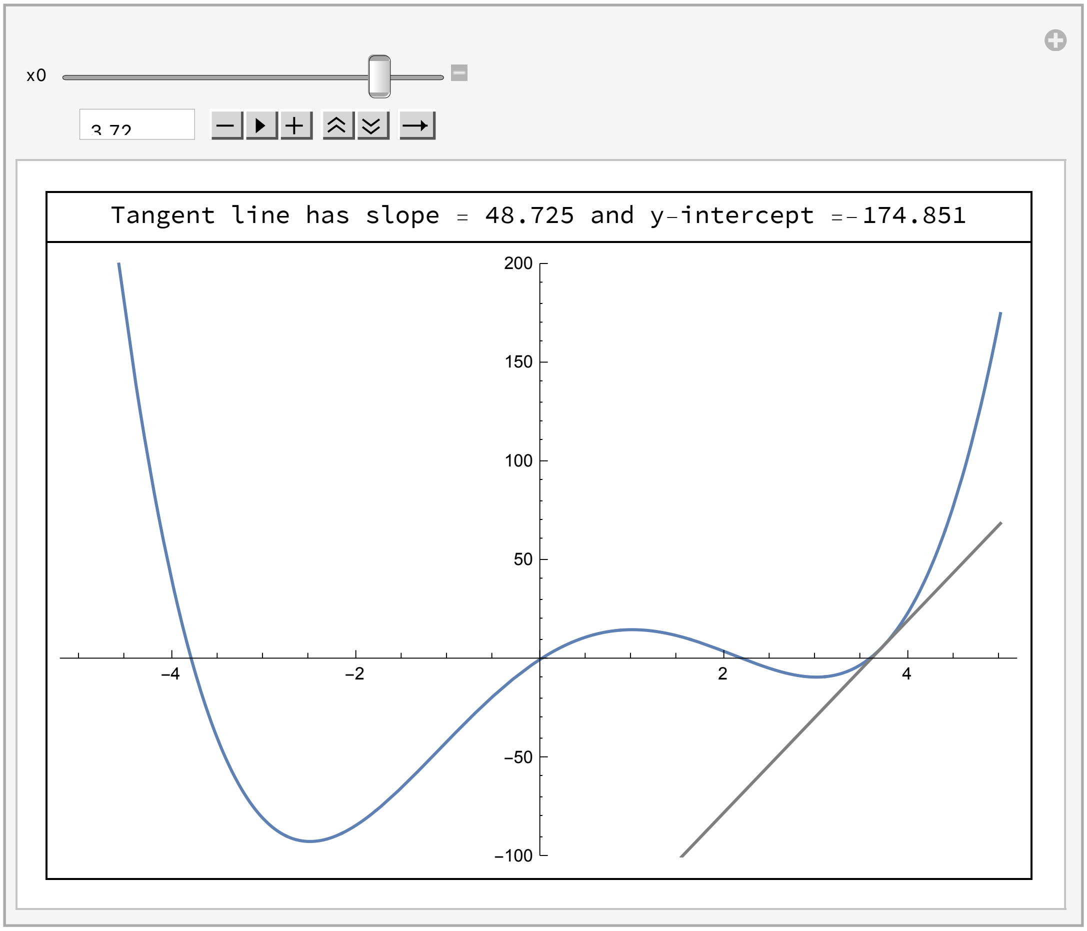
```
(*Video 1: Increasing/decreasing *)
```

In[285]:=
```
f[x_] := 30 x - 14 x^2 - 2 x^3 + x^4;
Manipulate[
 Grid[{{Row[{"Tangent line has slope = ", f'[x0], " and y-intercept =", f[x0] -
       x0 f'[x0]}]}, {Plot[{f[x], f'[x0] (x - x0) + f[x0]}, {x, -5, 5},
     PlotStyle → {Normal, Gray}, PlotRange → {-100, 200}, ImageSize → 500]}},
  Spacings → {1, 1}, Frame → All], {{x0, 1}, -5, 5}]
```

Out[286]=



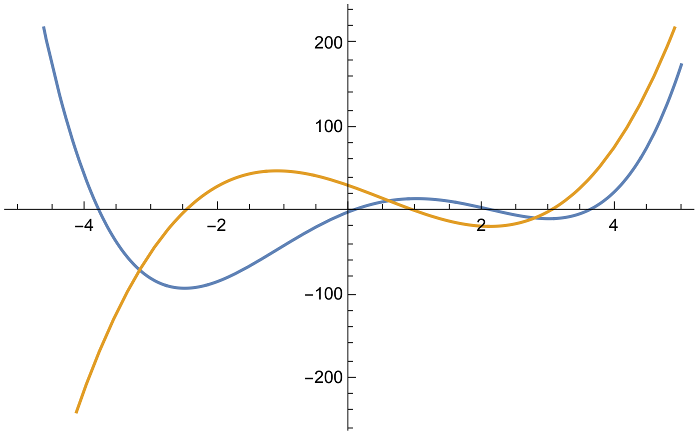

```
(*Slope of tangent line is positive (value of derivative) ---
 function is locally increasing*)
(*Slope of tangent line is negative (value of derivative) ---
 function is locally decreasing*)
```

In[299]:=
```
f[x]
f'[x]
```

Out[299]= $30 x - 14 x^2 - 2 x^3 + x^4$

Out[300]= $30 - 28 x - 6 x^2 + 4 x^3$

In[298]:= Plot[{f[x], f'[x]}, {x, -5, 5}]

Out[298]=


(*f[x] is increasing in the region where f'[x]>0*)

In[291]:= f'[x] > 0

Out[291]= $30 - 28x - 6x^2 + 4x^3 > 0$

Factor[f'[x]](*To determine by hand where f'[x] is positive, we factor it:*)

Out[292]= $2(-3+x)(-1+x)(5+2x)$

Reduce[f'[x] > 0](*With Mathematica we can use Reduce*)

Out[293]= $-\frac{5}{2} < x < 1 \,||\, x > 3$

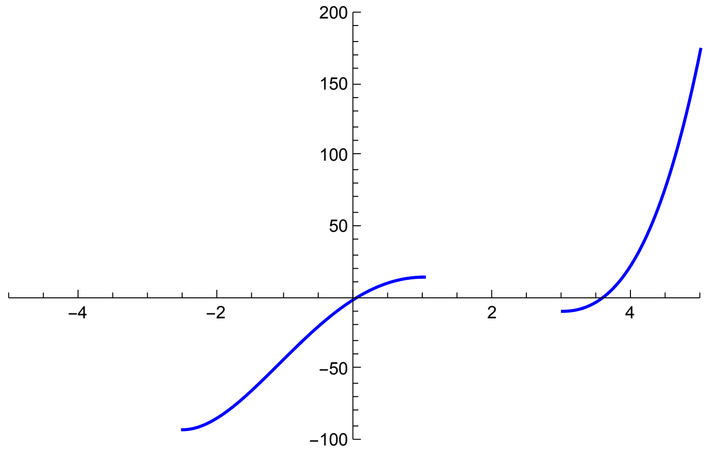
(*f[x] is decreasing in the region where f'[x]<0*)

In[295]:= Reduce[f'[x] < 0]

Out[295]= $x < -\frac{5}{2} \,||\, 1 < x < 3$

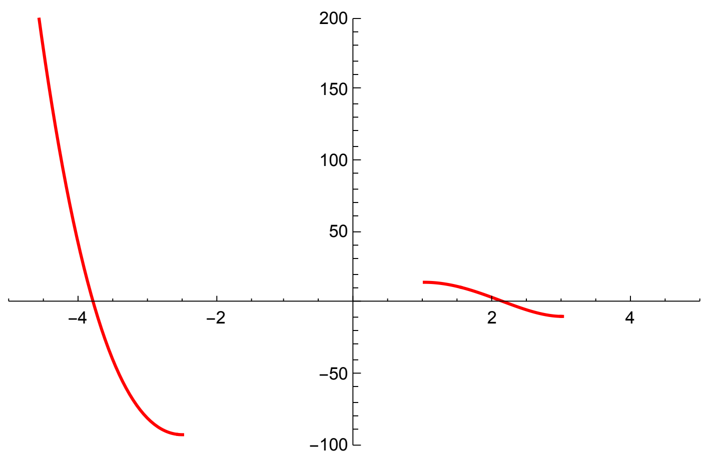
In[296]:=
```
increase = Show[
  Plot[f[x], {x, -5 / 2, 1}, PlotRange → {{-5, 5}, {-100, 200}}, PlotStyle → Blue],
  Plot[f[x], {x, 3, 5}, PlotRange → {{-5, 5}, {-100, 200}}, PlotStyle → Blue]]
```

Out[296]=



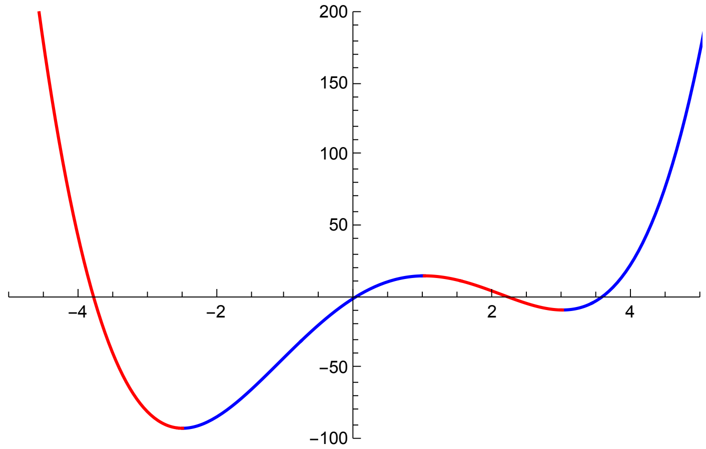
In[263]:=
```
decrease = Show[
  Plot[f[x], {x, -5, -5 / 2}, PlotRange → {{-5, 5}, {-100, 200}}, PlotStyle → Red],
  Plot[f[x], {x, 1, 3}, PlotRange → {{-5, 5}, {-100, 200}}, PlotStyle → Red]]
```

Out[263]=



In[265]:= Show[increase, decrease]

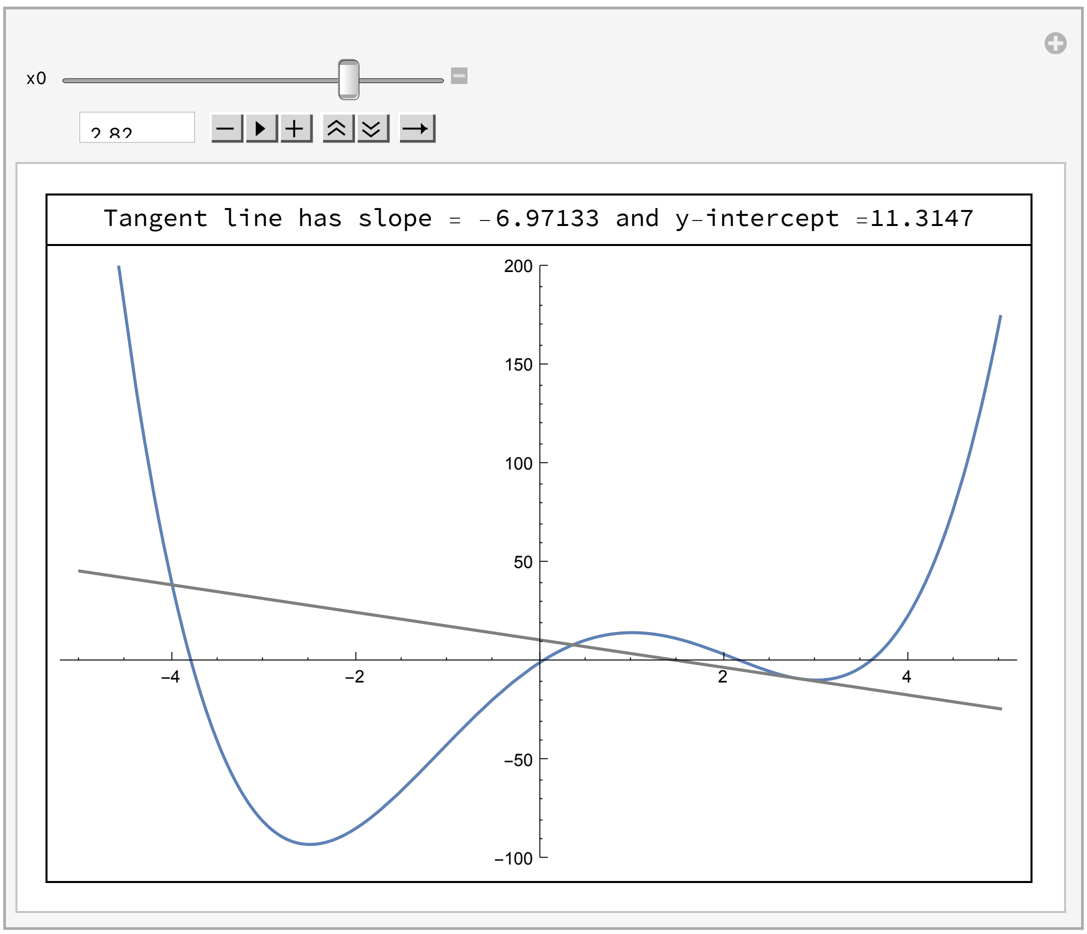
Out[265]=



(*Video 2: Extrema *)

In[312]:= f[x]

Out[312]= $30 x - 14 x^2 - 2 x^3 + x^4$



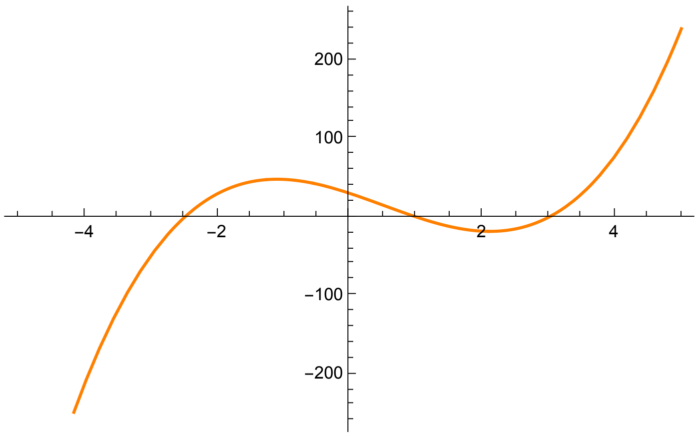
```
(*Points where f[x] goes from increasing to decreasing are local maxima,
i.e. f'[x] goes from being positive to being negative *)
(*Points where f[x] goes from decreasing to increasing are local minimum,
i.e. f'[x] goes from being negative to being positive.*)

f'[x] == 0(*solutions to this equation are called critical points*)
```

In[313]:= Solve$[30 - 28x - 6x^2 + 4x^3 == 0]$

Out[313]= $\left\{\left\{x \to -\frac{5}{2}\right\}, \{x \to 1\}, \{x \to 3\}\right\}$

In[315]:= `Plot[f'[x], {x, -5, 5}, PlotStyle → Orange]`

Out[315]=



In[323]:= (*x=$-\frac{5}{2}$ is a local minimum:*)

f'$\left[-\frac{5}{2} - 0.1\right]$

f'$\left[-\frac{5}{2}\right]$

f'$\left[-\frac{5}{2} + 0.1\right]$

Out[323]= -8.064

Out[324]= 0

Out[325]= 7.344

```
(*x=1 is a local maximum:*)
f'[1 - 0.1]
f'[1]
f'[1 + 0.1]
```

Out[326]= 2.856

Out[327]= 0

Out[328]= -2.736

```
(*x=1 is a local minimum:*)
f'[3 - 0.1]
f'[3]
f'[3 + 0.1]
```

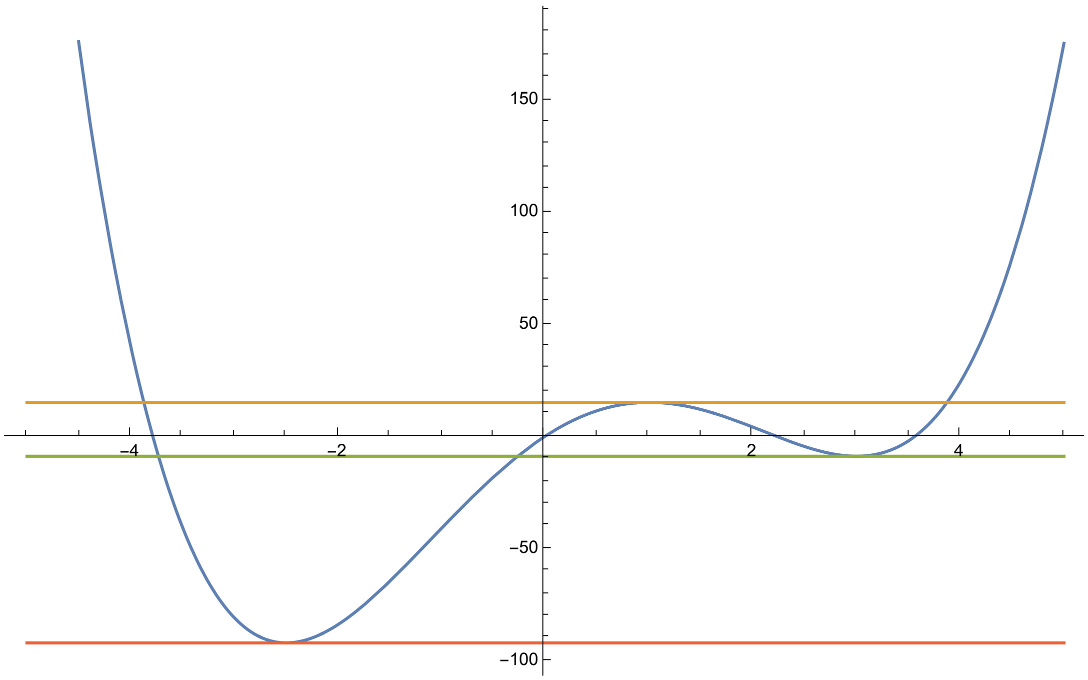
Out[329]= -4.104

Out[330]= 0

Out[331]= 4.704

In[332]:=
```
Plot[{f[x], f'[x0] (x - x0) + f[x0] /. {x0 → 1},
  f'[x0] (x - x0) + f[x0] /. x0 → 3, f'[x0] (x - x0) + f[x0] /. x0 → -5 / 2}, {x, -5, 5}]
```

Out[332]=




```
(*In graph above, all critical points (values of x such that f'[x]=0) are either
 local minima or local maxima. These are called local extrema for f[x]! *)
```

```
(*For global minima and maxima on an interval [a,b],
check values at critical points as well as the endpoints x=a and x=b*)
```

```
(*Values of f[x] at endpoints of [-5,5] *)f[-5]
```

Out[333]= 375

In[334]:=
```
f[5]
```

Out[334]= 175

In[341]:=
```
(*Values of f[x] at critical points in the interior of [-5,5] *)
N[f[-5 / 2]]
f[1]
f[3]
```

Out[341]= -92.1875

Out[342]= 15

Out[343]= -9

```
(*Global maximum for f[x] is 375, achieved at x=-5*)
(*Global minimum for f[x] is -92.1875 achieved at x=-5/2*)
```

```
(*Video 3: Mean Value Theorem *)
```

```
(*If f[x] is continuous on [a,b] and differentiable on (a,b),
then there exists (at least one) value x=c such that
   f'[c](b-a) = f(b)-f(a),
equivalently:
   f'[c] = (f(b)-f(a))/(b-a),
i.e., tangent line to y=
 f[x] at x=c has same slope as line through (a,f[a]) and (b,f[b]).
*)
```

In[598]:=
```
f[x]
```

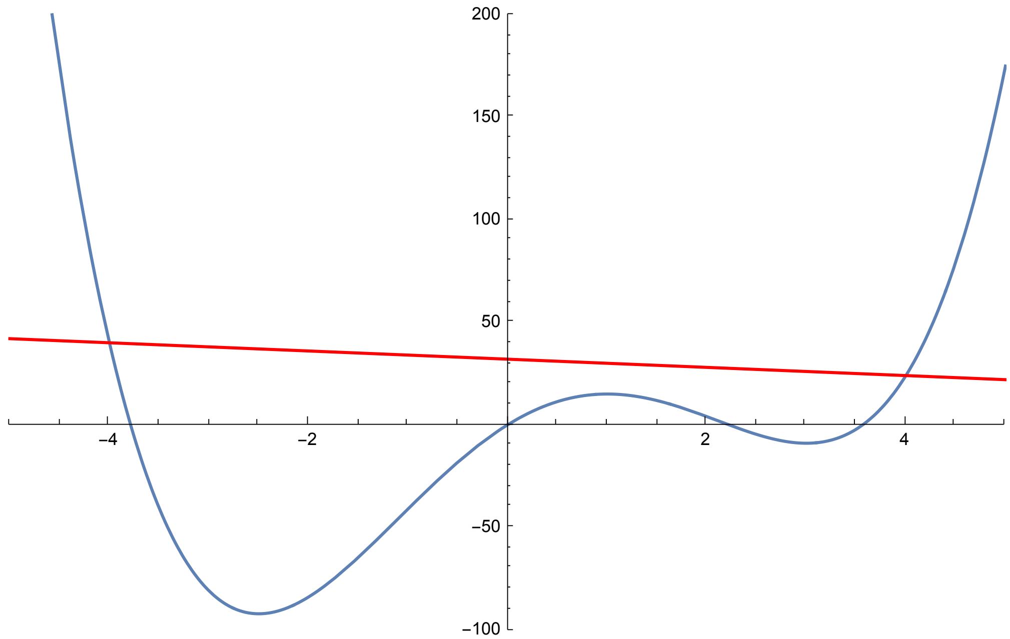
Out[598]= $30\,x - 14\,x^2 - 2\,x^3 + x^4$

In[642]:=
```
(*let's take some different values of a,b to see these lines:*)
a = -4; b = 4;
(f[b] - f[a]) / (b - a)
```

Out[643]= -2

In[610]:= **secantline**

Out[610]=

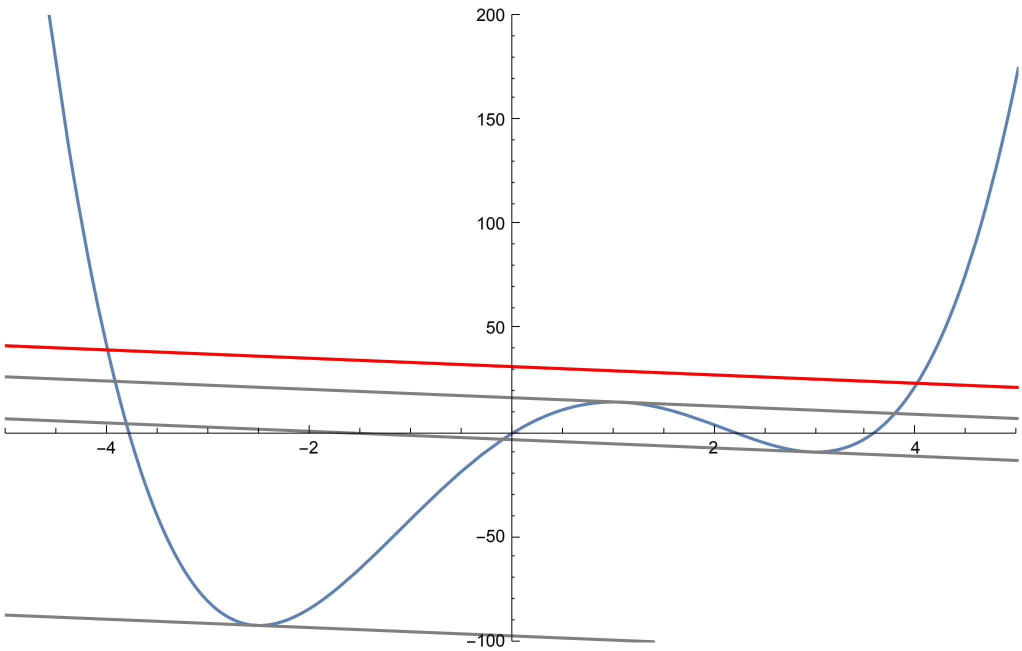


In[611]:= **pts = NSolve[f'[x0] == (f[b] - f[a]) / (b - a)]**

Out[611]= {{x0 → -2.52567}, {x0 → 1.07261}, {x0 → 2.95305}}

In[612]:=
```
secantline = Plot[{f[x], (f[b] - f[a]) / (b - a) (x - a) + f[a]}, {x, -5, 5},
   PlotRange → {{-5, 5}, {-100, 200}}, PlotStyle → {Normal, Red}];
tangentlines = Plot[{f'[x0] (x - x0) + f[x0] /. pts[[1, 1]],
    f'[x0] (x - x0) + f[x0] /. pts[[2, 1]], f'[x0] (x - x0) + f[x0] /. pts[[3, 1]]},
   {x, -5, 5}, PlotRange → {{-5, 5}, {-100, 200}}, PlotStyle → Gray];
Show[secantline, tangentlines]
```

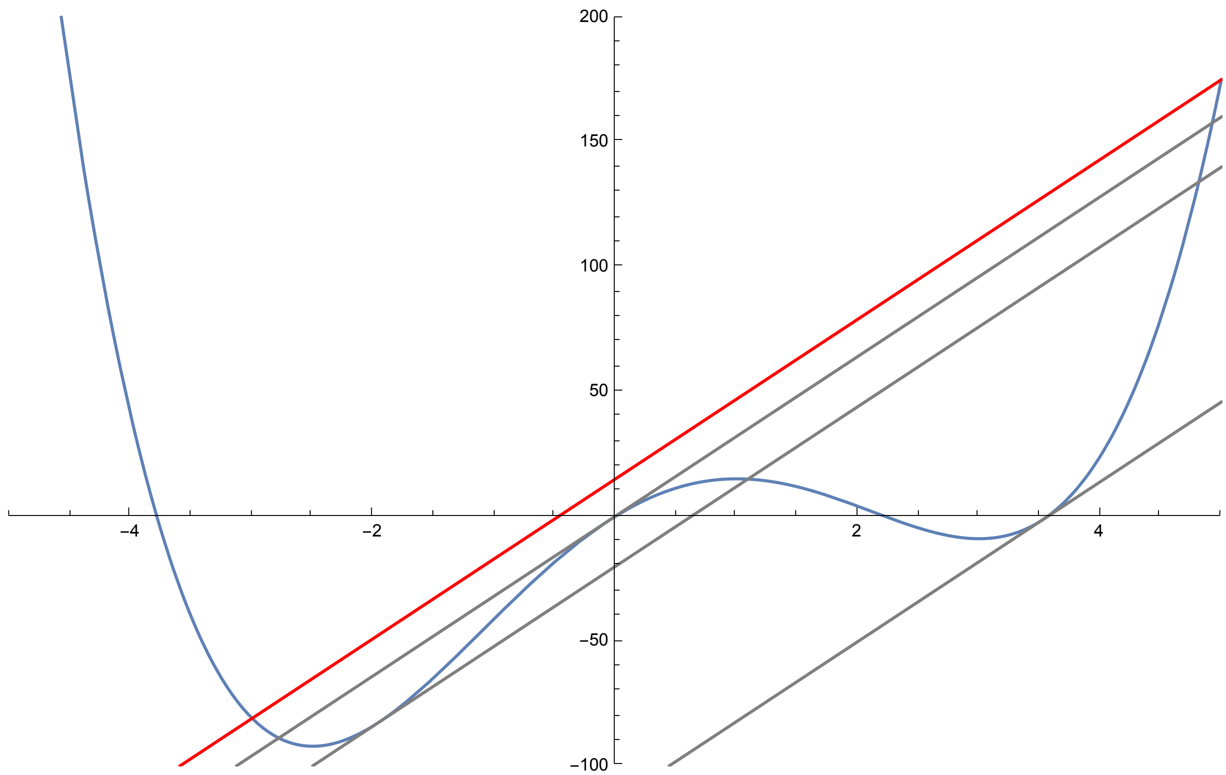
Out[614]=



In[628]:=
```
a = -3; b = 5;
(f[b] - f[a]) / (b - a)

pts = NSolve[f'[x0] == (f[b] - f[a]) / (b - a)];
secantline = Plot[{f[x], (f[b] - f[a]) / (b - a) (x - a) + f[a]}, {x, -5, 5},
   PlotRange → {{-5, 5}, {-100, 200}}, PlotStyle → {Normal, Red}];
tangentlines = Plot[{f'[x0] (x - x0) + f[x0] /. pts[[1, 1]],
    f'[x0] (x - x0) + f[x0] /. pts[[2, 1]], f'[x0] (x - x0) + f[x0] /. pts[[3, 1]]},
   {x, -5, 5}, PlotRange → {{-5, 5}, {-100, 200}}, PlotStyle → Gray];
Show[secantline, tangentlines]
```

Out[629]= 32

Out[633]=



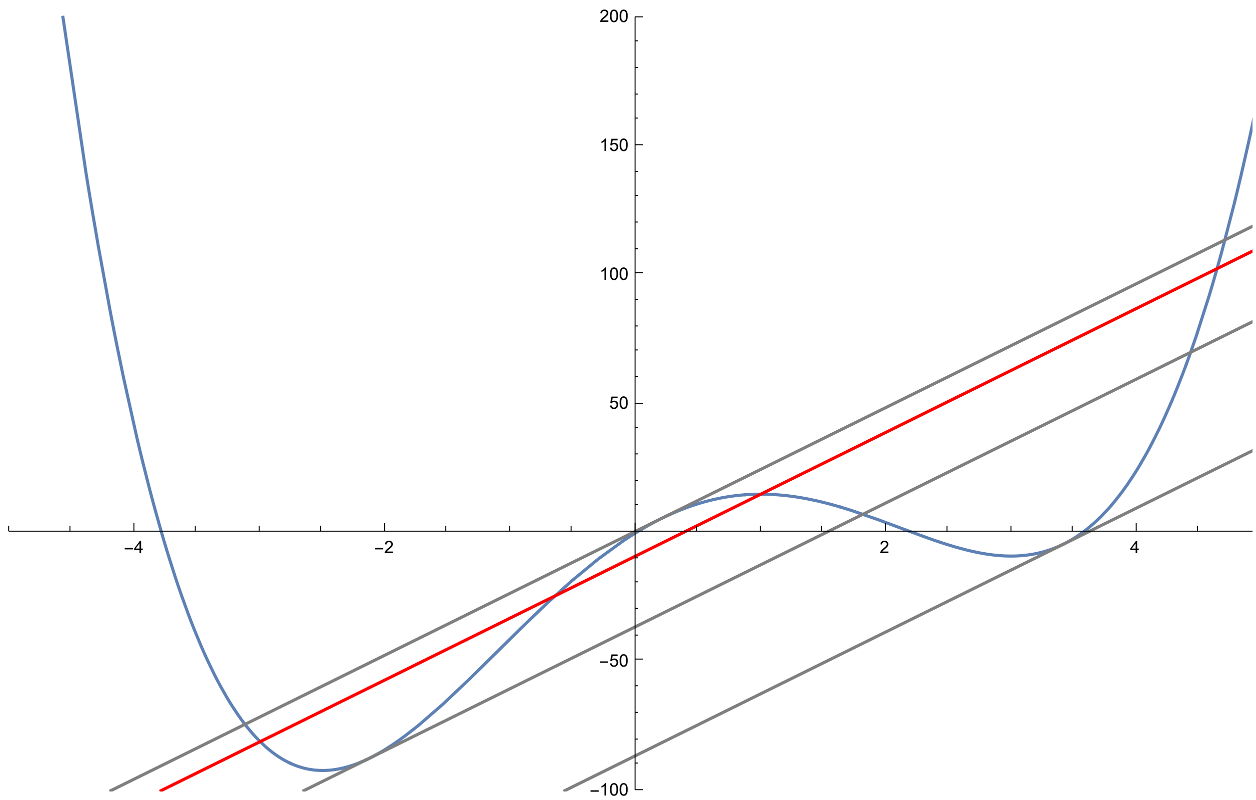
In[627]:=
```
pts
```

Out[627]= {{x0 → -1.95305}, {x0 → -0.0726131}, {x0 → 3.52567}}

In[634]:=
```
a = -3; b = 1;
(f[b] - f[a]) / (b - a)

pts = NSolve[f'[x0] == (f[b] - f[a]) / (b - a)];
secantline = Plot[{f[x], (f[b] - f[a]) / (b - a) (x - a) + f[a]}, {x, -5, 5},
   PlotRange → {{-5, 5}, {-100, 200}}, PlotStyle → {Normal, Red}];
tangentlines = Plot[{f'[x0] (x - x0) + f[x0] /. pts[[1, 1]],
    f'[x0] (x - x0) + f[x0] /. pts[[2, 1]], f'[x0] (x - x0) + f[x0] /. pts[[3, 1]]},
   {x, -5, 5}, PlotRange → {{-5, 5}, {-100, 200}}, PlotStyle → Gray];
Show[secantline, tangentlines]
```

Out[635]= 24

Out[639]=



In[640]:=
```
pts
```

Out[640]= {{x0 → -2.12545}, {x0 → 0.206412}, {x0 → 3.41904}}