6 – Work and Energy

**Work** - The concept following from the analysis of simple machines

- Simple machines allow to gain in force: achieve a greater output force with a smaller input force
- Gaining in force one loses in distance
- Concept of work:
  \[
  \text{Work} = \text{force} \times \text{distance}
  \]
is conserved, (work input) = (work output)
**Definition of Work**

- **Force** \(F\)
- **Displacement** \(d\)

**Work:**

\[ W = F_x d = F d \cos \theta = F \cdot d \]

Force components perpendicular to displacement do not produce work!

Work is negative, if \(\cos \theta < 0\)

**Unit of work:** J(oule)

\[ J = \text{N(ewton)} \times \text{m} = \text{kg m}^2/\text{s}^2 \]

**Example:**

- **Pull**
- \(F_x\), \(F_y\), \(F_{fr}\), \(F_N\)

**Problem:**

- \(m = 50 \text{ kg}\), \(F = 100 \text{ N}\), \(F_{fr} = 50 \text{ N}\)
- \(\theta = 37^\circ\), \(d = 40 \text{ m}\)

Work done by each force - ?

**Solution**

- \(W_G = mgd \cos 90^\circ = 0\)
- \(W_N = F_N d \cos 90^\circ = 0\)
- \(W_{\text{pull}} = F d \cos \theta = 100 \text{ N} \times 40 \text{ m} \times \cos 37^\circ = 3200 \text{ J}\)
- \(W_{\text{fr}} = F_{\text{fr}} d \cos 180^\circ = 50 \text{ N} \times 40 \text{ m} \times (-1) = -2000 \text{ J}\)
**Power** - Rate of doing work

\[ P = \frac{W}{t} \quad \text{or} \quad P = \frac{W}{t} = F \frac{d}{t} = Fv \]

*Car*

For \( P = \text{const} \)

- Greater \( v \)  \( \rightarrow \) Smaller \( F \)  \( \rightarrow \) smaller acceleration \( a = \frac{F}{m} \)

Unit of power: \( W(\text{att}) \)

\[ W = \text{J(oule)}/s = \text{kg m}^2/\text{s}^3 \]
(Mechanical) Energy - Work stored in a body or ability of a body to do work

Mechanical energy = Kinetic energy + Potential energy

\[ E = E_{\text{kin}} + E_{\text{pot}} \]

\[ E_{\text{kin}} = \frac{mv^2}{2}, \quad E_{\text{pot}} \text{ - different forms} \]

Work done \quad \text{ Increase of energy}

Illustration for the linear motion with constant acceleration \((v_0=0)\)

\[ W = F \times d = ma \times \frac{1}{2}at^2 = \frac{m(at)^2}{2} = \frac{mv^2}{2} \quad \text{- Kinetic energy} \]

In general:

\[ W_{21} = E_2 - E_1 \]

Work of external forces done on the way from position 1 to position 2
Equals the change of energy of the system
**Potential Energy**

- Work needed to bring a system into another state quasistatically ($v \to 0$)

**Gravitational energy**

\[ E_{\text{potential}} = W = Fh = mgh \]

\[ \begin{align*}
F &= mg \\
\text{Ground level} \\
d &= h \\
a &= 0
\end{align*} \]

**Elastic energy**

\[ F = kx \]

\[ k \text{ – stiffness of spring} \\
x \text{ – elongation/kompression} \]

\[ E_{\text{potential}} = W = \frac{1}{2} kx \times x = \frac{1}{2} kx^2 \]

\[ \begin{align*}
\text{force} \\
kx \\
\text{distance} \\
x
\end{align*} \]
Conservation of Energy

In the absence of dissipation (friction) the total energy of an isolated system is conserved:

\[ E \equiv E_{\text{tot}} = E_{\text{pot}} + E_{\text{kin}} = \text{const} \]

Energies of the two different kinds can be transformed into each other:
- potential energy can be released into kinetic energy
- kinetic energy can be absorbed into potential energy

Oscillations!

\[ E_{\text{potential}} = mgh \]
\[ E_{\text{kinetic}} = \frac{mv^2}{2} \]

**Problem**

At 1: \( m=0.5 \) kg, \( h=12 \) cm, \( v=0 \)
Speed at 2?

**Solution**

\[ E_1 = E_{\text{kin},1} + E_{\text{pot},1} \]
\[ = E_2 = E_{\text{kin},2} + E_{\text{pot},2} \]
\[ E_{\text{kin},1} = E_{\text{pot},2} = 0, \]
thus
\[ E_{\text{pot},1} = mgh = E_{\text{kin},2} = \frac{mv_2^2}{2} \]
\[ v_2 = \sqrt{2gh} = \sqrt{2 \times 9.8 \text{ m/s}^2 \times 0.12 \text{ m}} = 1.53 \text{ m/s} \]
**Problem**

A dart of a mass 0.100 kg is pressed against the spring of a toy dart gun. The spring with spring stiffness $k=250 \text{ N/m}$ is compressed 6.0 cm and released. If the dart detaches from the spring when the spring is reaching its natural length ($x=0$) what speed does the dart acquire?

**Known:** $m = 0.1 \text{ kg}$, $k = 250 \text{ N/m}$, $x_1 = 6 \text{ cm} = 0.06 \text{ m}$

**To find:** $v_2 \rightarrow ?$

**Solution:**

The total energy of the system spring + dart is conserved

State 1: Deformed spring, potential energy
State 2: Flying dart, kinetic energy

$$E_1 = E_2 \quad \Rightarrow \quad \frac{1}{2}kx_1^2 = \frac{mv_2^2}{2} \quad \Rightarrow \quad v_2 = \sqrt{\frac{kx_1^2}{m}} = x_1 \sqrt{\frac{k}{m}}$$

More accurately:

$$\sqrt{x^2} = \sqrt{(x^2)^{1/2}} = x^{2 \times 1/2} = x^1 = x$$

Plugging numbers:

General analytical result

$$v_2 = 0.06 \text{ m} \sqrt{\frac{250 \text{ N/m}}{0.1 \text{ kg}}} = 0.06 \sqrt{2500} = 0.06 \times 50 = 3 \text{ m/s}$$

Check units separately: $m \sqrt{\frac{\text{N/m}}{\text{kg}}} = m \sqrt{\frac{\text{kg m/s}^2 / \text{m}}{\text{kg}}} = m \sqrt{1/\text{s}^2} = \text{m/s}, \text{ OK}$
Homework:

Giancoli Chapter 6, Problems 1, 5, 9, 15, 27, 39, 59