Solutions to Physics I Gravity and Kepler's Laws Practice Problems

1.) Titan, the largest moon of Saturn, has a mean orbital radius of $1.22x10^9$ m. The orbital period of Titan is 15.95 days. Hyperion, another moon of Saturn, orbits at a mean radius of $1.48x10^9$ m. Use Kepler's third law of planetary motion to predict the orbital period of Hyperion in days.

$r_T = 1.22x10^9\ m$

$T_T = 15.95\ days$

$r_H = 1.48x10^9\ m$

$T_H = ?$

$$\left(\frac{T_T}{T_H}\right)^2 = \left(\frac{r_T}{r_H}\right)^3$$

$$\left(\frac{15.95\ days}{T_H}\right)^2 = \left(\frac{1.22x10^9\ m}{1.48x10^9\ m}\right)^3$$

$$\left(\frac{15.95\ days}{T_H}\right)^2 = 0.824^3$$

$$\frac{254.4\ days^2}{{T_H}^2} = 0.560$$

$$T_H = \sqrt{\frac{254.4\ days^2}{0.560}}$$

$T_H = 21.3\ days$

2.) The mass of Earth is $5.97x10^{24}$ kg, the mass of the Moon is $7.35x10^{22}$ kg, and the mean distance of the Moon from the center of Earth is $3.84x10^{5}$ km. Use these data to calculate the magnitude of the gravitational force exerted by Earth on the Moon.

$m_E = 5.97x10^{24}\ kg$

$m_M = 7.35x10^{22}\ kg$

$r = 3.84x10^{5}\ km = 3.84x10^{8}\ m$

$G = 6.673x10^{-11}\ N \cdot m^2/kg^2$

$F_{ME} = ?$

$$F_{ME} = G\frac{m_E m_M}{r^2}$$

$$F_{ME} = (6.673x10^{-11}\ N \cdot m^2/kg^2)\left[\frac{(5.97x10^{24}\ kg)(7.35x10^{22}\ kg)}{(3.84x10^{8}\ m)^2}\right]$$

$$F_{ME} = (6.673x10^{-11}\ N \cdot m^2/kg^2)\left(\frac{4.39x10^{47}\ kg^2}{1.47x10^{17}\ m^2}\right)$$

$$F_{ME} = (6.673x10^{-11}\ N \cdot m^2/kg^2)(2.98x10^{30}\ kg^2/m^2)$$

$$F_{ME} = 1.99x10^{20}\ N$$

3.) The planet Mercury travels around the Sun with a mean orbital radius of $5.8x10^{10}$ m. The mass of the Sun is $1.99x10^{30}$ kg. Use Newton's version of Kepler's third law to determine how long it takes Mercury to orbit the Sun. Give your answer in Earth days.

$r_M = 5.810x10^{10}\ m$

$m_S = 1.99x10^{30}\ kg$

$T_M = ?$

$$T_M{}^2 = \left(\frac{4\pi^2}{Gm_S}\right)r^3$$

$$T_M{}^2 = \left[\frac{39.5}{(6.673x10^{-11}\ N\cdot m^2/kg^2)(1.99x10^{30}\ kg)}\right](5.810x10^{10}\ m)^3$$

$$T_M{}^2 = \left[\frac{39.5}{1.33x10^{20}\ N\cdot m^2/kg}\right](1.96x10^{32}\ m^3)$$

$$T_M{}^2 = (2.96x10^{-19}\ s^2/m^3)(1.96x10^{32}\ m^3)$$

$$T_M{}^2 = 5.82x10^{13}\ s^2$$

$$T_M = \sqrt{5.82x10^{13}\ s^2}$$

$$T_M = 7.63x10^6\ s\left(\frac{1\ hour}{3600\ s}\right)\left(\frac{1\ day}{24\ hours}\right) = 88.3\ days$$

4.) Earth has an orbital period of 365 days and its mean distance from the Sun is 1.495x10^{8} km. The planet Pluto's mean distance from the Sun is 5.896x10^{9} km. Using Kepler's third law, calculate Pluto's orbital period in Earth days.

$T_E = 365\ days$

$r_E = 1.495x10^8\ km$

$r_P = 5.896x10^9\ km$

$T_P = ?$

$$\left(\frac{T_E}{T_P}\right)^2 = \left(\frac{r_E}{r_P}\right)^3$$

$$\left(\frac{365\ days}{T_P}\right)^2 = \left(\frac{1.495x10^8\ km}{5.896x10^9\ km}\right)^3$$

$$\left(\frac{365\ days}{T_P}\right)^2 = (2.54x10^{-2})^3$$

$$\left(\frac{1.32x10^5\ days^2}{{T_P}^2}\right) = 1.63x10^{-5}$$

$$T_P = \sqrt{\frac{1.32x10^5\ days^2}{1.63x10^{-5}}}$$

$$T_P = 9.00x10^4\ days$$

5.) The planet Venus orbits the Sun with a mean orbital radius of 1.076×10^{11} m. The mass of the Sun is 1.99×10^{30} kg. Using Newton's version of Kepler's third law, calculate the orbital period of Venus.

$r_V = 1.076x10^{11}\ m$

$m_S = 1.99x10^{30}\ kg$

$T_V = ?$

$$T_V{}^2 = \left(\frac{4\pi^2}{Gm_S}\right) r_V{}^3$$

$$T_V{}^2 = \left[\frac{4\pi^2}{(6.673x10^{-11}\ N \cdot m^2/kg^2)(1.99x10^{30}\ kg)}\right](1.076x10^{11}\ m)^3$$

$$T_V{}^2 = \left[\frac{39.5}{1.33x10^{20}\ N \cdot m^2/kg}\right](1.25x10^{33}\ m^3)$$

$$T_V{}^2 = (2.97x10^{-19}\ s^2/m^3)(1.25x10^{33}\ m^3)$$

$$T_V{}^2 = 3.17x10^{14}\ s^2$$

$$T_V = \sqrt{3.17x10^{14}\ s^2}$$

$$T_V = 1.93x10^7\ s$$