Congratulation and best wishes!

INTERACTION LOCALIZATION PARADIGM:

WEAKLY INTERACTING BOSE GAS IN DISORDERED ENVIRONMENT

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OUTLINE

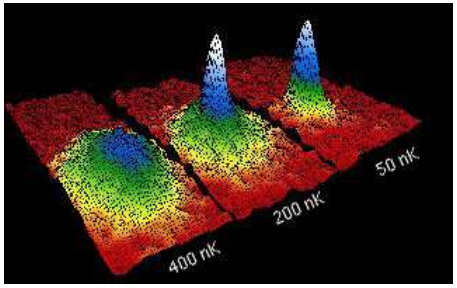
- Introduction
  - Single-atom levels in a random potential
  - Weakly repulsive gas in an uncorrelated random potential
  - Strongly correlated random potential
  - Conclusions

Purpos: Semiquantitative analysis of the phase states of the gas in a box through geometrical and physical description of deeply localized states

Introduction

Bose-Einstein-condensation: finite part of atoms in the state with minimal energy.

Examples: Superfluid ^{4}He, laser cooled atoms in a trap



Disorder: Superfluid He in a porous media: J. Reppy and coworkers, J. Low Temp. Phys. **87**, 205 (1992) and references therein: Breakdown of super-fluidity at strong disorder.

^{3}He C. L. Vicente, H. C. Choi, J. S. Xia, W. P. Halperin, N. Mulders, and Y. Lee, Phys. Rev. B **72**, 094519 (2005).

Bose-Einstein Condensate in a Random Potential

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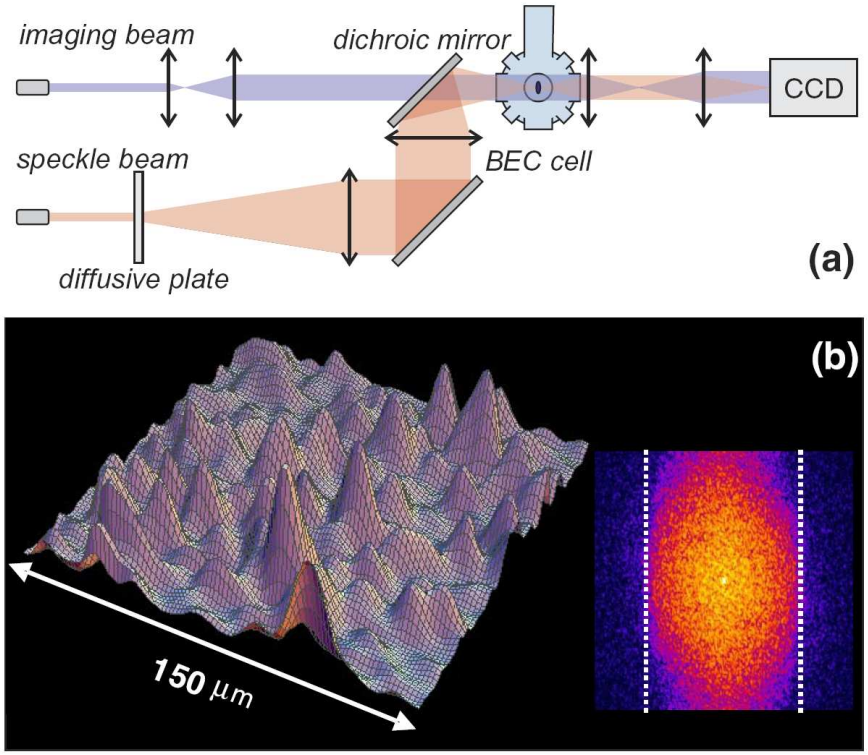


FIG. 1 (color online). (a) Optical setup for both the speckle potential and the imaging beam for the BEC. The axial direction of the magnetic trap is in the vertical direction of the figure. (b) 3D representation of the speckle potential (left) and its Fourier transform (right). The dotted lines correspond to a length scale of about 10 μm in the axial direction.

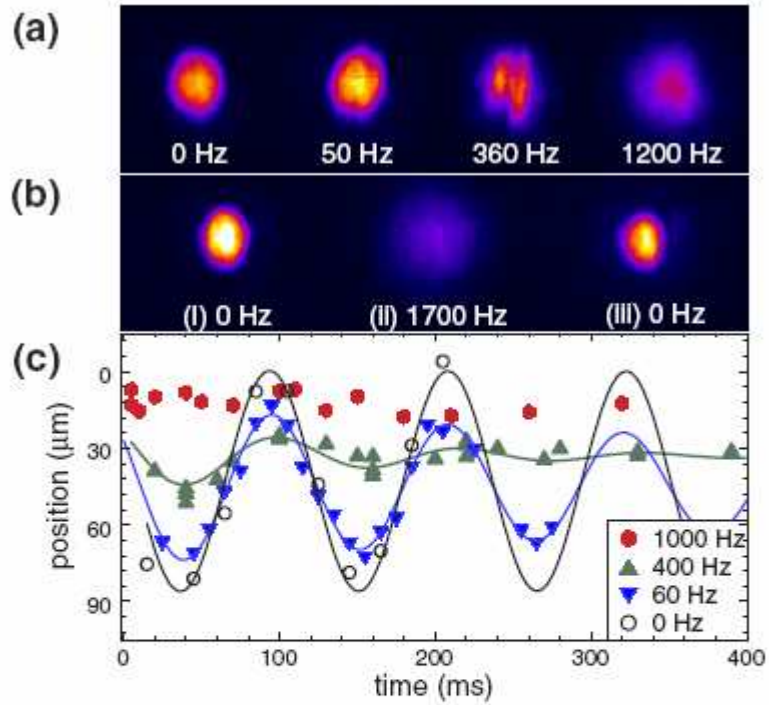


FIG. 2 (color online). (a) Density profile of the BEC after 28 ms of expansion from the combined magnetic and speckle potential for varying speckle potential intensities V_s (indicated in bottom part). To demonstrate that the transfer to the speckle is adiabatic (b) shows the density profiles with (i) no speckle potential, (ii) when the speckle potential is abruptly switched off, and (iii) when it is adiabatically ramped off. (c) Dipole oscillations in the combined magnetic and speckle potential for varying V_s.

Theoretical works:

Corrections to the Bogolyubov theory caused by a weak disorder

K. Huang and H.F. Meng, Phys. Rev. Lett. **69,** 644 (1992);
S. Giorgini, L.P. Pitaevsky and S. Stringari, Phys. Rev. B **49**, 12938 (1994)
A. V. Lopatin and V.M. Vinokur, Phys. Rev. Lett. **88**, 235503 (2002)

Possible Bose-glass state

M.P.A. Fisher et al., Phys. Rev. B **50**, 546 (1989)

R.T. Scalettar et al., Phys. Rev. Lett. **66,** 3144 (1991)
W. Krauth et al., Phys. Rev. Lett. **67**, 2307 (1991)

Spin model, transition from the normal state to superfluid

M. Ma, B.I. Halperin and P.A. Lee, Phys. Rev. B **34**, 3136 (1986)

Bose Hubbard model, scaling near transition point

M.P.A. Fisher et al., Phys. Rev. B **50**, 546 (1989)

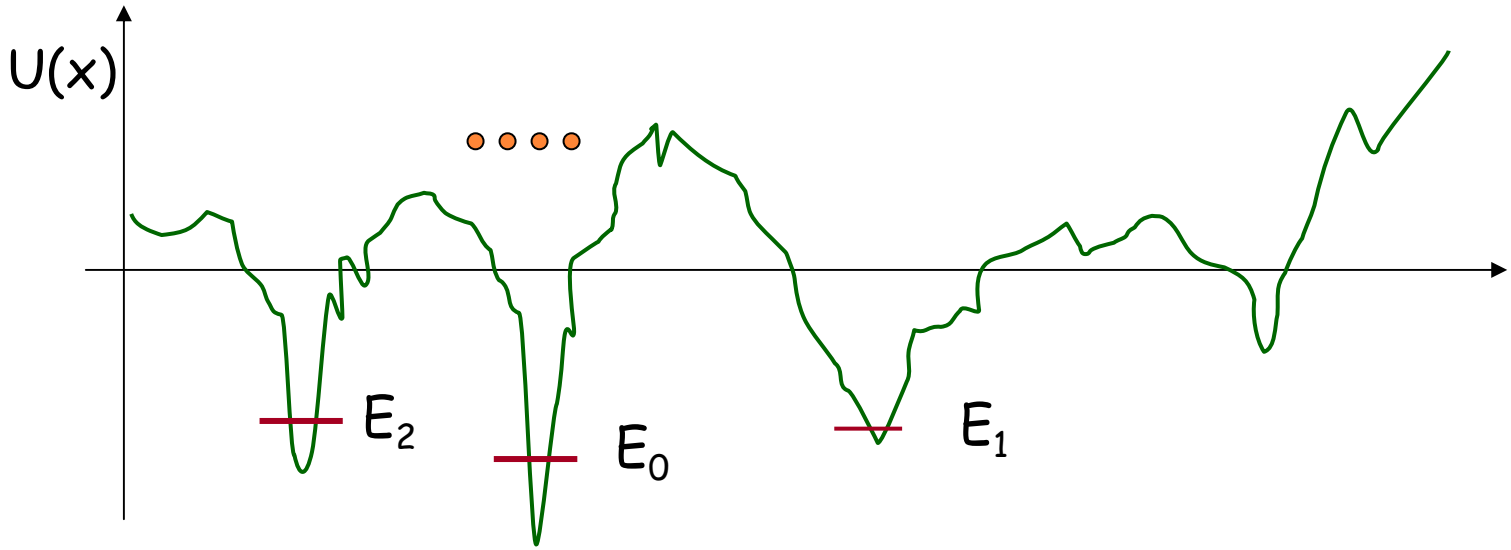
One-dimensional Hubbard model

T. Giamarchi and H. Schulz, Europhys. Lett. **3**, 1287 (1987)

Single atom levels in a random potential

Ideal Bose gas

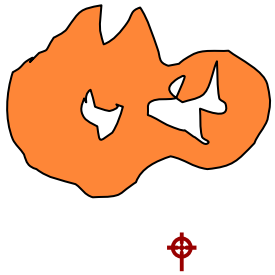
Ground state



Non-ergodic ground state or quenched distribution dependent on the cooling speed

Gaussian white noise

$$\langle U(\mathbf{x})U(\mathbf{x}')\rangle = \kappa^2\delta(\mathbf{x}-\mathbf{x}')$$



Independent probability distribution in each point:

$$dW\left[U(\mathbf{x}),\Omega\right] = \exp\left[-\frac{1}{2\kappa^2}\int_\Omega U^2(\mathbf{x})\,d\Omega\right]\prod_{\mathbf{x}\in\Omega}\frac{\sqrt{\Delta\Omega}\,dU(\mathbf{x})}{\sqrt{2\pi}\,\kappa}$$

Energy levels in the random potential:

$$-\frac{\hbar^2}{2m}\nabla^2\psi + U(\mathbf{x})\psi = E[U]\psi$$

Density of states: $\nu(E,\Omega) = \int \delta\left(E - E[U]\right) dW\left[U(\mathbf{x}),\Omega\right]$

$$dW\left[U(\mathbf{x}),\Omega\right] = \exp\left[-\frac{1}{2\kappa^2}\int_\Omega U^2(\mathbf{x})\,d\Omega\right]\prod_{\mathbf{x}\in\Omega}\frac{\sqrt{\Delta\Omega}\,dU(\mathbf{x})}{\sqrt{2\pi}\kappa}$$

Deep levels: $E < 0; |E| \quad E_L = \frac{m^2\kappa^4}{\hbar^2}$

I.M. Lifshitz, Zittartz and Langer, Halperin and Lax, 1966

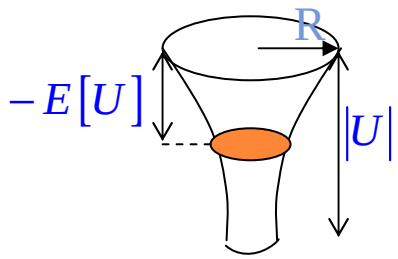
Minimize $\int_\Omega U^2(\mathbf{x})\,d\Omega$ at fixed $E[U]$

The optimal fluctuation potential well is spherically symmetric.
The optimal level is the only bound state in this well.

$U(\mathbf{x}) = \lambda|\psi(\mathbf{x})|^2$ Ginzburg-Landau equation

Fluctuation potential well has the same linear size as the wave function

Semiquantitative approach



Typical potential well with a deep level

All calculations for 3d space

Balance of kinetic and potential energy: $|E| \quad |U| \quad \frac{\hbar^2}{2mR^2}$

Probability to find energy less than $-E$ or part of space occupied by the potential wells with the size of the quantum state less than R:

$$P(R) = \exp\left(-\frac{U^2\Omega}{2\kappa^2}\right) = \exp\left(-\frac{\pi\hbar^4}{6m^2\kappa^2 R}\right)$$

Explanation: $\int_\Omega U^2(\mathbf{x})\,d\Omega \approx U^2\Omega \qquad \Omega \approx \frac{4}{3}\pi R^3$

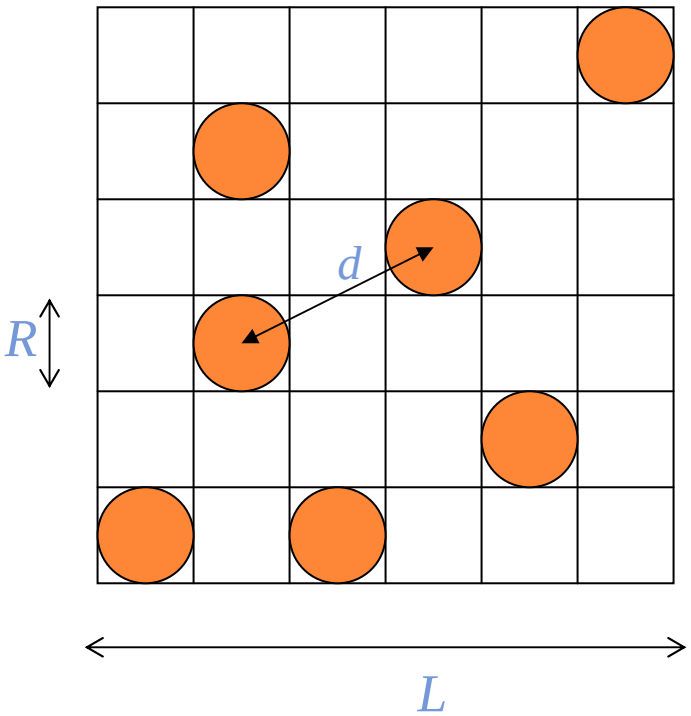
Larkin length: $\mathsf{L} = \frac{\pi\hbar^4}{6m^2\kappa^2}$

Probability: $P(R) = \exp\left(-\frac{\mathsf{L}}{R}\right)$

Preexponent: inessential

Valid at $R \quad \mathsf{L} \qquad P(\mathsf{L}) \quad 1 \qquad E(\mathsf{L}) \equiv \mathsf{E} = \frac{\hbar^2}{2m\mathsf{L}^2}$

Levels of Larkin size percolate



Density of cells with the quantum states of the size R

$$n_w(R) = R^{-3}\exp\left(-\frac{\mathsf{L}}{R}\right)$$

Average distance between the states of the size R:

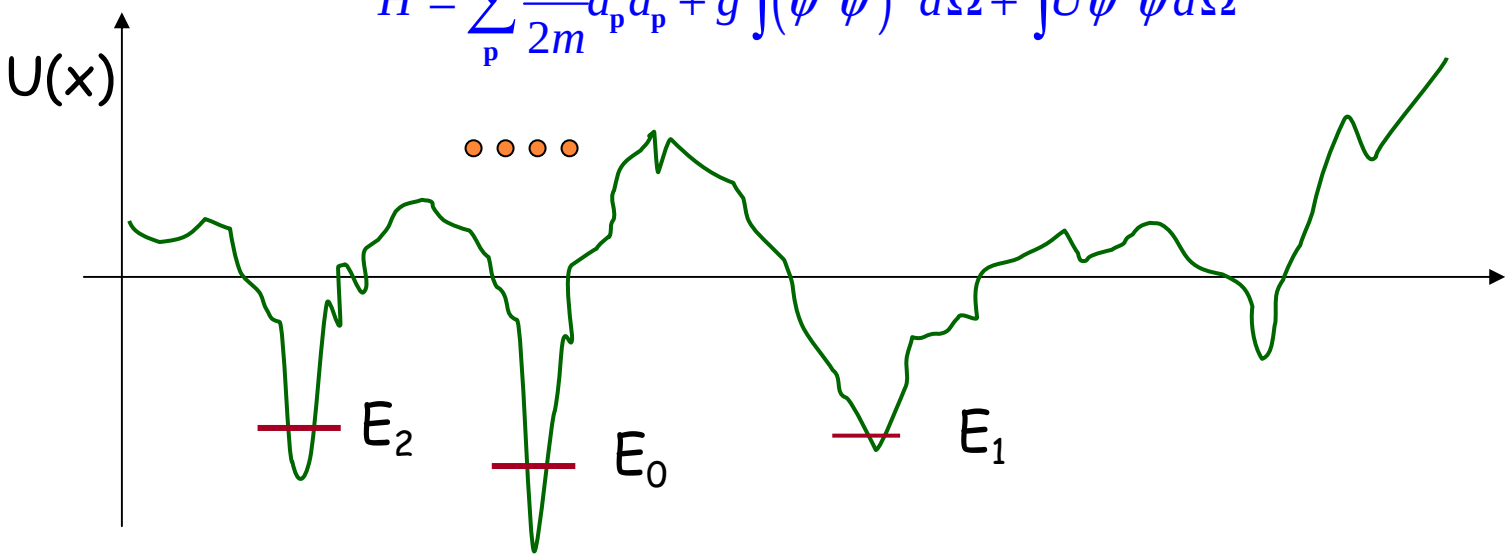
$$d(R) = \left[n_w(R)\right]^{-1/3} = R\exp\left(\frac{\mathsf{L}}{3R}\right)$$

Tunneling amplitude between neighboring states of the size R

$$t(R) = \exp\left(-\frac{\sqrt{2m|U|}\,d(R)}{\hbar}\right) = \exp\left[-\exp\left(\frac{\mathsf{L}}{3R}\right)\right]$$

Weakly repulsive gas in a random potential

$$H = \sum_{\mathbf{p}} \frac{\mathbf{p}^2}{2m} a_{\mathbf{p}}^{\dagger} a_{\mathbf{p}} + g \int \left(\psi^{\dagger}\psi\right)^2 d\Omega + \int U \psi^{\dagger}\psi d\Omega$$



Due to repulsion atoms are distributed between different wells

Repulsion restores ergodicity

Repulsion energy per particle is $gn_p(R)$

$n_p(R)$ is the density of particles in the well of the size R

Let the potential wells with the radii up to R are filled. Then:

$$n_p(R) = \frac{n}{n_w(R)} \frac{3}{4\pi R^3} = \frac{3n}{4\pi} \exp\left(\frac{\mathsf{L}}{R}\right)$$

Number of particles per well

Energy per particle: $$\mu = gn_p(R) - \frac{\hbar^2}{2mR^2}$$

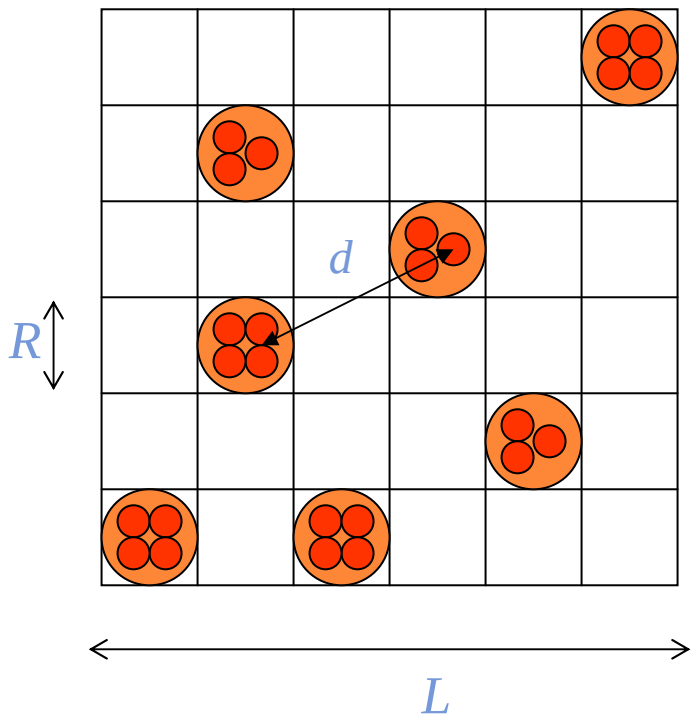
Energy of the random well

Mininization over R: $$R(n) = \frac{\mathsf{L}}{\ln \frac{n_c}{n}} \qquad n_c = \left(3\mathsf{L}^2 a\right)^{-1} \qquad g = \frac{4\pi\hbar^2 a}{m}$$

Typical radius reached *Critical density* *Scattering length*

Chemical potential: $$\mu = -\frac{\hbar^2}{2m\mathsf{L}^2}\left(\ln\frac{n_c}{n}\right)^2 = -\mathsf{E}\left(\ln\frac{n_c}{n}\right)^2$$

Confirmed by exact calculation. A. and V. Babichenko, Dec. 2008



Maximal size of the well occupied by atoms:

$$R(n) = \mathrm{L} / \ln \frac{n_c}{n} \qquad n_c = \left(3\mathrm{L}^2 a\right)^{-1}$$

Distance between occupied wells:

$$r(n) = \mathrm{L}\left(\frac{n_c}{n}\right)^{1/3} \ln \frac{n_c}{n} \qquad R(n)$$

Tunneling amplitude between wells:

$$t(n) = \exp\left[-\left(\frac{n_c}{n}\right)^{1/3}\right]$$

Number of particles per well: $\mathrm{N} = \dfrac{\mathrm{L}}{3a \ln\left(n_c / n\right)}$

Density of particles in the wells: n_c

Interaction between deeply localized states

At n much less than n_c different wells are exponentially weakly coupled and have well defined number of particles. The phase in each well is uncertain. Random singlet. No coherence, no superfluidity, Bose glass?

At n approaching n_c distances between the wells become of the same order of magnitude as their radius, the overlapping factor t becomes of the order of 1. Strong growth of the correlation between phases.

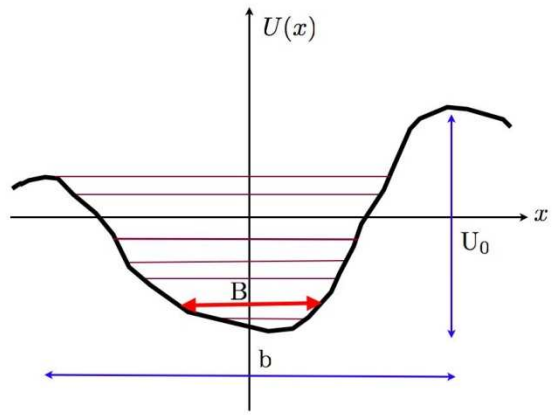
At $n=n_c$ quantum phase transition from disordered singlet state to disordered superfluid. Scaling near transition was studied by Matthew Fisher *et al*. Marginal dimensionality is 1 (Giamarchi and Schulz). Is it the same transition?

n n_c-- almost homogeneous superfluid, small corrections to Bogolyubov formulae from disorder (Huang and Meng)

Model description: random quantum XY-model:

$$H_{XY} = \sum_j K_j \left(N_j - \bar{N}_j\right)^2 - \sum_{ij} t_{ij} \cos\left(\varphi_i - \varphi_j\right)$$

Strongly correlated potential



Correlation length b

Average square potential U_0^2

Larkin length $L = \dfrac{\hbar^4}{m^2 U_0^2 b^3}$ b

Many levels: $N_l = \left(b\sqrt{2mU_0}/\hbar\right)^3 / 6\pi^2$

Deep optimal fluctuation has the shape of correlation function and the depth E

Single-particle level probability $p(E) = \exp\left(-\dfrac{E^2}{2U_0^2}\right)$

Bose gas with average density n fills the potential wells of the depth $E = -U_0\sqrt{\ln(n_c/n)}$ occupying the levels near the bottom up to the radius $R = b/\sqrt{\ln(n_c/n)}$

$n_c = \dfrac{mU_0}{\hbar^2 a}$ Critical density does not depend on b

Conclusions

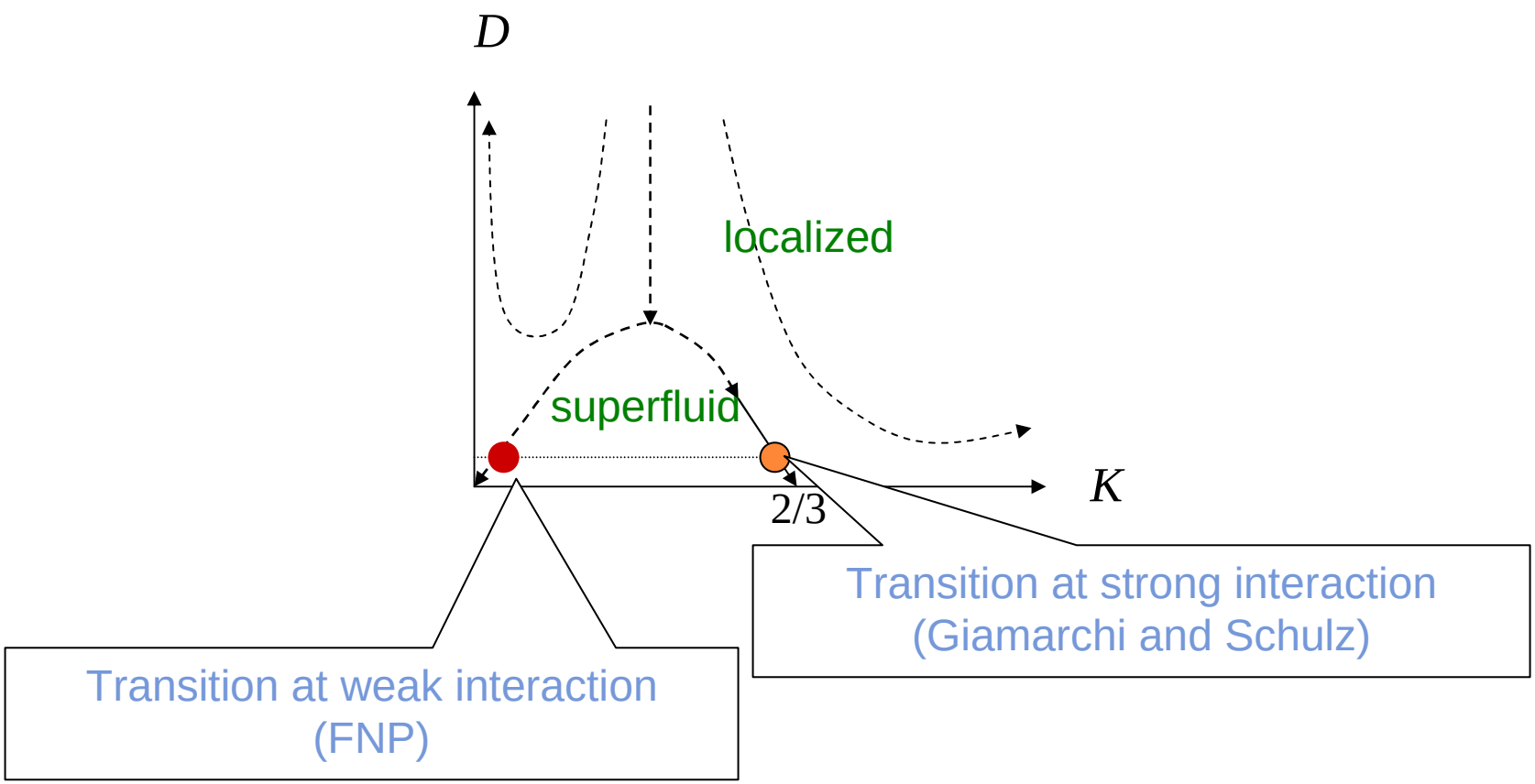
- Physical and geometrical properties of weakly interacting Bose-gas in random Gaussian potential depends on three characteristic lengths: the Larkin length and correlation length of disorder and the scattering amplitude
- The ground state of ideal Bose gas in random potential in thermodynamic limit is non-ergodic
- Even an infinitosimal interaction restores ergodicity in thermodynamic limit
- At average particle density much less than a critical one, the Bose particles occupy deep potential wells up to some size and exponentially weakly tunnel to other wells. The number of particles in each well is defined, but phases are uncertain. It is a disordered singlet state.
- At average particle density equal to its critical value determined by interaction and disorder, the transition to the inhomogeneous superfluid proceeds.
- Critical density is inverse proportional to the square of Larkin length and the scattering amplitude
- In the deeply localized state the density of particles in each fragment is approximately equal to critical.
- In a strongly correlated potential deep fluctuation wells have the linear size equal to the correlation length and contain many levels. At density much less than critical Bose-particles reside near the bottom of such wells. The transition to superfluid state is reached not far from the percolation threshold of classical trajectories.

What did not enter to this presentation

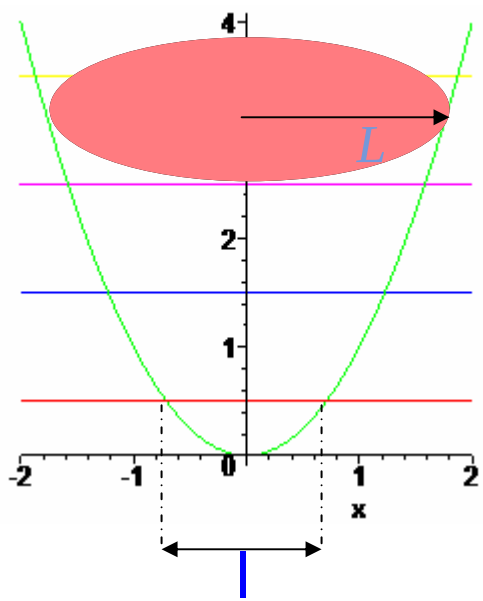
- Two- and one- dimensional traps
- Bose-gases with attraction in a disordered trap
- Fermi-gases and admixtures
- Dipolar oscillations of the clouds in traps
- Thermodynamics ?
- Coulomb forces (M. Müller and B.I. Shklovskii, arXiv:0812.4134)
- Exact treatment (A. Babichenko and V. Babichenko, arXiv: 0812.0966)

Acknowledgements: DOE, grant DE-FG02-06ER 46278 and DFG project NA222/5-2

Giamarchi-Schulz phase diagram



Weakly repulsive gas in a harmonic trap



Single particle states in a trap

$$V(\mathbf{x}) = \frac{m\omega^2 \mathbf{x}^2}{2}$$

Energy levels: $E_n = \left(n + \frac{1}{2}\right)\hbar\omega$

$$l = \sqrt{\frac{\hbar}{m\omega}}$$ Magnetic length

Weakly repulsive N particles in a harmonic trap, no disorder, form a spherical cloud of the size R

$$E(R,N) = N\left(\frac{\hbar^2}{2mR^2} + \frac{m\omega^2 R^2}{2} + gn\right) \qquad n = \frac{3N}{4\pi R^3}; \qquad g = \frac{4\pi\hbar^2 a}{m}$$

Kinetic energy *Trap energy* *Interaction energy*

$$E(L,N) = N\frac{\hbar^2}{2m}\left(\frac{1}{L^2}+\frac{L^2}{\ell^4}+\frac{3Na}{L^3}\right)$$

Must be minimized over R

Weak interaction $\quad 3Na \qquad \ell \Rightarrow L = \ell$

Strong interaction $\quad 3Na \qquad \ell \Rightarrow L = \left(\frac{9}{2}Na\ell^4\right)^{1/5}$

Thomas-Fermi approximation

Disorder potential $\langle U(\mathbf{x})U(\mathbf{x}')\rangle = \kappa^2\delta(\mathbf{x}-\mathbf{x}')$

L l -- weak disorder; small corrections to the previous results

Strong disorder, weak interaction

a Na L l -- *non-ergodic situation*; in equilibrium all particles occupy a single random potential well with minimal energy

Size of the cloud is determined by the energy balance:

$$\mathsf{E}_{\mathsf{L}} = \frac{\hbar^2}{2m\mathsf{L}^2} \qquad \frac{m\omega^2 L^2}{2} \quad \Rightarrow \quad L = \frac{\mathsf{l}^2}{\mathsf{L}}$$

Energy of disorder (logarithmic accuracy)

Energy of trap

Typical size of the well: $R = \dfrac{\mathsf{L}}{6\ln(\mathsf{l}/\mathsf{L})}$

Strong disorder, moderately strong interaction: $a \quad L \quad Na \quad l$

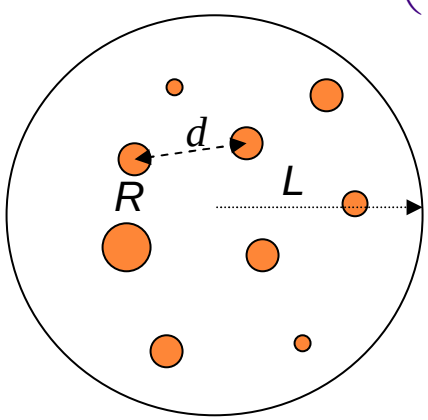
Ergodic situation

$$N \quad \left(l^6 / L^5 a\right) \qquad \text{or} \qquad \Gamma = l^6 / \left(3L^5 a N\right) \quad 1$$

Multiple random potential wells (fragmentation). Average particle density *n* is a slow function of coordinate *r*

$$\mu = g n_p(r) - \frac{\hbar^2}{2mR^2(r)} + \frac{m\omega^2 r^2}{2} \approx -E_L \ln^2 \frac{n_c}{n(r)} + \frac{m\omega^2 r^2}{2}$$

$$n(r) = n_c \exp\left(-\sqrt{\frac{L^2 r^2}{l^4} - \frac{\mu}{E_L}}\right) \approx n_c \exp\left(-\sqrt{-\mu / E_L}\right) \exp\left(-\frac{L^2 r^2}{2 l^4 \sqrt{-\mu / E_L}}\right)$$



$$R = L / \ln \Gamma \qquad d = L \Gamma^{1/3} / \ln \Gamma \qquad L = \left(l^2 / L\right) \sqrt{\ln \Gamma}$$

No coherence, random singlet or Bose glass

Counter-intuitive dependence of *L* on *N*

$$N \quad \left(l^6 / L^5 a\right) \qquad \text{or} \qquad \Gamma = l^6 / \left(3L^5 a N\right) \quad 1$$

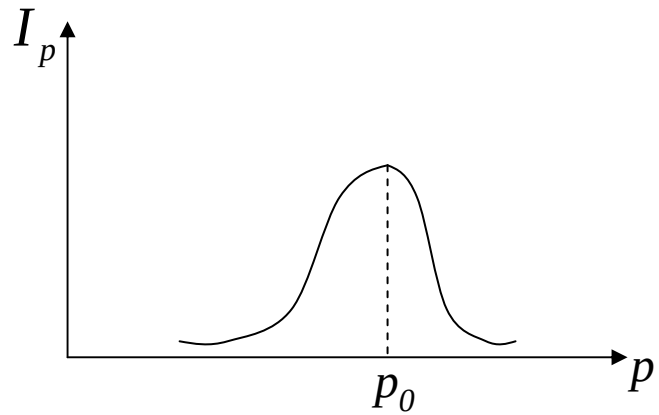
Small corrections of the order of ☝🗁✍🗎 to the state without disorder

Superfluid

Phase transition line: $\Gamma = 1$

Distribution of momentum in fragmented state

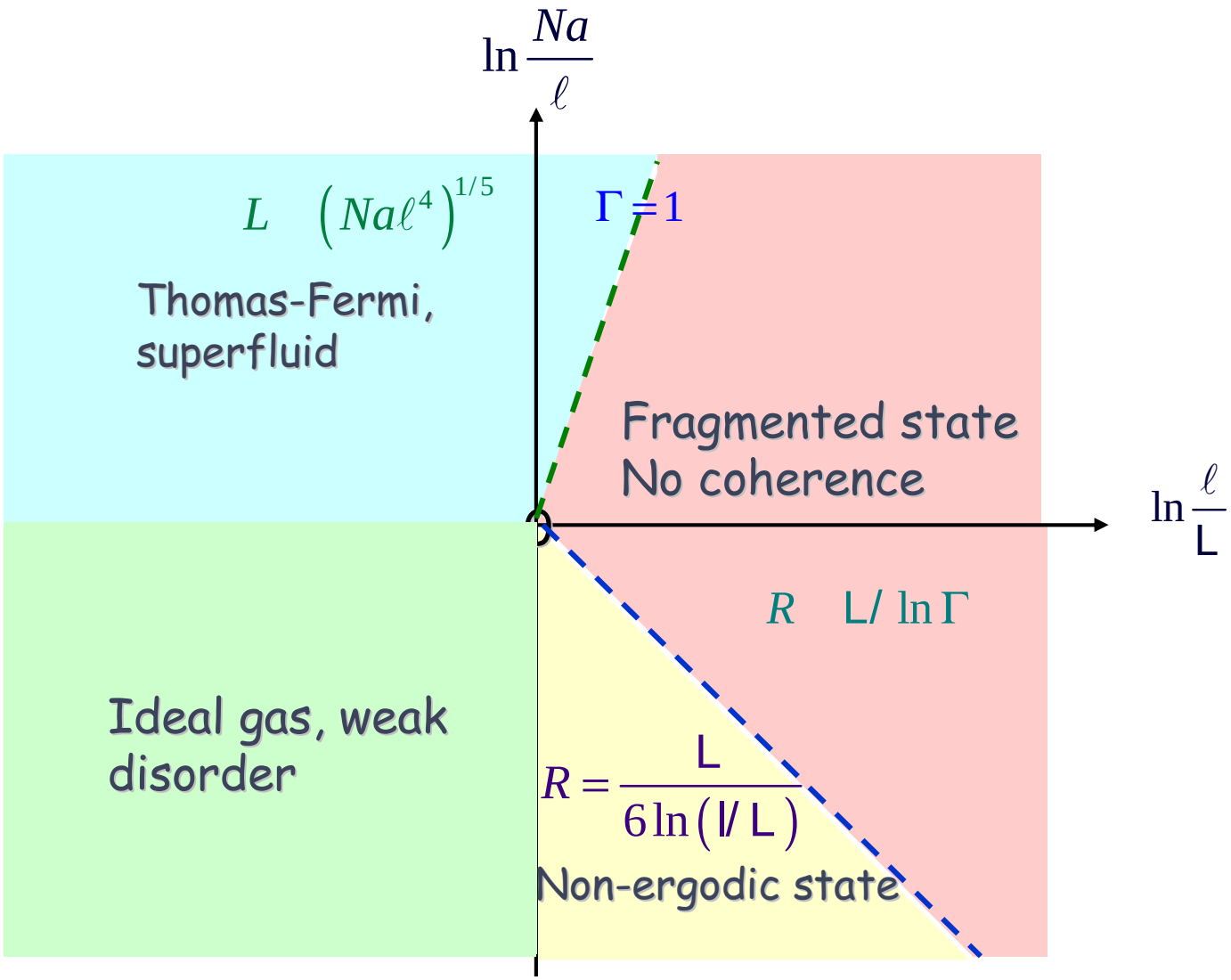
$$p_0 = \left(\hbar / L\right) \ln \Gamma$$



Number of fragments

$$N_F \quad \frac{12 N a}{L} \left(\ln \Gamma\right)^{9/2}$$

Phase diagram



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