2 – Motion in one dimension

In Lecture 1 we have seen that each vector (say, position or radius-vector r) in a three-dimensional space can be represented by its three components (\(r_x\), \(r_y\), and \(r_z\)) that can be considered independently of each other. In this Lecture we will concentrate on the behavior of one of these components, say \(r_x\). Other components either do not exist (as is the case for the motion in one dimension) or are ignored.

Let us repeat the definitions for the x-components of the velocity and acceleration

\[
\begin{align*}
\Delta x & = x_2 - x_1, \\
\Delta t & = t_2 - t_1 \\
\Delta x & = \frac{\Delta x}{\Delta t}, \\
a_x & = \frac{\Delta v_x}{\Delta t}
\end{align*}
\]

Here 1 is the initial state and 2 is a final state. Note that \(v_x\) and \(a_x\) can be both positive and negative.
Graphical representation of motion

Coordinate $x$ depending on time $t$, that is, $x(t)$ can be represented graphically. Velocity $v_x$ can be interpreted geometrically as the slope of the curve $x(t)$.

The instantaneous velocity defines a straight line that is tangential to the curve $x(t)$ at a given point (shown on the right of the graph).
Motion with a constant velocity $v_x$ is geometrically described as a straight line (see the graph on the left). Its analytical representation is

$$x = x_0 + v_x t, \quad v_x = \text{const}$$

Task: Prove that $v_x$ in the above formula is indeed velocity.

Constant velocity plotted as a function of time is obviously a horizontal line (see the graph on the right). One can see that the change of the coordinate $x$ during the elapsed time $\Delta t$ is given by the area under the velocity curve: $\Delta x = v_x \Delta t$. Task: Prove that the latter relation yields the framed formula above. Note: If $v_x < 0$, the straight line representing $v_x$ on the plot goes below the $t$-axis. In this and similar cases the area under the curve is defined as negative.
In the general case when the velocity $v_x$ changes with time $t$, one can split the time interval $t_2-t_1$ into many small subintervals $\Delta t_i$ and define the displacement $x_2-x_1$ as the area under the curve representing $v_x(t)$:

Practical application of the above requires calculus!
Acceleration $a_x$ can be geometrically defined as the slope of the curve $v_x(t)$, similarly to the definition of the velocity $v_x$ from the graph $x(t)$.

In turn, the change of the velocity $v_x$ during a time interval can be represented as the area under the curve $a_x(t)$. 

![Graphical representation of acceleration](image)
This important kind of motion is represented by the formula

\[ v_x = v_{x0} + a_x t, \quad a_x = \text{const} \]

where \( v_x \) is a shortcut for the function \( v_x(t) \) and the constant \( v_{x0} \) is the velocity at zero time, \( v_{x0} = v_x(0) \). The time dependence of the x-coordinate \( x(t) \) can be found as the area under the „curve“ \( v_x(t) \). This leads to

\[ x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2 \]

\( \frac{1}{2} \) in the formula above appears because the area of the triangle is half of the area of the corresponding rectangle.