Analytical Dynamics - Graduate Center CUNY - Fall 2008

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Problem set 01 Due: We, Nov 19

1 Moments of inertia

(10 points) Calculate tensors of inertia with respect to the principal axes of the following bodies:

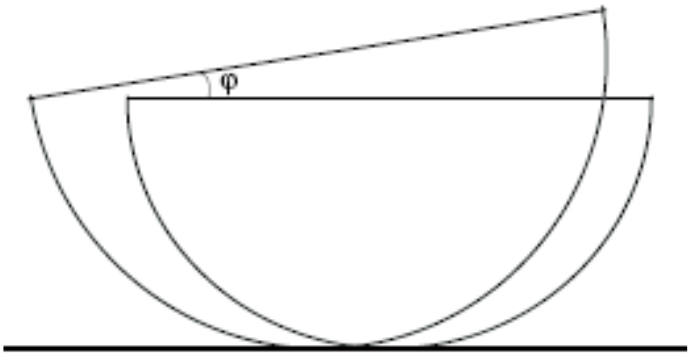
a) Hollow sphere of mass M and radius R

b) Cone of the height h and radius of the base R; both with respect to the apex and to the center of mass

c) Body of a box shape with sides a, b, and c

2 Half-cylinder

(10 points) Consider a half-cylinder of mass M and radius R on a horizontal plane.



a) Find the position of its center of mass (CM) and the moment of inertia with respect to CM.

b) Write down the Lagrange function in terms of the angle φ (see figure)

c) Find the frequency of cylinder's oscillations in the linear regime, $\varphi \ll 1$.

3 Rod on the axis

(10 points) Rod of length l and mass M is mounted on an axis at its center.

a) If the angle θ between the rod and the axis is fixed and the rod rotates with the angular velocity $\omega = \dot{\varphi}$ around the axis, what is the (i) kinetic energy of the rod; (ii) breaking torque acting from the rod on the axis?

b) Set up the Lagrange equations for the rod in the case where both θ and φ can freely change. Find integrals of motion. If you have access to mathematical software, you can try to produce numerical solutions with particular initial conditions such as $\theta(0) = \theta_0$, $\dot{\theta}(0) = 0$, $\varphi(0) = 0$, $\dot{\varphi}(0) = \omega_0$

c) Consider the motion of this system confined to the vicinity of $\theta = \pi/2$ and try to integrate Lagrange equations analytically