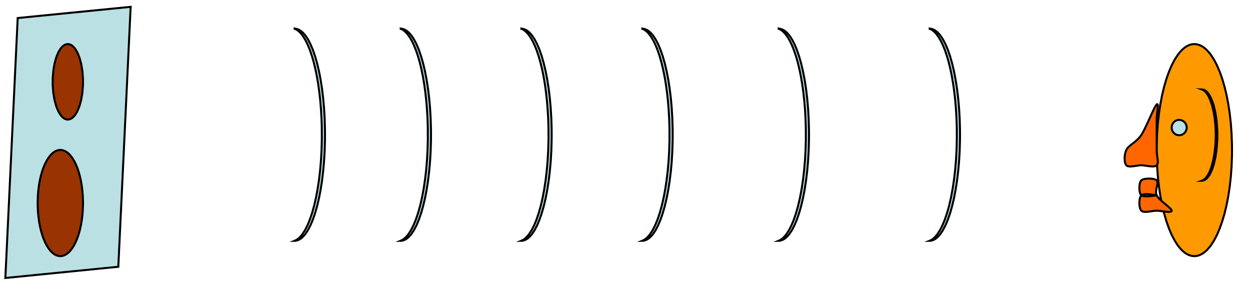
Introduction

Sound, semantically, is what we hear. As a physical phenomenon, sound is propagation of waves, acoustic or sound waves. As a whole, sound includes (i) the source (e.g., the speaker), (ii) propagating sound waves, and (iii) receptor (e.g., human ear). Source and receptor perform <u>oscillations</u>, between source and receptor are <u>waves</u>. This chapter is devoted to oscillations.



Preliminary: Physical quantities

Quantity	Time, period	Frequency	Length, displacement	Area	Volume	Velocity, speed	Mass	Density	Force	Pressure
Symbol, formula	t, T	$f = \frac{1}{T}$	l	A, S	V	$v = \frac{l}{t}$	m	$\rho = \frac{m}{V}$	F	$P = \frac{F}{S}$
Unit (SI)	s	s^{-1}	m	m^2	m^3	m/s	kg	kg/m^3	N	N/m^2

1 – Oscillations

Simple harmonic oscillations

Oscillations in general are <u>periodic</u> motions, like the motion of the membrane of a speaker or that of the human ear. Displacement of the membrane returns to its initial value after the time T has elapsed. The same happens for any multiples of T that is called the period of oscillations.

<u>Simple harmonic motion</u> is a special kind of periodic motion that is described by <u>sinusoidal</u> functions of time t

$$Q(t) = A\sin(2\pi f t + \varphi_0), \qquad A\cos(2\pi f t + \varphi_0).$$

<u>Any physical quantity that oscillates (Pressure, displacement)</u>

<u>Amplitude</u> (strength of oscillations)

<u>Frequency</u> (pitch)

<u>Phase</u> (shift along time axis)

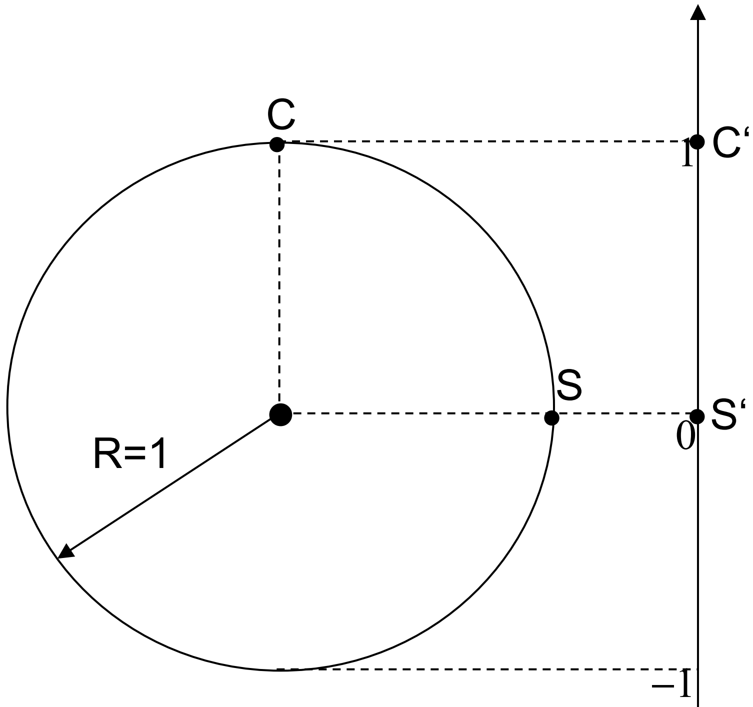
Unit of frequency: s^{-1} or Hz.
f shows the number of oscillations per second

Sinusoidal functions above are periodic with <u>period</u> T, that is, their values remain the same if the time is increased by T.

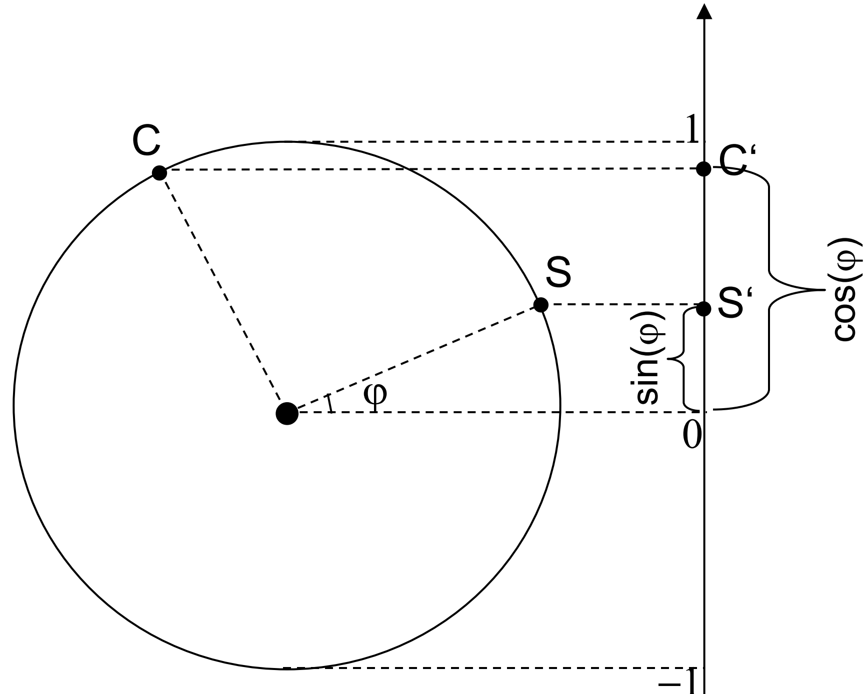
Relation between frequency and period:

$$f = \frac{1}{T}, \qquad T = \frac{1}{f}, \qquad fT = 1$$

Geometrical definition of sin and cos

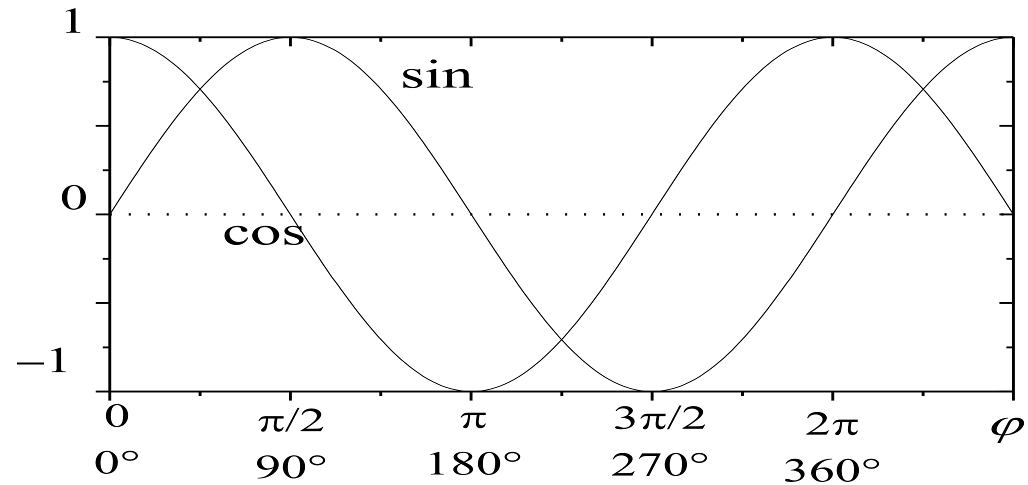


Initial state: $\varphi=0$, $\sin(\varphi)=0$, $\cos(\varphi)=1$



General state: the circle is rotated by φ

Plot of sinusoidal functions:



Their period is 360° or 2π raduans

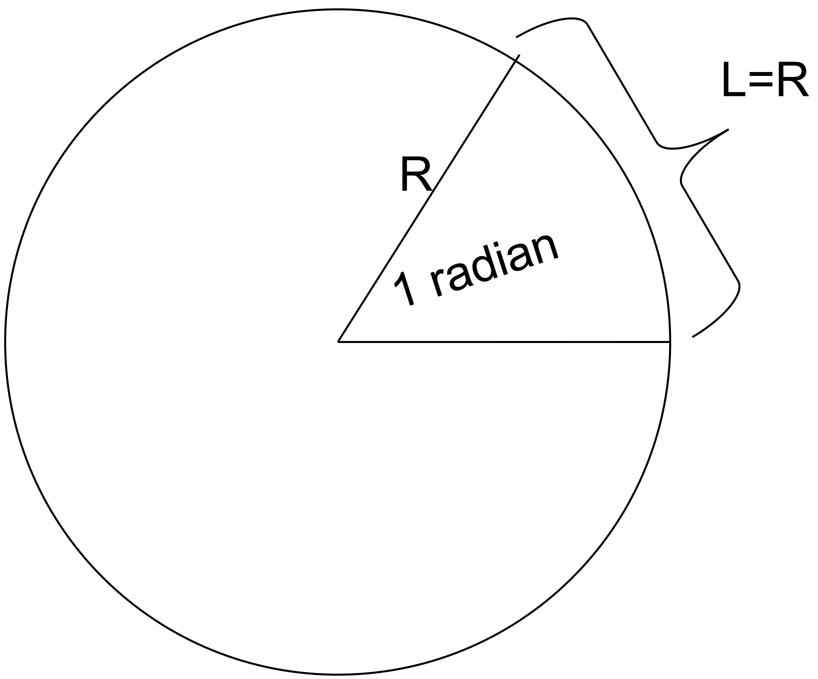
sin and cos only differ by a phase:
$\sin(\varphi+\varphi_0)=\cos(\varphi)$ with $\varphi_0=\pi/2$

Angles

Angles φ can be measured in

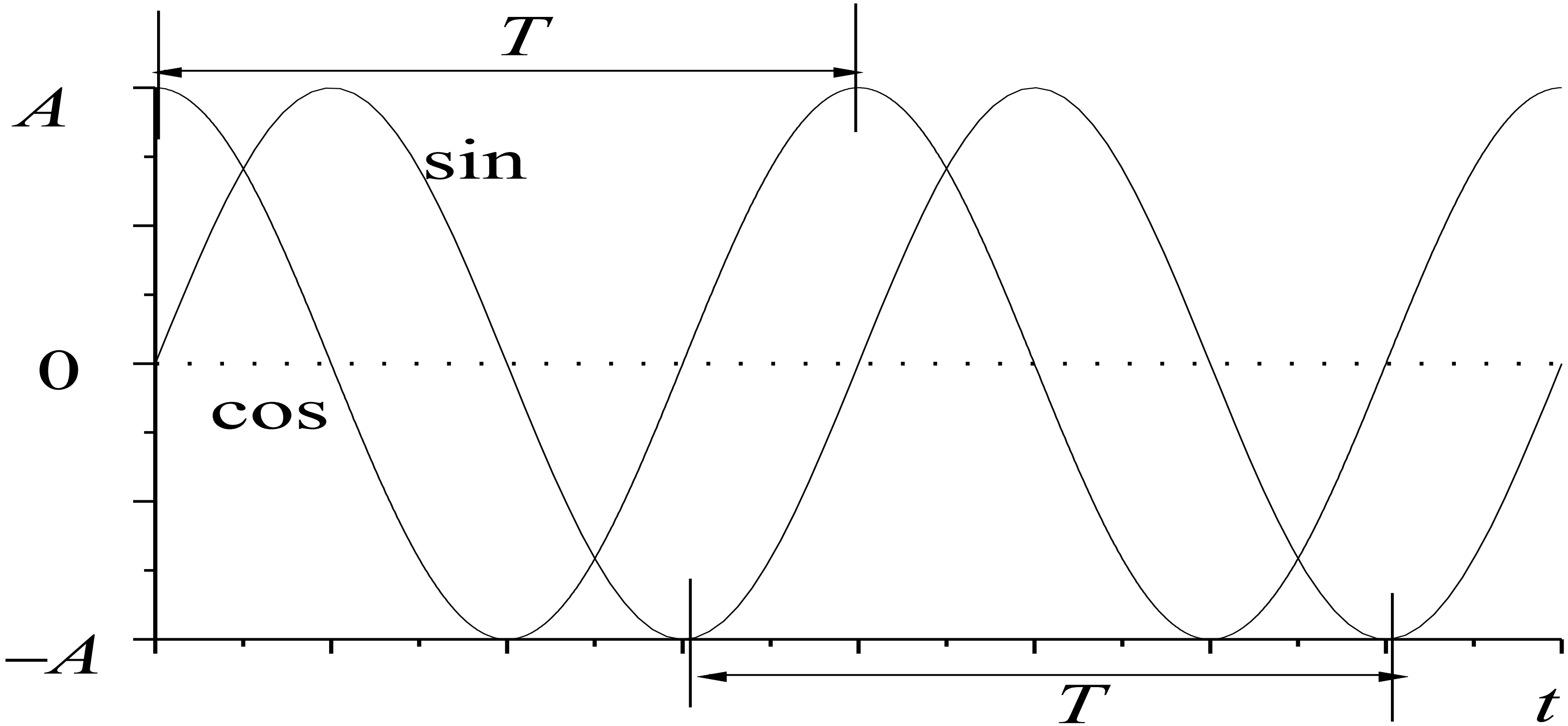
- degrees (used in everyday life)
- revolutions (360°) (used in engineering)
- radians (used in physics)

Radian is such an angle, for which the length of the arc is equal to the radius. In other words, the angle in radians is given by L/R and it is dimensionless. Revolution corresponds to $L=2\pi R$, thus $360° = 2\pi$ radians, thus



$$\boxed{1 \text{ radian} = 360°/(2\pi) = 57.3°}$$

Graphical representation of oscillations



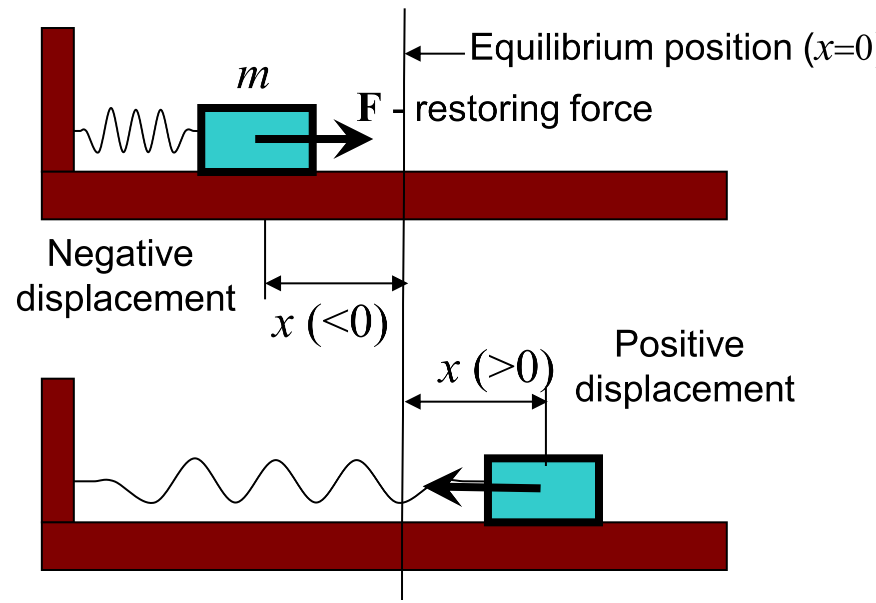
Period T is the time between the two neighboring crests of the sinusoudal function.

Physical realizations of oscillations

Physically periodic functions can be of many different kinds: Displacement of the membrane, pressure of the air, voltage and current in electric circuits etc. This defines what is the amplitude A and what is its unit.

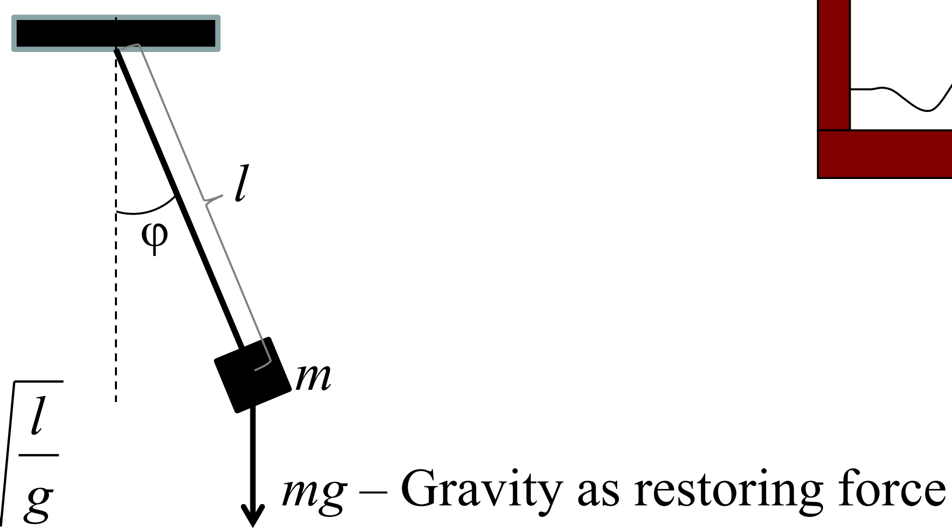
Harmonic oscillations require linear restoring force, that is a force that is opposite to the displacement out of an equilinrium state and depends linearly on it. Examples: Pendulum, mass on a spring

Mass on a spring (no friction)



$$T = 2\pi\sqrt{\frac{m}{k}}$$

Pendulum



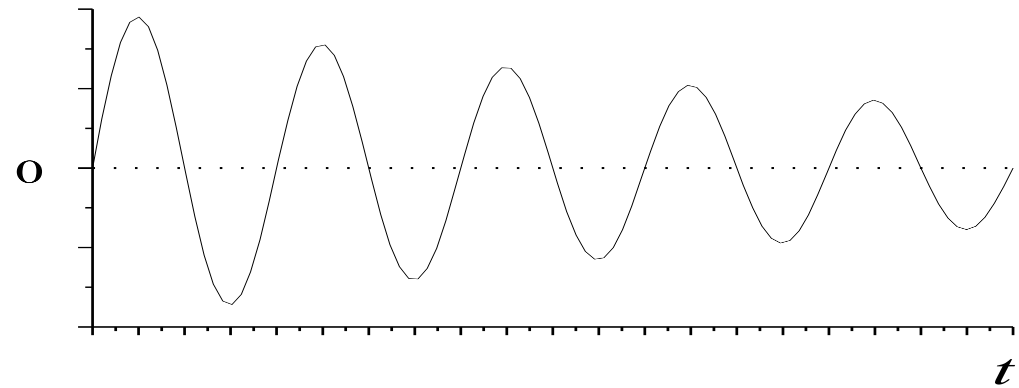
$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$|\varphi| << 1$$

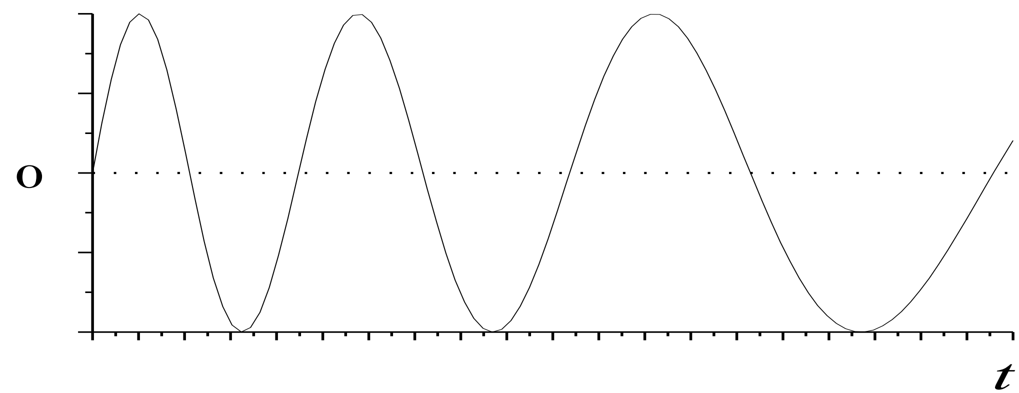
Simple harmonic oscillations with slowly changing parameters

If the parameters of the oscillation such as frequency and amplitude are slowly changing in time, in comparison to the frequency itself, the oscillation remains close to simple harmonic.

Amplitude decreases
(damped oscillation)



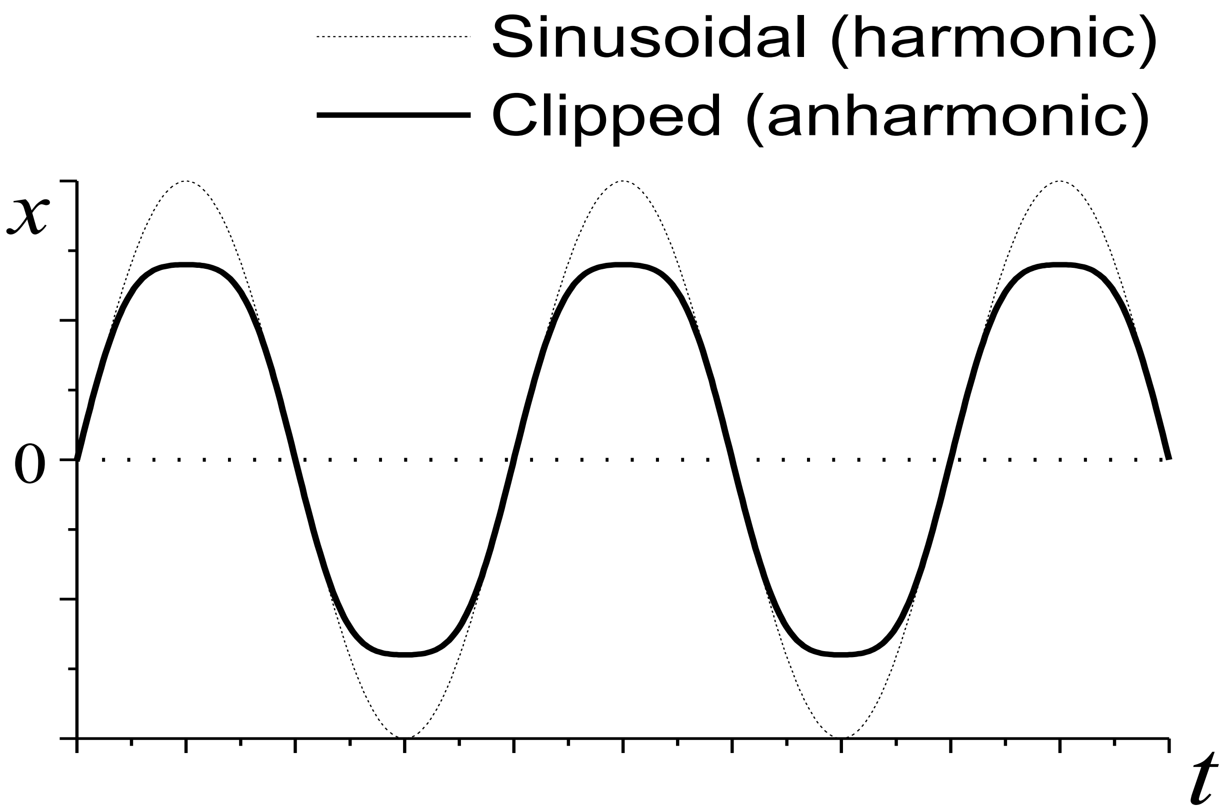
Frequency (pitch) decreases,
period increases



Anharmonic oscillations

Every oscillation that is not sinusoidal (harmonic) is called anharmonic.

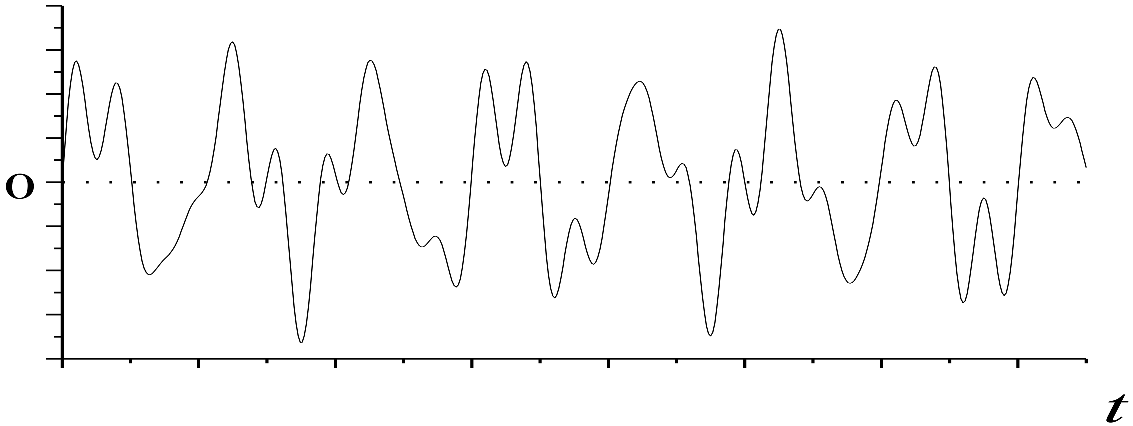
Example: Clipping as a result of overload in (analog) amplifiers



In digital circuits clipping is even more severe, it creates regions where $x(t)$ is completely flat.

Combinations of harmonic oscillations

A sum (superposition) of several harmonic oscillations can look wild. Below is the plot of a function that is a sum of three sinusoidal functions with different frequencies and amplitudes:



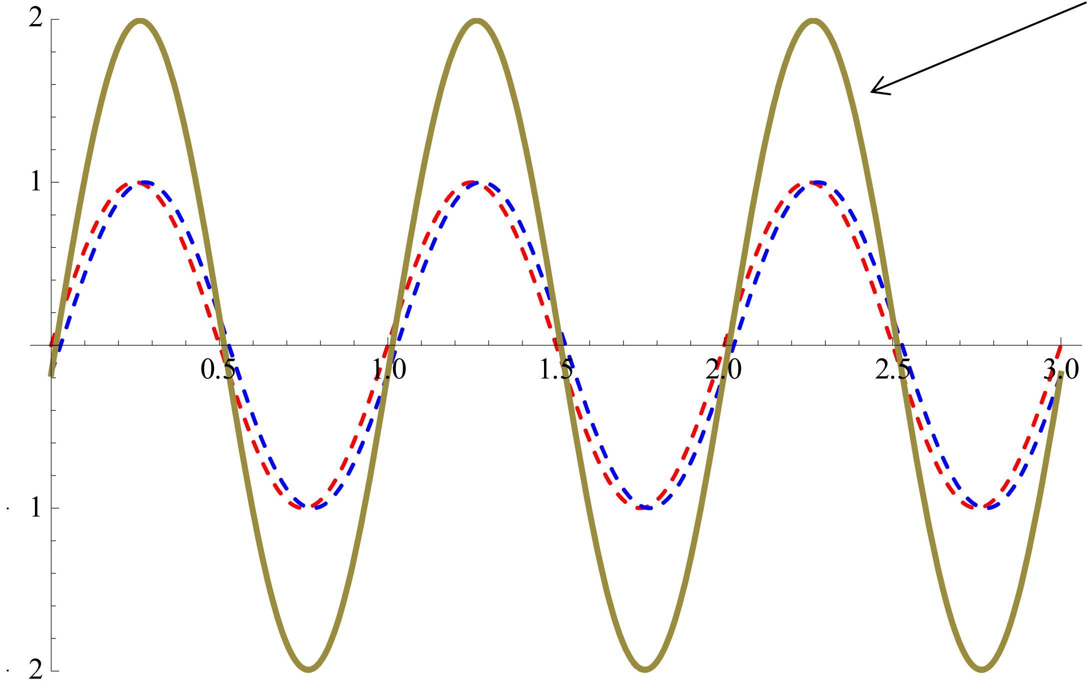
This is not a periodic function, thus one cannot speak of (a single) oscillation. We will see that simple anharmonic oscillations can be represented as sums of many harmonic oscillations in the particular case in which motion remains periodic. In particular, the anharmonic oscillation in the previous page is a sum of three harmonic functions.

Phase relations

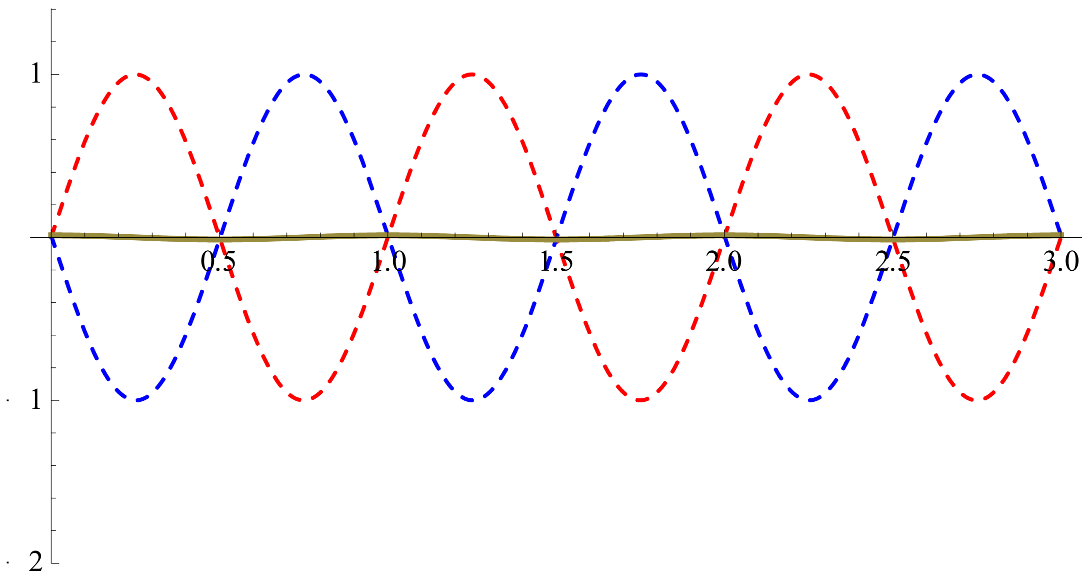
Two oscillations are said to be <u>in-phase</u> if they are both sin or both cos and their phases φ_0 are the same. Otherwise the oscillations are said to be <u>out of phase</u>. If the phases differ by 180°=π, the oscillations are said to be <u>anti-phase</u>. The sum of two anti-phase oscillations with equal amplitudes A is zero, that is, these two oscillations cancel each other.

Interference of oscillations

Sum of the two oscillations



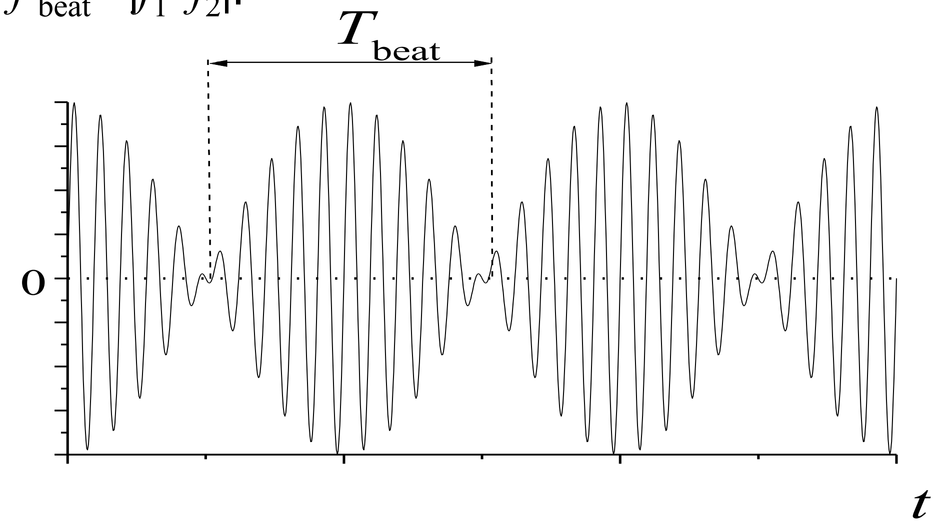
Two oscillations in-phase,
constructive interference



Two oscillations anti-phase,
destructive interference

Beats

Sum (combination, superposition) of two sinusoidal functions with close amplitudes A_1 and A_2 and slightly different frequencies f_1 and f_2 shows the so-called beats with the difference frequency $f_{\text{beat}} = |f_1 - f_2|$.



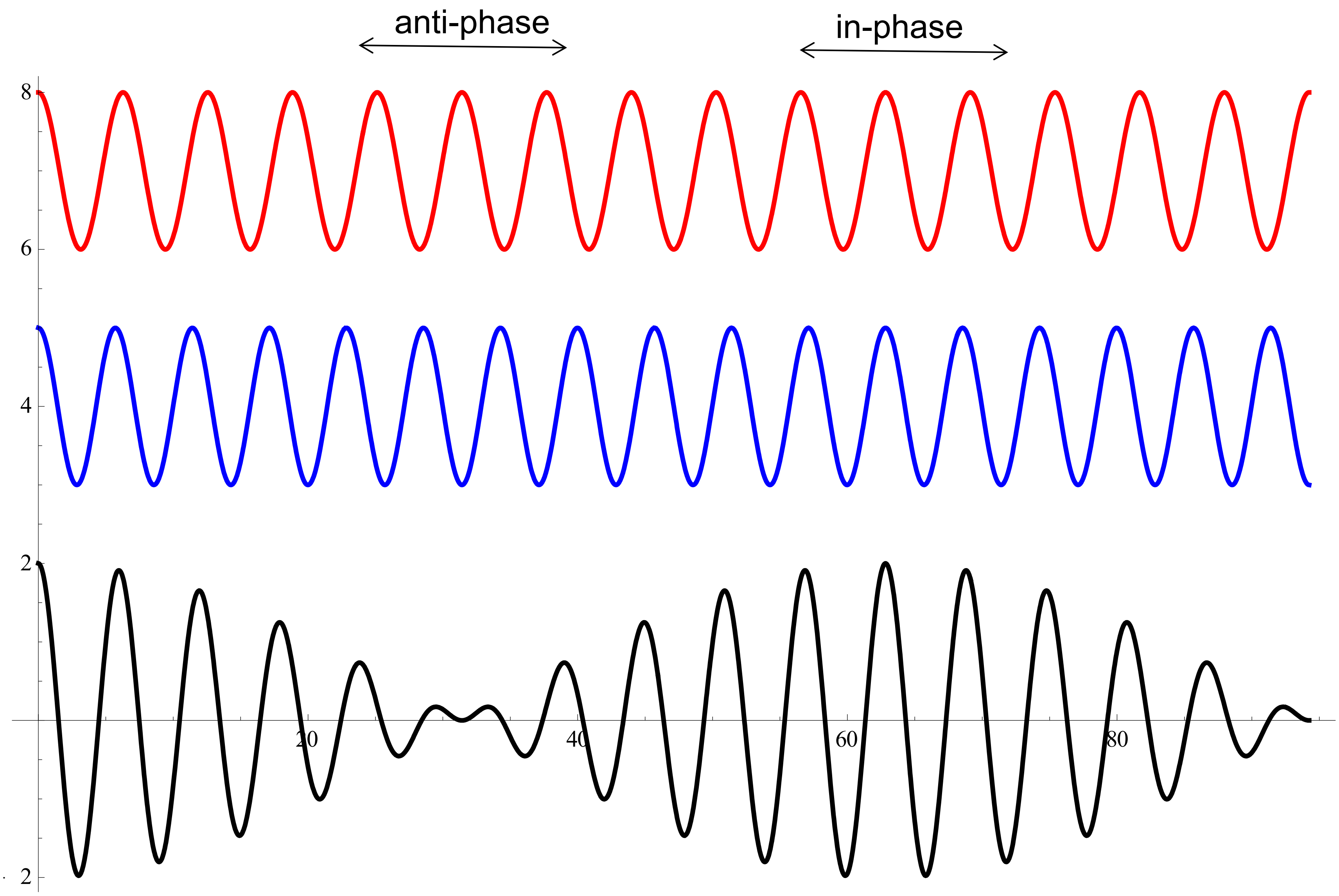
The period of the beats is given by

$$T_{\text{beat}} = \frac{1}{f_{\text{beat}}} = \frac{1}{|f_1 - f_2|}$$

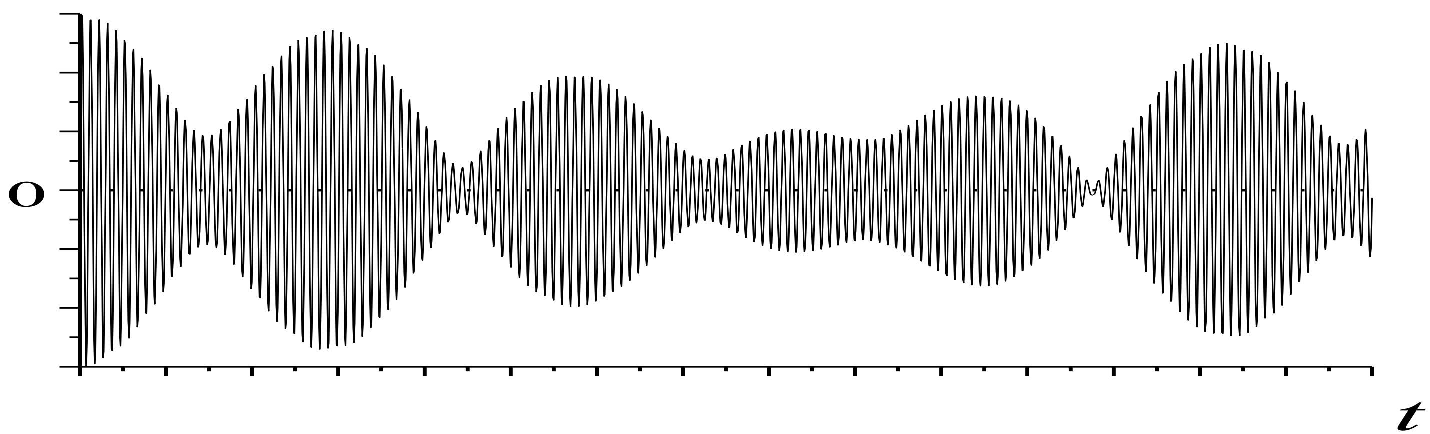
For instance, if $f_1 = 440$ Hz and $f_2 = 442$ Hz, then the beat frequency is $f_{\text{beat}} = 2$ Hz and the beat period is $T_{\text{beat}} = 0.5$ s.

Beats can be also described by single sinusoidal functions with slowly changing amplitudes.

Beats as interference



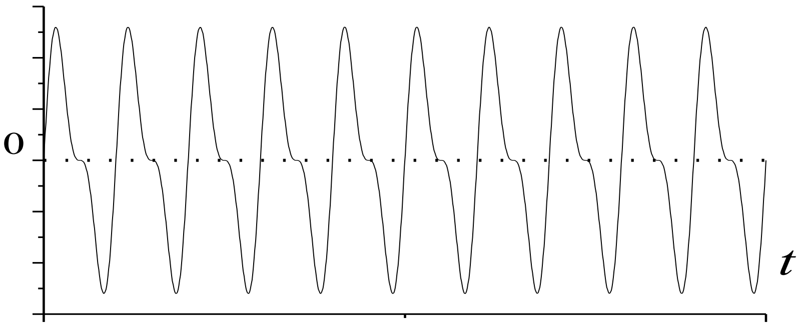
In pianos, each tone is created by three strings tuned in unison. Since this unison is never perfect, the tone becomes richer because of very slow beats. Note that three strings are needed to prevent the intensity of sound from periodically turning to zero. If the piano is out of tune, the frequencies of the beats become fast enough to irritate the listener. Here is an example of beats between three sinusoidal oscillations with the same amplitudes and slightly different frequencies.



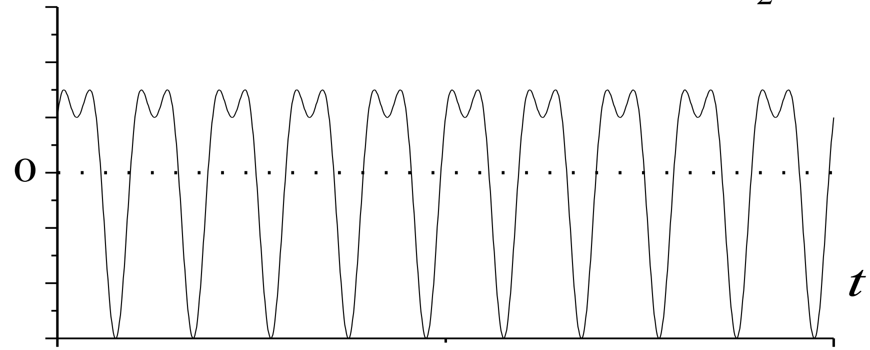
Phase relations, Ohm's law, phase beats

If the frequencies of two or more adding oscillations are multiples of each other, the resulting oscillation depends on the <u>phase</u> of the summands, that is, of their shift along the time axis. For instance, these two oscillations look very different:

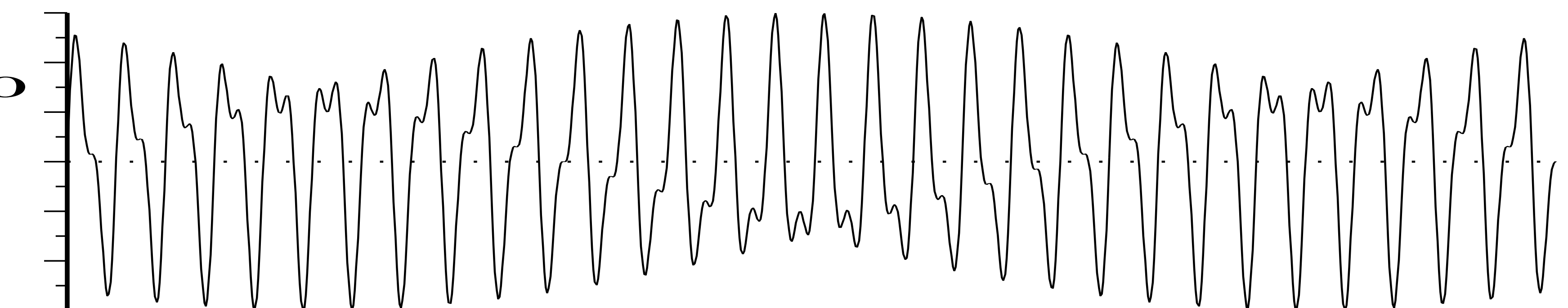
$$x(t) = A\sin(2\pi f t) + A\sin(4\pi f t)$$



$$x(t) = A\sin(2\pi f t) + A\sin(4\pi f t + \tfrac{\pi}{2})$$

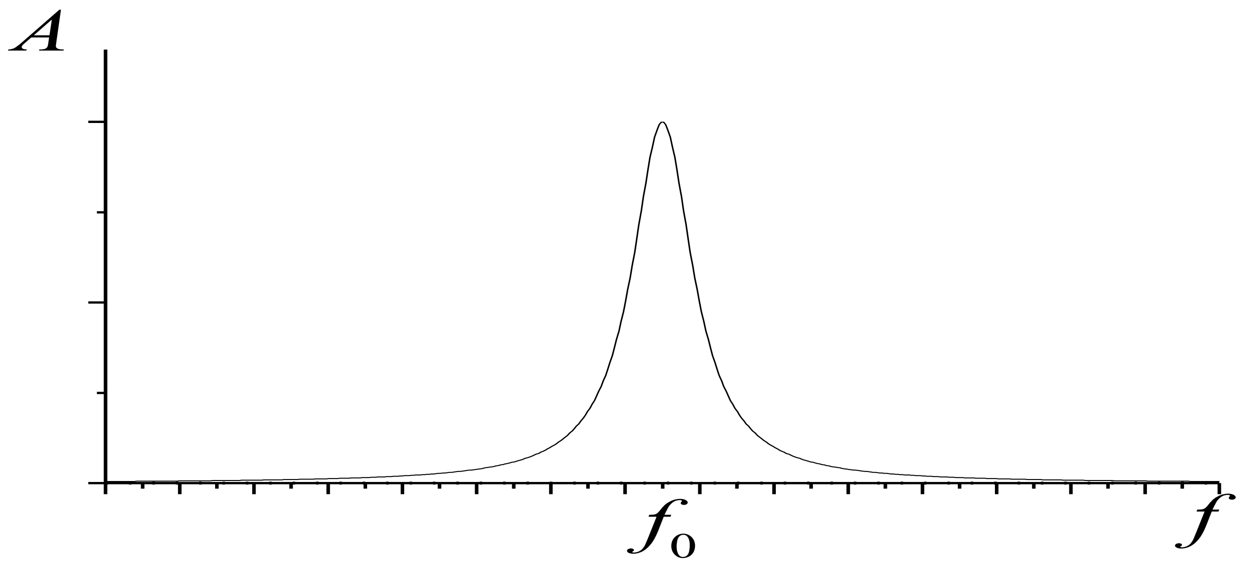


However, human ear cannot distinguish between the sound produced by these two oscillations. This is the phychoacoustical Ohm's law: *The perceived sound depends on frequencies and amplitudes but not on the phases*. On the other hand, if the frequencies of the two oscillations in the sum differ a little, the shape of the resulting sum will slowly change with time, similarly to the <u>beats</u>. Now human ear can detect the change of the sound on time due to these <u>phase beats</u>.

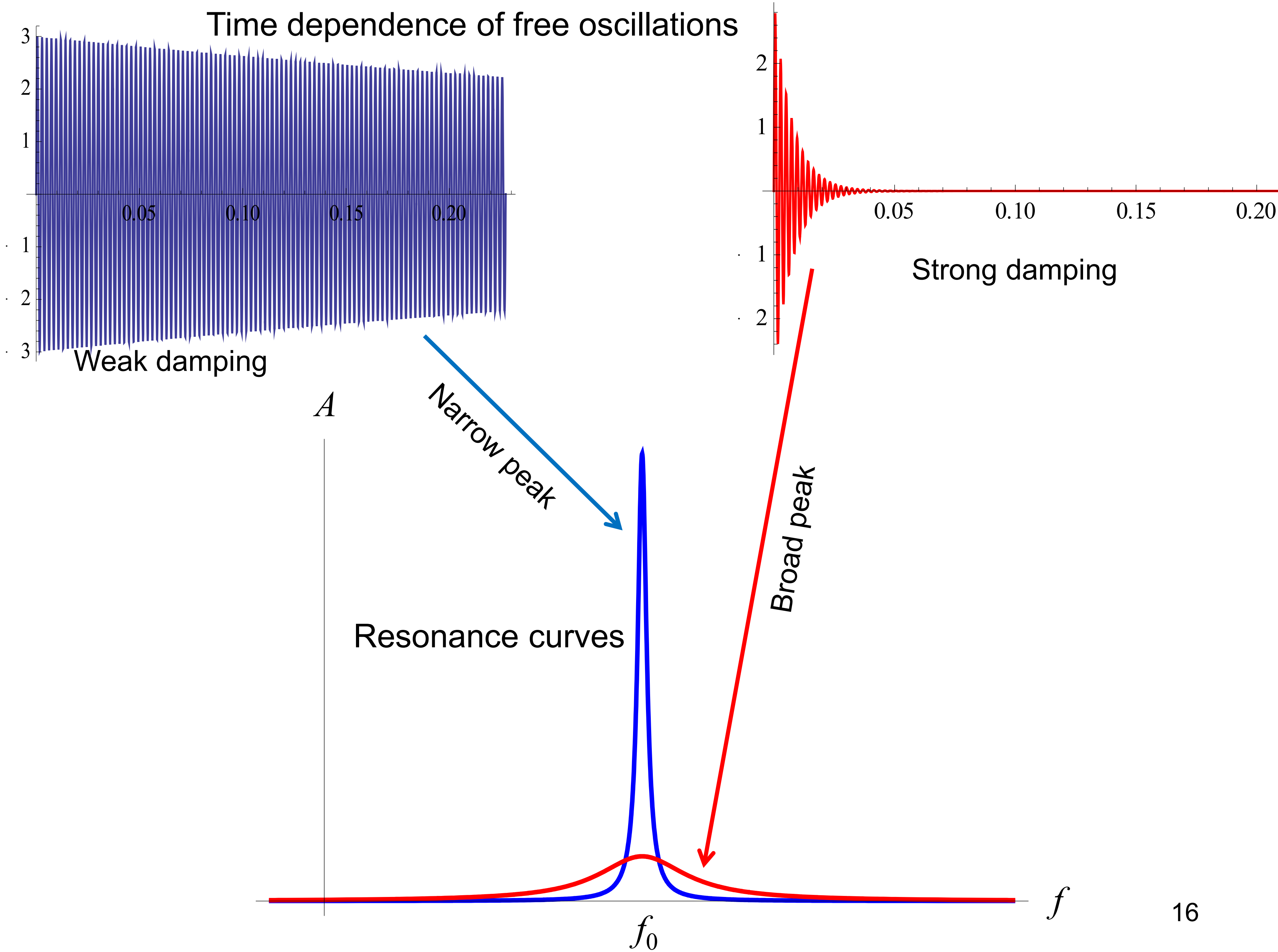


Damped oscillations and resonance

In most of realistic situations there is a friction in the system that leads to damping of oscillations (see page 3). On the other hand, if a periodic force is applied to an oscillator, it oscillates with the frequency of the external force f with some amplitude A. This amplitude if proportional to the amplitude of the force and depends on f and on the frequency of the oscillator f_0.



One can see that the response of the oscillator to the periodic external force is of a resonance character with a maximum at $f = f_0$. There is no or very little response if the external frequency is far from the resonance. On the other hand, at resonance, $f = f_0$, the response can be very large even for a weak external force. This was the reason for some bridge crashes, caused by the wind or marching soldiers.



Time dependence of free oscillations
3
2
1
0.05
0.10
0.15
0.20
1
2
3
Weak damping
2
1
0.05
0.10
0.15
0.20
1
2
Strong damping
Narrow peak
Broad peak
A
Resonance curves
f
f_0

Example of resonance: Tachoma Narrows Bridge breakdown