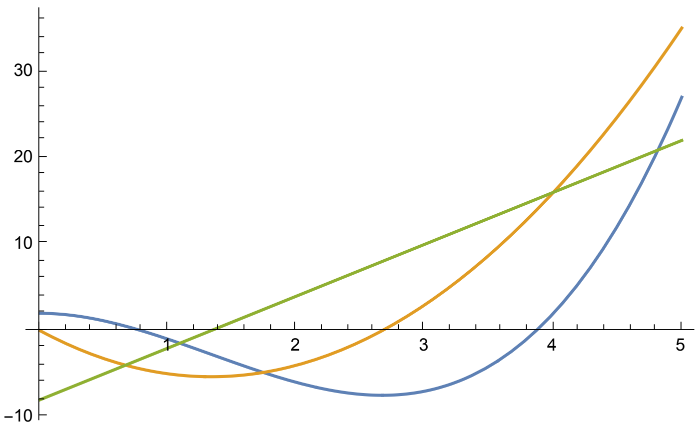
```
(*Video 1: Velocity *)
```

In[170]:=
```
x[t_] := t^3 - 4 t^2 + 2
(* Physical interpretation;
x[t] = position at time t,
x'[t] = velocity at time t,
x''[t] = acceleration at time t *)
Plot[{x[t], x'[t], x''[t]}, {t, 0, 5}]
(*units: x = meters; t = second;
x/t = x'[t] = meters/second,  x''[t] = meters/second^2.
*)
```

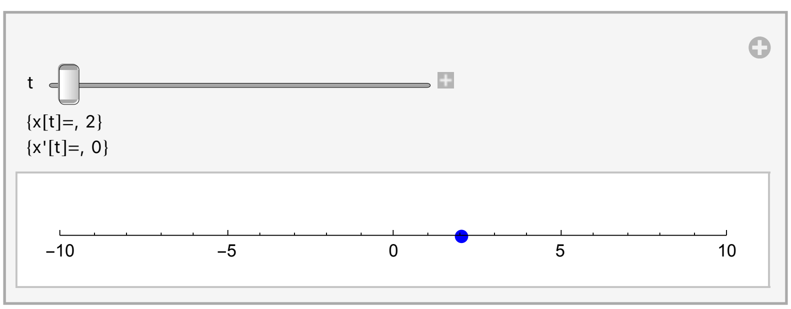
Out[171]=




```
(*Auxiliary function to plot motion*)
plot1dmotion[x_] := {Blue, PointSize[0.02], Point[{x, 0}]};
```

In[172]:=
```
Manipulate[
 Graphics[plot1dmotion[x[t]], Axes → {True, False}, PlotRange → {{-10, 10}, {0, 1}}],
 {t, 0, 5}, Dynamic[{"x[t]=", x[t]}], Dynamic[{"x'[t]=", x'[t]}]]
```

Out[172]=




```
(*Average velocity from t0 to t1: ratio between displacement in x, and in t*)
(x[t1] - x[t0]) / (t1 - t0)

(* (Instantaneous) velocity: limit of the above as t1-->t0: x'[t0] = dx/dt [t0]*)
```

In[74]:=
```
Limit[(x[t1] - x[t0]) / (t1 - t0), t1 → t0]
```

Out[74]= $-8\,t0 + 3\,t0^2$

```
x'[t](*Velocity at time t*)
```

Out[∘]= $-8 t + 3 t^2$

```
Solve[-8 t + 3 t^2 == 0, t]
(*When does the particle change direction? At time t=8/3 s.*)
```

Out[∘]= $\{\{t \to 0\}, \{t \to \frac{8}{3}\}\}$

```
(*Acceleration: x''[t]*)
```

In[85]:=
```
x''[t]
```

Out[85]= $-8 + 6 t$

```
(*Video 2: Product and Quotient rules *)
```

In[117]:=
```
(*Product Rule*)
TraditionalForm[HoldForm[(f[x] × g[x])'] == Simplify[D[f[x] × g[x], x]]]
```

Out[117]//TraditionalForm=

$$(f(x)\, g(x))' = g(x)\, f'(x) + f(x)\, g'(x)$$

In[118]:=
```
(*Quotient Rule*)
TraditionalForm[HoldForm[(f[x] / g[x])'] == Simplify[D[f[x] / g[x], x]]]
```

Out[118]//TraditionalForm=

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{g(x)\, f'(x) - f(x)\, g'(x)}{g(x)^2}$$

In[142]:=
```
(*Example*)
f[x_] := x^4 + 3 x^2 + 1
g[x_] := x^3 - x + 3
```

In[149]:=
```
f'[x]
g'[x]
```

Out[149]= $6 x + 4 x^3$

Out[150]= $-1 + 3 x^2$

In[146]:=
```
f[x] × g[x]
Expand[f[x] × g[x]]
```

Out[146]= $(3 - x + x^3)\, (1 + 3 x^2 + x^4)$

Out[147]= $3 - x + 9 x^2 - 2 x^3 + 3 x^4 + 2 x^5 + x^7$

In[155]:=
```
D[f[x] × g[x], x](*using the product rule*)
D[3 - x + 9 x^2 - 2 x^3 + 3 x^4 + 2 x^5 + x^7, x](*expanding the product of polynomials*)
```

Out[155]= $(3 - x + x^3)\, (6 x + 4 x^3) + (-1 + 3 x^2)\, (1 + 3 x^2 + x^4)$

Out[156]= $-1 + 18 x - 6 x^2 + 12 x^3 + 10 x^4 + 7 x^6$

```
Expand[(3 - x + x^3) (6 x + 4 x^3) + (-1 + 3 x^2) (1 + 3 x^2 + x^4)] (*It matches!*)
```

Out[157]= $-1+18\,x-6\,x^2+12\,x^3+10\,x^4+7\,x^6$

```
In[158]:= Simplify[D[f[x] / g[x], x]]
```

Out[158]= $$\frac{1+18\,x-6\,x^2+12\,x^3-6\,x^4+x^6}{\left(3-x+x^3\right)^2}$$

```
In[159]:= Simplify[f'[x] × g[x] - f[x] × g'[x]]
```

Out[159]= $1+18\,x-6\,x^2+12\,x^3-6\,x^4+x^6$

```
(*Note: quotient rule is a particular case of the product rule!*)
```

```
In[162]:= f[x] / g[x] == f[x] (1 / g[x])
```

Out[162]= True

```
(*We need the following fact,
which follows from the Chain rule -- to be seen next class
*)
```

```
In[169]:= D[1 / (g[x]), x]
```

Out[169]= $$-\frac{g'[x]}{g[x]^2}$$

```
(*Applying product rule to f[x](1/g[x]) we then recover the quotient rule:*)
Simplify[f'[x] (1 / g[x]) + f[x] × D[1 / (g[x]), x]]
```

Out[167]= $$\frac{g[x]\,f'[x]-f[x]\,g'[x]}{g[x]^2}$$

```
In[168]:= Simplify[D[f[x] / g[x], x]]
```

Out[168]= $$\frac{g[x]\,f'[x]-f[x]\,g'[x]}{g[x]^2}$$