is greater than the speed of sound in the air ➡ path C

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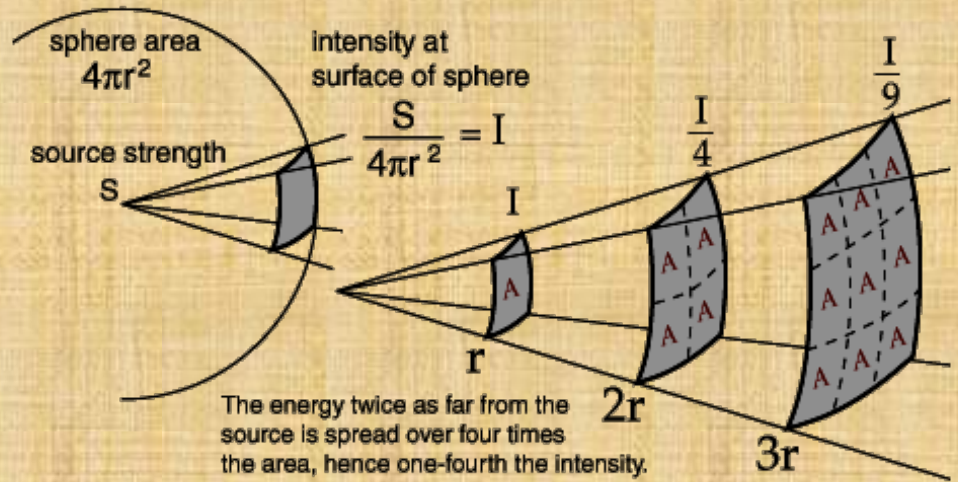
Wave intensity

Wave transports energy from one place to another

As waves travel through a medium the energy is transferred as vibrational energy from a particle to particle in the medium

If a point source emits waves uniformely in all directions then the energy at a distance r from the source is distributed uniformly on a spherical surface of radius r and area $A = 4\pi\, r^2$



The average power per unit area that is incident perpendicular to the direction of propagation is called the intensity

$$I = \frac{P_{av}}{A}$$

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Wave intensity (cont'd)

Sound waves from a telephone handset spreading out in the air
The wave have been made visible by sweeping out the space in front of the handset with a light source whose brightness is controlled by a microphone

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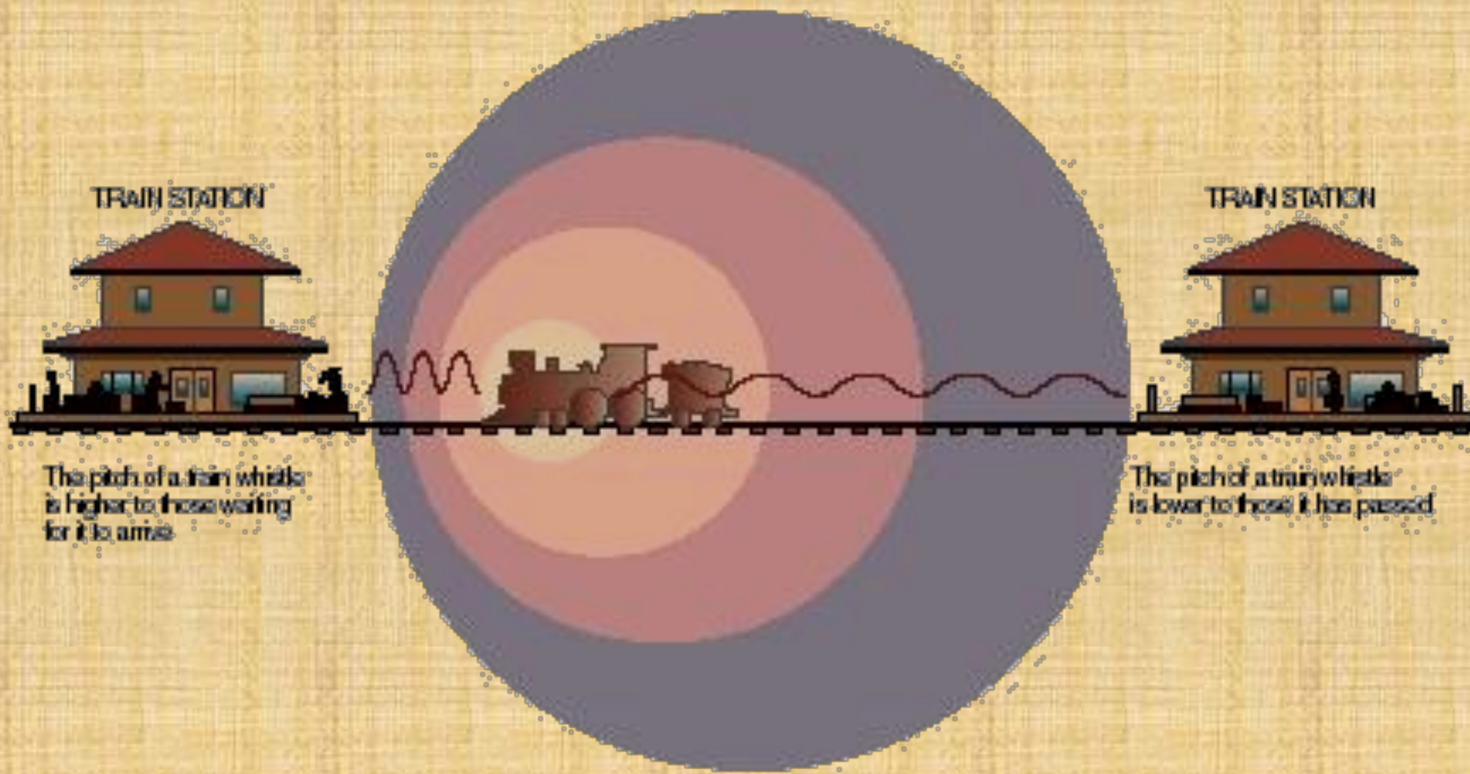
Doppler Effect

You may have noticed that you hear the pitch of the whistle on a speeding train dropped abruptly as it passes you



This phenomenon is known as Doppler effect

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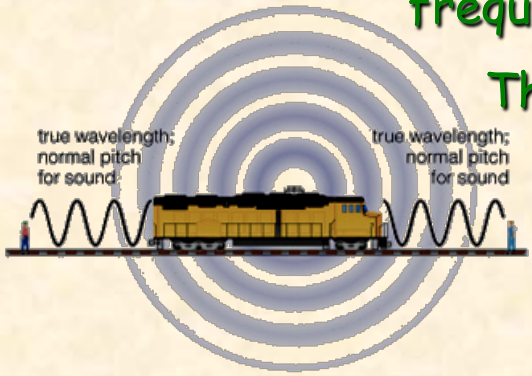
Doppler effect (cont'd)

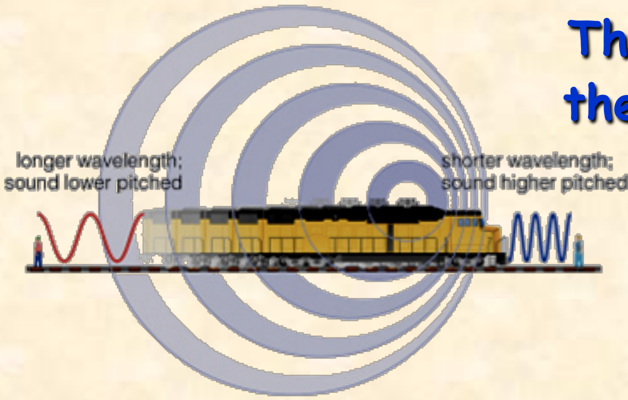
Consider the whistle of a train at rest which is emitting sound of a particular frequency in all directions as shown in the figure



The sound waves are moving at the speed of sound in air which is independent of the velocity of the source or observer

If the our source is moving then whistle emits sounds at the same frequency as it does at rest

The sound wavefronts it emits forward are closer together than when the train is at rest



This is because the train as it moves is "chasing" the previously emitted wavefronts and emits each crest closer to the previous one

Thus an observer in front of the train will detect more wave crests passing per second so the frequency heard is higher

The wavefronts emitted behind the train are farther apart than when the train is at rest because the train is speeding away from them

Fewer wave crest per second pass by an observer behind the moving train and the perceived pitch is lower

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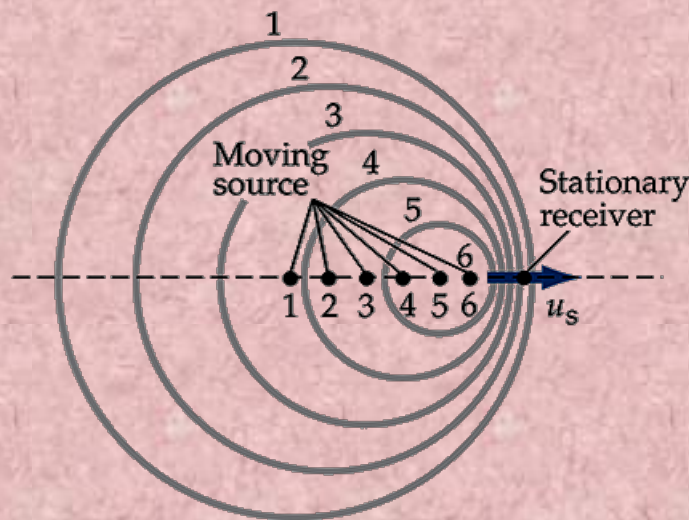
Doppler effect (cont'd)

In the following discussion all motions are relative to the medium

Consider a source moving with speed u_s and a stationary receiver



The source has frequency f_s

The received frequency

(the number of crests passing the receiver per unit time)

is related to the wavelength λ (distance between successive crests)

and the wave speed v by $f_r \, \lambda = v$

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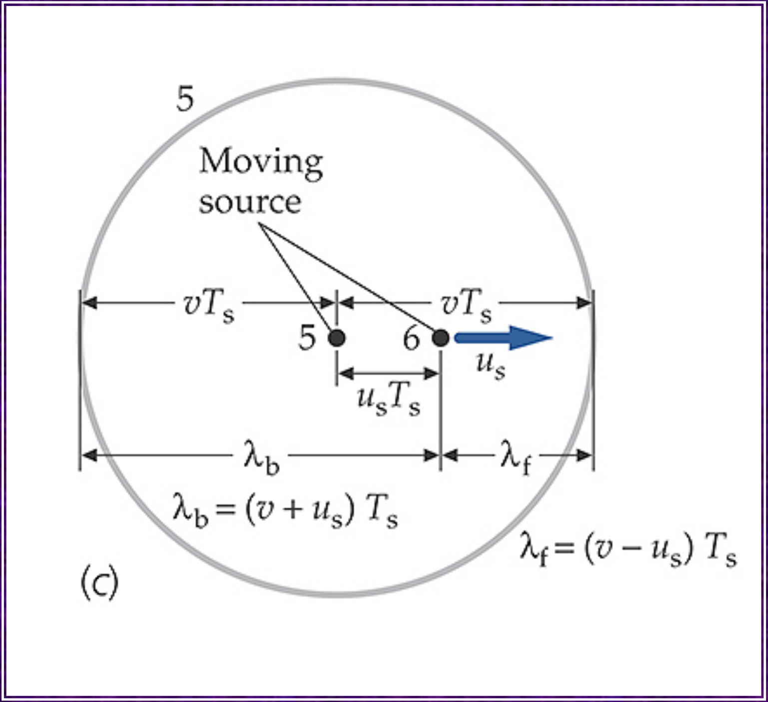
Doppler effect (cont'd)



A wave crest leaves the source at time t_1 and the next wave crest leaves the source at time t_2

The time between these two events is $T_s = t_2 - t_1$ and during this time the source and crest leaving the source at time t_1 travel distances u_sT_s and vT_s , respectively

⇓

At time t_2 the distance between the source and the crest leaving at time t_1 equals the wavelength λ

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Doppler effect (cont'd)

If $u_s < v$

$\Downarrow$

Behind the source

$$\lambda = \lambda_b = (v + u_s)\, T_s$$

In front of the source

$$\lambda = \lambda_f = (v - u_s)\, T_s$$

We can express both λ_b and λ_f as

$$\lambda = (v \pm u_s)\, T_s = \frac{v \pm u_s}{f_s}$$

Substituting for our expression for λ and rearranging

$$f_r = \frac{v}{\lambda} = \frac{v}{v \pm u_s}\, f_s \quad \text{(stationary receiver)}$$

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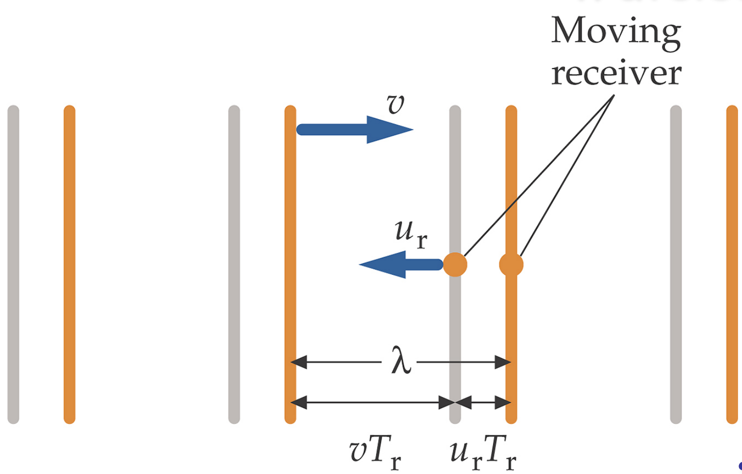
Doppler effect (cont'd)

When the receiver moves with respect to the medium the received frequency is different because the receiver moves past more or fewer wave crests in a given time

Let T_r denote the time between arrivals of succesive crests for a receiver moving with speed u_r

During the time between the arrivals of two succesive crests each crest will have traveled a distance vT_r and during the same time the receiver will have traveled a distance $u_r T_r$



If the receiver moves in the direction opposite to the wave during a time T the distance a wave moves + the distance the receiver moves equals the wavelength

$$vT_r + u_r T_r = \lambda \Longrightarrow T_r = \lambda/(v + u_r)$$

If the receiver moves in the same direction as the wave

$$vT_r - u_r T_r = \lambda \Longrightarrow T_r = \lambda/(v - u_r)$$

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Doppler effect (cont'd)

Because $f_r = \lambda / T$ we have

$$f = \frac{1}{T_r} = \frac{v \pm u_r}{\lambda}$$

Substituting for λ

$$f_r = \frac{v \pm u_r}{v \pm u_s} f_s$$

In a reference frame in which the medium is moving (for example the reference frame of the ground if air is the medium and there is a wind blowing)

The wave speed v is replaced by $v' = v \pm u_w$ where u_w is the velocity of the wind relative to the ground

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Batman has sent a signal to the batcave calling for his batfriends to cover his escape.
Answering the signal, a bat which is nearby starts flying at 5 m/s.
As it flies, the bat emits an ultrasonic sound wave with frequency 30 kHz towards the tall wall of the building.
What frequency does the bat hear in the reflected wave?
$f = 3.09 \times 10^4$ Hz
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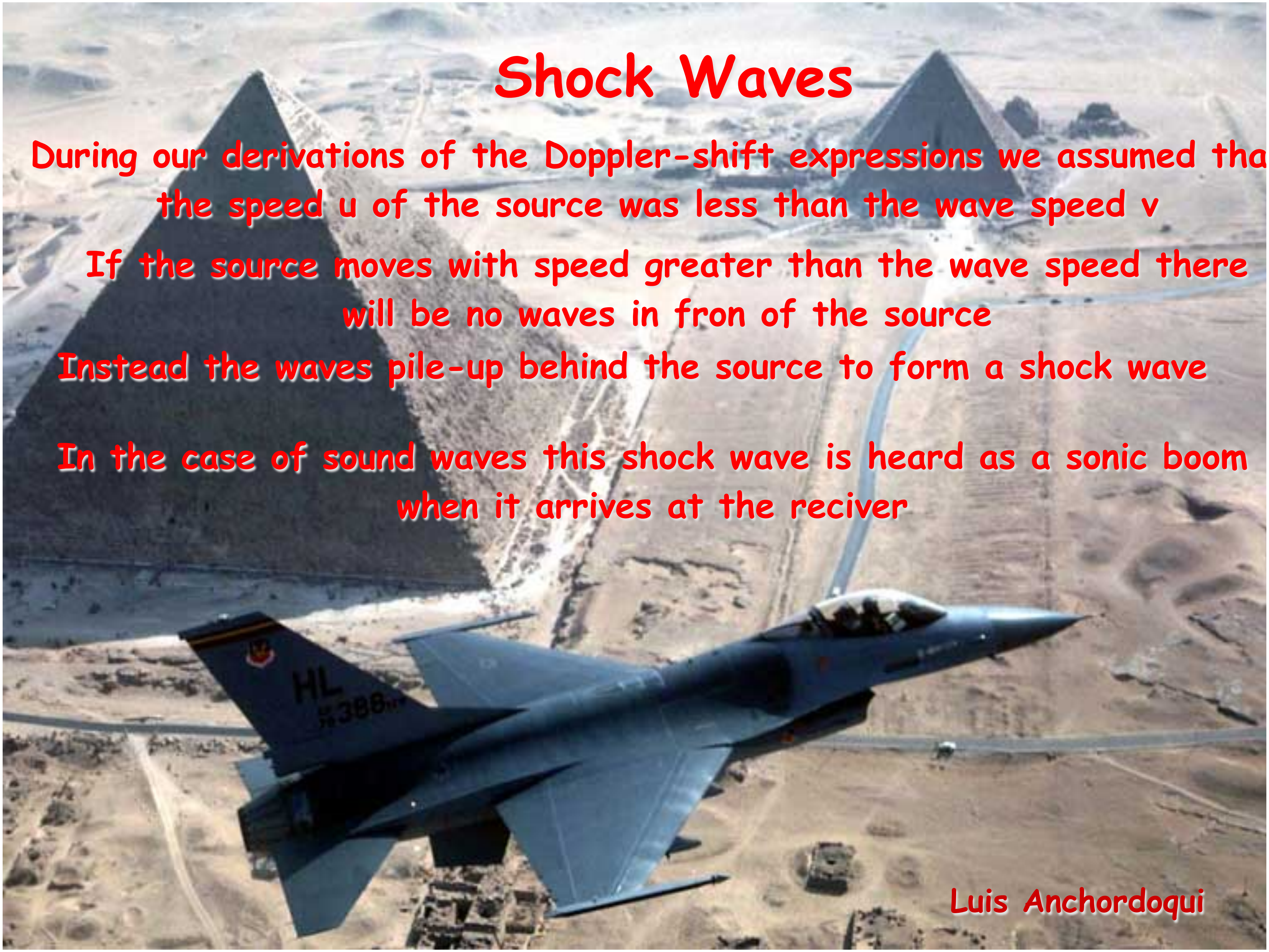
Shock Waves

During our derivations of the Doppler-shift expressions we assumed tha the speed u of the source was less than the wave speed v

If the source moves with speed greater than the wave speed there will be no waves in fron of the source

Instead the waves pile-up behind the source to form a shock wave

In the case of sound waves this shock wave is heard as a sonic boom when it arrives at the reciver



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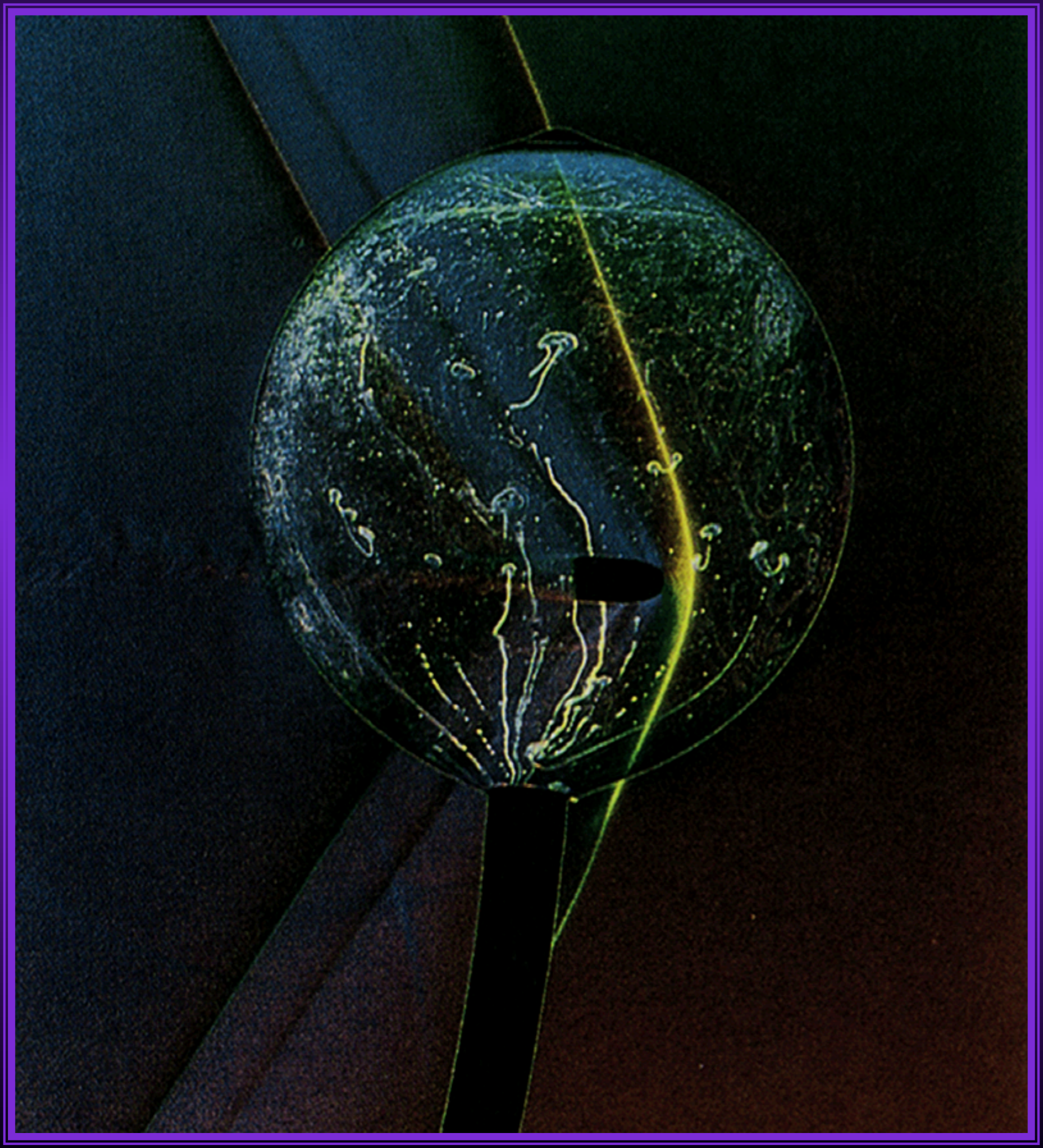
Shock waves produced by a bullet traversing a helium balloon



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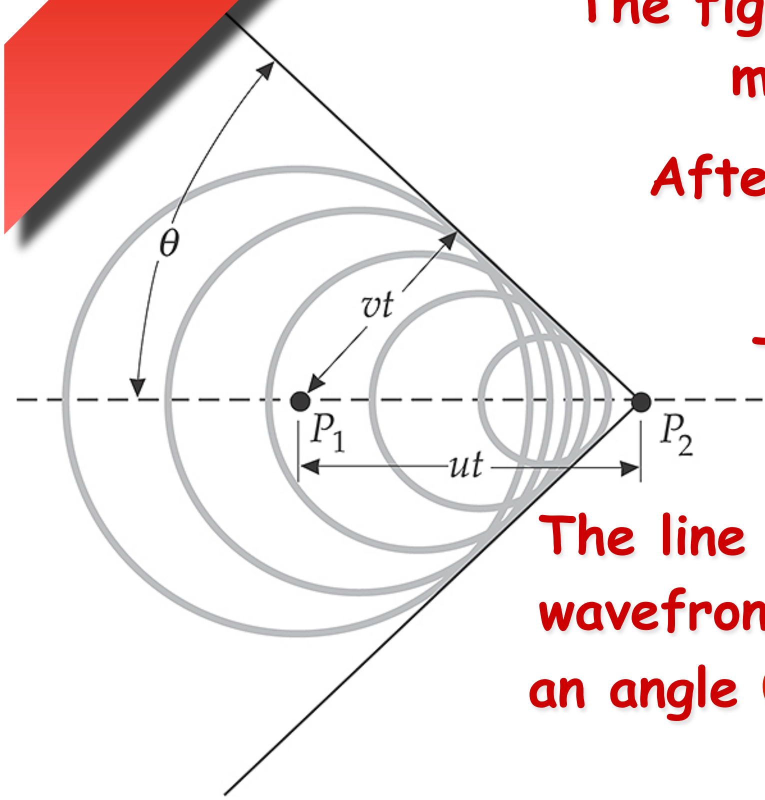
Mach Number

The figure shows a source originally at point P_1 moving to the right with velocity u



After some time t the wave emitted from point P_1 has traveled a distance vt

The source has traveled a distance ut and will be at point P_2

The line from this new position of the source to the wavefront emitted when the source was at P_1 makes an angle θ with the path of the source known as the Mach angle +

$$\sin\theta = \frac{v\,t}{u\,t} = \frac{v}{u}$$

The shock wave is confined to a cone that narrows as u increases

The ratio of the source speed u to the wave speed v is called the Mach number

$$\text{Mach number} = \frac{u}{v}$$

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Sonic Boom
Shock waves from a supersonic plane
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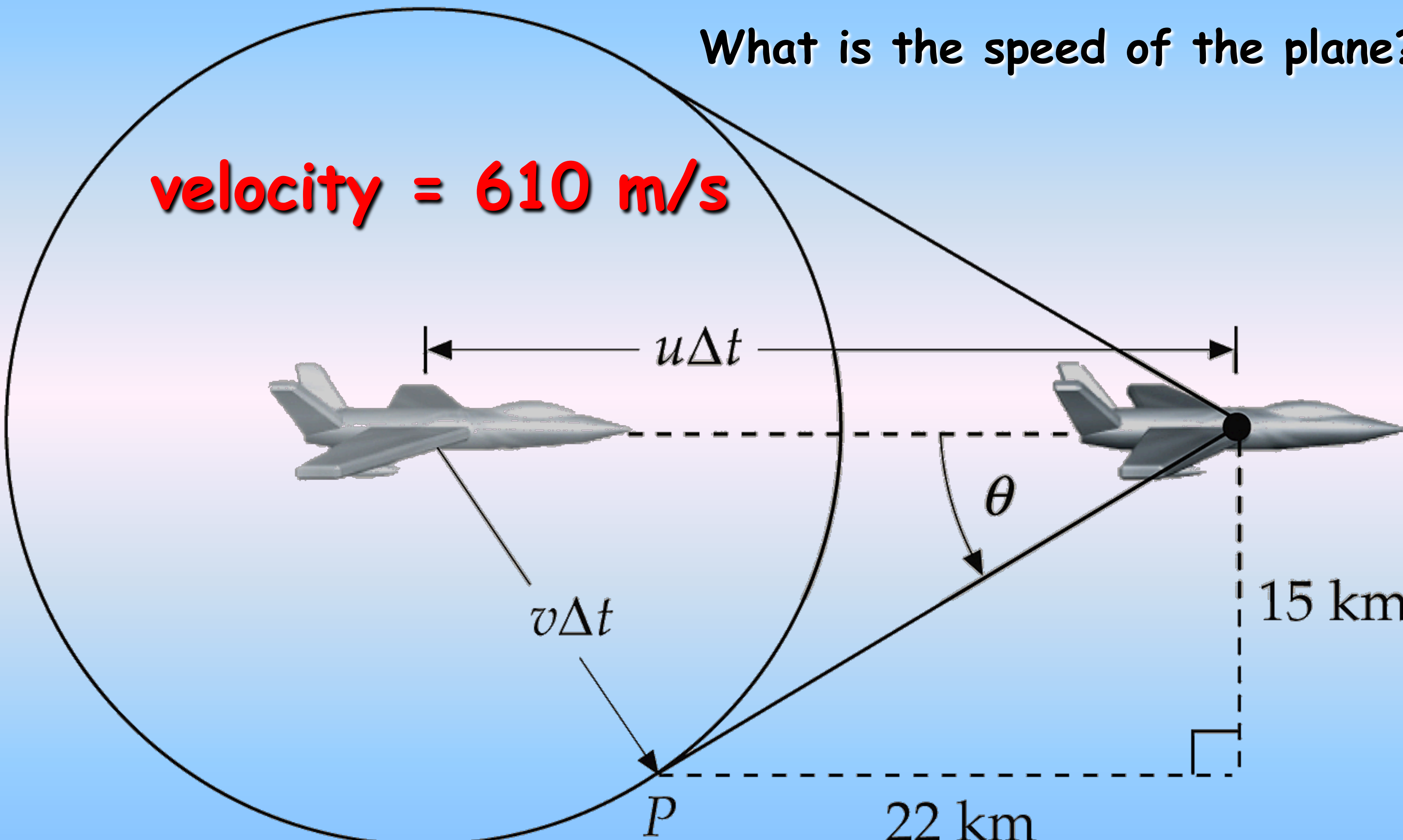
A supersonic plane flying due east at an altitude of 15 km passes directly over point P.

The sonic boom is heard at point P when the plane is 22 km east of point P.

What is the speed of the plane?



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Doppler Effect (summary)

Stationary Sound Source

•

Doppler Shift

Source moving with $v_{source} < v_{sound}$

•

Breaking the Sound Barrier ⟶ Sonic Boom

Source moving with $v_{source} > v_{sound}$

•

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Physics 209

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