Sample Final Exam – MAT 176

(12/26/2019)

This exam should be taken without text, notes, or electronic devices. Show your work, and indicate answers clearly. Cross out all work that you do not want to be graded.

1. (5pts.) Evaluate the following indefinite integral

   $$\int te^t \, \mathrm{d}t =$$

2. (5pts.) Evaluate the following definite integral, simplifying the answer as much as possible

   $$\int_0^{\pi/2} \frac{\cos x}{1+\sin^2 x} \mathrm{d}x =$$

3. (5pts.) Use that $\cos^2\theta = \dfrac{1+\cos(2\theta)}{2}$ in order to evaluate the following definite integral

   $$\int_0^{\pi/12} \frac{\cos^2(3t)}{4} \, \mathrm{d}t =$$

4. (5pts.) Use that $\displaystyle\int \frac{\mathrm{d}u}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + C$ in order to evaluate the following indefinite integral

   $$\int \frac{5}{\sqrt{9-4x^2}} \, \mathrm{d}x =$$

5. (5pts.) Sketch the region R in the xy-plane bounded between the graphs $y = \sqrt{x}$ and $y = x^2$. Set up an integral that computes the area of the region R. Do not evaluate this integral.

6. (5pts.) Set up an integral that computes the volume of the solid obtained by revolving the region R in the previous problem around the x-axis. Do not evaluate this integral.

7. (10pts.) Evaluate the improper integrals (using a limit to compute their value or to show they diverge)

   (a) $\displaystyle\int_1^{\infty} \frac{1}{\sqrt{x}} \, \mathrm{d}x$

   (b) $\displaystyle\int_0^{\infty} e^{-y} \, \mathrm{d}y$

8. (15pts.) Compute the limit of the sequence, or show that it diverges

   (a) $\displaystyle\lim_{k\to\infty} \frac{e^k}{k^2}$

   (b) $\displaystyle\lim_{n\to\infty} \frac{\cos n}{n}$

   (c) $\displaystyle\lim_{n\to\infty} \sum_{k=0}^{n} \frac{3}{2^k}$

9. (10pts.) Suppose 300mg of painkiller is given to a patient precisely at 11:00am every day, and only 20% of the drug remains in the body after 1 day, since the other 80% are excreted.

   a) What amounts of painkiller are in the patient's body (measured at 10:59am, immediately before that day's dose is given) after 1 day and after 2 days of treatment?

   b) Use a geometric series to compute the amount of painkiller in the body after a very long time of treatment (measured at 10:59am, immediately before that day's dose is given).

10. (10pts.) Use a convergence test to determine if each of the following series converges or diverges

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}+100}{n^2+1}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{n^2-1}{n^2+1}$

11. (5pts.) Find the interval of convergence of the following power series (remember to check the endpoints!)

$$\sum_{n=2}^{\infty} \frac{5(x-2)^n}{n-1}$$

12. (5pts.) Find the degree 4 Maclaurin polynomial $P_4(x) = \displaystyle\sum_{k=0}^{4} \frac{f^{(k)}(0)}{k!} x^k$ for the function $f(x) = 3 + \cos x$.

13. (5pts.) Use the Maclaurin series $e^x = \displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$ to find the Maclaurin series for the function $g(x) = 2e^{3x}$.

14. (5pts.) Integrate the Maclaurin series $\displaystyle\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n$ to find the Maclaurin series for the function $L(x) = \ln(1+x)$.

15. (5pts.) The rate of change in the number of squirrels $S(t)$ that live on Lehman College campus is directly proportional to $60 - S(t)$, where t is the time in years. When $t = 0$, the population was 20, and when $t = 2$, the population increased to 50. Find the population when $t = 3$.