

(\*Video 1: Antiderivatives / Indefinite integral \*)

In[40]:=  $F[x_] := x^3$

In[41]:=  $F'[x]$

Out[41]=  $3x^2$

$\text{Integrate}[3x^2, x]$  (\*Command to find an antiderivative\*)

Out[42]=  $x^3$

(\*Are antiderivatives unique?

No! All antiderivatives of the same function differ by constants\*)

In[48]:=  $D[x^3 + c, x]$

Out[48]=  $3x^2$

(\*Antiderivative of a polynomial:\*)

$\text{Integrate}[x^n, x]$  (\*  $n \neq -1$ \*)

Out[59]=  $\frac{x^{1+n}}{1+n}$

In[60]:=  $D\left[\frac{x^{1+n}}{1+n}, x\right]$

Out[60]=  $x^n$

In[67]:=  $\text{Integrate}[4x^5 + 2x^3 + 1/2x + 1, x]$

Out[67]=  $x + \frac{x^2}{4} + \frac{x^4}{2} + \frac{2x^6}{3}$

In[70]:=  $\text{Integrate}[6x^6 + 4x^8 + 1/3x^2 + 12, x]$

Out[70]=  $12x + \frac{x^3}{9} + \frac{6x^7}{7} + \frac{4x^9}{9}$

In[72]:=  $D\left[12x + \frac{x^3}{9} + \frac{6x^7}{7} + \frac{4x^9}{9} + 15, x\right]$

Out[72]=  $12 + \frac{x^2}{3} + 6x^6 + 4x^8$

$\text{Integrate}[x^2, y]$

(\*Careful: second argument given is the variable of integration!\*)

Out[76]=  $x^2 y$

In[77]:=  $D[x^2 y, y]$

Out[77]=  $x^2$

In[81]:= `Integrate[a x^3 + b x^2 + c x + d, x]`

$$\text{Out[81]} = d x + \frac{c x^2}{2} + \frac{b x^3}{3} + \frac{a x^4}{4}$$

In[82]:= `D[d x + \frac{c x^2}{2} + \frac{b x^3}{3} + \frac{a x^4}{4}, x]`

$$\text{Out[82]} = d + c x + b x^2 + a x^3$$

(\*Video 2: Definite integrals\*)

`HoldForm[Integrate[x^2, x]] == Integrate[x^2, x]`

(\*Antiderivative of  $x^2$ , also known as indefinite integral of  $x^2$  \*)

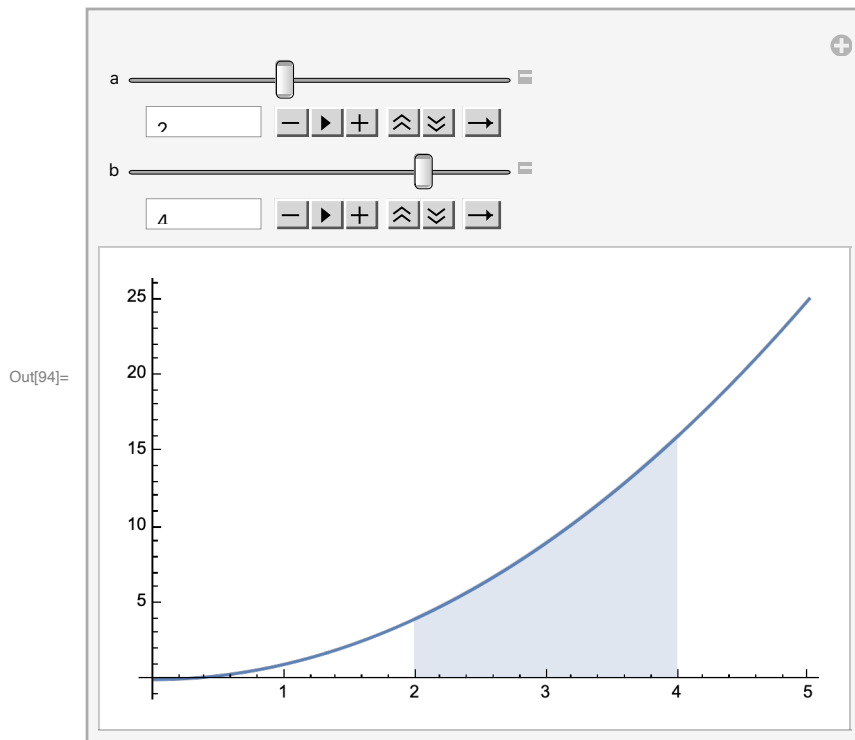
$$\text{Out[105]} = \int x^2 dx = \frac{x^3}{3}$$

`HoldForm[Integrate[x^2, {x, a, b}]]` (\*Definite integral of  $x^2$ , from  $x=a$  to  $x=b$ \*)

$$\text{Out[96]} = \int_a^b x^2 dx$$

(\*The above definite integral, by definition, is the area under the graph of  $y=x^2$  that lies between  $x=a$  and  $x=b$ \*)

In[94]:= `Manipulate[Show[Plot[x^2, {x, 0, 5}], Plot[x^2, {x, a, b}], Filling -> Bottom, PlotRange -> {{0, 5}, {0, 25}}], {a, 0.01, 5}, {b, 0.02, 5}]`



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In[109]:= Integrate[x^2, {x, 2, 4}]
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Out[109]=  $\frac{56}{3}$ 
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