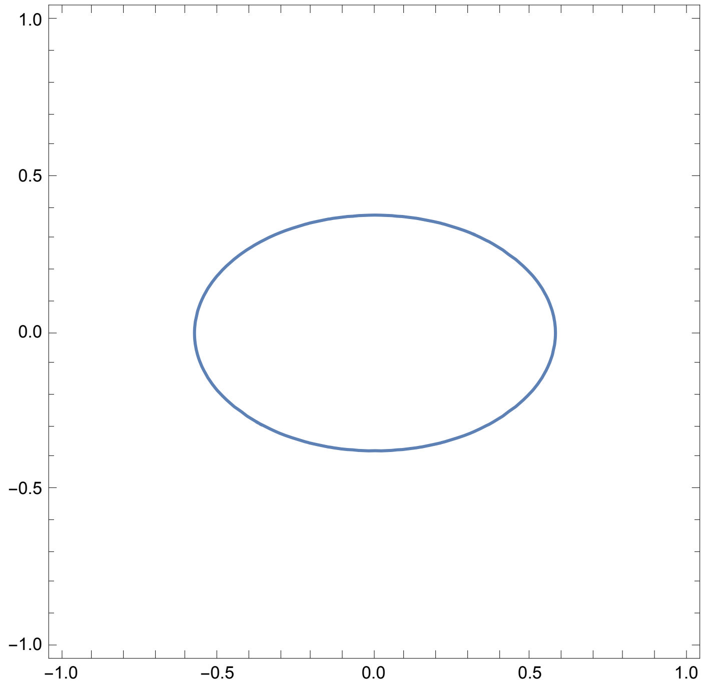
```
(*Video 1: Implicit differentiation *)

f[x]^4(*f is an unspecified function of x*)

Out[15]= f[x]^4

D[f[x]^4, x](*we can differentiate f^4 without knowing what f is:*)

Out[16]= 4 f[x]^3 f′[x]

In[38]:= f[x_] := 2 x + 3

In[39]:= Expand[4 f[x]^3 f′[x]]

Out[39]= 216 + 432 x + 288 x^2 + 64 x^3

In[40]:= Expand[D[f[x]^4, x]]

Out[40]= 216 + 432 x + 288 x^2 + 64 x^3

In[41]:= Clear[f]

(*the same logic applies to more complicated examples: *)
D[f[x]^4 + 2 x f[x] + Cos[2 f[x]] + x^3, x]

Out[42]= 3 x^2 + 2 f[x] + 2 x f′[x] + 4 f[x]^3 f′[x] - 2 Sin[2 f[x]] f′[x]

In[43]:= D[3 x^2 + 7 y[x]^2, x](* y is a function of x'*)

Out[43]= 6 x + 14 y[x] y′[x]

D[a[x] × b[x] + a[x]^2 + E^(b[x]), x](*a and b are functions of x*)

Out[37]= 2 a[x] a′[x] + b[x] a′[x] + e^b[x] b′[x] + a[x] b′[x]

(*Video 2: Application of implicit differentiation*)
```

In[82]:=
```
ellipse = ContourPlot[3 x^2 + 7 y^2 == 1, {x, -1, 1}, {y, -1, 1}]
```

Out[82]=



(*Find the tangent line to the ellipse above at the point $\left\{x\to\frac{\sqrt{3}}{4}, y\to\frac{1}{4}\right\}$ *)

In[71]:= $3\,x^\wedge 2 + 7\,y^\wedge 2\ /.\ \left\{x \to \frac{\sqrt{3}}{4},\ y \to \frac{1}{4}\right\}$

Out[71]= 1

In[74]:=
```
D[3 x^2 + 7 y[x]^2, x] == 0
```

Out[74]=
```
6 x + 14 y[x] y′[x] == 0
```

In[73]:=
```
Solve[6 x + 14 y[x] y′[x] == 0, y′[x]]
```

Out[73]= $\left\{\left\{y'[x] \to -\frac{3\,x}{7\,y[x]}\right\}\right\}$

In[76]:= (*Slope of the tangent line at the given point:*)

$m = -\frac{3\,x}{7\,y}\ /.\ \left\{x \to \frac{\sqrt{3}}{4},\ y \to \frac{1}{4}\right\}$

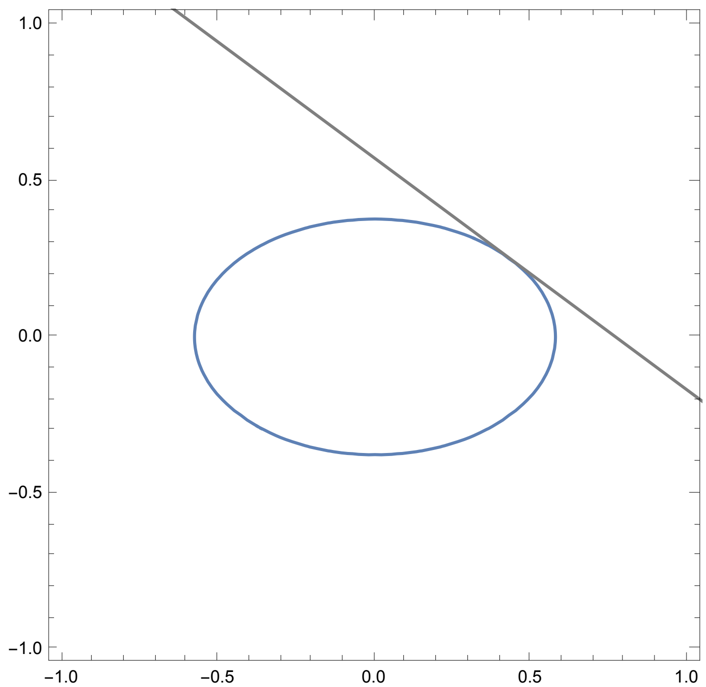
Out[76]= $-\frac{3\sqrt{3}}{7}$

In[78]:=
```
(*Tangent line to graph of y[x] at (x0,y0) is given by:*)
Collect[y0 + m (x - x0) /. {x0 → √3/4, y0 → 1/4}, x]
```

Out[78]= $\frac{4}{7} - \frac{3\sqrt{3}\,x}{7}$

In[91]:=
```
line = Plot[4/7 - (3 √3 x)/7, {x, -3, 3}, PlotStyle → Gray];
Show[ellipse, line]
```

Out[92]=

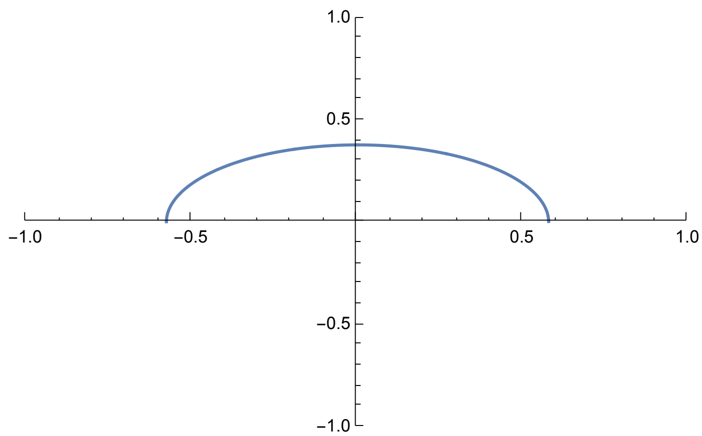



```
(*Can also do it explicitly:*)
Solve[3 x^2 + 7 y^2 == 1, y]
```

Out[84]= $\left\{\left\{y \to -\frac{\sqrt{1-3x^2}}{\sqrt{7}}\right\}, \left\{y \to \frac{\sqrt{1-3x^2}}{\sqrt{7}}\right\}\right\}$

In[87]:=
```
Plot[Sqrt[1 - 3 x^2]/Sqrt[7], {x, -1, 1}, PlotRange → {{-1, 1}, {-1, 1}}]
```

$$\text{Plot}\left[\frac{\sqrt{1-3x^2}}{\sqrt{7}}, \{x, -1, 1\}, \text{PlotRange} \to \{\{-1, 1\}, \{-1, 1\}\}\right]$$

Out[87]=


In[93]:= $$D\left[\frac{\sqrt{1-3x^2}}{\sqrt{7}}, x\right]$$

Out[93]= $$-\frac{3x}{\sqrt{7}\sqrt{1-3x^2}}$$

(*So we find, once again, the slope of the tangent line to be:*)

$$-\frac{3x}{\sqrt{7}\sqrt{1-3x^2}} \text{ /. } x \to \frac{\sqrt{3}}{4}$$

Out[89]= $$-\frac{3\sqrt{3}}{7}$$

m (*agrees with earlier computation!*)

Out[90]= $$-\frac{3\sqrt{3}}{7}$$