

Solutions to HW8

#1 $X \sim \text{Exponential}(\lambda)$ this number changes $E(X) = \frac{1}{\lambda} = \frac{1}{\# \text{ years tree lived}}$

$$P(X > 100) = 1 - P(0 < X < 100)$$

$$= 1 - \int_0^{100} \lambda e^{-\lambda x} dx = 1 - (1 - e^{-100\lambda}) = \underline{\underline{e^{-100\lambda}}}$$

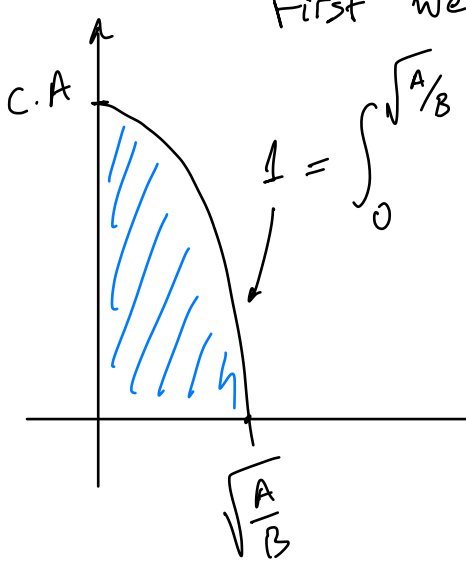
#2 $\lambda(t) = at + b$ these numbers change

$$F(t) = 1 - e^{-\int_0^t as + b ds} = 1 - e^{-\left(\frac{as^2}{2} + bs\right)\Big|_0^t} = 1 - e^{-\frac{at^2}{2} - bt}$$

$$\Rightarrow P(X \leq \frac{1}{2}) = F(\frac{1}{2}) = \underline{\underline{1 - e^{-\frac{a}{8} - \frac{b}{2}}}}$$

#3 $f(x) = C(A - Bx^2)$ these numbers change

First we find C:



$$1 = \int_0^{\sqrt{A/B}} C(A - Bx^2) dx = C \left(Ax - \frac{Bx^3}{3} \right) \Big|_0^{\sqrt{A/B}} = C \left(A\sqrt{\frac{A}{B}} - \frac{B}{3} \frac{A}{B} \sqrt{\frac{A}{B}} \right)$$

$$\Rightarrow C = \frac{1}{A\sqrt{\frac{A}{B}} - \frac{A}{3}\sqrt{\frac{A}{B}}} = \frac{1}{A\sqrt{\frac{A}{B}}(1 - \frac{1}{3})} = \boxed{\frac{3\sqrt{B}}{2A\sqrt{A}}}$$

$$E(X) = \int_0^{\sqrt{A/B}} C(A - Bx^2)x dx = \dots = \frac{A^2 C}{4B} = \boxed{\frac{3\sqrt{A}}{8\sqrt{B}}}$$

$$E(X^2) = \int_0^{\sqrt{\frac{A}{B}}} C(A - Bx^2)x^2 dx = \dots = \frac{2}{15} \frac{A^2 \sqrt{A}}{B\sqrt{B}} \cdot C = \boxed{\frac{A}{5B}}$$

$$\text{Var}(X) = E(X^2) - E(X)^2 = \frac{A}{5B} - \frac{9A}{64B} = \boxed{\frac{19A}{320B}}$$