Solutions to HW 7

#1 X = # cars arriving at traffic light

these numbers change each time.

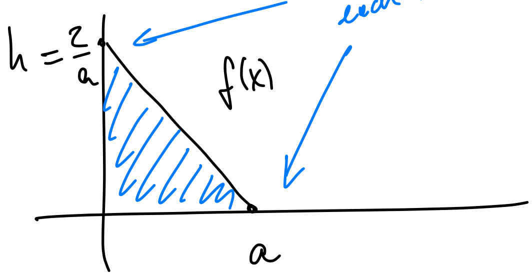
$$X \sim \text{Poisson}(\lambda), \qquad \lambda = E(X) = \underbrace{r}_{\text{rate}}\ \underbrace{t}_{\text{time interval}}$$

$$\left(r = \frac{\#\text{cars}}{\#\text{seconds}} \right)$$

$$\mathbb{P}(X=3) = \frac{e^{-\lambda}\lambda^3}{3!} = \frac{r^3t^3e^{-rt}}{6}$$

#2

these change each time.



area of a triangle

$$\int_0^a f(x)\,dx = \frac{ah}{2} = 1 \Rightarrow h = \frac{2}{a}$$

$$\text{Slope} = -\frac{\Delta y}{\Delta x} = -\frac{2/a}{a} = -\frac{2}{a^2}$$

$$f(x) = \begin{cases} \frac{2}{a} - \frac{2}{a^2}x, & x \in [0,a] \\ 0, & \text{otherwise} \end{cases}$$

$$E(X) = \int_0^a x f(x)\,dx = \int_0^a \frac{2}{a}x - \frac{2}{a^2}x^2\,dx = \frac{\cancel{2}}{a}\frac{x^2}{\cancel{2}} - \frac{2}{a^2}\frac{x^3}{3}\Big|_0^a$$

$$= \frac{a^2}{a} - \frac{2}{3}\frac{a^3}{a^2} = a - \frac{2}{3}a = \boxed{\frac{a}{3}}$$

(**Bonus:** $Var(X) = E(X^2) - E(X)^2$

$$E(X^2) = \frac{a^2}{6}, \quad E(X)^2 = \frac{a^2}{9} \Rightarrow Var(X) = a^2\left(\frac{1}{6} - \frac{1}{9}\right) = \boxed{\frac{a^2}{18}}$$)

#3 $X \sim \text{Uniform}([0,1])$

$$f(x) = \begin{cases} 1 & \text{if } 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

$Y = X^n$ ← this number changes.

$$E(XY) = E(X \cdot X^n) = E(X^{n+1}) = \int_{-\infty}^{+\infty} x^{n+1} f(x)\,dx = \int_0^1 x^{n+1}\,dx = \frac{x^{n+2}}{n+2}\Big|_0^1$$

$$= \frac{1}{n+2}$$

$$E(Y) = \int_0^1 x^n \, dx = \left. \frac{x^{n+1}}{n+1} \right|_0^1 = \frac{1}{n+1}$$

$$E(X) = \frac{1}{2}$$

$$Cov(X, Y) = E(XY) - E(X)E(Y) = \frac{1}{n+2} - \frac{1}{2} \frac{1}{n+1} = \boxed{\frac{n}{2n^2+6n+4}}$$