

## CUNY Common Core Course Submission Form

Instructions: All courses submitted for the Common Core must be liberal arts courses. Courses may be submitted for only one area of the Common Core. All courses must be 3 credits/3 contact hours unless the college is seeking a waiver for another type of Math or Science course that meets major requirements. Colleges may submit courses to the Course Review Committee at any time. Courses must also receive local campus governance approval for inclusion in the Common Core.

<b>College</b>	Lehman College
<b>Course Prefix and Number (e.g., ANTH 101, if number not assigned, enter XXX)</b>	MAT 124
<b>Course Title</b>	Algebraic Thinking and Functions for Educators
<b>Department(s)</b>	Mathematics
<b>Discipline</b>	Mathematics
<b>Credits</b>	3
<b>Contact Hours</b>	3
<b>Pre-requisites (if none, enter N/A)</b>	Departmental permission
<b>Co-requisites (if none, enter N/A)</b>	n/a
<b>Catalogue Description</b>	Using generalization, algebraic structures, and reasoning to represent and analyze mathematical situations. In-depth attention given to functions, modeling, and the transition from arithmetic to algebra. Note. Intended for pre-service elementary and middle school teachers.
<b>Special Features (e.g., linked courses)</b>	
<b>Sample Syllabus</b>	Syllabus must be included with submission, 5 pages max recommended

**Indicate the status of this course being nominated:**

current course   
 revision of current course   
 a new course being proposed

### CUNY COMMON CORE Location

**Please check below the area of the Common Core for which the course is being submitted. (Select only one.)**

**Required**

- English Composition  
 Mathematical and Quantitative Reasoning  
 Life and Physical Sciences

**Flexible**

- World Cultures and Global Issues     Individual and Society  
 US Experience in its Diversity     Scientific World  
 Creative Expression

### Waivers for Math and Science Courses with more than 3 credits and 3 contact hours

Waivers for courses with more than 3 credits and 3 contact hours will only be accepted in the required areas of "Mathematical and Quantitative Reasoning" and "Life and Physical Sciences." Three credit/3-contact hour courses must also be available in these areas.

**If you would like to request a waiver please check here:**

Waiver requested

**If waiver requested:**

Please provide a brief explanation for why the course will not be 3 credits and 3 contact hours.

**If waiver requested:**

Please indicate whether this course will satisfy a major requirement, and if so, which major requirement(s) the course will fulfill.

## Learning Outcomes

In the left column explain the course assignments and activities that will address the learning outcomes in the right column.

### B. Mathematical and Quantitative Reasoning: Three credits

A course in this area must meet all the learning outcomes in the right column. A student will:

**SLO 5** is assessed through a combination of assignments, quizzes, and examinations. Students are expected to read, interpret, understand, and appropriately utilize multiple mathematical representations—including algebraic expressions, equations, graphs, diagrams, tables, and formulas—to solve quantitative problems. Students are also expected to use correct function notation as part of their problem-solving approach.

#### Examples of Assessment Tasks:

##### a) In class discussions and written assignments

The following representative tasks require students to engage with and make connections among multiple representations.

- **Functions (Topic 4: Constant & Linear Functions):**  
Fill in the missing coordinates so that the points lie on the graph of the function  $y = -2x + 1$ , and explain your reasoning:  
 $(3, \underline{\quad})$ ,  $(\underline{\quad}, -13)$ ,  $(a, \underline{\quad})$ ,  $(\underline{\quad}, b)$ .
- **Variables and Expressions (Topic 2: Structure of Expressions):**  
Draw, label, and shade a rectangle that represents the equivalent expressions  $(x + 3)(y + 4)$  and  $xy + 4x + 3y + 12$ . Explain how the visual model supports the algebraic equivalence.
- **Real Number System (Topic 1: Properties of Real Numbers):**  
Examine the decimal representations of the fractions  $\frac{1}{4}$ ,  $\frac{1}{3}$ ,  $\frac{5}{8}$ , and  $\frac{2}{7}$ . Identify which decimals terminate and which repeat and infer the relationship between a fraction's denominator and the type of decimal representation.

##### b) Sample final exam alignment

In the attached sample final examination, SLO 1 is assessed through the following items:

- **Question 1 (Topic 1: Real Number Systems & Properties):**  
Students differentiate between rational and irrational numbers and make inferences about irrationality. Specifically, students infer that if a number  $P$  satisfies  $\left(\frac{1}{3}\right)^2 < P < \left(\frac{1}{2}\right)^2$  (equivalently,  $\frac{1}{9} < P < \frac{1}{4}$ ), and  $P$  is not a perfect square, then  $\sqrt{P}$  is irrational. Students may also infer that the average of two rational numbers is always a rational number.
- **Question 2 (Topic 1: Real Number Systems & Properties):**  
Students correctly order a set of rational numbers and justify their reasoning using decimal representations, equivalent forms, or radicals, drawing clear and valid inferences from these representations.

**SLO 1:** Interpret and draw appropriate inferences from quantitative representations, such as formulas, graphs, or tables.

<ul style="list-style-type: none"> <li>• Question 8 (<b>Topic 4: Functions – Domain and Range</b>): Students interpret a graph to correctly identify the domain and range of a function.</li> <li>• Question 10a (<b>Topic 4: Functions – Linear, Quadratic, and Polynomial Functions</b>): Students read, examine, and interpret a graph in order to identify key features and perform a function transformation using appropriate algebraic notation.</li> </ul> <p><b>SLO 1</b> is also partially assessed through Question 7 (<b>Topic 5: Modeling with Functions – Real World Applications</b>) which requires students to interpret and make connections among quantitative information presented in multiple representations, including:</p> <ul style="list-style-type: none"> <li>• A table of paired data (e.g., temperature and coffee sales),</li> <li>• A linear regression equation generated by the student,</li> <li>• A numerical correlation coefficient.</li> </ul>	
<p><b>SLO 2</b> is assessed through small group projects, in class discussions, and on written assignments. Students will be expected to create algebraic/function models to express written and/or verbal problems in an appropriate mathematical format</p> <p><b>Examples of Assessment Tasks:</b></p> <p><b>a) In class discussions and written assignment</b></p> <p>Several examples of problems covering this objective are included below.</p> <ul style="list-style-type: none"> <li>• At a yogurt shop, frozen yogurt is 45 cents for each ounce; a waffle cone to hold the yogurt is \$1. Create a table to describe the cost of buying a frozen yogurt cone for different ounces. Then, create an equation for the situation and graph it. Be sure to clearly define any and all variables you use. (<b>Topic 3: Equality Relations &amp; Equations – Solving Equations &amp; Inequalities</b>)</li> <li>• Consider the sequence given by 1, 4, 7, 10, 13, 16, ... Find an expression for the Nth entry in this sequence and explain why your expression is valid. (<b>Topic 4: Functions, Their Representations and Features – Sequences &amp; Series</b>)</li> <li>• A company collects data on advertising spending (<math>x</math>) and sales revenue (<math>y</math>). Represent the relationship with a linear regression equation. (<b>Topic 5: Modeling Functions &amp; Predicting Change-Regression Equations</b>)</li> </ul> <p><b>b) Sample final exam alignment</b></p> <ul style="list-style-type: none"> <li>• Question 4a (<b>Topic 2: Variables &amp; Expressions- Algebraic Notations, Symbols, Variables &amp; order of Operations</b>) requires students to examine a pattern represented visually by a set of tiles and algebraically represent how many tiles there are for the <math>n</math>th tile</li> </ul>	<p>SLO 2: Represent quantitative problems expressed in natural language in a suitable mathematical format.</p>

<p>to be able to solve for the number of tiles needed for any given diagram number.</p> <ul style="list-style-type: none"> <li>• Question 6c (<b>Topic 3: Equality Relation and Equations – Solving Equations and Inequalities</b>) prompts students to solve an absolute value inequality algebraically and represent the solution set using an appropriate set notation.</li> <li>• Question 7 (<b>Topic 4: Functions – Transformations of Functions; Topic 2: Variables – Algebraic Notations and Symbols</b>) requires students to represent quantitative information symbolically and numerically by constructing and using a linear regression model derived from a real-world data table. Question 10b requires students to represent horizontal translation symbolically using appropriate function notations and explain reasoning.</li> </ul>	
<p>SLO 4 is assessed through in-class discussions and presentations, as well as written assignments, quizzes, and examinations. Students are expected to explain, describe, and clearly communicate either orally or in writing, fundamental concepts of algebra and functions to audiences of varied mathematical maturity, including K–8 learners, Grades 9–12 students, and college-level peers.</p> <p>In addition, students are expected to identify, interpret, and explain common mathematical errors made by elementary and middle school students when working with algebraic and functional concepts.</p> <p><b>Examples of Assessment Tasks</b></p> <p>a) In class discussions and written assignment</p> <p>The representative tasks below are designed to elicit clear mathematical explanations, multiple representations, and pedagogically sound reasoning.</p> <ul style="list-style-type: none"> <li>• <b>Mathematical Properties (Topic 1: Real Number Systems &amp; Properties)</b> State the commutative property of addition. Explain what it means for two expressions to be equal, and provide at least two different ways to justify why the expressions are equal.</li> <li>• <b>Equation Solving (Multiple Representations) – (Topic 3: Equality Relation and Equations)</b> Solve the equation <math>3x + 2 = x + 8</math>. in two ways: (1) using properties of equality, and (2) using a visual model such as a pan-balance representation.</li> <li>• <b>Error Analysis (Topic 3: Equality Relations and Equations)</b>  Analyze the following chain of reasoning and explain the mathematical error(s) that lead to the incorrect conclusion that the equation <math>x - 1 = 0</math> has no solutions:   <math display="block">x - 1 = 0 \rightarrow \frac{x - 1}{x - 1} = 0 \rightarrow 1 = 0</math> </li> <li>• <b>Function Behavior (Topic 4: Functions, Their Representations and Features-Exponential Function)</b> Explain how doubling time affects the graph of an exponential</li> </ul>	<p>SLO 4. Effectively communicate quantitative analysis or solutions to mathematical problems in written or oral form.</p>

<p>function, using appropriate mathematical language and representations.</p> <p><b>b) Sample Final Exam Alignment</b></p> <p>In the attached sample final examination, SLO 4 is assessed through the following items:</p> <ul style="list-style-type: none"> <li>• Question 7 (<b>Topic 5: Modeling with Functions – Real-World Applications</b>): Students translate a computed statistic into a verbal conclusion using appropriate mathematical vocabulary (e.g., “strong negative linear relationship”) and justify their claim using evidence from the calculation. In Question 7, students must clearly communicate the meaning of a statistical measure (the correlation coefficient) by explaining whether it indicates a strong linear relationship, using appropriate quantitative language and justification.</li> <li>• Questions 1a and 1b (<b>Topic 1: Real Number Systems &amp; Properties</b>): Students explain how they determine two rational numbers and two irrational numbers between two given rational numbers, articulating their reasoning clearly and precisely.</li> <li>• Additional Written Justifications (Questions 2 and 4b; <b>Topic 4: Functions – Sequences and Series; ; Topic 2: Variables &amp; Expressions- Algebraic Notations, Symbols, Variables &amp; order of Operations</b>): Students are required to justify their solutions in writing, demonstrating clear mathematical communication and logical reasoning.</li> </ul>	
<p>SLO 3 is assessed through whole-class and small-group discussions, as well as written assignments, quizzes, and examinations. Students are expected to understand and utilize the relationship between algebraic representations and function graphs to solve problems both symbolically and geometrically.</p> <p><b>Examples of Assessment Tasks</b></p> <p><b>a) In-class discussions and written assignments</b></p> <p>Representative tasks require students to move fluently between algebraic expressions, equations, and graphical representations.</p> <ul style="list-style-type: none"> <li>• Linear Functions (<b>Topic 4: Functions – Constant &amp; Linear</b>): A student earns \$5 per hour babysitting. Write an algebraic expression that represents the student’s total earnings after <math>h</math> hours, and explain how the expression relates to the graph of the function.</li> <li>• Quadratic Functions (<b>Topic 4: Functions – Quadratic</b>): An object is dropped from the top of a building. After <math>t</math> seconds, the height <math>h</math> of the object (in feet) is given by</li> </ul> $h = 16(13 + t)(13 - t).$ <p>Sketch the graph of the function, reason about the structure of the graph in relation to the physical context, and determine when the object hits the ground.</p>	<p>SLO 3: Use algebraic, numerical, graphical, or statistical methods to draw accurate conclusions and solve mathematical problems.</p>

<ul style="list-style-type: none"> <li>• Solving Equations (<b>Topic 3: Equality Relations &amp; Equations – Solving Equations and Inequalities</b>): Represent and solve the following problem algebraically, clearly defining the variable used: At a frog exhibit, <math>\frac{3}{5}</math> of the frogs are bullfrogs. Of the remaining frogs, <math>\frac{2}{3}</math> are tree frogs, and the rest are river frogs. There are 36 bullfrogs in the exhibit. How many river frogs are there?</li> </ul> <p><b>b) Sample Final Exam Alignment</b></p> <p>In the attached sample final examination, SLO 3 is assessed through the following items:</p> <ul style="list-style-type: none"> <li>• Questions 6a and 6b (<b>Topic 4: Functions – Quadratic and Polynomial Functions</b>): Students algebraically solve a quadratic equation and a polynomial equation for all real values of <math>x</math>. This requires students to factor polynomials completely and to recognize appropriate solution strategies, including square-rooting, factoring, and use of the quadratic formula.</li> <li>• Question 9b (<b>Topic 4: Functions – Exponential Functions</b>): Students use an exponential function to determine an output value for a given input. Students must understand the structure of the exponential function and correctly identify and interpret the dependent and independent variables within the function rule.</li> </ul>	
<p><b>SLO 5</b> is assessed through in-class discussions and graded written work. Students are expected to explain and justify why and how function models appropriately represent a given situation, recognize when a proposed model is not appropriate, and determine or propose a more suitable alternative model when feasible.</p> <p><b>Examples of Assessment Tasks:</b></p> <p><b>a) In class discussions and written assignment</b></p> <p>Representative tasks are intentionally designed to require students to evaluate the appropriateness of function models, articulate limitations of proposed models, and justify alternative modeling choices based on contextual constraints.</p> <ul style="list-style-type: none"> <li>• Modeling with Functions (<b>Topic 4: Functions – Other Types</b>): <b>Problem:</b> The level of a certain toxin in a lake is observed to increase and decrease over time. Biologists are interested in studying the relationship among the toxin level in the lake, the number of freshwater mussels present, and any relationship between these variables.</li> </ul> <p>i) Explain why the proposed rule below may not define a function: <i>Assign to each amount of toxin in the lake the number of mussels present when there is that amount of toxin.</i></p>	<p>SLO 5. Evaluate solutions to problems for reasonableness using a variety of means, including informed estimation.</p>

<p>ii) Describe how the situation could be revised to develop a valid and meaningful function model.</p> <ul style="list-style-type: none"> <li>Features of a Quadratic Function (<b>Topic 4: Functions, Their Representations and Features</b>) A student claims <math>f(x) = (x - 2)^2 + 3</math> that the highest value of <math>f(x)</math> is 3. Do you agree or disagree? Justify.</li> </ul> <p><b>b) Sample final exam alignment</b></p> <p>SLO 5 is also primarily addressed by:</p> <ul style="list-style-type: none"> <li>Question 10(c) (<b>Topic 4: Functions – Transformations of Functions</b>) because students analyze a claim about <math>h(x)</math> which requires them to evaluate a proposed functional model (the transformed function), identify inaccuracies in the claim (misinterpreted reflection and translation) and correctly justify an appropriate alternative understanding of the function's behavior.</li> </ul> <p>SLO 5 is addressed in part by the following questions:</p> <ul style="list-style-type: none"> <li>Question 7 (<b>Topic 5: Modeling with Functions – Real World Applications- Regression Equation</b>) requires students to draw conclusions and make data-driven predictions using a quantitative model derived from real-world information. By calculating the correlation coefficient <math>r</math> and interpreting its value, students must assess the strength of the linear relationship and determine whether the model is appropriate for making predictions.</li> <li>Question 9b (<b>Topic 4: Functions – Exponential Functions</b>) Students explain how the exponential function models bacterial growth by recognizing the assumptions and structure of the model (doubling time, initial value) and apply the model to make predictions and justify their conclusions. Thus the question engages students in the evaluation, justification, and application of a function model in a real-world context.</li> </ul>	
<p><b>SLO 6</b> is assessed through in-class discussions and graded written assignments. Students are expected to apply algebraic representations and function models to solve real-world problems, including problems that involve linear, piecewise, and exponential relationships, as well as various regression models.</p> <p><b>Examples of Assessment Tasks:</b></p> <p><b>a) In-class discussions and written assignments</b></p> <p>Representative tasks are designed to connect algebraic and functional reasoning to authentic, contextual situations across disciplines.</p> <ul style="list-style-type: none"> <li><b>Piecewise/Linear Cost Modeling (Business Context)</b> At a store that sells fences, if you buy 15 feet of fencing or less, the total cost, including delivery is \$200. Each additional foot of fencing costs an additional \$10. Let <math>F</math> be the number of feet of fencing in an order and let <math>C</math> be the cost (in dollars) of the order.</li> </ul>	<p>SLO 6. Apply mathematical methods to problems in other fields of study.</p>

- What restriction should be made on F so that the relationship between C and F is linear? Explain.
  - Without writing an equivalent equation, explain how to interpret each side of the equation below and explain why the equation describes the relationship between F and C:
  - $C-200=10(F-15)$
- **Real-Life Application:** Give an example of two variables that have a positive linear relationship. Give an example of two variables that have a negative linear relationship. Explain your reasoning.

**b) Sample final exam alignment**

The questions below require students to construct, interpret, and apply algebraic and functional models in multiple real-world contexts. By translating contextual information into equations or functions, analyzing relationships, and making predictions or interpretations, students demonstrate mastery of SLO 6's focus on applied function modeling.

- **Question 5 (Manufacturing / Business Mathematics- Topic 3: Equality Relation and Equations – Solving Equations; Topic 4: Functions- Polynomial; Other Functions)** Students model a real-world scenario with algebra, interpret dimensions, and solve a practical problem.
- **Question 7 (Coffee Sales / Financial Mathematics- Topic 5: Modeling with Functions – Real World Applications)** Students analyze data, build a regression model, interpret the correlation coefficient, and make predictions based on the model.
- **Question 9 (Biology - Topic 4: Functions –Exponential Growth):** Students construct and apply an exponential function to model bacterial population growth and predict future values, which are questions frequently discussed in Biology courses.
- **Question 4b (Visual Pattern / Arts- Topic 4: 4. Functions – Sequences and Series):** Students apply algebraic representation to a visual pattern (shaded vs. white tiles), connecting numeric models to a concrete context.