10 – Temperature & Kinetic theory

**Definition of temperature**

Temperature \( T \) is the measure of physical objects to be hot, warm and cold. Experience shows that when most objects become warmer, they increase their length. This property of thermal expansion is used to build thermometers. The height of a column of mercury or colored alkohol in a tube measures the temperature.

To exactly define the temperature scale, one needs two anchor points. Both Farenheit (F) and Celcius (C) scales use freezing and boiling points of water.

- Freezing point of water: 32°F or 0°C
- Boiling point of water: 212°F or 100°C
- Differences: 180°F or 100°C – F-Degree is finer

Relations between Farenheit and Celcius temperatures:

\[
T_C = \frac{5}{9}(T_F - 32)
\]

\[
T_F = \frac{9}{5}T_C + 32
\]
**Molecular nature of temperature, Kelvin scale**

Physically, temperature measures kinetic energy of molecules and atoms in bodies. As there is a huge quantity of the latter and they are all moving randomly in different directions that change because of interactions and collisions, heated bodies do not move as the whole. This form of kinetic energy is not mechanical, from the macroscopic point of view, and it is a part of the so-called internal energy.

For an ideal gas (very weak interaction between molecules), pressure at a constant volume of volume at a constant pressure are proportional to temperature

\[ PV = RT \]

**Absolute zero of temperature, no molecular motion**

\[ -273^\circ C \]

\[ 0 = T - 273.2 \]

Kelvin temperature scale: uses the same intervals as the Celsius scale but is shifted:

\[ T = T_c + 273.2, \quad T_c = T - 273.2 \]

Unit: K(elvin), like \( T = 273 \) K

Physicists use the Kelvin scale in which for a ideal gas is

\[ PV = RT, \quad R = \text{const} \]
Thermal expansion

Change of temperature $T$ causes change of the length $L$ of physical bodies

$$\Delta L = \alpha L_0 \Delta T \quad \text{or} \quad L = L_0 (1 + \alpha \Delta T) \quad \text{if} \quad \Delta L = L - L_0$$

Here $\alpha$ is the thermal expansion coefficient.
For most of solids $\alpha$ is positive and $\alpha \sim 10^{-5} \text{ K}^{-1}$ – very small but important in the engineering.

Example: Bridge expansion. The steel bed of a suspension bridge is 200 m long at 20°C. If the extremes of temperature to which it might be exposed are -30°C to 40°C, how much will it contract and expand? For the steel $\alpha = 12 \times 10^{-6} \text{ K}^{-1}$

Solution: The difference in temperatures (the working temperature interval) is $\Delta T = 70 \text{°C} = 70 \text{ K}$. One obtains

$$\Delta L = \alpha L_0 \Delta T = 12 \times 10^{-6} \times 200 \times 70 = 0.168 \text{ m} \approx 17 \text{ cm} \quad - \text{a substantial change of length!}$$

Volume thermal expansion is considered similarly to the linear thermal expansion:

$$\Delta V = \beta V_0 \Delta T \quad \text{or} \quad V = V_0 (1 + \beta \Delta T) \quad \text{if} \quad \Delta V = V - V_0$$

If linear expansion in all three directions is the same, one obtains from $L = L_0 (1 + \alpha \Delta T)$

$$V = V_0 (1 + \alpha \Delta T)^3 \equiv V_0 (1 + 3\alpha \Delta T) = V_0 (1 + \beta \Delta T) \quad \text{where} \quad \beta = 3\alpha$$
Equation of state of the ideal gas

\[ PV = Nk_B T \]

- \( N \) – number of molecules in the gas
- \( k_B \) – Boltzmann constant, \( k_B = 1.38 \times 10^{-23} \text{ J/K} \)

Example: How many molecules are on average in 1 liter air at the atmospheric pressure and room temperature?

Formulation: Room temperature, \( T = 300 \text{ K} \); \( V = 1 \text{ liter} = 1000 \text{ cm}^3 = 10^{-3} \text{ m}^3 \), \( P = 10^5 \text{ N/m}^2 \). \( N = ? \)

Solution:

\[
N = \frac{PV}{k_B T} = \frac{10^5 \times 10^{-3}}{1.38 \times 10^{-23} \times 300} = 2.4 \times 10^{22}
\]

- A huge number!
Density of the ideal gas

Density of an ideal gas \( (M – \text{mass of the whole gas}, \ m – \text{mass of one molecule}) \)

\[
\rho = \frac{M}{V} = \frac{Nm}{V} = \frac{Pm}{k_BT} = \rho - \text{does not depend on the amount of gas}
\]

Example: The density of air is 1.29 kg/m\(^3\) at atmospheric pressure and \(T=0\,^\circ\text{C}=273.2\,\text{K}\). What is the average mass of the molecule of air?

Solution:

\[
m = \frac{\rho k_B T}{P} = \frac{1.29 \times 1.38 \times 10^{-23} \times 273.2}{10^5} = 4.86 \times 10^{-26}\,\text{kg}
\]

Check: Air consists mostly of Nitrogen that forms molecules consisting of two atoms. Each Nitrogen atom has a nucleus that consists of 14 protons and neutrons, combined. The mass of proton and neutron is \(1.67 \times 10^{-27}\,\text{kg}\). Thus the mass of a Nitrogen molecule is

\[
m = 2 \times 14 \times 1.67 \times 10^{-27} = 4.68 \times 10^{-26}\,\text{kg}
\]

The mass of an average air molecule is slightly higher because the air also contains Oxygen whose molecule contains \(2 \times 16 = 32\) protons and neutrons, combined.
Three main types of processes: Isothermic, Isochoric, and Isobaric

1. Isothermic process
   \[ T = \text{const} \]

2. Isochoric process
   \[ V = \text{const} \]

3. Isobaric process
   \[ P = \text{const} \]

Volume thermal expansion coefficient at constant pressure:

\[ \Delta V = \frac{Nk_B}{P} \Delta T = \beta V \Delta T \quad \Rightarrow \quad \beta = \frac{Nk_B}{PV} = \frac{Nk_B}{Nk_B T} = \frac{1}{T} \]
Kinetic interpretation of temperature

The microscopic origin of the pressure of the gas on the walls of the container surrounding it is the mechanical impact of molecules that hit the walls and rebound. Calculating the momentum transferred from molecules to the wall allows to calculate the pressure on the walls. The result is

$$PV = \frac{2}{3} N \langle \varepsilon_K \rangle$$

where $$\langle \varepsilon_K \rangle = \frac{\langle mv^2 \rangle}{2}$$ is the average translational kinetic energy of the molecule.

Comparing this result with $$PV = N k_B T$$ yields the relation

$$\langle \varepsilon_K \rangle = \frac{3}{2} k_B T$$

Between the temperature of the gas and kinetic energy of its molecules.

Example: Speed of air molecules. We calculate the root-mean-square speed $$v_{\text{rms}} = \sqrt{\langle v^2 \rangle} = \sqrt{\frac{3k_B T}{m}}.$$ For Nitrogen the molecular mass is $$m = 4.68 \times 10^{-26} \text{ kg}$$ thus

$$v_{\text{rms}} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 300}{4.68 \times 10^{-26}}} = 515 \text{ m/s}$$

For Oxygen the molecular mass is somewhat larger and thus $$v_{\text{rms}}$$ is somewhat smaller.