Solutions to HW2

#1 $P\left((A \cup B \cup C)^c\right) = 1 - P(A \cup B \cup C)$

"no dish at all"

inclusion exclusion

$$= 1 - \left(P(A) + P(B) + P(C)\right) + \left(P(AB) + P(AC) + P(BC)\right) - P(ABC)$$

"exactly one"

#2 $P\left(A \cap B^c \cap C^c\right) + P\left(A^c \cap B \cap C^c\right) + P\left(A^c \cap B^c \cap C\right) =$
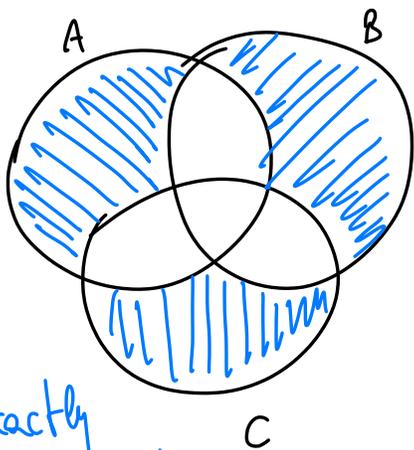
"Exactly one dish"

$$= P(A \cup B \cup C) - P(AB) - P(AB) - P(BC) + 2P(ABC)$$

$$= P(A) + P(B) + P(C) - \left(P(AB) + P(AC) + P(BC)\right)$$
$$+ P(ABC)$$

$$- P(AB) - P(AB) - P(BC) + 2P(ABC)$$

$$= P(A) + P(B) + P(C)$$

$$- 2\left(P(AB) + P(AC) + P(BC)\right)$$
$$+ 3P(ABC)$$

#3   A = # is divisible by 3

      B = # ——————ᴗ——————— 5.

$$P(A) = \frac{\lfloor \frac{100}{3} \rfloor}{100} = \frac{33}{100}, \quad P(B) = \frac{\lfloor \frac{100}{5} \rfloor}{100} = \frac{20}{100}, \quad P(AB) = \frac{\lfloor \frac{100}{15} \rfloor}{100} = \frac{6}{100}$$

$$P\left((A \cup B)^c\right) = 1 - P(A \cup B) = 1 - P(A) - P(B) + P(A \cap B)$$

not divisible by 3 nor 5.

$$= 1 - \frac{33}{100} - \frac{20}{100} + \frac{6}{100} = \boxed{\frac{53}{100}} \quad (53\%)$$

#4   We recognize this to be the __matching problem__
      with $n = 4$ objects (cf. Video 3 of Lecture 4).
      Thus, the probability that __no characters__ are typed
      in their correct placement is

$$\sum_{j=0}^{4} \frac{(-1)^j}{j!} = 1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} = \frac{3}{8} = \boxed{0.375}$$