```
(*Video 1: Evaluating limits with Mathematica*)
```

In[212]:=
```
TraditionalForm[HoldForm[Limit[Sqrt[x + 3], x → 1]]]
```

Out[212]//TraditionalForm=

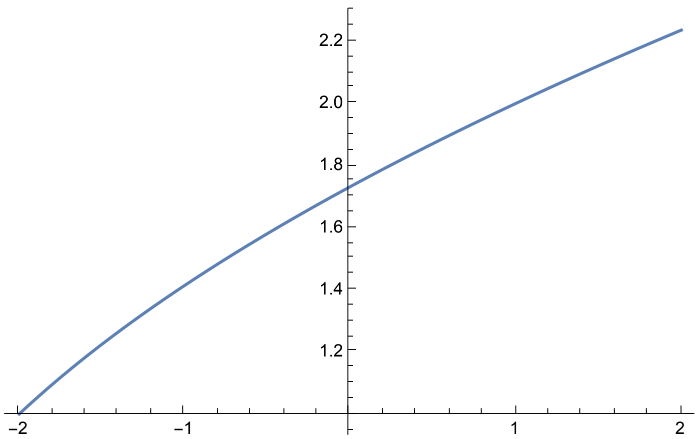
$$\lim_{x\to 1} \sqrt{x+3}$$

In[242]:=
```
Limit[Sqrt[x + 3], x → 1]
```

Out[242]= 2

In[244]:=
```
Plot[Sqrt[x + 3], {x, -2, 2}]
```

Out[244]=



In[245]:=
```
f[x_] := Sqrt[x + 3];
a = -2; b = 2;
```

In[236]:=
```
Manipulate[Grid[{{Row[{"f[t]  = ", f[t]}]}},
   {Show[Plot[f[x], {x, a, b}], Plot[f[t], {x, a, b}, PlotStyle → Red],
     ParametricPlot[{t, y}, {y, f[a], f[b]}, PlotStyle → Gray], ImageSize → 500]}},
  Spacings → {1, 1}, Frame → All], {{t, 1}, a, b}, TrackedSymbols :→ t]
```

Out[236]=

```
t
1.02

f[t]  = f[1.02]

Show[Plot[f[x], {x, a, b}], Plot[f[FE`t$$573], {x, a, b}, PlotStyle → Red],
 ParametricPlot[{FE`t$$573, y}, {y, f[a], f[b]}, PlotStyle → Gray],
 ImageSize → 500]
```

In[233]:=
```
TraditionalForm[HoldForm[Limit[(x^3 - 1) / (x - 1), x → 1]]]
```

Out[233]//TraditionalForm=

$$\lim_{x\to 1} \frac{x^3-1}{x-1}$$

```
In[256]:= Factor[x^3 - 1]
Out[256]= (-1 + x) (1 + x + x^2)

In[257]:= Simplify[(x^3 - 1) / (x - 1)]
Out[257]= 1 + x + x^2

In[258]:= Limit[(x^3 - 1) / (x - 1), x → 1]
Out[258]= 3

In[259]:= Clear[a, b, f]
          f[x_] := (x^3 - 1) / (x - 1);
          a = 0; b = 2;
          Manipulate[Grid[{{Row[{"f[t] = ", f[t]}]},
              {Show[Plot[f[x], {x, a, b}], Plot[f[t], {x, a, b}, PlotStyle → Red],
                ParametricPlot[{t, y}, {y, f[a], f[b]}, PlotStyle → Gray], ImageSize → 500]}},
            Spacings → {1, 1}, Frame → All], {{t, 1}, a, b}, TrackedSymbols :→ t]
```

t

1.065

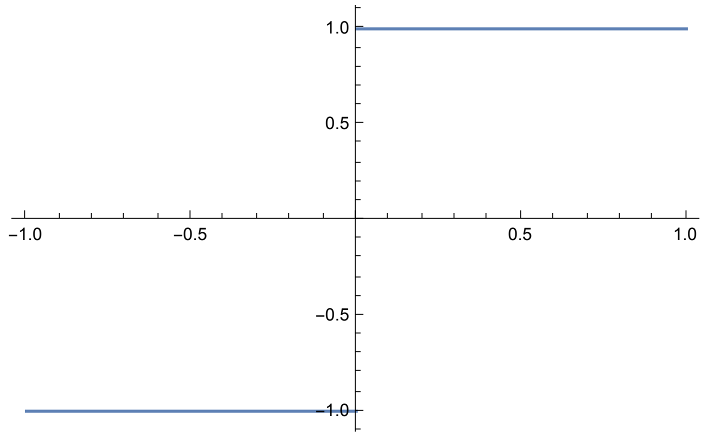
f[t] = f[1.065]
Show[Plot[f[x], {x, a, b}], Plot[f[FE`t$$641], {x, a, b}, PlotStyle → Red], ParametricPlot[{FE`t$$641, y}, {y, f[a], f[b]}, PlotStyle → Gray], ImageSize → 500]

```
In[265]:= Clear[a, b, f]

          (*Video 2: Directional limits*)

In[301]:= TraditionalForm[HoldForm[Abs[x] / x]]
Out[301]//TraditionalForm=
```

$$\frac{|x|}{x}$$

In[268]:=
```
Plot[Abs[x] / x, {x, -1, 1}]
```

Out[268]=



In[372]:=
```
TraditionalForm[HoldForm[Limit[Abs[x] / x, x → 0, Direction → "FromAbove"]]]
TraditionalForm[HoldForm[Limit[Abs[x] / x, x → 0, Direction → "FromBelow"]]]
```

Out[372]//TraditionalForm=

$$\lim_{x\to 0^+} \frac{|x|}{x}$$

Out[373]//TraditionalForm=

$$\lim_{x\to 0^-} \frac{|x|}{x}$$

In[369]:=
```
Limit[Abs[x] / x, x → 0, Direction → "FromAbove"]
```

Out[369]= 1

In[371]:=
```
Limit[Abs[x] / x, x → 0, Direction → "FromBelow"]
```

Out[371]= −1

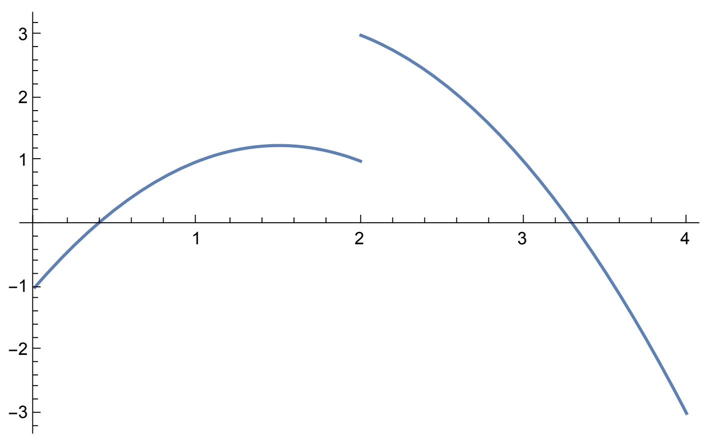
In[375]:=
```
TraditionalForm[HoldForm[Abs[x - 2] / (x - 2) - x^2 + 3 x]]
```

Out[375]//TraditionalForm=

$$\frac{|x-2|}{x-2} - x^2 + 3\,x$$

In[377]:=
```
Plot[Abs[x - 2] / (x - 2) - x^2 + 3 x, {x, 0, 4}]
```

Out[377]=



In[297]:=
```
TraditionalForm[
 HoldForm[Limit[Abs[x - 2] / (x - 2) - x^2 + 3 x, x → 2, Direction → "FromAbove"]]]
```

Out[297]//TraditionalForm=

$$\lim_{x\to 2^{+}}\left(\frac{|x-2|}{x-2}-x^{2}+3\,x\right)$$

In[379]:=
```
Limit[Abs[x - 2] / (x - 2) - x^2 + 3 x, x → 2, Direction → "FromAbove"]
```

Out[379]= 3

In[300]:=
```
TraditionalForm[
 HoldForm[Limit[Abs[x - 2] / (x - 2) - x^2 + 3 x, x → 2, Direction → "FromBelow"]]]
```

Out[300]//TraditionalForm=

$$\lim_{x\to 2^{-}}\left(\frac{|x-2|}{x-2}-x^{2}+3\,x\right)$$

In[380]:=
```
Limit[Abs[x - 2] / (x - 2) - x^2 + 3 x, x → 2, Direction → "FromBelow"]
```

Out[380]= 1

In[381]:=
```
f[x_] := Abs[x - 2] / (x - 2) - x^2 + 3 x;
a = 0; b = 4;
c = Minimize[{f[x], a ≤ x ≤ b}, x][[1]];
d = Maximize[{f[x], a ≤ x ≤ b}, x][[1]];
```

In[384]:=
```
Manipulate[Grid[{{Row[{"f[t] = ", f[t]}]},
   {Show[Plot[f[x], {x, a, b}], Plot[f[t], {x, a, b}, PlotStyle → Red],
     ParametricPlot[{t, y}, {y, c, d}, PlotStyle → Gray], ImageSize → 500]}},
  Spacings → {1, 1}, Frame → All], {{t, 2.2}, a, b}, TrackedSymbols :→ t]
```

Out[384]=

t

1.98

```
f[t] = f[1.98]
Show[Plot[f[x], {x, a, b}], Plot[f[FE`t$$895], {x, a, b}, PlotStyle → Red],
 ParametricPlot[{FE`t$$895, y}, {y, c, d}, PlotStyle → Gray],
 ImageSize → 500]
```

```
Clear[a, b, c, d, f]
```

```
(*Video 3: Squeeze Theorem*)
```

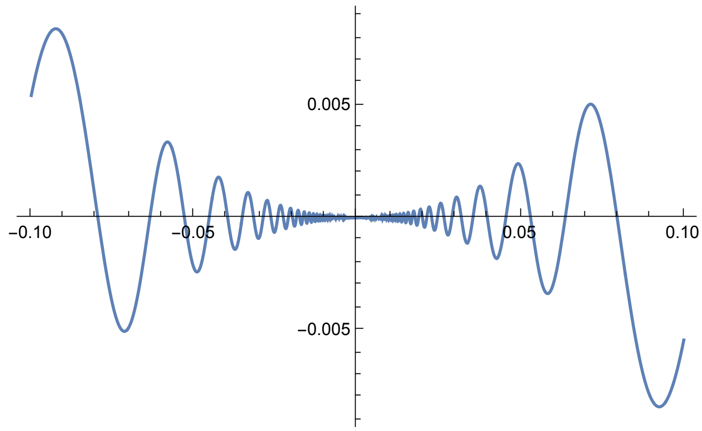
In[404]:=
```
TraditionalForm[HoldForm[x^2 Sin[1 / x]]]
```

Out[404]//TraditionalForm=

$$x^2 \sin\left(\frac{1}{x}\right)$$

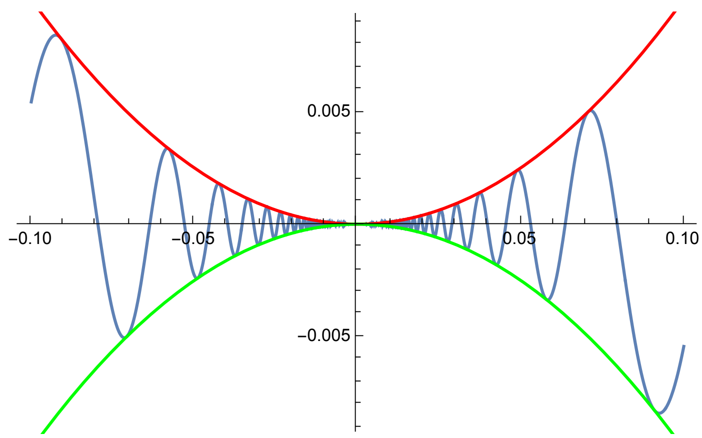
In[413]:=
```
Plot[x^2 Sin[1 / x], {x, -.1, .1}]
```

Out[413]=



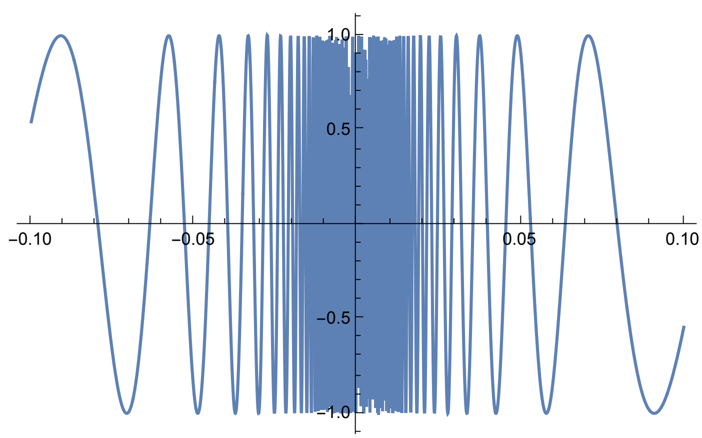
In[402]:=
```
Show[Plot[x^2 Sin[1 / x], {x, -.1, .1}],
 Plot[{x^2, -x^2}, {x, -.1, .1}, PlotStyle → {Red, Green}]]
```

Out[402]=




```
Plot[Sin[1 / x], {x, -.1, .1}]
```

Out[417]=



```
(* For all x ≠ 0, we know that x^2>0 *)

(* Therefore, as -1≤Sin[1/x]≤1, we conclude that for all x≠0,
-x^2 ≤ x^2 Sin[1/x] ≤ x^2
*)
```

In[418]:=
```
Limit[x^2 , x → 0]
Limit[-x^2 , x → 0]
```

Out[418]= 0

Out[419]= 0

In[409]:=
```
TraditionalForm[HoldForm[Limit[x^2 Sin[1 / x], x → 0]]]
```

Out[409]//TraditionalForm=

$$\lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right)$$

In[422]:=
```
TraditionalForm[
 HoldForm[Limit[-x^2 , x → 0] <= Limit[x^2 Sin[1 / x], x → 0] <= Limit[x^2 , x → 0]]]
```

Out[422]//TraditionalForm=

$$\lim_{x \to 0} -x^2 \leq \lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right) \leq \lim_{x \to 0} x^2$$

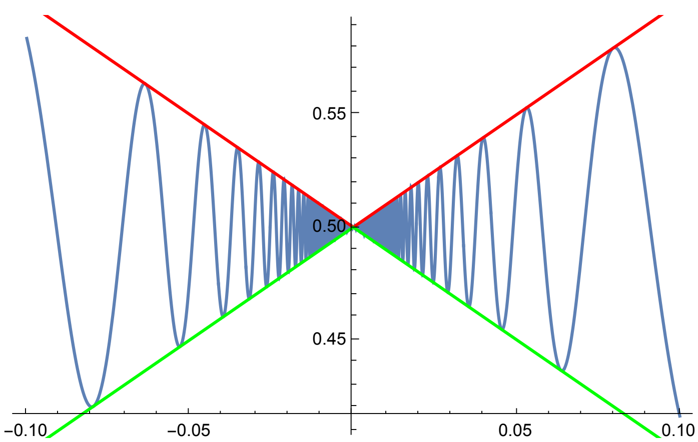
In[424]:=
```
Limit[x^2 Sin[1 / x], x → 0]
```

Out[424]= 0

```
(*Try to argue this one on your own: *)
```

In[432]:=
```
Show[Plot[x Cos[1 / x] + 1 / 2, {x, -.1, .1}],
 Plot[{Abs[x] + 1 / 2, -Abs[x] + 1 / 2}, {x, -.1, .1}, PlotStyle → {Red, Green}]]
```

Out[432]=



In[433]:=
```
Limit[x Cos[1 / x] + 1 / 2, x → 0]
```

Out[433]= $\frac{1}{2}$