Lecture 2

All problems in today's lecture are from the textbook "Elementary Linear Programming with Applications", by Kolman and Beck.

Exercise 1.

Equipment purchasing problem. A container manufacturer is considering the purchase of two different types of cardboard-folding machines: model A and model B. Model A can fold 30 boxes per minute and requires 1 attendant, whereas model B can fold 50 boxes per minute and requires 2 attendants. Suppose the manufacturer must fold at least 320 boxes per minute and cannot afford more than 12 employees for the folding operation. If a model A machine costs \$15,000 and a model B machine costs \$20,000, how many machines of each type should be bought to minimize the cost?

Solution to Exercise 1. From the statement of the problem we have:

	Machine A	Machine B
Box/min	30	50
Attendants	1	2

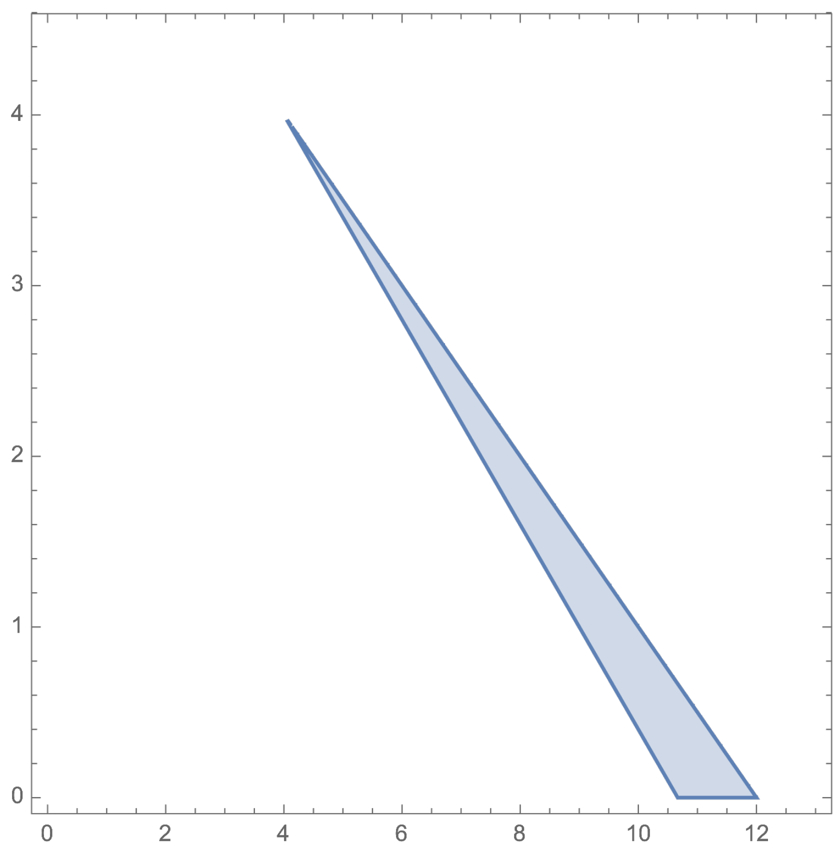
If we use x units of Machine A and y units of Machine B, then the constraints are

$$\begin{aligned} 30x + 50y &\geq 320 \\ x + 2y &\leq 12 \\ x \geq 0,\ y &\geq 0 \end{aligned}$$

We want to find the minimum value of total cost $c(x,y) = 15\,x + 20\,y$ (in thousands) among the points (x,y) satisfying the constraints. In other words, we want to solve the optimization problem

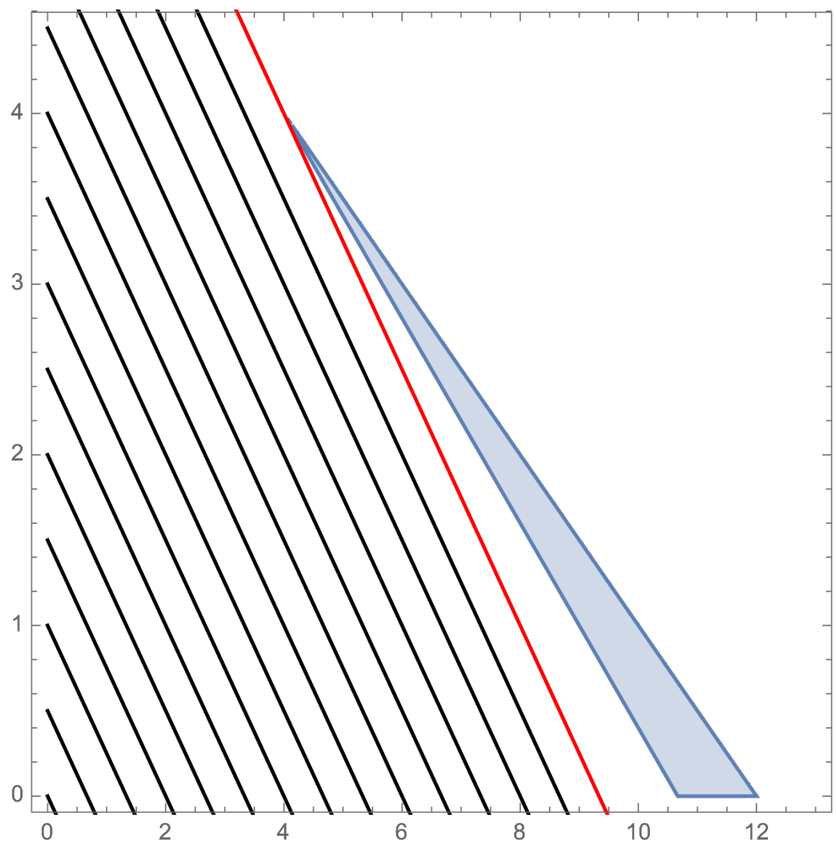
$$\min \quad 15\,x + 20\,y \quad \text{s.t.} \quad \begin{aligned} &30x + 50y \geq 320 \\ &x + 2y \leq 12 \\ &x \geq 0,\ y \geq 0 \end{aligned}$$

The corresponding feasible region is:



Overlapping the feasible region with plots of the levelsets $\{c(x,y) = t\}$ of the cost function, which are the black parallel lines shown in the next page, we see that the minimum must be achieved at the vertex $(4,4)$, where $c(4,4) = 140$. That optimal levelset is shown in red.

Thus, the optimal purchase strategy is to buy 4 machines of each type.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Exercise 2.

Disease treatment problem. Dr. R. C. McGonigal treats cases of tactutis with a combination of the brand-name compounds Palium and Timade. The Palium costs \$0.40/pill and the Timade costs \$0.30/pill. Each compound contains SND plus an activator. The typical dosage requires at least 10 mg of SND per day. Palium contains 4 mg of SND and Timade contains 2 mg of SND. In excessive amounts the activators can be harmful. Consequently Dr. McGonigal limits the total amount of activator to no more than 2 mg per day. Palium and Timade each contain 0.5 mg of activator per pill. How many of each pill per day should Dr. McGonigal prescribe to minimize the cost of the medication, provide enough SND, and yet not exceed the maximum permissible limit of activator?

Solution to Exercise 2. From the statement of the problem we have:

	Palium	Timade
SND	4	2
Activator	0.5	0.5

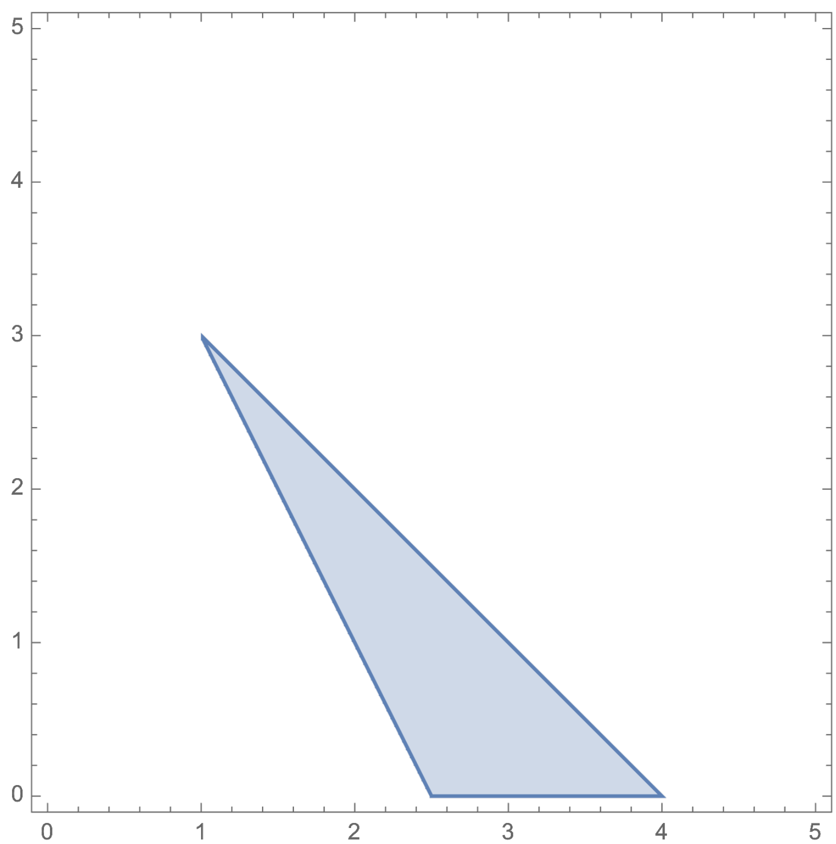
If we use x Palium pills and y Timade pills, then the constraints are

$$\begin{aligned} 4x + 2y &\geq 10 \\ 0.5x + 0.5y &\leq 2 \\ x \geq 0,\ y &\geq 0 \end{aligned}$$

We want to find the minimum value of total cost $c(x, y) = 0.4\,x + 0.3\,y$ among the points (x, y) satisfying the constraints. In other words, we want to solve the optimization problem

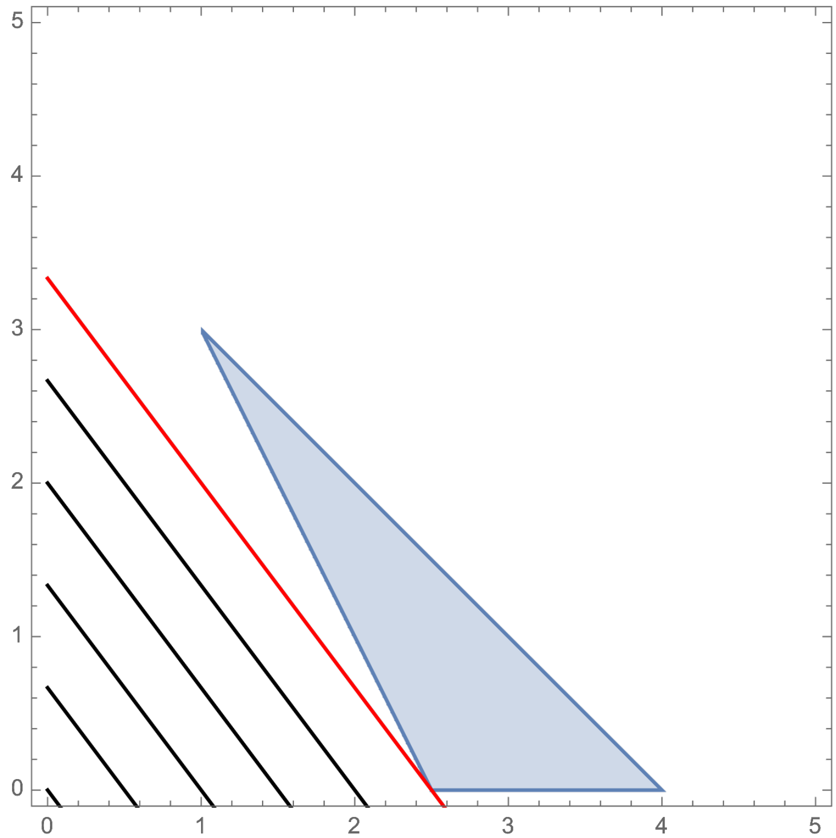
$$\min \quad 0.4\,x + 0.3\,y \quad \text{s.t.} \quad \begin{aligned} &4x + 2y \geq 10 \\ &0.5x + 0.5y \leq 2 \\ &x \geq 0,\ y \geq 0 \end{aligned}$$

The corresponding feasible region is:



Overlapping the feasible region with plots of the levelsets $\{c(x, y) = t\}$ of the cost function, which are the black parallel lines shown in the next page, we see that the minimum must be achieved at the vertex $(2.5, 0)$, where $c(2.5, 0) = 1$. That optimal levelset is shown in red.

Thus, the optimal prescription is 2.5 pills of Palium per day and no pills of Timade.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Exercise 3.
Agricultural problem. A farmer owns a farm that produces corn, soybeans, and oats. There are 12 acres of land available for cultivation. Each crop that is planted has certain requirements for labor and capital. These data along with the net profit figures are given in the accompanying table.

	Labor (hr)	*Capital (\$)*	*Net profit (\$)*
Corn (per acre)	6	36	40
Soybeans (per acre)	6	24	30
Oats (per acre)	2	18	20

The farmer has \$360 available for capital and knows that there are 48 hr available for working these crops. How much of each crop should be planted to maximize profit?

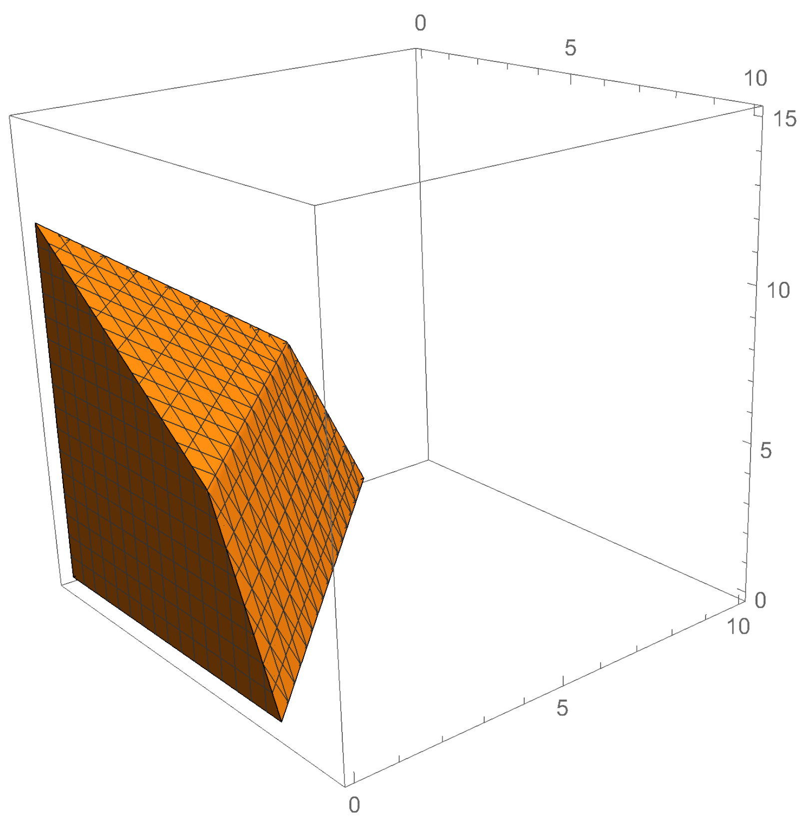
Solution to Exercise 3. If the farmer plants x acres of corn, y acres of soy, and z acres of oats, then the constraints are

$$\begin{aligned} x + y + z &\leq 12 \\ 6x + 6y + 2z &\leq 48 \\ 36x + 24y + 18z &\leq 360 \\ x \geq 0,\ y \geq 0,\ z &\geq 0 \end{aligned}$$

We want to find the maximum value of profit $c(x, y) = 40\,x + 30\,y + 20\,z$ among the points (x, y, z) satisfying the constraints. In other words, we want to solve the optimization problem

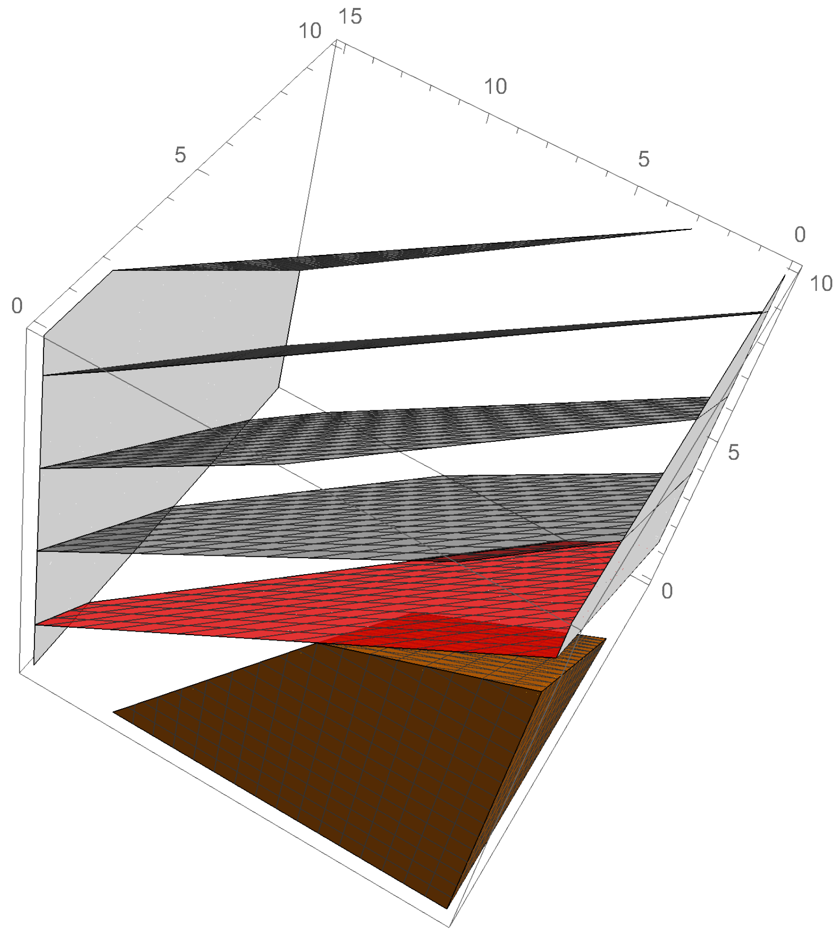
$$\max \quad 40\,x + 30\,y + 20\,z \quad \text{s.t.} \quad \begin{aligned} & x + y + z \leq 12 \\ & 6x + 6y + 2z \leq 48 \\ & 36x + 24y + 18z \leq 360 \\ & x \geq 0,\ y \geq 0,\ z \geq 0 \end{aligned}$$

The corresponding feasible region is:



Overlapping the feasible region with plots of the levelsets $\{c(x, y, z) = t\}$ of the profit function, which are the black parallel planes shown in the next page, we see that the maximum must be achieved at the vertex $(6, 0, 6)$, where $c(6, 0, 6) = 360$. That optimal levelset is shown in red.

Thus, the optimal allocation is to plant 6 acres of corn, no soy, and 6 acres of oats.



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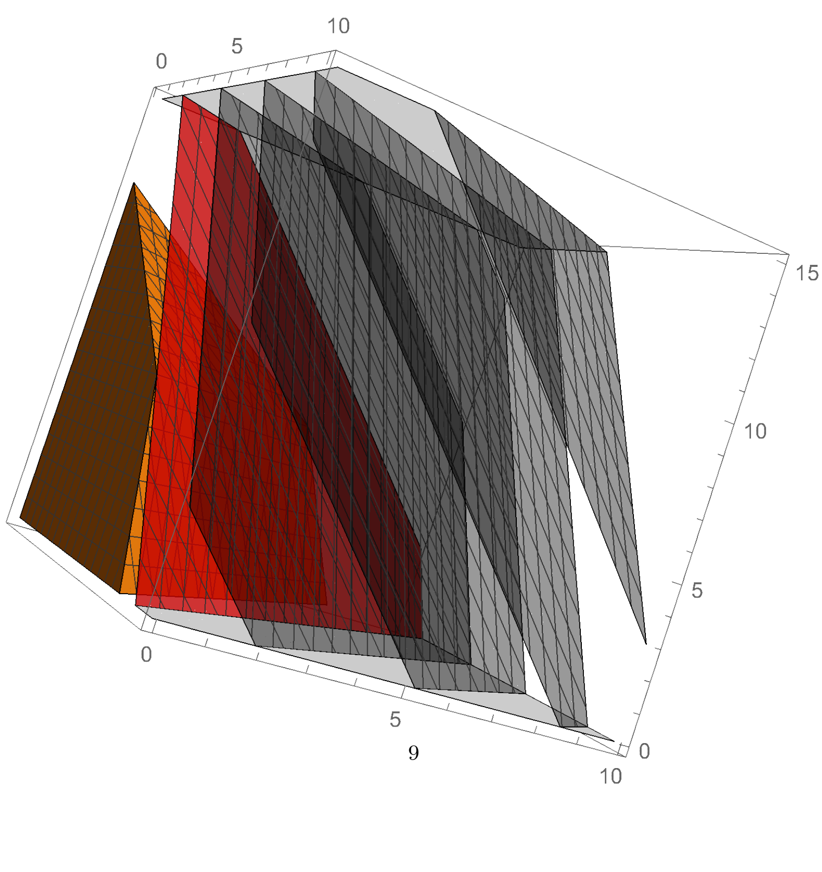

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