$$\mathbf{A}_{\text{dip}}(\mathbf{r}) = \frac{\mu_0 I}{4\pi r^2} \oint r' \cos\theta \, dl = \frac{\mu_0 I}{4\pi r^2} \oint (\hat{r} \cdot \mathbf{r}') \, dl \tag{5.76}$$

This integral can be cast in a more useful form by the following unlovely manipulations: First note that

$$d[(\hat{r}\cdot\mathbf{r}')\mathbf{r}'] = (\hat{r}\cdot d\mathbf{r}')\mathbf{r}' + (\hat{r}\cdot\mathbf{r}')d\mathbf{r}'$$

so that

$$\oint [(\hat{r}\cdot d\mathbf{r}')\mathbf{r}' + (\hat{r}\cdot\mathbf{r}')d\mathbf{r}'] = \oint d[(\hat{r}\cdot\mathbf{r}')\mathbf{r}'] = 0$$

(the total change in $[(\hat{r}\cdot\mathbf{r}')\mathbf{r}']$ around a *closed* loop is zero), and hence

$$\oint (\hat{r}\cdot d\mathbf{r}')\mathbf{r}' = -\oint (\hat{r}\cdot\mathbf{r}')\, d\mathbf{r}' \tag{5.77}$$

Therefore, using the BAC-CAB rule,

$$\hat{r} \times \oint (\mathbf{r}' \times d\mathbf{r}') = \oint [\mathbf{r}'(\hat{r}\cdot d\mathbf{r}') - d\mathbf{r}'(\hat{r}\cdot\mathbf{r}')] = -2\oint (\hat{r}\cdot\mathbf{r}')\, d\mathbf{r}'$$

Now $d\mathbf{r}'$ is just the infinitesimal displacement vector we have always called $d\mathbf{l}$ (Fig. 5.49); so

$$\oint (\hat{r}\cdot\mathbf{r}')\, d\mathbf{l} = -\tfrac{1}{2}\hat{r} \times \oint (\mathbf{r}' \times d\mathbf{l}) \tag{5.78}$$

With this the dipole potential (5.76) becomes

$$\mathbf{A}_{\text{dip}}(\mathbf{r}) = \frac{\mu_0 I}{4\pi r^2}\left[-\frac{1}{2}\hat{r} \times \oint (\mathbf{r}' \times d\mathbf{l})\right]$$

or, more compactly,

$$\boxed{\mathbf{A}_{\text{dip}} = \frac{\mu_0}{4\pi}\frac{\mathbf{m}\times\hat{r}}{r^2}} \tag{5.79}$$

where $\mathbf{m}$, the **magnetic dipole moment** of the loop, is defined by

$$\boxed{\mathbf{m} = \tfrac{1}{2} I \oint (\mathbf{r}' \times d\mathbf{l})} \tag{5.80}$$

In the special case of a *plane* loop, the integral in (5.80) admits a nice geometrical interpretation: $\frac{1}{2}(\mathbf{r}' \times d\mathbf{l})$ is the area of the shaded triangle, in Fig. 5.50, so the integral is the area of the whole loop:

$$\tfrac{1}{2}\oint (\mathbf{r}' \times d\mathbf{l}) = \mathbf{a} \tag{5.81}$$