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PHY131 – Conceptual physics – Lecture notes

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1. Kinematics

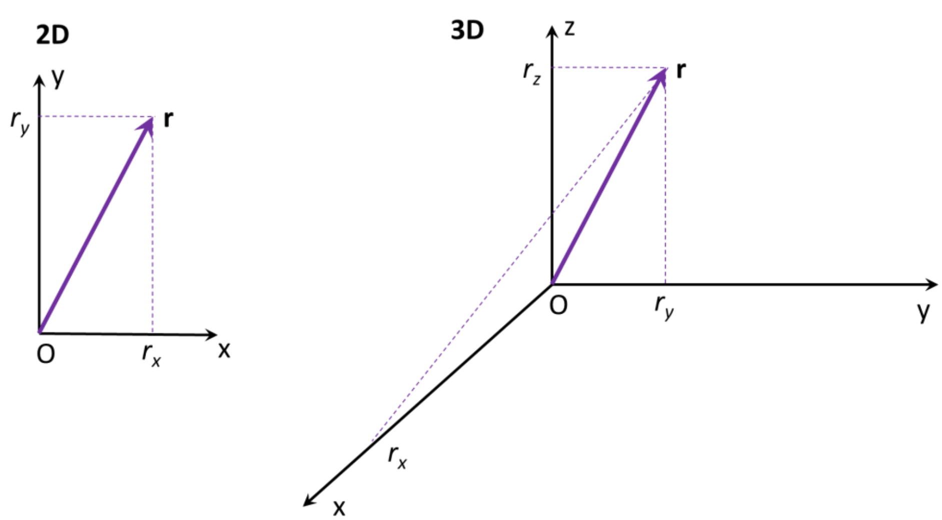
Most of the physical quantities are scalars or vectors.

Scalars are just numbers (mass, temperature, pressure, electric charge, speed, etc.)

Vectors are mathematical objects that can be envisioned as arrows having a direction and a length (magnitude). Examples of vectors: Position vector in 3D space, displacement, velocity, acceleration, force, electric and magnetic fields, etc. Vectors are denoted as symbols with arrows in handwriting, such as $\vec{r}$, and as bold symbols in print, such as $\mathbf{r}$.

Vectors in 3D can be represented via their components (*x,y,z*) in a coordinate system of three mutually perpendicular axes. If the object is in the plane (as we are on the surface of the Earth), then the coordinate system has only two axes and there are only two components of a vector: (*x,y*). These correspond to geographical latitude and longitude or to the set of two GPS coordinates.

Vectors & coordinate systems: position vector and its components



Displacement $\mathbf{d}$ of an object is defined as a vector that goes from the object's initial position $\mathbf{r}_i$ to its final position $\mathbf{r}_f$. Of course, not only the amount (length or magnitude) of the displacement is important, but also its direction. This is why vectors are so important.

Distance d is a scalar equal to the magnitude of the displacement $\mathbf{d}$ from one position to the other.

An object (or a body, a person, a material point, etc.) can move from the initial to the final position along a trajectory that is not necessarily straight. The length of the trajectory will be denoted as L. As the straight line is the shortest way, the length of the trajectory cannot be smaller than the distance, $d \leq L$.

An object (or a system of objects) is static if its position (positions) does (do) not depend on time. If there is a time dependence, we speak of kinematics or, more generally, of dynamics. Important quantities are velocity and speed.

Average velocity is defined as the displacement divided by the time interval of the motion,

$$\mathbf{v} = \frac{\mathbf{r}_f - \mathbf{r}_i}{t} = \frac{\mathbf{d}}{t}.$$

This formula is familiar to everybody but one should understand that the velocity is a vector and it also describes the direction of the motion. The magnitude of the velocity is just distance over time, $v = d/t$.

Average speed is the trajectory length divided by the time interval of the motion,

$$s = \frac{L}{t}.$$

Since $d \le L$, the speed is no less than the magnitude of the velocity, $v \le s$.

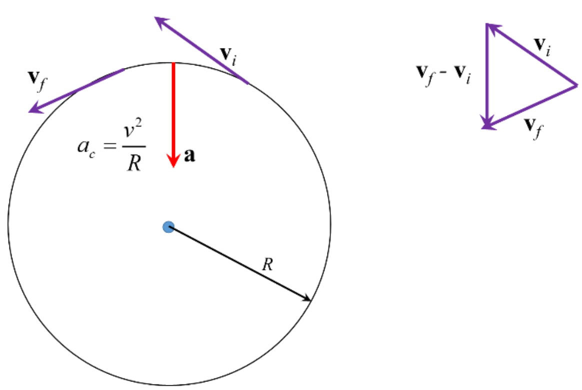
Velocity and speed may change over time. In this case, it is convenient to speak of the instantaneous velocity and speed. They are defined in the same way, only the time interval should be made very short.

Acceleration is defined as the change of the velocity divided by the time interval:

$$\mathbf{a} = \frac{\mathbf{v}_f - \mathbf{v}_i}{t}.$$

Acceleration is a vector. Acceleration arises if the velocity changes its magnitude, its direction, or both. If a car is accelerating from the rest, the direction of the acceleration is the same as the direction of the velocity. If a car is braking, the acceleration is directed opposite to the velocity. In this case, one can speak of deceleration. If an object is moving along a circular trajectory with a constant speed, the velocity still changes its direction with time. Thus, there is an acceleration. This acceleration is called centripetal acceleration (a_c) and it is directed toward the center of the circle.

Circular trajectory. Centripetal acceleration



Physicists use the international system of units called SI (French *Système International*). The basic units of mechanics are meter (m), second (s), and kilogram (kg). The etalons of meter and kilogram are kept in Paris. The main principle of the SI is its decimal character: everything goes by powers of 10. For instance, 1 km = 1000 m, 1 cm = 0.01 m, 1 mm = 0.001 m, etc. Using this system of units is much easier than that of the archaic systems.

All other units are derivatives of the basic units and they can be figured out from the definitions of the physical quantities. For instance, velocity and speed are defined as distance-over-time, thus their unit is m/s. Similarly, the unit of the acceleration is m/s^2.

2. Newtonian dynamics and gravity

The main law of mechanics is Newton's second law that states that forces applied to the object cause its acceleration. The formula reads *Force = Mass x Acceleration* or, in the algebraic form,

$$\mathbf{F} = m\mathbf{a}.$$

As the acceleration, the force is a vector. The mass of the body m is a scalar so that the directions of the force and the acceleration coincide. In Newton's law, the force $\mathbf{F}$ stands for the sum of all forces acting on the body. From the formula above, it follows that the unit of force is kg m/s^2. This unit has a special name, newton, that is, N = kg m/s^2. Newton is a derivative, not a basic, unit.

If the force is zero, the acceleration is also zero, so that the object is not accelerating and its velocity does not change (is a constant). This particular case constitutes Newton's first law.

The third Newton's law is an auxiliary law saying that when two bodies interact exerting forces on each other these forces are opposite vectors of the same magnitude.

One of the most important forces is the gravity force that everyone feels. The gravity force is directed down and its magnitude is given by

$$F_G = mg.$$

Here g is the free-fall acceleration, g = 9.8 m/s^2. The weight of the body is defined as the magnitude of the gravity force acting on it and is measured in newtons. In the everyday language weight and mass mean the same.

The formula above is just a special case of the attractive gravity force acting between two bodies of masses M and m that also was established by Newton:

$$F_G = G\frac{Mm}{R^2}.$$

Here G is the gravitation constant and R is the distance between the two bodies. Comparing the two formulas, one can identify $g = GM/R_E^2$, where R_E is now the radius or the Earth, 6400 km.

Centripetal acceleration is given by $a_c = v^2/R$, thus there has to be a force $F_c = ma_c$. The role of the centripetal force is played by such physical forces as the tension of the cord, friction between the road and car's tires, as well as the gravity force in the planetary mechanics.

Another example of the force is the elastic force, that is, the force needed to deform an elastic object. As the simplest case, the force needed to compress or expand a spring is given by the Hooke's law

$$F = kx,$$

Where k is the so-called spring constant and x is the change of the spring's length.

3. Energy

Energy has become a major concept in physics well after Newton in the course of the industrial progress that introduced machines doing useful work. It was found that the energy, that can be understood as the stored work or ability to do work, is conserved in all physical processes, including functioning of machines and life. Work is defined as *Work = Force x Displacement*, that is

$$W = Fd.$$

It should be stressed, that the work depends on the relative directions of the vectors $\mathbf{F}$ and $\mathbf{d}$. For instance, if these vectors are perpendicular, the work is zero. In particular, centripetal forces are not doing work. If the force and the displacement are opposite, the work is negative. The unit of the work is N m = kg m^2/s^2. It has a special name joule (J), that is, J = N m = kg m^2/s^2. As the energy is defined by work, its unit is the same, J.

Power is defined as work done per unit of time,

$$P = \frac{W}{t}.$$

The unit of power is watt, W = J/s = kg m^2/s^3.

There are several other special units of work and energy, such as kilowatt-hour, kWh, calorie, etc. The unit kWh is usually used in measuring the generated and consumed electrical energy, 1 kWh = 3600 kJ = 3.6×10^6 J.

Calorie is used to describe thermal processes, as well as the energy contained in food. Initially, calorie was defined as an amount of heat needed to increase the temperature of 1 g water by one degree Celsius. However, the heat is just a form of energy transferred between the objects by non-mechanical (thermal) way. Joule has established the relation between the units as 1 cal = 4.184 J in his famous experiment. Kilocalorie (kcal, Cal) is used in many cases instead of the calorie. The food calorie is, in fact, the kilocalorie. Another relation is 1 kWh = 860 Cal.

The law of conservation of energy does not preclude transforming the energy into forms that cannot be used. From the practical perspective, this energy gets lost. This always happens if the friction forces are acting. For instance, work done on wiping a blackboard or a window is converted into the internal energy of these objects, that is, into the energy of their disorderly moving atoms and molecules. This microscopic motion defines the temperature of these bodies that slightly increases in the process. One can confirm this picture experimentally by just rubbing hands. This is an example of conversion of mechanical work into a non-mechanical form of energy.

Mechanical energy is just one form of energy. There are two kinds of mechanical energy: kinetic energy and potential energy. Kinetic energy is the energy of the motion and is defined by

$$E_k = \frac{mv^2}{2}.$$

For a system of bodies, their kinetic energies add up. Potential energy is related to the object's position. There are so many types of potential energy E_p as there are different forces. However, there is no potential energy corresponding to the friction force. The gravitational energy of bodies near the Earth's surface is given by

$$E_G = mgh,$$

where h is the height above the chosen reference level. The potential energy of attracting celestial bodies is

$$E_G = -G\frac{Mm}{R}.$$

The potential energy of a deformed spring is given by

$$E_p = \frac{kx^2}{2}.$$

If there are no friction forces, the total mechanical energy is conserved: $E_k + E_p$ = const. In the presence of weak friction forces, the total mechanical energy is slowly decreasing, and finally, the system is reaching the state with zero kinetic energy and minimal potential energy. Note that the total energy including the internal energy of other objects is always conserved.

Other forms of energy are thermal energy, the energy of the electromagnetic field (photons), chemical energy (including the energy stored in batteries, in food, and in the muscles), nuclear energy, etc.

Machines spend energy to produce useful work W. However, some part of the spent energy is wasted. The energy balance looks like

$$E_{spent} = W + E_{wasted}.$$

One can introduce the energy efficiency as the ratio of the useful work to the total energy spent:

$$\eta = \frac{W}{E_{spent}} = \frac{E_{spent} - E_{wasted}}{E_{spent}}.$$

All machines have $h < 1$, although the efficiency is increasing with the progress in the engineering.

4. Density, pressure, flow

Above we considered objects as point masses, that is a good approximation in many cases. In other cases, one needs to take into account the distribution of masses in space. An obvious example is a water in the sea. If the mass is distributed, then the force acting on this mass is distributed, too.

In this section, we consider solids, liquids, and gases. Solids retain their shape. Liquids retain their volume but not shape, taking the shape of their container. Gases retain neither shape nor volume, taking the volume of their container. Liquids and gases are called fluids.

Distribution of the mass in space is described by the density ρ (Greek *rho*)

$$\rho = \frac{m}{V}.$$

Here m is the total mass of the body and V is its volume. Of course, this formula defines the average density. To obtain the density at a given point, one has to consider a very small volume V around this point. Solids and liquids are practically incompressible and they have the same density everywhere. To the contrary, gases are compressible, and their density can change from point to point. For instance, the density of the air in the atmosphere is decreasing with the altitude.

The density of the water is 1000 kg/m^3 or 1 g/cm^3. The latter uses another system of units, CGS, in which basic units are centimeter, gram, and second. This system of units is most popular in physics but it is ceding its terrain to SI. The density of the air at normal conditions is about 1 kg/m^3, much smaller than water. Gold has the highest density of metals, 19.3 g/cm^3. Uranium is only a little bit lighter, 19.1 g/cm^3. It is used in missiles to increase their mechanical impact on the armor. Aluminum is a light metal having the density 2.7 g/cm^3.

In the case of distributed forces, it is convenient to introduce pressure as the perpendicular force per unit area of the surface,

$$P = \frac{F}{A}.$$

The unit of pressure is N/m^2 that has the special name pascal: 1 Pa = 1 N/m^2. Here A is the area to which the force F is applied. Pressure is denoted by the same symbol P as the power above but this should not lead to a confusion. If the force is applied to a very small area (which is the case with sharp objects), pressure can become very high and the surface can be destroyed (cut, pierced).

Within a liquid, pressure increases with the depth h because of the gravity. Any area inside the liquid feels the force from the column of the liquid above it. As the result, pressure in the liquid is given by

$$P = \rho g h + P_0,$$

where P_0 is the atmospheric pressure. In fact, atmospheric pressure also arises from the column of the air above us. However, it is difficult to calculate because of the density of the air changes with the height. At normal conditions, atmospheric pressure is 10^5 Pa = 1 atm.

If the gravity can be neglected than the pressure everywhere in the liquid is the same. This is the Pascal principle.

If an object is submerged in a liquid, it experiences pressure from all directions. Since the pressure around the bottom of the body is higher than the pressure at its top, there is a net force directed upward, the so-called buoyant force. Archimedes' law states that the buoyant force F_b is equal to the weight of the liquid displaced by the object, that is,

$$F_b = m_{liq} g = \rho_{liq} g V.$$

The apparent weight of the submerged object is its actual weight (gravity force acting on it) minus buoyant force, that is

$$F_{G,app} = F_G - F_b = \left(\rho - \rho_{liq}\right) g V.$$

If the object is lighter than the liquid, the apparent weight according to this formula is negative. Physically this means that the object will be moving upward instead of drowning until it reaches the surface of the liquid. There it comes to the stationary state of floating.

Above we considered the part of physics studying liquids at rest, hydrostatics. Hydrodynamics studying the motion of liquids is much more complicated and requires such mathematical tool as calculus. In fact, hydrodynamics includes gases in its part called aerodynamics. The specific of gases is their compressibility, as mentioned above. Here we will talk about only a couple of basic notions of hydrodynamics.

The motion of a liquid is stationary if the velocity of the liquid at any point does not depend on time. An example is a water flowing in a pipe or in a quiet river. Waves in the sea is an example of a nonstationary motion.

The flow of a liquid can be laminar (smooth) or turbulent. Laminar motion is characterized by smooth lines of the stream. If a droplet of an ink is put into a laminar flow, it will travel along the stream line (this is a method of visualizing stream lines). In a turbulent flow, the velocity of the liquid chaotically changes in time and space. A droplet of ink put into a turbulent flow will quickly mix with the surrounding liquid because different parts of this droplet will be pulled in different directions. As the result, the ink droplet disappears and it becomes impossible to follow stream lines that simply do not exist in this case. Turbulence usually occurs when the speed of the flow increases that leads to its instabilities.

An important basic property of any flow is its continuity, a consequence of the conservation of matter. If we consider some volume within a flowing fluid, the change of the mass in this volume is equal to the amount of mass entering through its surfaces. Liquids are incompressible, thus the amount of mass in any given volume is a constant. This means that the total amount of mass entering this volume is zero. The implication is that if some amount of mass enters the volume from one side, the same amount has to exit from the other side. For instance, the amount of liquid going through any cross-section A of a pipe during a unit of time is constant:

$$vA = \text{const} \text{ or } v_1 A_1 = v_2 A_2.$$

This form of the continuity equation means that if the pipe becomes narrower, the speed increases. Everyone who was watering a garden using a garden hose knows this: making the exit from the hose narrow, one can increase the speed of the water jet to make it travel a longer distance in the air.

Another very important law is Bernoulli's law (a form of the energy conservation law) that states that along stream lines (or along a pipe) pressure, height, and the fluid's speed satisfy the relation

$$P + \rho g h + \frac{\rho v^2}{2} = \text{const.}$$

Here h is the height (not the depth, as above). For the liquid at rest, $v = 0$, one recovers the hydrostatic result that pressure decreases with the height (and increases with the depth). The most interesting implication of Bernoulli's law is the relation between the speed of the liquid and pressure. If the speed increases, as it happens if the pipe narrows, pressure decreases. This may seem to be counter-intuitive to the people who have an intuition. However, it can be qualitatively explained using the Newtonian dynamics. When a portion of the fluid enters the constriction, it accelerates according to the continuity equation above. This means that the forces acting on its back are stronger than those acting on its front, that is, that the pressure behind this portion of the fluid is greater than the pressure before it. Thus the pressure in the wide part of the pipe is greater than the pressure in the constriction.

Bernoulli's law has one great application: aviation. The cross-section of the airplane's wing is more bulged at the top than at the bottom. This effectively creates a kind of constriction for the air flowing above the wing. According to Bernoulli's law, the pressure above the wing becomes lower than that below the wing, and a lifting force arises, overcoming the weight of the plane. This happens if the plane's speed is high enough.

The even more important implication of Bernoulli's law is that it is working in voice bands producing speech via instability of the air flow through them. In the absence of speech, human brain could be unable to develop, discover Bernoulli's law, and build airplanes.

5. Temperature, gases, thermodynamics

In this part of physics, thermal phenomena are considered. In the name thermodynamics, "dynamics" is somewhat misleading, as is the case with lots of physical terminology. The real subject is not motion but rather transformation between different forms of energy into each other and work done in different processes. In particular, thermodynamics explains how heat machines such as refrigerators are working.

The central concepts of thermodynamics are internal energy, temperature, and heat.

Above, for a physical object as a whole, the kinetic and potential energies were introduced. Kinetic energy is related to the motion of the object as the whole ($mv^2/2$) and potential energy is related to the object's position (e.g., mgh). It was stressed that if friction forces are present, kinetic and potential energy transforms into the energy of chaotically moving atoms and molecules of the body. This is also a kinetic energy, however, unseen by the eye and not related to the motion of the object as the whole. This internal kinetic energy is a part of the body's internal energy and defines its temperature.

Another part of the internal energy is the potential energy of interaction between atoms and molecules inside the body, including the energy of chemical bonds. This energy resembles the energy of the spring considered above. Thus, the internal energy is also a sum of kinetic and potential energies but it is microscopic, unlike the macroscopic energies of the body as a whole.

Internal potential energy is important in solids and gases since here atoms and molecules are close to each other so that the forces of attraction and repulsion are strong. Uncompressed gases, such as the air at normal condition, are rarified so that the molecules are far away from each other and do not interact. Such gases are referred to as ideal gases. In ideal gases, there is only kinetic internal energy. The average kinetic energy per molecule is directly related to the temperature T in degrees of Kelvin (in kelvins or in K) as

$$\langle\frac{mv^2}{2}\rangle = \frac{3}{2}k_BT.$$

Here k_B is the Boltzmann constant, the transformation factor between kelvins and joules: $k_B = 1.38\times10^{-23}$ J/K.

The Kelvin temperature scale (used in physics and part of SI) is related to the Celsius scale as $T = T(°C) + 273.15$. For instance, $0°C$ (melting of ice) corresponds to 273 K while $100°C$ (boiling of water) corresponds to 373 K. The so-called normal conditions correspond to the temperature 300 K and the pressure 1 atm. This temperature corresponds to 300-273 = 27°C. Kelvin temperature scale is fundamental because the temperature becomes zero when the thermal motion of atoms and molecules freezes out completely. This is called the absolute zero of the temperature, in Celsius -273 degrees.

The Fahrenheit temperature scale used in the USA is related to the Celsius scale as

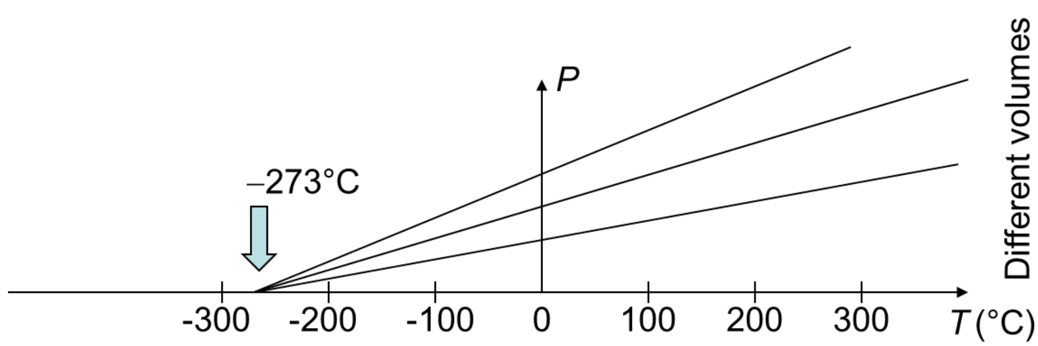
$$T(°\mathrm{F}) = \frac{9}{5} T(°\mathrm{C}) + 32, \qquad T(°\mathrm{C}) = \frac{5}{9} \left(T(°\mathrm{F}) - 32\right).$$

In the Fahrenheit scale, the absolute zero is -460 degrees, while the normal temperature is 81 degrees.

Ideal gases obey the equation of state that relates pressure, volume, and the temperature as a simple law

$$PV = Nk_B T.$$

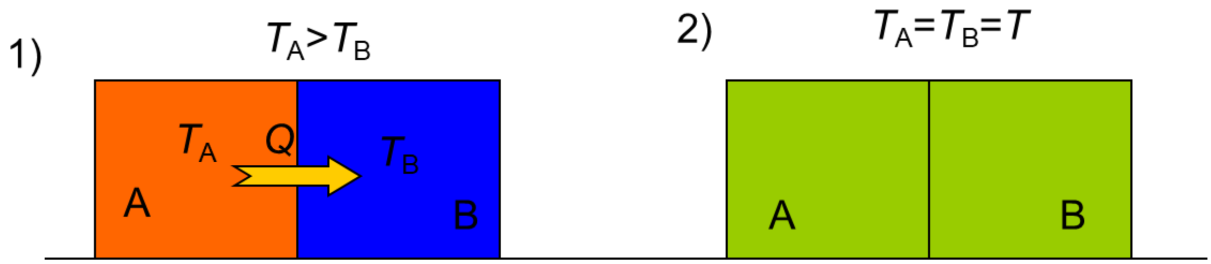
Here N is the number of molecules in the container of the volume V. We have seen that the temperature is proportional to the kinetic energy of the molecules flying around. These molecules exert a mechanical impact on the walls of the container as they rebound from the walls. Since the number of molecules in the gas is huge ($N = 2.4 \times 10^{22}$ in one liter at normal conditions!) impacts of individual molecules merge into a constant pressure on the walls. The equation of state of the ideal gas clearly shows the absolute zero as such temperature at which pressure or volume of the gas turns to zero. For instance, one can plot dependences $P(T)$ at different constant volumes that are all straight lines with different slopes. All these lines cross the horizontal axis at the temperature -273 degrees Celsius. Of course, at low temperatures, the gas will liquefy and the gas equation of state will become invalid. However, the straight lines can be extrapolated to the absolute zero.



Processes including the internal energy of the body U satisfy the energy conservation law. This important law is called the First law of thermodynamics and has the form

$$\Delta U = Q + W.$$

Here $\Delta U = U_{final} - U_{initial}$ is the change of the body's internal energy, Q is the heat received by the body and W is work done on the body, for instance, by compressing it. If, to the contrary, the body is expanding, $W < 0$ that means the body itself is doing a positive work on the environment, losing its internal energy. The heat Q is the energy received by the body in a non-mechanical way, for instance, via a thermal contact of two bodies with different temperatures.



Experimentally it is established that the heat is always flowing from the hotter body to the colder body and this process ends in the equilibration of the temperatures. This is the Second law of thermodynamics.

It is important to realize that one cannot speak of the "amount of heat in the body", as one cannot speak of the "amount of work in the body". Both heat and work can only be given away or received. Only the internal energy is the so-called function of the state, that is, it has a particular value in a particular state of the system. All thermodynamic machines (e.g., refrigerators) work in cycles. At the end of each cycle, the system returns to the initial state with the same internal energy. But the net heat and work in each cycle are nonzero, satisfying $Q + W = 0$.

In many cases, when a thermodynamic system (a body) is receiving heat, its temperature increases:

$$Q = C\Delta T = mc\Delta T.$$

Here $\Delta T = T_{final} - T_{initial}$ is the change of the temperature, C is the heat capacity of the whole body, m is the mass of the body, and c is the specific heat capacity (that is, the heat capacity per unit mass). For solids and liquids the heat capacity is a constant. Before Joule's experiments it was unknown that heat is a form of energy, and the unit of heat, the calorie, was introduced as the amount of heat needed to raise the temperature of 1 g water by 1 degree Celsius (or Kelvin). Thus the specific heat capacity of the water is c = 1 cal/(g K). Using the relations between different energy units given in Chapter *Energy,* one can calculate, say, the amount of electrical energy in kWh needed to warm up a particular amount of water.

When the system is receiving heat, it can do work losing a part of its internal energy and thus making the temperature change smaller or even negative. Thus, in general, the heat capacity depends on the process. However, this happens only for gases. Solids and liquids are expanding only very weakly with increasing their temperature, and their heat capacity is practically constant.

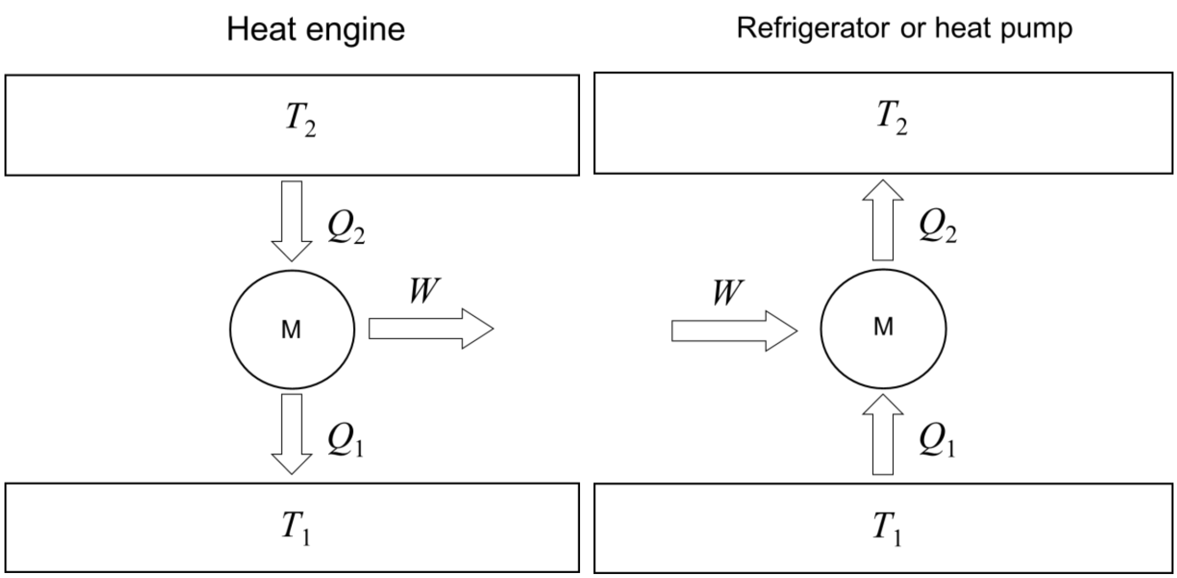
Let us consider heat machines. The first (historical) type of them is the heat engine that, in one cycle, does work W by taking heat Q_2 from a thermostat (reservoir, heat bath) at the temperature T_2 and gives away a smaller amount of heat Q_1 to another thermostat at the temperature $T_1 < T_2$. The first law of thermodynamics for one cycle reads $Q_2 = W + Q_1$. The energy (or heat) efficiency of this machine is the ratio of the (useful) work to the received heat,

$$\eta = \frac{W}{Q_2} = \frac{Q_2 - Q_1}{Q_2}.$$

French engineer Sadi Carnot with the help of the second law of thermodynamics has proven a theorem according to which the efficiency of a heat engine cannot exceed that of the so-called Carnot cycle done on the ideal gas, for which the efficiency can be calculated. The resulting maximal possible efficiency of the heat engine is given by

$$\eta_{max} = \frac{T_2 - T_1}{T_2}.$$

It is always smaller than 1 and approaches 1 in the case when the temperature of the cold thermostat approaches the absolute zero. Currently, heat engines are not used since receiving and giving away heat are slow processes. What is used is combustion engines in which the chemical energy of the fuel is converted into work.



The most popular among heat machines is the refrigerator that per cycle receives the heat Q_1 from the cold reservoir (the interior of the refrigerator) and ejects the heat Q_2 into the hot reservoir (the environment). To make it function, work has to be done on the system via the electric motor. The energy efficiency of the refrigerator is defined as the ratio of the heat extracted to the work spent:

$$\eta = \frac{Q_1}{W} = \frac{Q_1}{Q_2 - Q_1} \leq \frac{T_1}{T_2 - T_1}.$$

In the last formula, Carnot's theorem was used. One can see that the efficiency of the refrigerator can be whatever large at the beginning of the process when the temperatures of the interior and the exterior of the refrigerator are close to each other. However, as T_1 decreases, the efficiency decreases as well and can become whatever small. This is why to obtain very low temperatures in physics research labs, different methods are used.

The third type of the heat machine is the so-called heat pump. It works with the same cycle as the refrigerator sucking the heat from the environment and releasing it in the interior (of a house, for instance). If the temperatures of the exterior and the interior are close to each other, the efficiency of the heat pump is high. If it is very cold outside, the efficiency becomes small, as intuitively expected. The heat pump is not a familiar machine in the household yet.