Solutions to HW 5

#1 $\Omega = \{B, D\}$, $P(D) = p$, $P(B) = 1 - p$.

$X : \Omega \rightarrow \mathbb{R}$ income from the bet.

$E(X) = p \cdot (+2) + (1-p) \cdot (-3) = \boxed{5p - 3}$

$\underline{\text{Note}}$: $E(X) > 0 \iff 5p - 3 > 0 \iff p > 3/5$.

#2 $\Omega = \{0, 1, 2, 3\}$, $X = $ Total \$ amount of fine

$X(n) = 125 \cdot n$:

$X(0) = 0$, $\quad X(2) = 250$,
$X(1) = 125$, $\quad X(3) = 375$.

Bernoulli process with prob. of $\underline{\text{success}}$ p, $n = 3$.

(# of successes)

$K = 0$: $\quad P(X = 0) = (1-p)^3$

$K = 1$: $\quad P(X = 125) = 3(1-p)^2 p$

$K = 2$: $\quad P(X = 250) = 3(1-p)p^2$

$K = 3$: $\quad P(X = 375) = p^3$

$$E(X) = 0 \cdot P(X=0) + 125 \cdot P(X=125)$$
$$+ 250 \cdot P(X=250) + 375 \cdot P(X=375)$$

$$= 125 \cdot 3(1-p)^2 \cdot p + 250 \cdot 3 \cdot (1-p)p^2 + 375 \cdot p^3$$

$$= \boxed{375p}$$

Alternative solution: ← using the fact that $E(\cdot)$ is linear

Let $Y = \$$ amount of fine <u>for each</u> movie $= \begin{cases} 0 & \text{if not caught} \\ 125 & \text{if caught.} \end{cases}$

$$P(Y=0) = 1-p$$
$$P(Y=125) = p$$

$$E(Y) = 0 \cdot P(Y=0) + 125 \cdot P(Y=125) = 125 \cdot p$$

$$X = 3 \cdot Y \implies E(X) = 3E(Y) = \boxed{375p.}$$

#3 ← problem reloads with new numbers.

This is Gambler's Ruin problem with

$$i = 5, \quad N = 10, \quad q = 1-p.$$

"Game" ends either when you lost \$5 or you earned \$5. So the total jackpot is <u>\$10</u> and your initial wealth is <u>\$5.</u>

$$P_5 = \frac{1 - (q/p)^i}{1 - (q/p)^N} = \boxed{\frac{1 - (q/p)^5}{1 - (q/p)^{10}}}$$

<u>Note</u>: If X = profit from this investment, then one may compute

$$E(X) = (+5) \cdot P_5 + (-5) \cdot (1 - P_5)$$ ← This is the <u>expected return</u> from this investment.