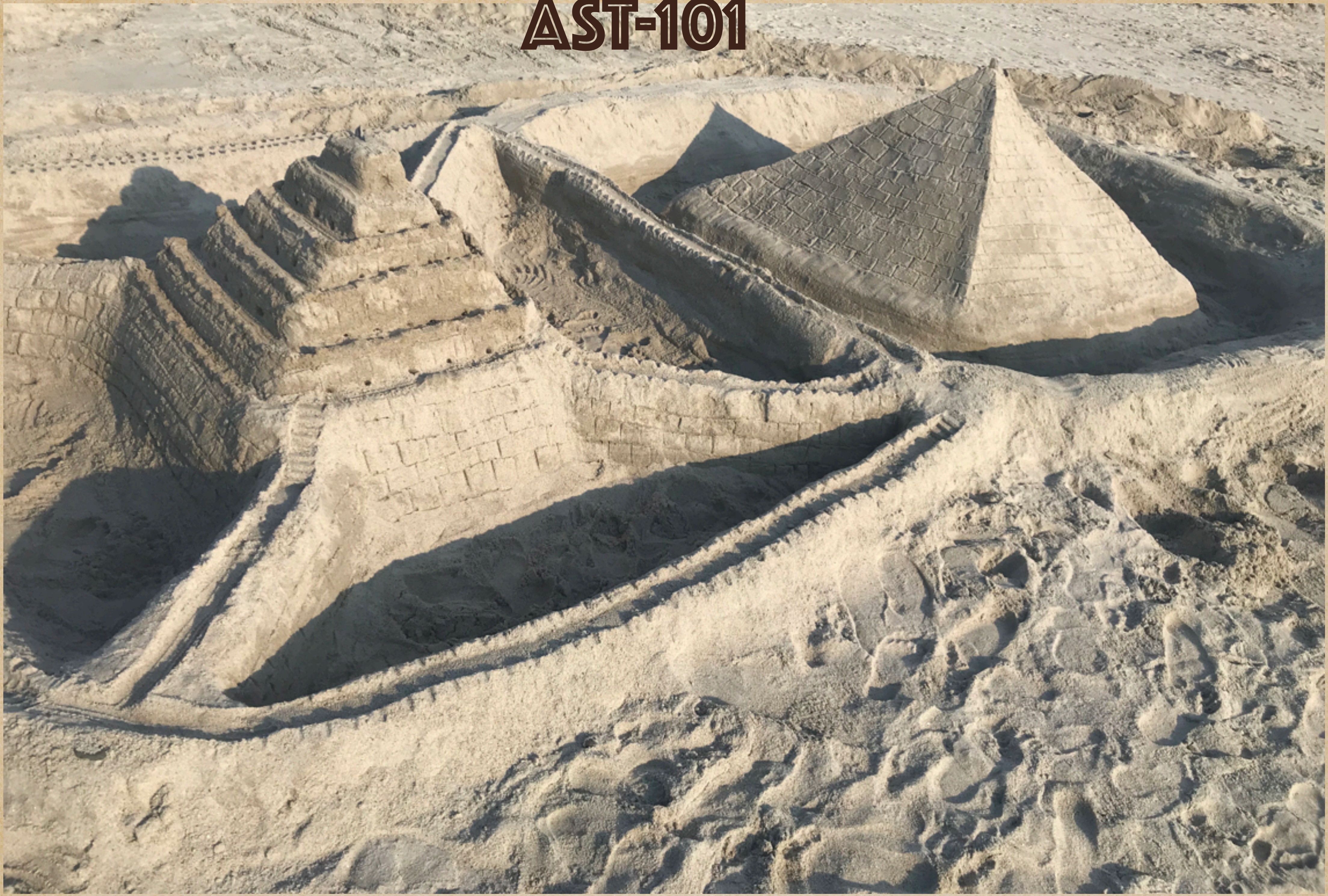
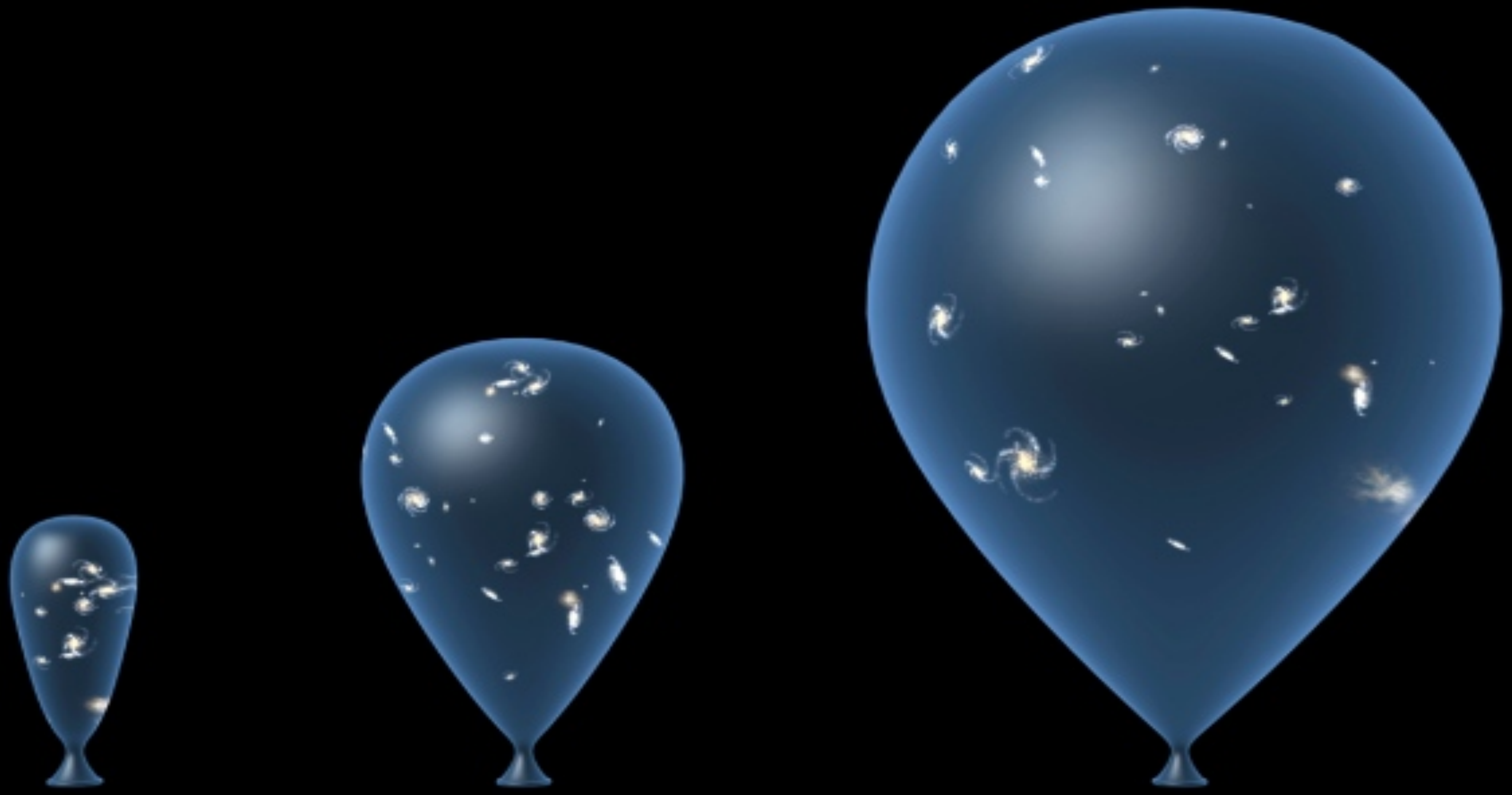


The Big-Bang theory

Luis Anchordoqui

Last class....



Hubble's Law

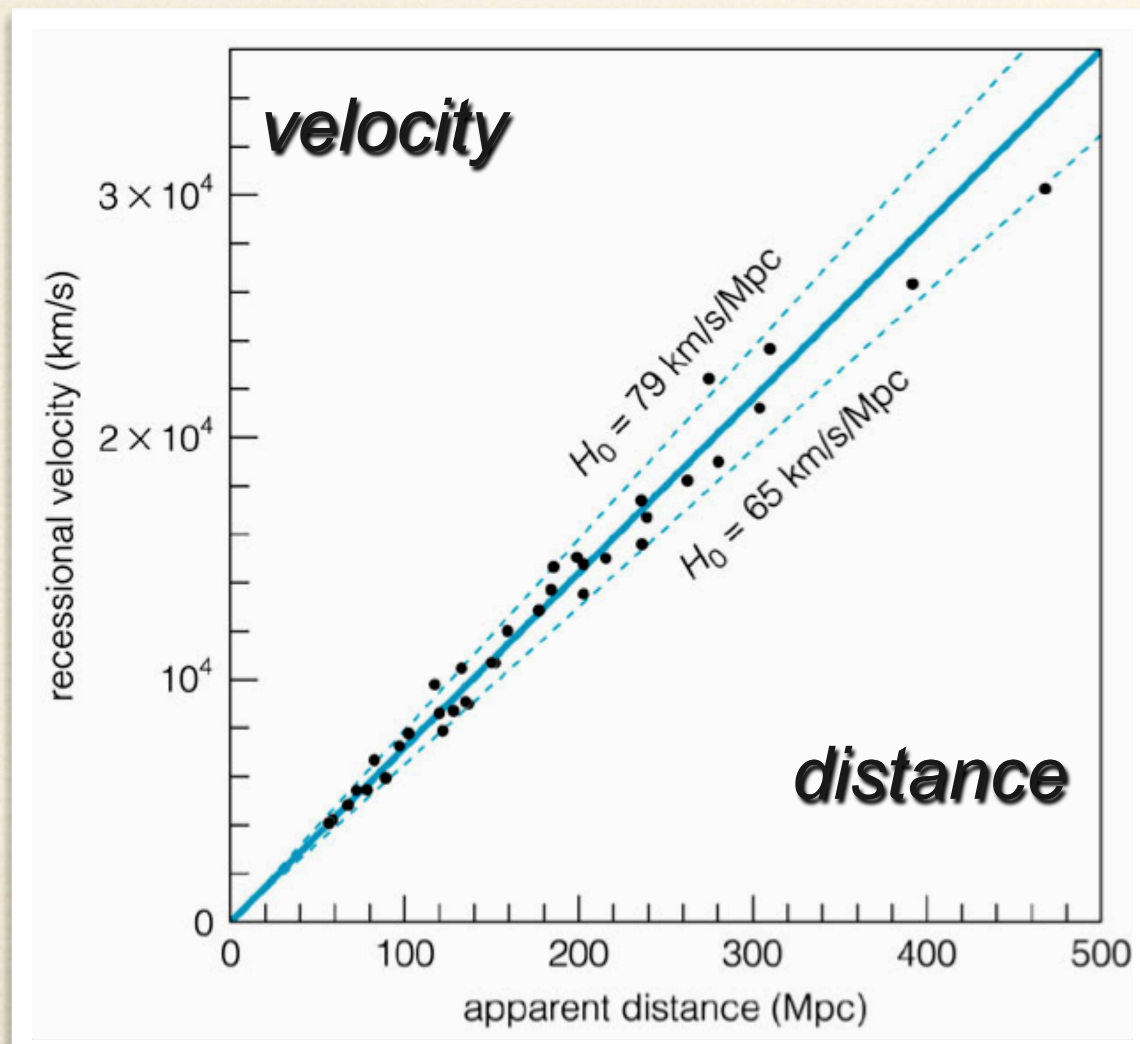
$v = H_0 \times d$

Velocity of Recession
(Doplpler Shift)
(km/sec)

Hubble's
Constant
(km/sec/Mpc)

Distance

(Mpc)



Implies the Expansion
of the Universe

The redshift of a Galaxy is:

A. The rate at which a Galaxy is expanding in size
B. How much reader the galaxy appears when observed at large distances
C. The speed at which a galaxy is orbiting around the Milky Way
D. The relative speed of the redder stars in the galaxy with respect to the blues stars
E. The recessional velocity of a galaxy, expressed as a fraction of the speed of light

The redshift of a Galaxy is:

A. The rate at which a Galaxy is expanding in size
B. How much reader the galaxy appears when observed at large distances
C. The speed at which a galaxy is orbiting around the Milky Way
D. The relative speed of the redder stars in the galaxy with respect to the blues stars
E. The recessional velocity of a galaxy, expressed as a fraction of the speed of light

To a first approximation, a rough maximum age of the Universe can be estimated using which of the following?

A. The age of the oldest open clusters

B. $1/H_0$ the Hubble time

C. The age of the Sun

D. The age of the Galaxy

E. There is no simple estimate

To a first approximation, a rough maximum age of the Universe can be estimated using which of the following?

A. The age of the oldest open clusters

B. $1/H_0$ the Hubble time

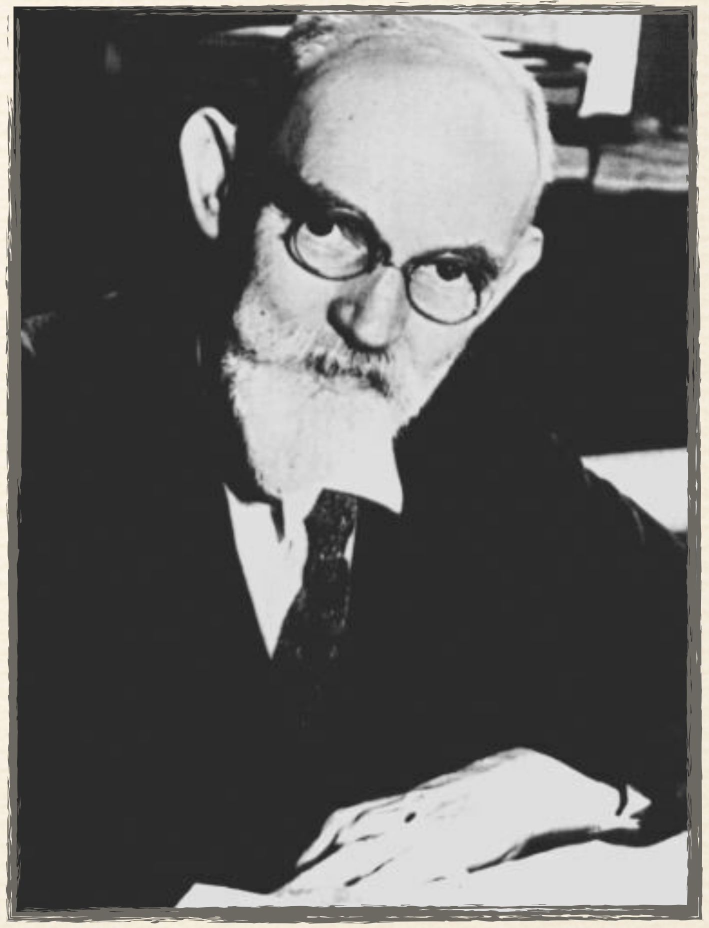
C. The age of the Sun

D. The age of the Galaxy

E. There is no simple estimate

The Early Cosmological Models

Einstein in 1917 constructed the first relativistic cosmological models. Thinking that the universe is static, he introduced the cosmological constant term to balance the force of gravity. This model was unstable.



Willem de Sitter in 1917 also developed a similar model, but also obtained solutions of Einstein equations for a nearly empty, expanding universe

In 1932, Einstein & de Sitter jointly developed another, simple cosmological model which bears their names

The Friedmann and Lemaitre Models

Alexander Friedmann

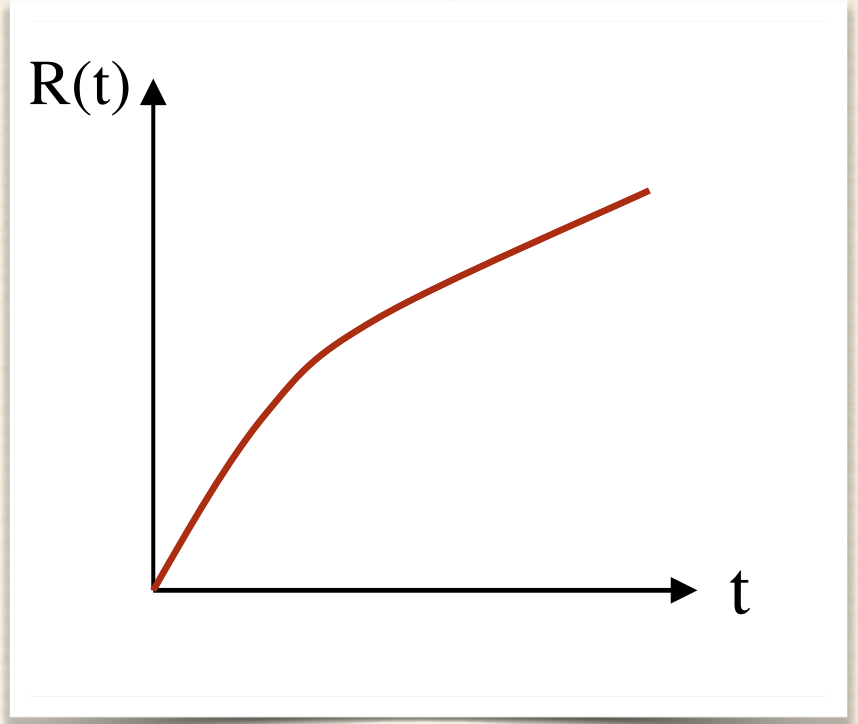
In 1922 developed the GR-based, expanding universe model. It was not taken very seriously at the time, since the expansion of the universe has not yet been established

Georges Lemaitre

In 1927 independently developed cosmological models like Friedmann's. In 1933, he ran the film backwards to a hot, dense, early state of the universe he called the cosmic egg. This early prediction of the Big Bang was largely ignored.

They used the homogeneity and isotropy to reduce the full set of 16 Einstein equations of GR to one: the Friedmann-Lemaitre eqn.

Kinematics of the Universe



We introduce a scale factor, commonly denoted as R(t) or a(t): a spatial distance between any two unaccelerated frames which move with their comoving coordinates

This fully describes the evolution of a homogeneous, isotropic universe

Computing R(t) and various derived quantities defines the cosmological models. This is accomplished by solving the Friedmann (or Friedmann-Lemaitre) Equation

The equation is parametrized (and thus the models defined) by a set of cosmological parameters

Cosmological Parameters

Cosmological models are typically defined through several handy key parameters:

1. The Hubble Parameter

The Hubble parameter is the normalized rate of expansion

$$H = \frac{1}{R}\frac{\Delta R}{\Delta t}$$

$$\Delta R = R_{final} - R_{initial}$$

$$\Delta t = t_{final} - t_{initial}$$

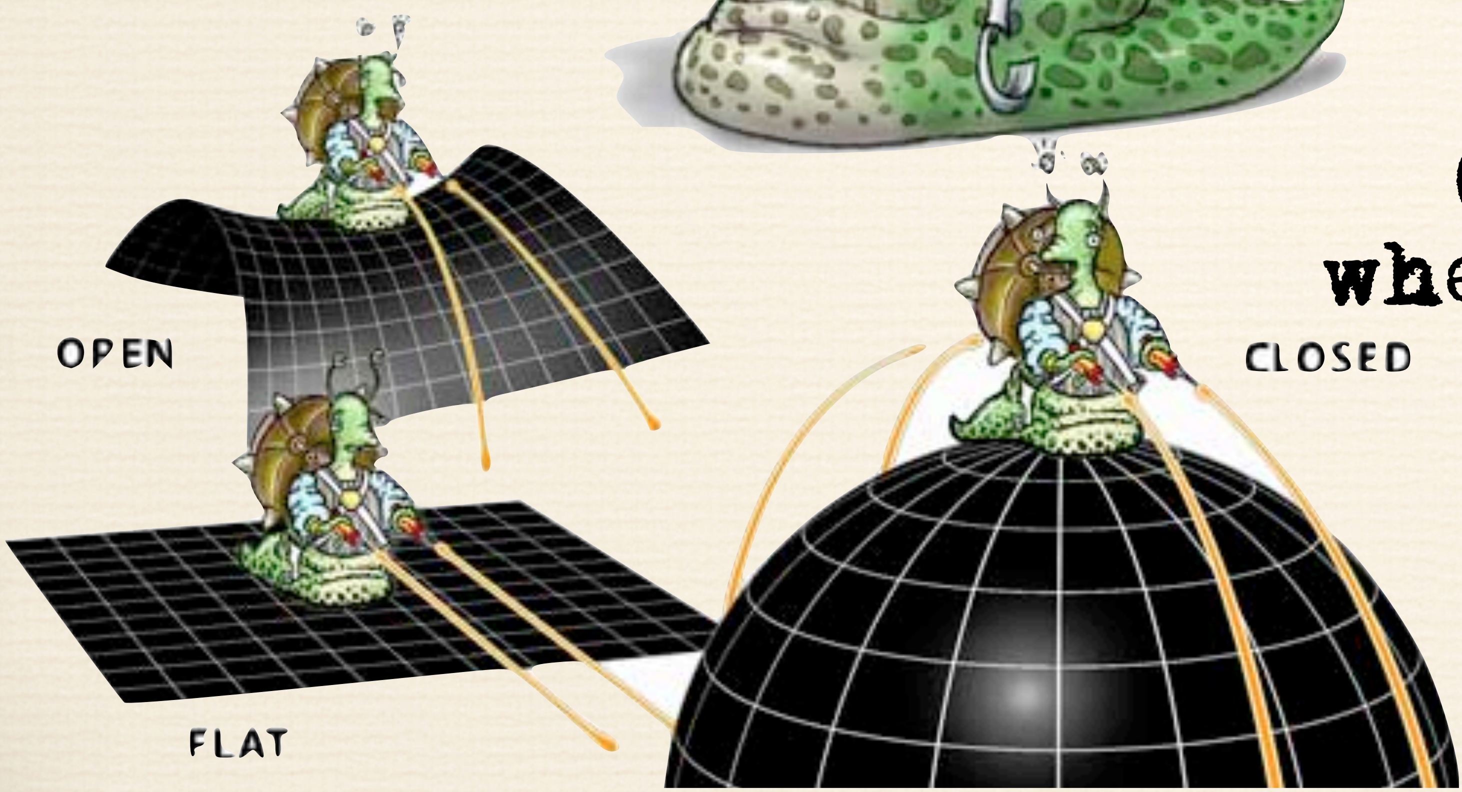
Note that the Hubble parameter is not a constant!

The Hubble constant is the Hubble parameter measured today - we denote its value by H_0

Current estimates are in the range of $H_0 = 65 - 75$ km/s/Mpc

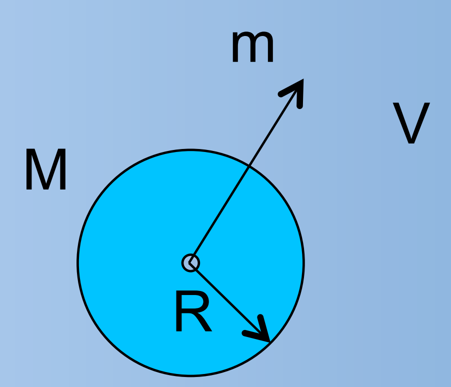
Mass-energy
determines geometry

Geometry determines
where mass-energy can go



Critical Mass Density for the Universe I

We can get an estimate of how much mass is needed to “close” the universe. More accurately, we calculate the mean density needed to close the universe.



Potential energy of mass m in gravitational field of M

$$\mathrm{PE} = -\frac{GMm}{R}$$

Kinetic energy of mass m

$$\mathrm{KE} = \frac{1}{2}mV^2$$

We balance gravitational potential energy and kinetic energy using simple Newtonian mechanics.

Total energy: $$E = \mathrm{KE} + \mathrm{PE} = \frac{1}{2}mV^2 - \frac{GMm}{R}$$

G = Gravitational constant

G = 6.67×10^{-8} cm^3 g^{-1}s^{-2} or G = 6.67×10^{-11} m^3 kg^{-1} s^{-2}

E = 0 corresponds to mass m having escape velocity from M

$$V_{\mathrm{esc}} = \left(\frac{2GM}{R}\right)^{1/2}$$

Example: Earth
R ~ 6371 km
M = 5.97×10^{24} kg

V_{esc} ~ 11.2 km/s

Critical Mass Density for the Universe II

We can get an estimate of how much mass is needed to "close" the universe. More accurately, we calculate the mean density needed to close the universe.

We balance gravitational potential energy and kinetic energy using simple Newtonian mechanics.

E = 0 corresponds to mass m having escape velocity from M

Volume of sphere = $(4\pi/3)R^3$

This gives a mass density ρ_c (g / volume)

We can also solve for E = 0 from

$$E = \mathrm{KE} + \mathrm{PE} = \frac{1}{2}mV^2 - \frac{GMm}{R}$$

to get $\frac{M}{R} = \frac{V^2}{2G}$

Now we consider m to be any object in the universe and we relate its velocity to the Hubble law, $V = H_0R$

Then: $\frac{M}{R} = \frac{V^2}{2G} = \frac{(H_0R)^2}{2G}$

Or: $\frac{M}{R^3} = \frac{H_0^2}{2G}$

→ $\frac{M}{(4\pi/3)R^3} = \frac{H_0^2}{2G(4\pi/3)} = \frac{3H_0^2}{8\pi G}$

$$\rho_c = \frac{3H_0^2}{8\pi G}$$

Critical mass density ~ 10^{-29} g cm^{-3}

Cosmological Parameters

2. The matter density parameter

Rewriting the Friedmann Eqn. using the Hubble parameter

$$H^2 - \frac{8}{3}\pi G\rho = -\frac{kc^2}{R}$$

The Universe is flat if k = 0, or if it has a critical density of

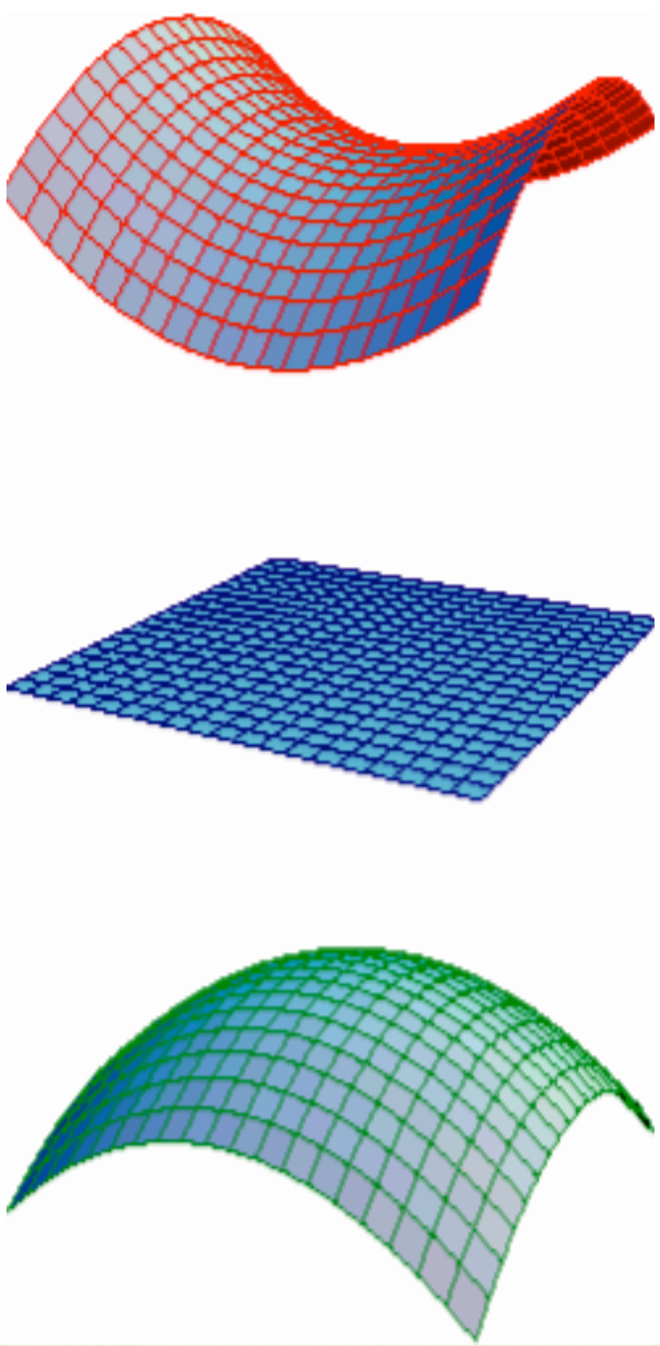
$$\rho_{crit} = \frac{3H^2}{8\pi G}$$

We define the matter density parameter as

$$\Omega_m = \frac{\rho}{\rho_{crit}}$$

Geometry and the Fate of the Universe

Matter and energy content of the universe determines its geometry (curvature of space), and the ultimate fate

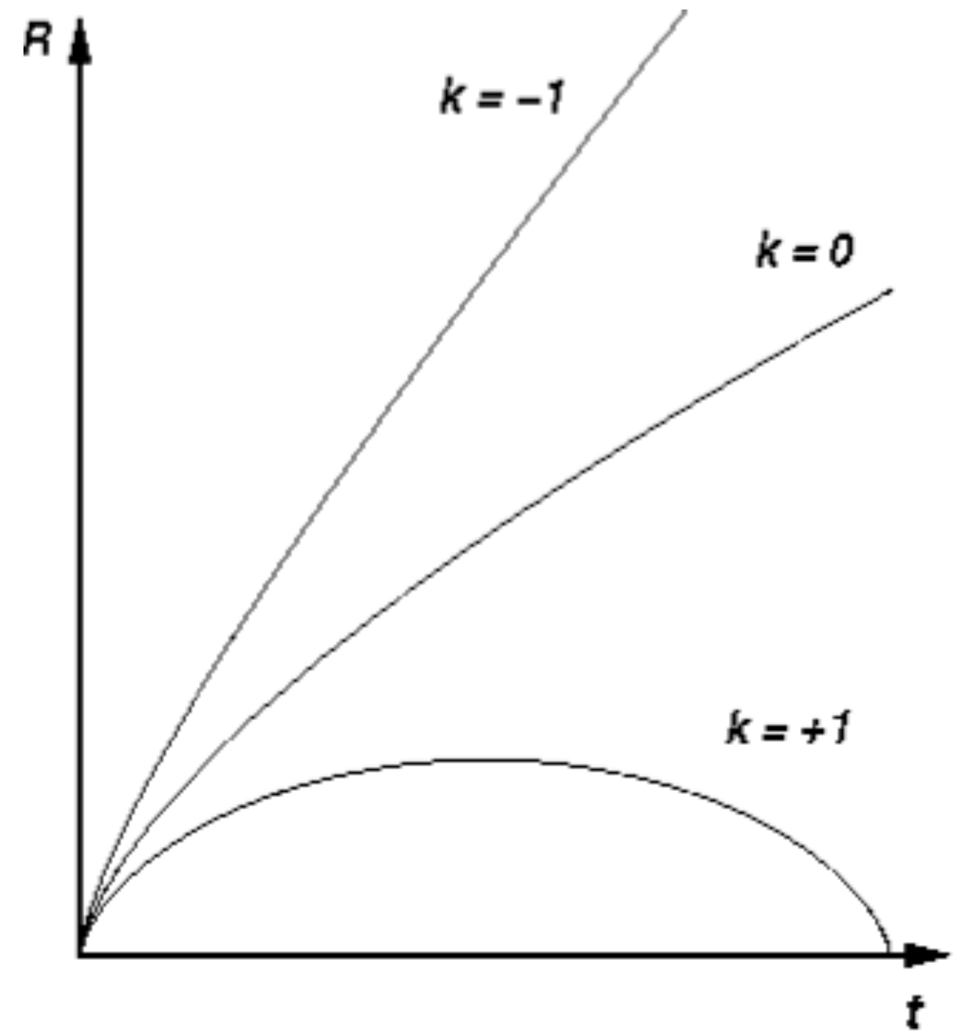


$\rho < \rho_{crit}$, $k = -1$
negative curvature
expands for ever

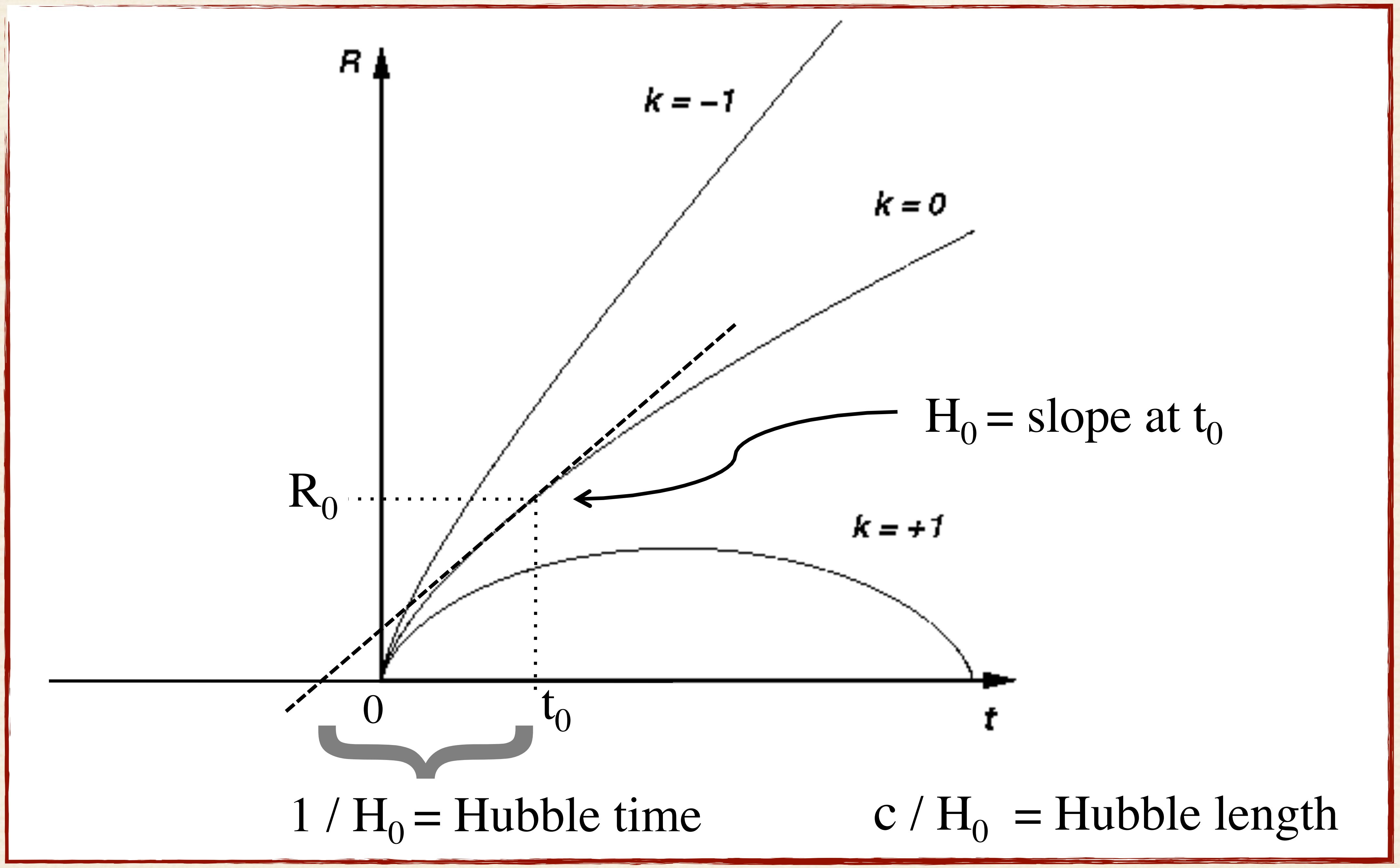
$\rho = \rho_{crit}$, $k = 0$
flat (Euclidean)
expands for ever

$\rho > \rho_{crit}$, $k = +1$
positive curvature
collapses

Possible expansion histories:



Hubble Constant Defines the Scale of the Universe



If the density of the Universe is less than critical, then the Universe:

A. Will ultimately collapse back in on itself

B. Will expand forever

C. Must be spherical

D. Will have a temperature of 2.73K forever

E. Must be static, with an unknown cause for the redshifts

If the density of the Universe is less than critical, then the Universe:

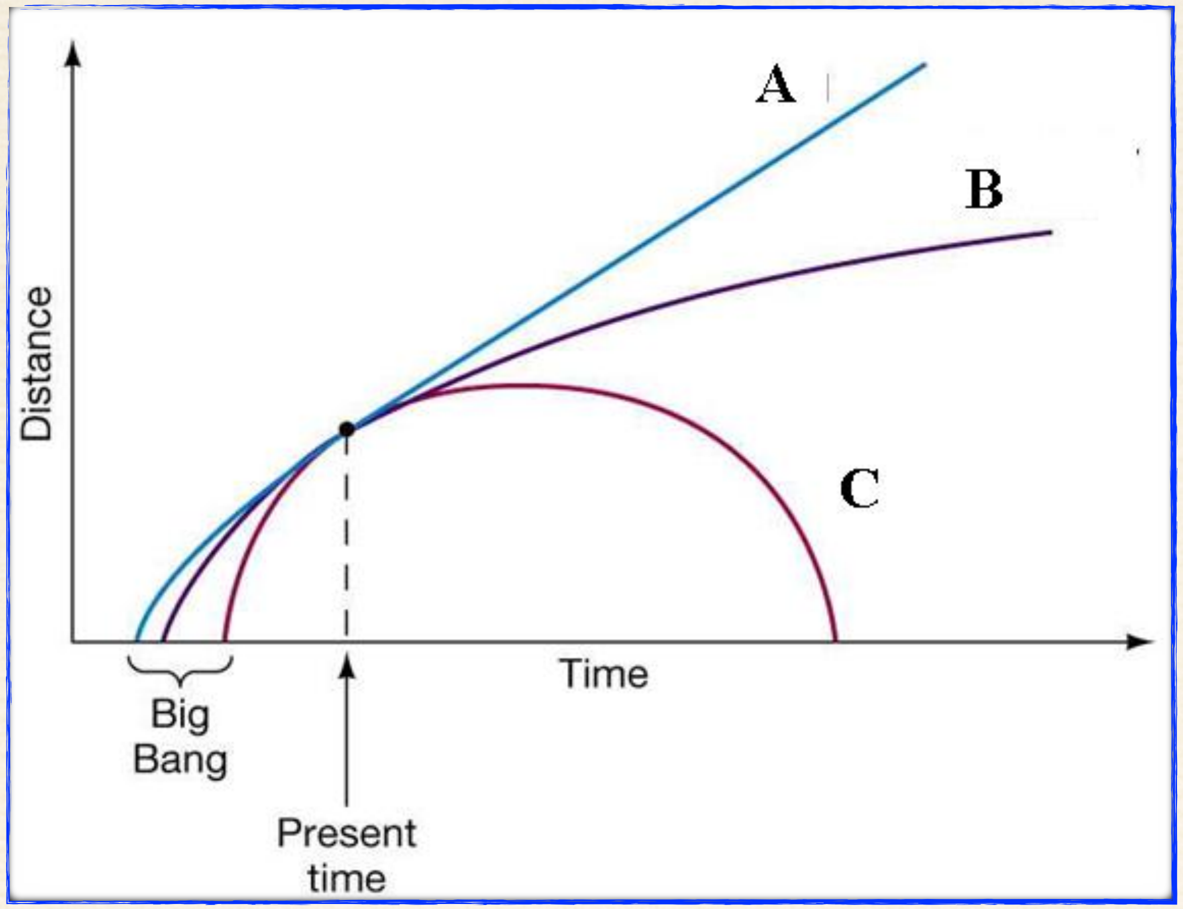
A. Will ultimately collapse back in on itself

B. Will expand forever

C. Must be spherical

D. Will have a temperature of 2.73K forever

E. Must be static, with an unknown cause for the redshifts

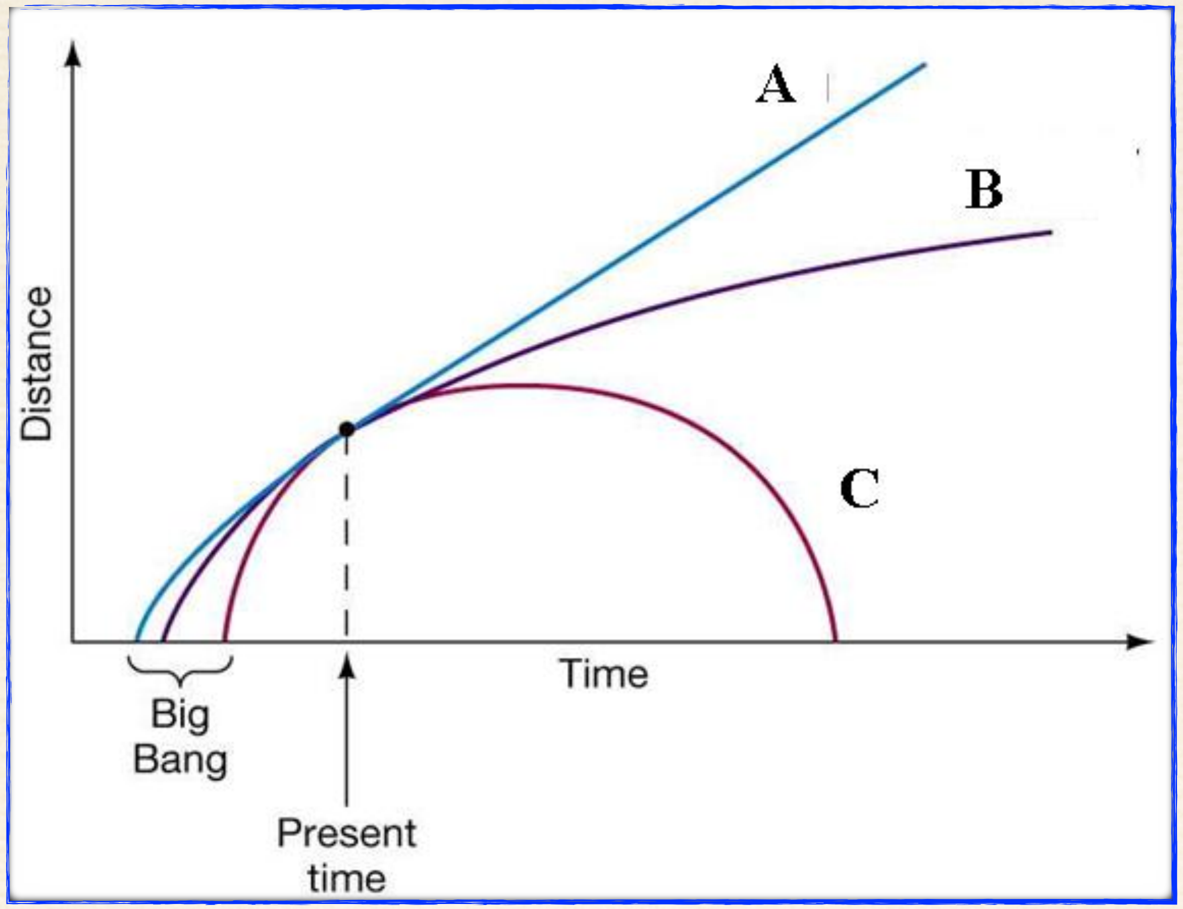


Curve A describes which type of Universe:

A. Open, where the Universe will continue to expand forever

B. Closed, where the Universe will ultimately re-collapse

C. Parallel, where multiple Universes parallel our own evolution

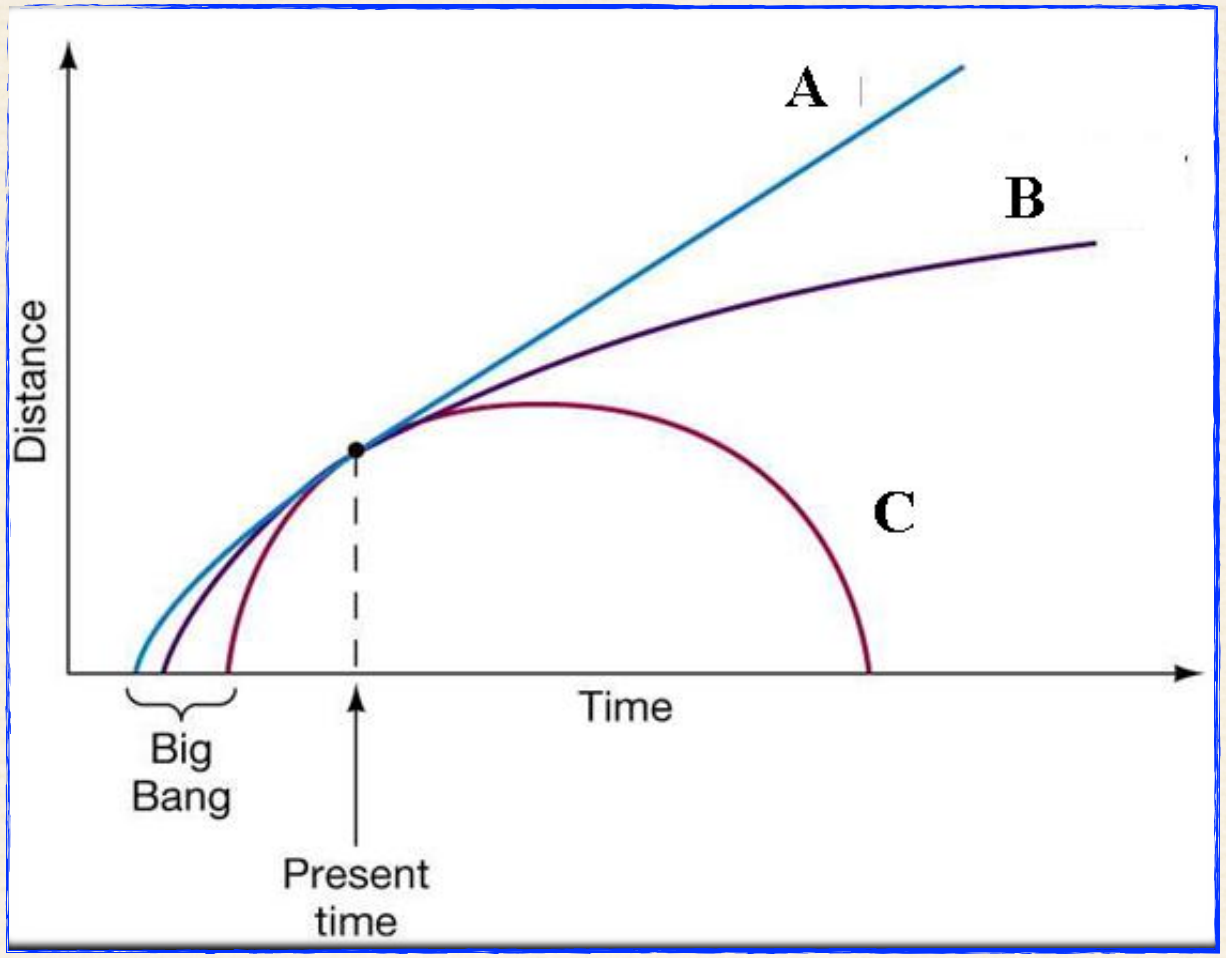


Curve A describes which type of Universe:

A. Open, where the Universe will continue to expand forever

B. Closed, where the Universe will ultimately re-collapse

C. Parallel, where multiple Universes parallel our own evolution

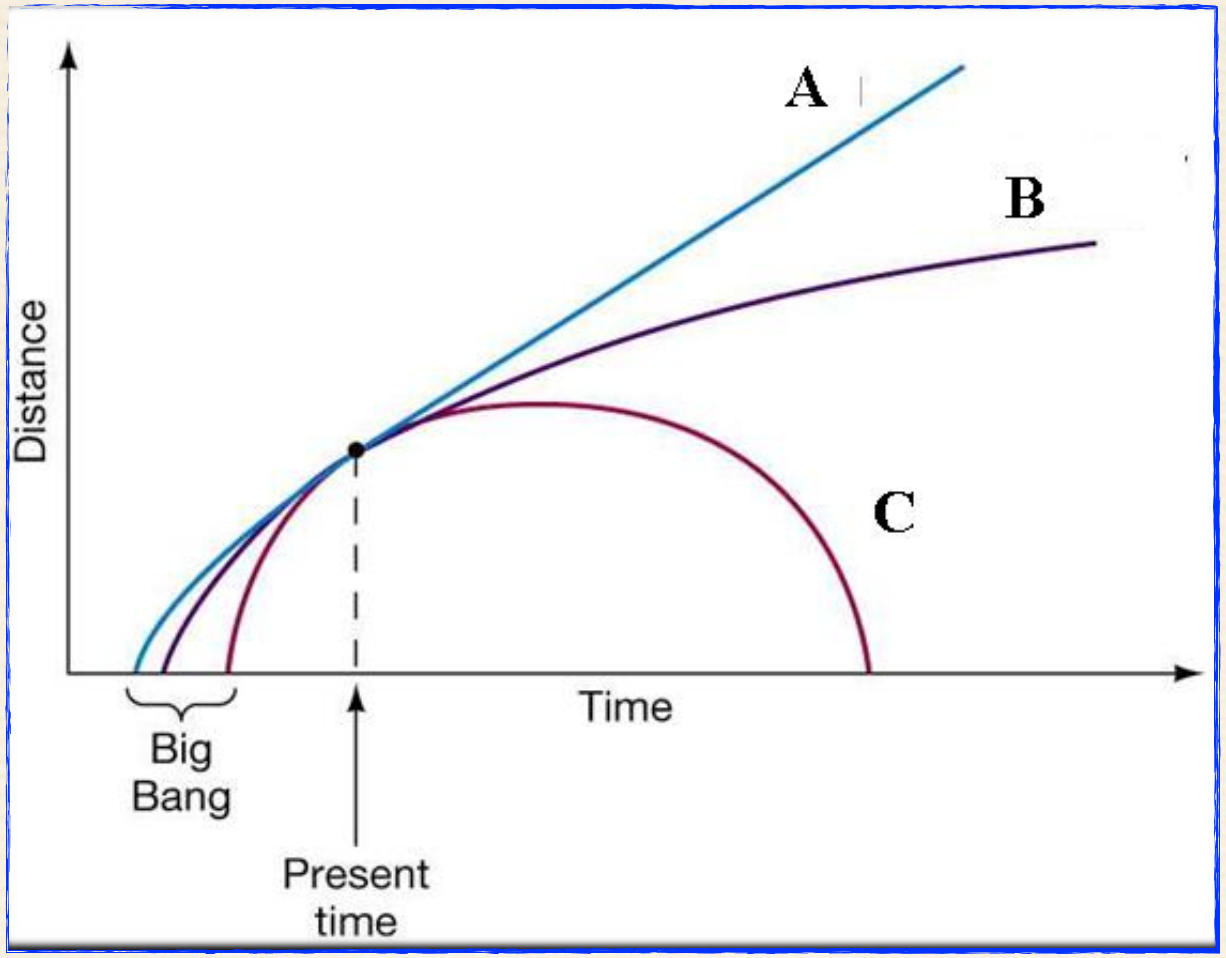


Curve C describes which type of Universe:

A. Parallel, where multiple Universes parallel our own evolution

B. Closed, where the Universe will ultimately re-collapse

C. Open, where the Universe will continue to expand forever



Curve C describes which type of Universe:

A. Parallel, where multiple Universes parallel our own evolution

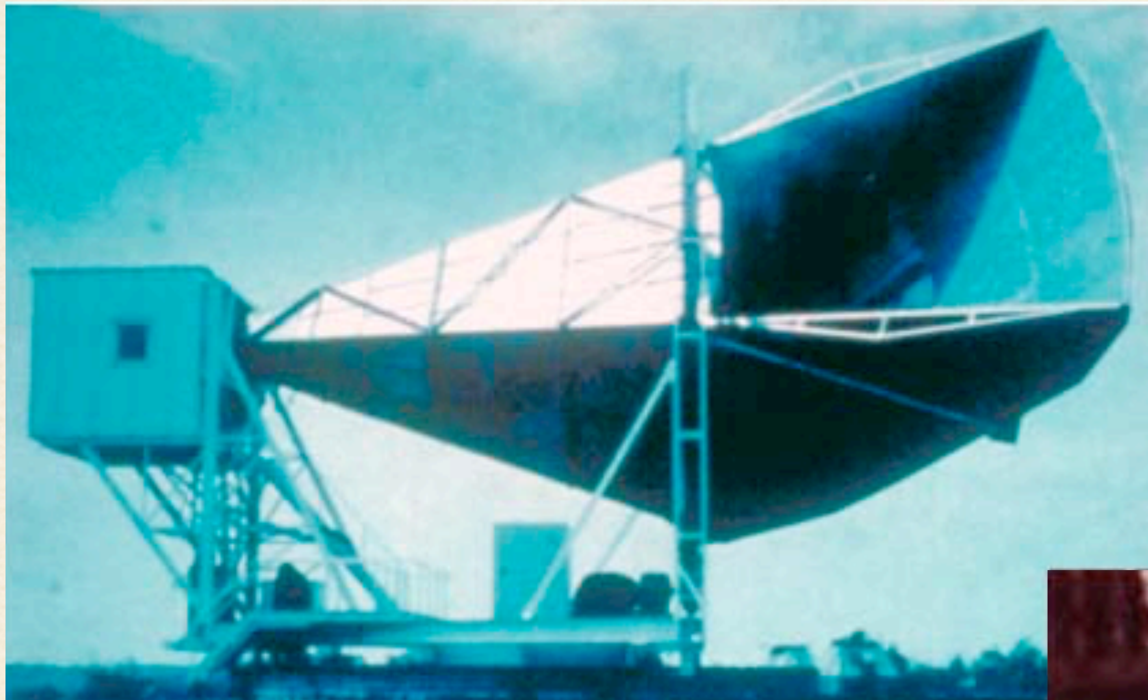
B. Closed, where the Universe will ultimately re-collapse

C. Open, where the Universe will continue to expand forever

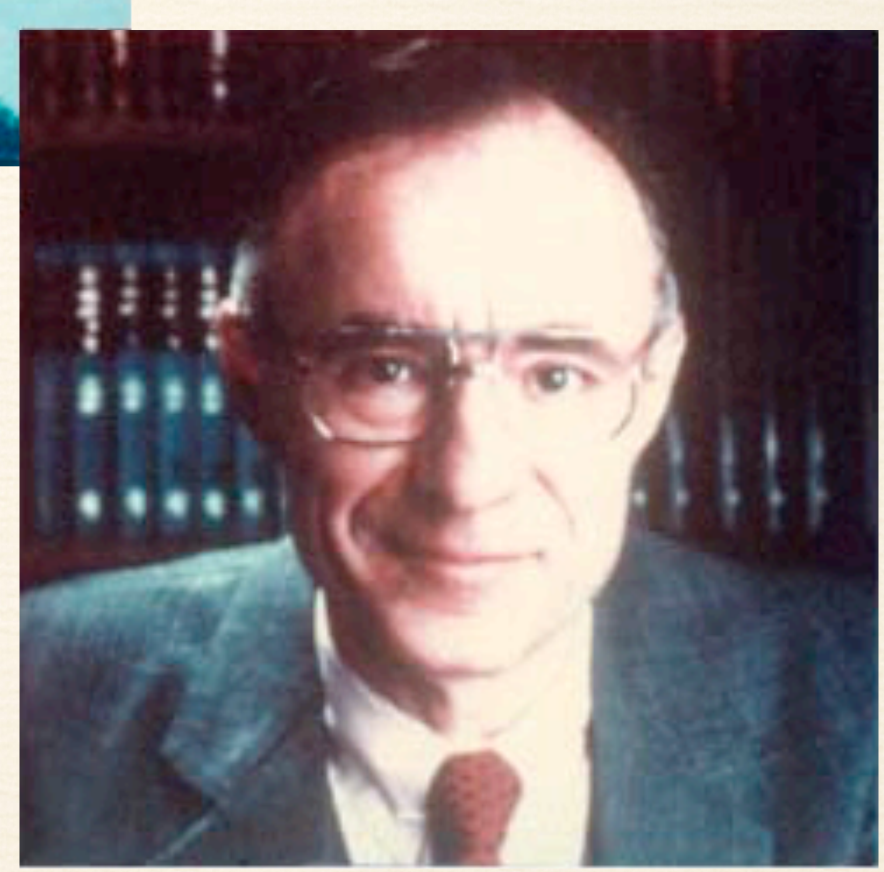
The Early Universe

and the Cosmic Microwave Background

Discovery of cosmic background



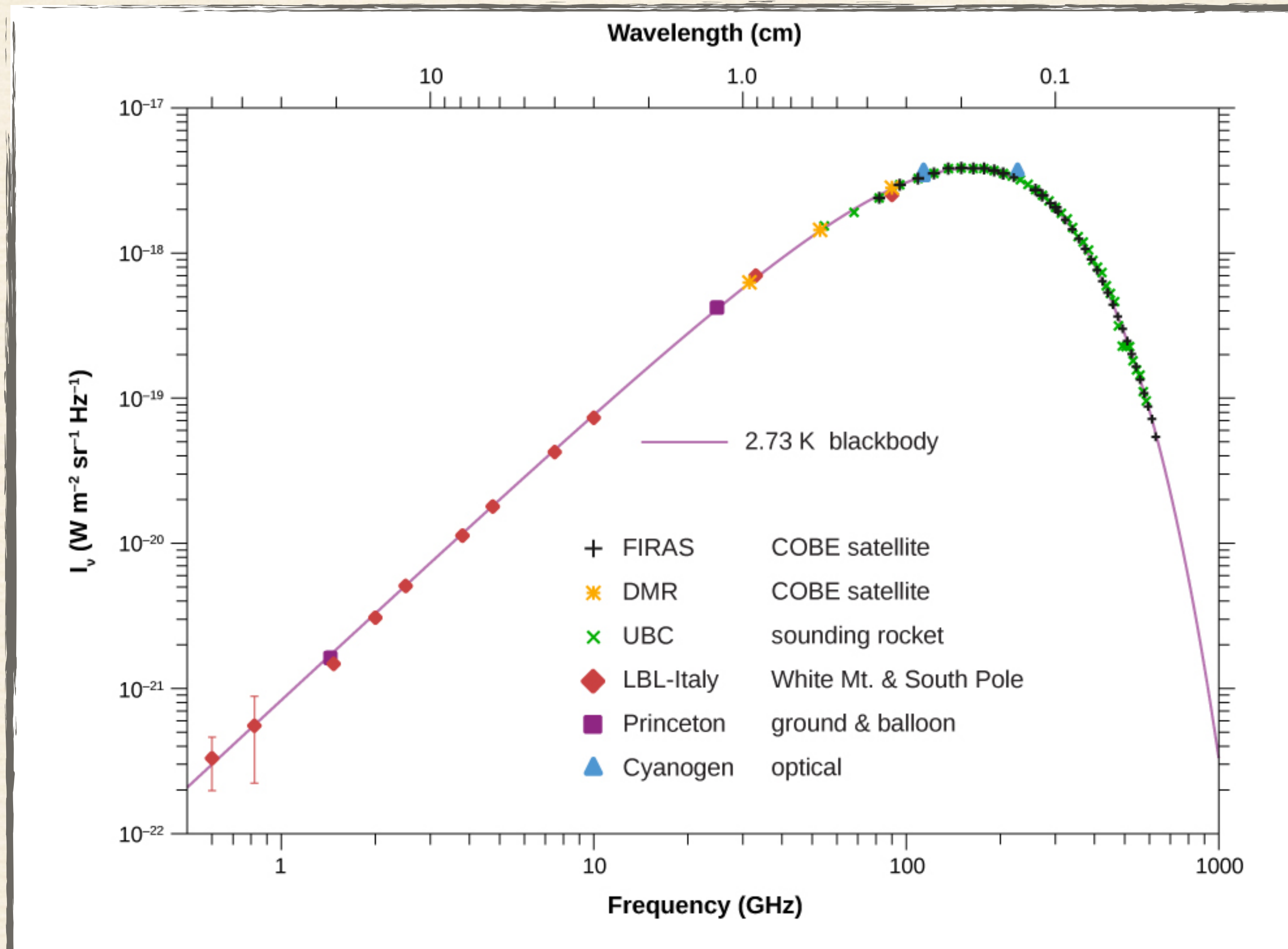
Microwave Receiver



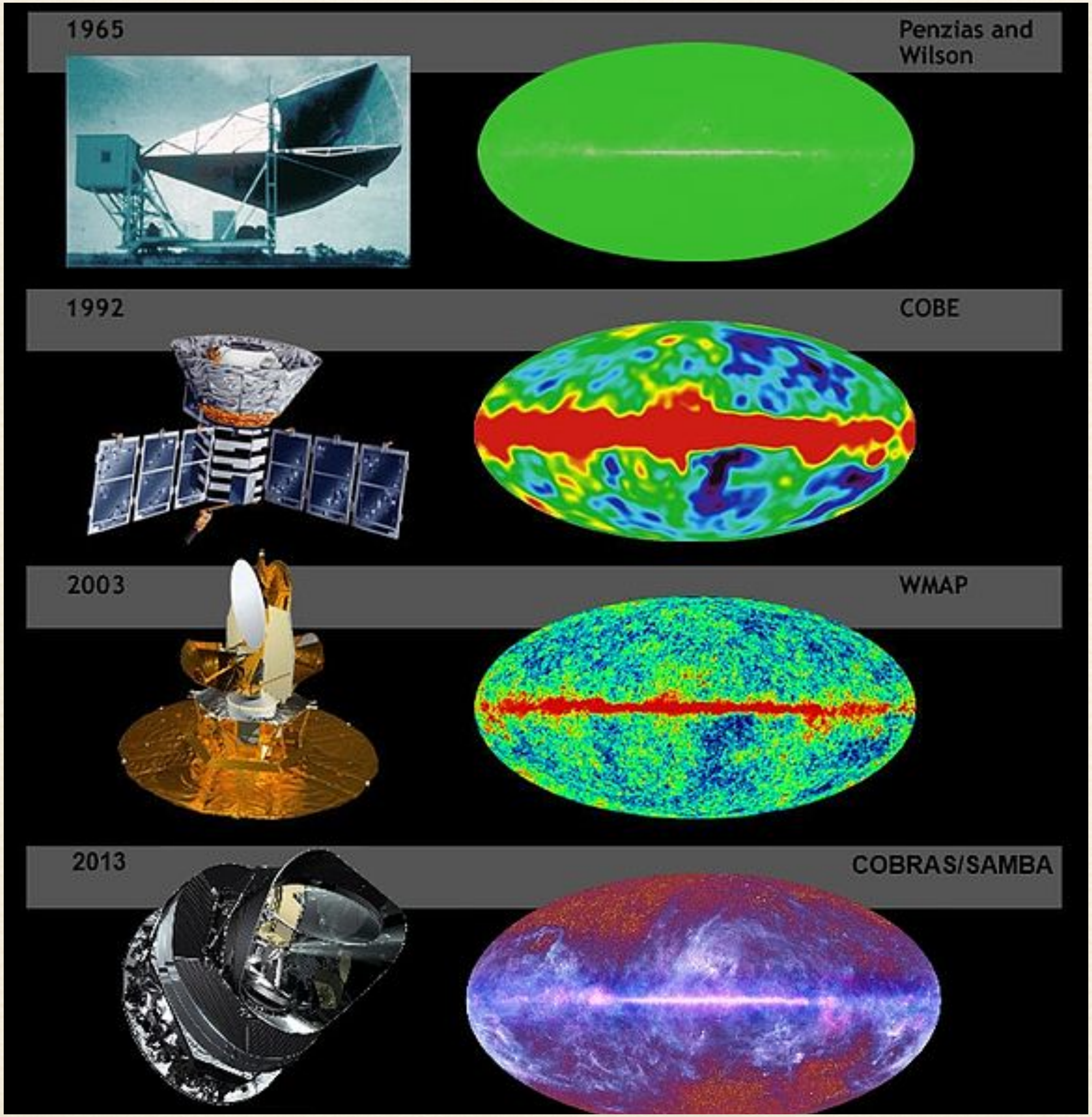
Arno Penzias

Robert Wilson

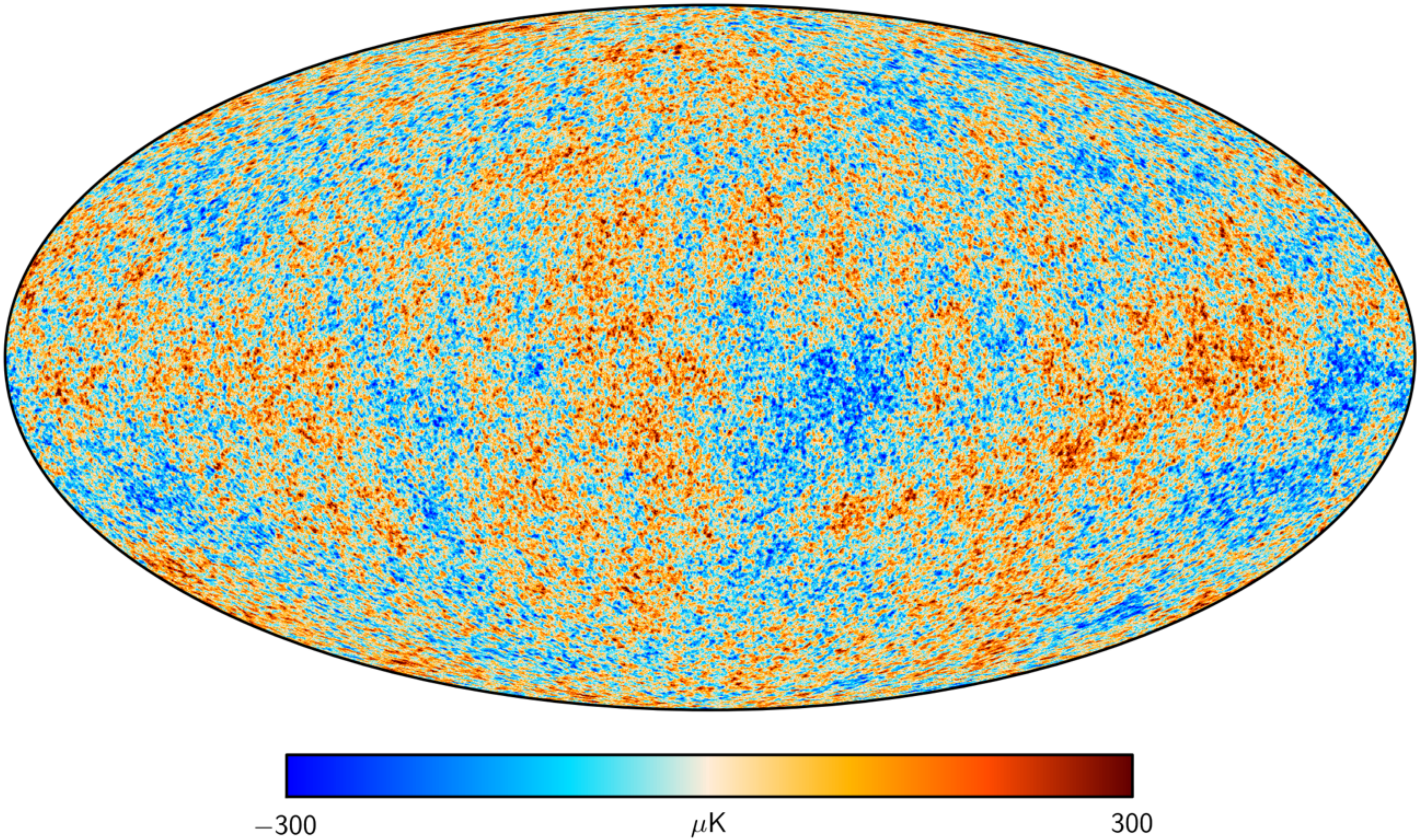
Spectrum of relic photons peaks at microwave frequencies



CMB temperature maps



Plancks mission



The cosmic microwave background is observed to be a 3K blackbody today, although in the past it must have been much hotter. We detect the peak of its emission today as microwaves. In the past, we would measure:

A. Nothing, because the photons would not have reached us yet

B. A much more energetic blackbody, peaking at shorter wavelengths

C. A much more energetic blackbody, peaking at longer wavelengths

D. A spectrum also in the microwave region, although it would not look like a blackbody

E. A much fainter blackbody, peaking at longer wavelengths

The cosmic microwave background is observed to be a 3K blackbody today, although in the past it must have been much hotter. We detect the peak of its emission today as microwaves. In the past, we would measure:

A. Nothing, because the photons would not have reached us yet

B. A much more energetic blackbody, peaking at shorter wavelengths

C. A much more energetic blackbody, peaking at longer wavelengths

D. A spectrum also in the microwave region, although it would not look like a blackbody

E. A much fainter blackbody, peaking at longer wavelengths

The Cosmological Principle

The cosmological principle is usually stated formally as

'Viewed on a sufficiently large scale, the properties of the Universe are the same for all observers.'

This amounts to the strongly philosophical statement that the part of the Universe which we can see is a fair sample, and that the same physical laws apply throughout. In essence, this in a sense says that the Universe is knowable and is playing fair with scientists

This is called the cosmological principle: the universe is homogeneous and isotropic

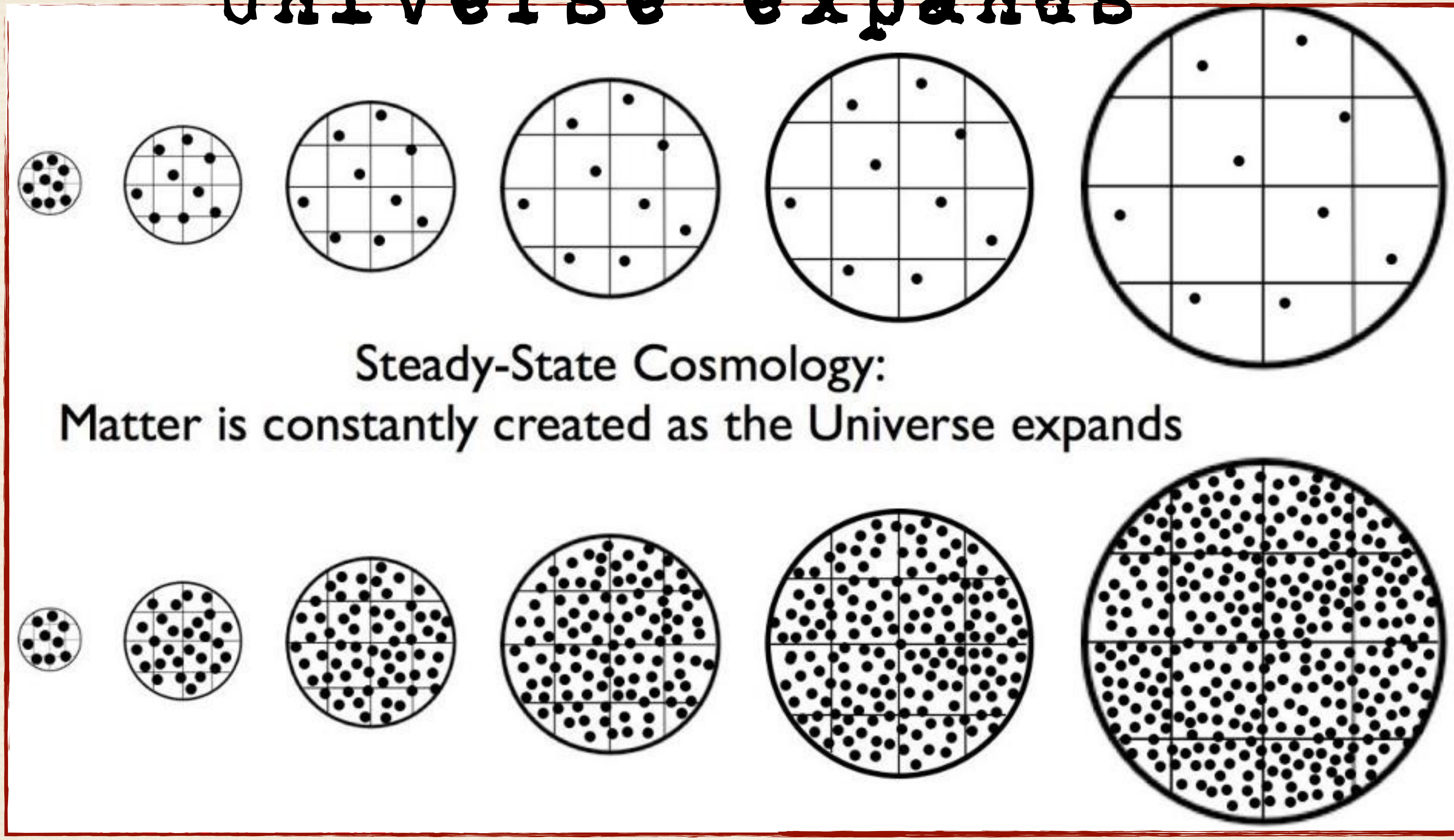
Recall that the Steady State Theory extends this to include time (universe invariant in time)

Is the Universe Static or Evolving?

Newton	1600s	Static
Einstein 1.0	1916	Static ("greatest blunder")
Hubble	1929	Expanding
Einstein 2.0 (Lemaitre 1927)	1929+	Expanding
	Underlying spatial symmetry of the universe (isotropy: it looks the same in all directions, statistically)	
Bondi, Hoyle, Gold	1948 Steady State Theory	Static
	Based on symmetry in time as well as space (more elegant)	
Gamov, Alpher, Herman	1948	Expanding
	Prediction of residual radiation from a hot initial universe	
Hoyle	1950	Coined "big bang" as a derogatory term
Refutation of SS Theory	1960s-1970s	Discovery of cosmic background radiation, Galaxy evolution.

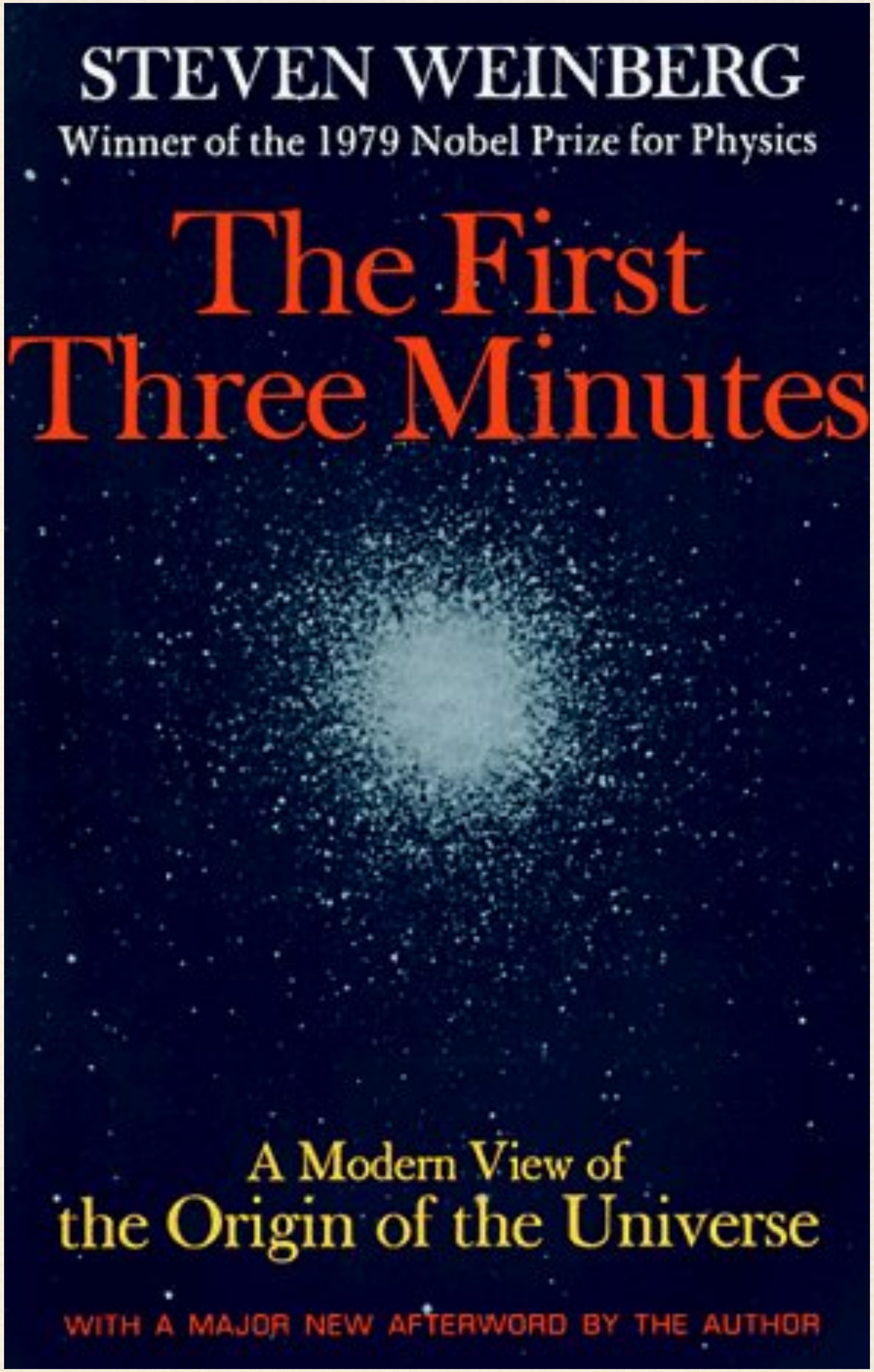
Big Bang Cosmology

Matter dilates as the Universe expands

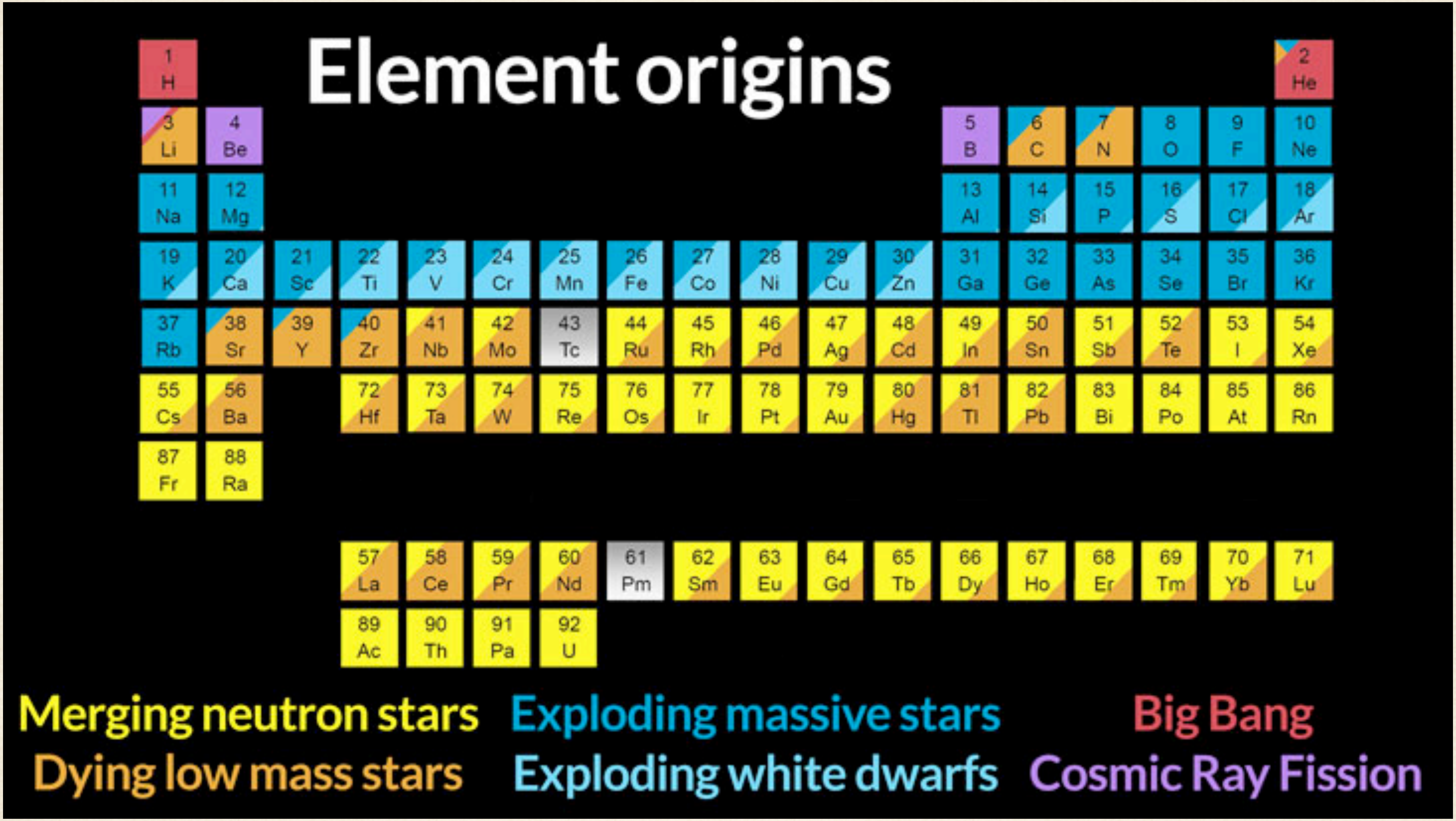


"Big Bang" *scenario from* The First Three Minutes by Steven Weinberg

Time	Temp	Energy kT	Density (water=1)	What's happening
.02 s	10^{11} K	8.6 MeV	4×10^{7}	The universe is mostly light. Electrons and positrons are created from light (pair-production) and destroyed at about equal rates. Protons and neutrons being changed back and forth, so about equal numbers. Only about one neutron or proton for each 10^{9} photons.
.11 s	3×10^{10} K	2.6 MeV		Free neutrons decaying into protons, so there begins to be an excess of protons over neutrons.
1.09 s	10^{10} K	860 keV	4×10^{5}	Primeval fireball becomes transparent to neutrinos, so they are decoupled. It is still opaque to light and electromagnetic radiation of all wavelengths, so they are still contained. Electron-positron annihilation now proceeding faster than pair-production.
13.8 s	3×10^{9} K	260 keV		Below pair-production threshold.
3 m 2 s	10^{9} K	86 keV		Electrons and positrons nearly all gone. Photons and neutrinos are main constituents of the universe in terms of energy. Neutron decay leaves 86% protons, 14% neutrons but these represent a small fraction of the energy of the universe.
3 m 46 s	0.9×10^{9} K	78 keV		Deuterium is now stable, so all the neutrons quickly combine to form deuterium and then helium. There is no more neutron decay since neutrons in nuclei are stable. Helium is about 26% by mass in the universe from this early time. Nothing heavier formed since there is no stable produce of mass 5.
34 m 40 s	3×10^{8} K	26 keV	10	Deuterium is now stable, so all the neutrons quickly combine to form deuterium and then helium. There is no more neutron decay since neutrons in nuclei are stable. Helium is about 26% by mass in the universe from this early time. Nothing heavier formed since there is no stable produce of mass 5.
7×10^{5} yrs	3000K	0.26 eV		Cool enough for hydrogen and helium nuclei to collect electrons and become stable atoms. Absence of ionized gas makes universe transparent to light for the first time.
10^{10} yrs	3 K			Living beings begin to analyze this process.




nature
THE INTERNATIONAL WEEKLY JOURNAL OF SCIENCE
ANATOMY OF A
KILONOVA
Aftermath of the merger between two neutron stars
PAGES 36, 64, 67, 71, 75, 80 & 85
HEALTH
LESSONS FROM SILICON VALLEY
Former funder seeks tech disruption for biomedicine
PAGE 23
MOLECULAR ECOLOGY
EVOLUTION IN ACTION
Tracing mutations in 60,000 generations of bacteria
PAGES 42 & 45
GENOMICS
CHROMOSOME COMPLEXITY
Two mechanisms that guide 3D structure of the genome
PAGES 38 & 51
NATURE.COM/NATURE
2 November 2017 £10
Vol. 551, No. 7678


Element origins
1 H
2 He
3 Li
4 Be
5 B
6 C
7 N
8 O
9 F
10 Ne
11 Na
12 Mg
13 Al
14 Si
15 P
16 S
17 Cl
18 Ar
19 K
20 Ca
21 Sc
22 Ti
23 V
24 Cr
25 Mn
26 Fe
27 Co
28 Ni
29 Cu
30 Zn
31 Ga
32 Ge
33 As
34 Se
35 Br
36 Kr
37 Rb
38 Sr
39 Y
40 Zr
41 Nb
42 Mo
43 Tc
44 Ru
45 Rh
46 Pd
47 Ag
48 Cd
49 In
50 Sn
51 Sb
52 Te
53 I
54 Xe
55 Cs
56 Ba
72 Hf
73 Ta
74 W
75 Re
76 Os
77 Ir
78 Pt
79 Au
80 Hg
81 Tl
82 Pb
83 Bi
84 Po
85 At
86 Rn
87 Fr
88 Ra
57 La
58 Ce
59 Pr
60 Nd
61 Pm
62 Sm
63 Eu
64 Gd
65 Tb
66 Dy
67 Ho
68 Er
69 Tm
70 Yb
71 Lu
89 Ac
90 Th
91 Pa
92 U
Merging neutron stars
Exploding massive stars
Big Bang
Dying low mass stars
Exploding white dwarfs
Cosmic Ray Fission

The fireball from the Big Bang explosion is seen today as what?

A. Nuclear ash, mostly iron and silicon

B. The cosmic microwave background

C. Occasional outbreaks of supernovae

D. A large Number of old stars

E. Northern lights

The fireball from the Big Bang explosion is seen today as what?

A. Nuclear ash, mostly iron and silicon

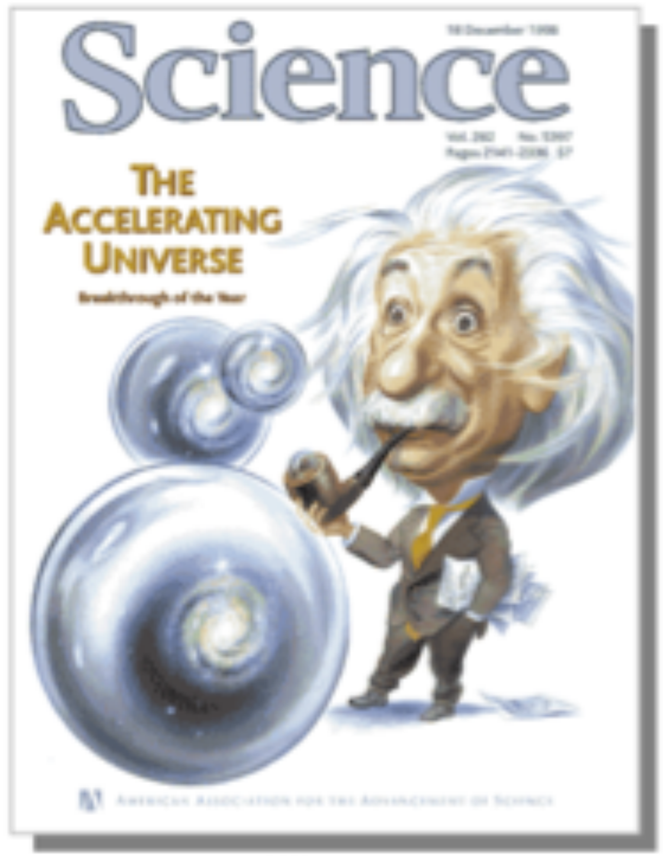
B. The cosmic microwave background

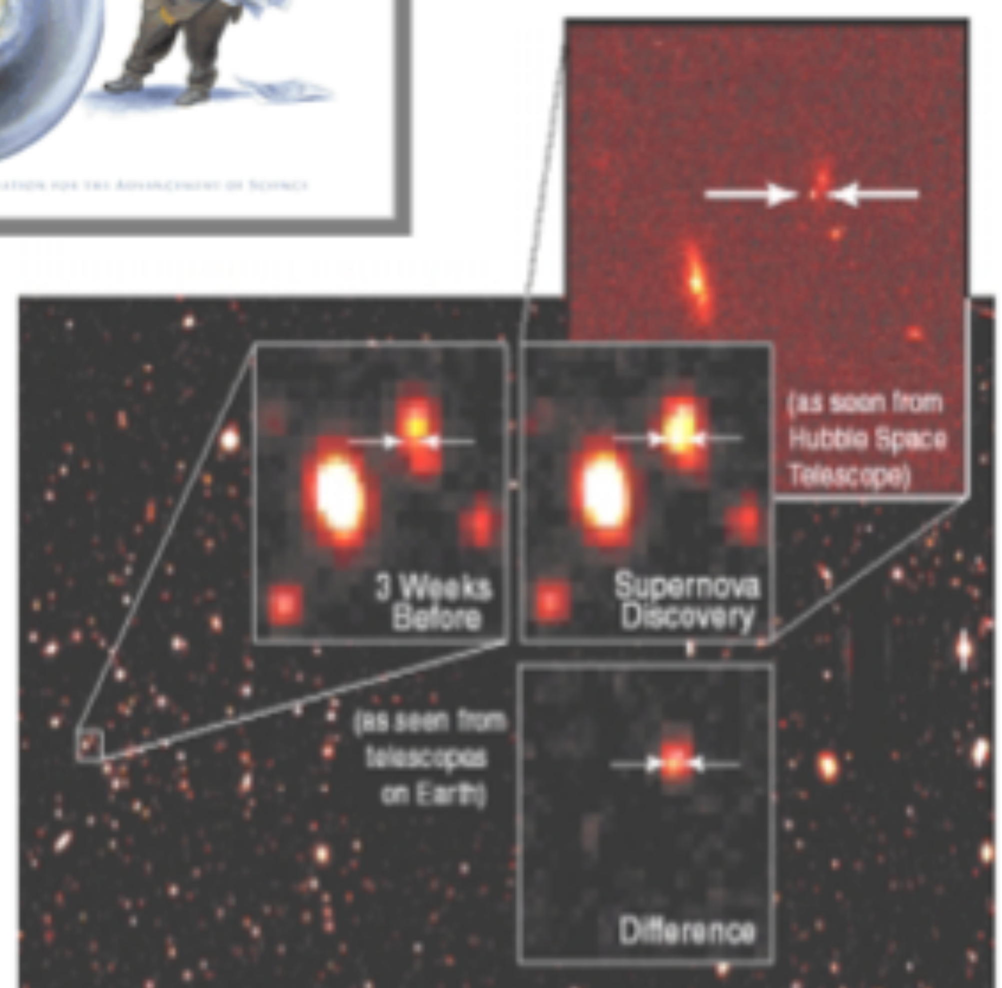
C. Occasional outbreaks of supernovae

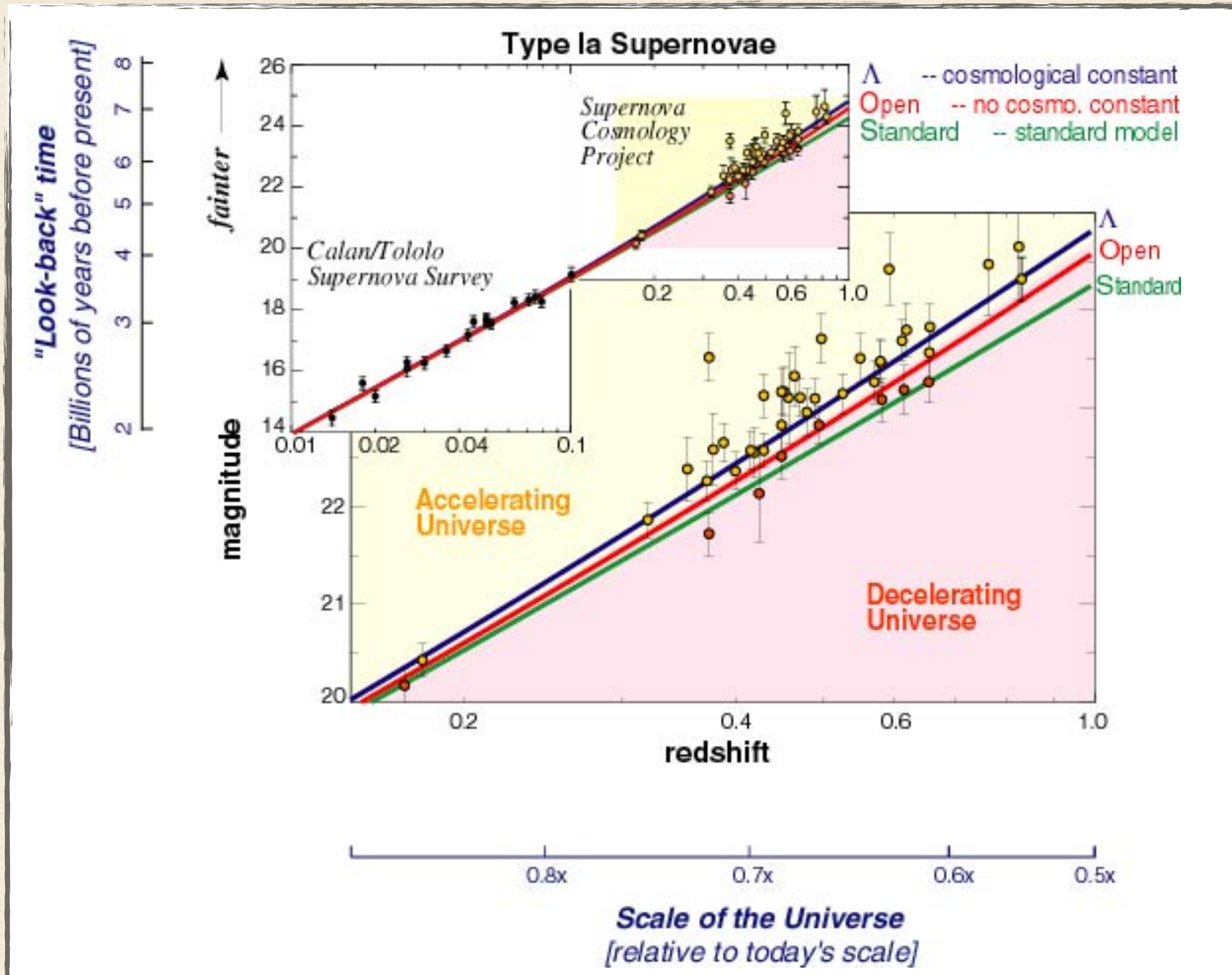
D. A large Number of old stars

E. Northern lights

Science's Breakthrough of the year: The accelerating Universe

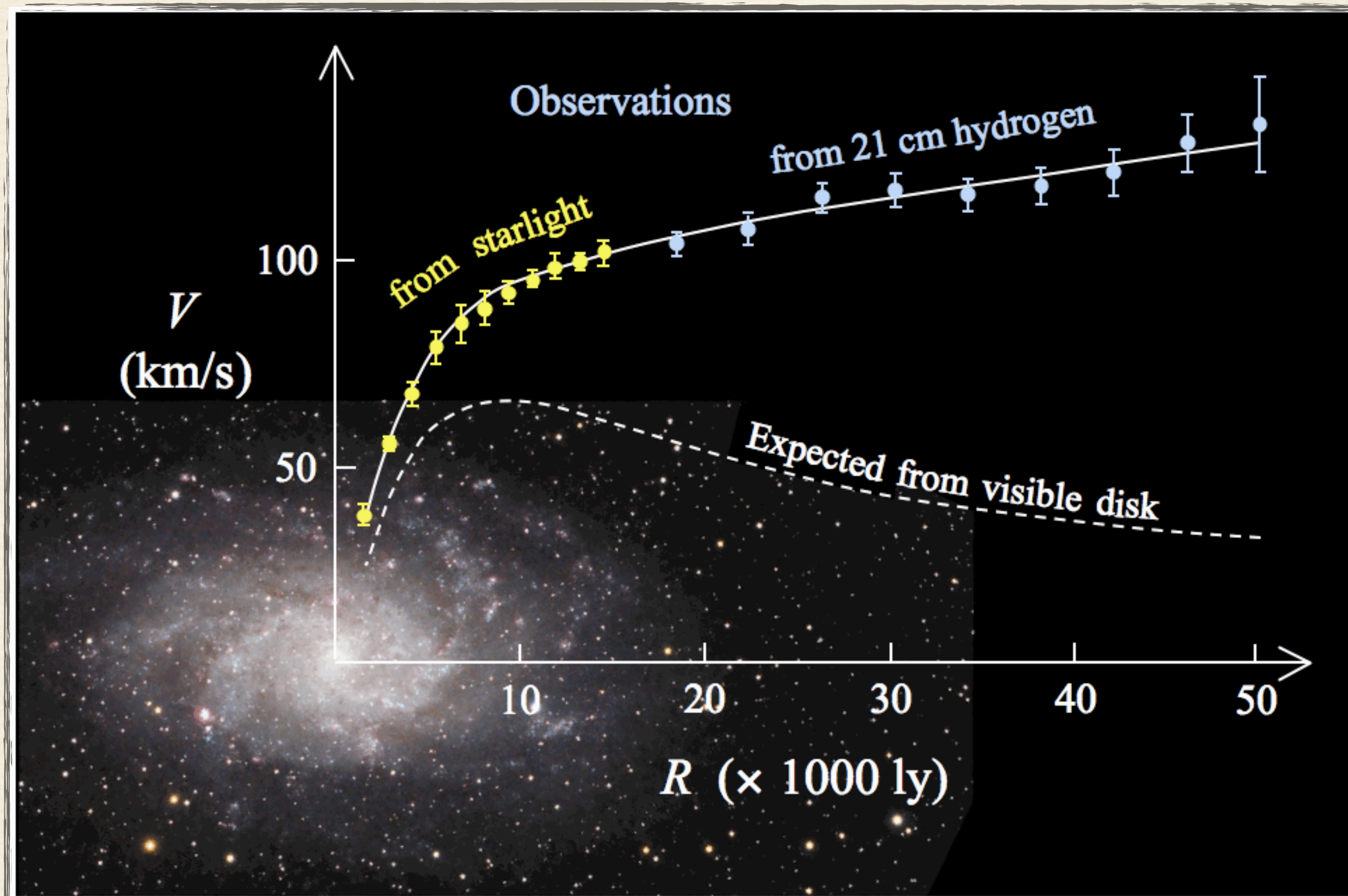






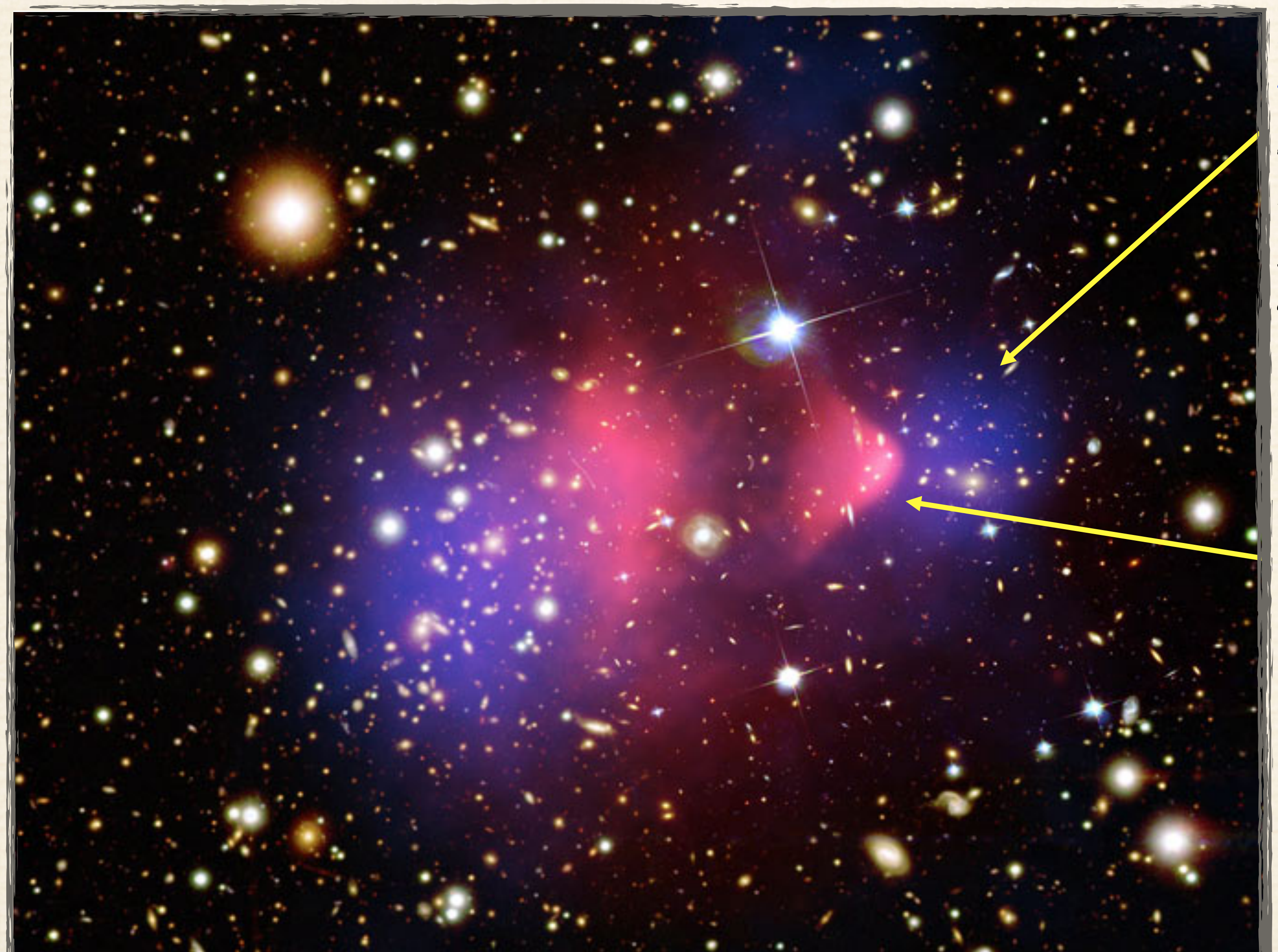
Type Ia Supernovae
Λ -- cosmological constant
Open -- no cosmo. constant
Standard -- standard model
Supernova Cosmology Project
Calan/Tololo Supernova Survey
fainter
magnitude
redshift
Accelerating Universe
Decelerating Universe
Λ
Open
Standard
"Look-back" time
[Billions of years before present]
Scale of the Universe
[relative to today's scale]
0.8x
0.7x
0.6x
0.5x

Rotation curve of M33



Dark Matter and X-Ray Gas in Cluster Mergers: The "Bullet Cluster" (1E 0657-56)

The dark matter clouds largely pass through each other, whereas the gas clouds collide and get shocked, and lag behind



Blue ☛ dark matter, as inferred from weak gravitational lensing

Pink ☛ X-ray gas

Recent observations of the very high-redshift Universe, using Supernovae Type Ia as standard candles, have revealed what ground-breaking discovery?

A. Dark matter is actually completely made up of brown dwarfs and Black holes
B. The Universe is not only expanding, but it is also accelerating
C. A planet just like Earth, orbiting a star just like the Sun
D. The Milky Way is actually at the center of the Universe
E. The first wormhole which might allow human travel over vast distances

Recent observations of the very high-redshift Universe, using Supernovae Type Ia as standard candles, have revealed what ground-breaking discovery?

A. Dark matter is actually completely made up of brown dwarfs and Black holes
B. The Universe is not only expanding, but it is also accelerating
C. A planet just like Earth, orbiting a star just like the Sun
D. The Milky Way is actually at the center of the Universe
E. The first wormhole which might allow human travel over vast distances

"Dark Matter" was first discovered through measurement of which of the following?

A. Galaxy rotation curves
B. Gravitational microlensing of compacto objects in the Milky Way halo
C. The velocity of the Milky Way toward the Virgo cluster
D. The velocity of stars deep within the Galactic center
E. The velocity of Andromeda toward the Milky Way

"Dark Matter" was first discovered through measurement of which of the following?

A. Galaxy rotation curves
B. Gravitational microlensing of compacto objects in the Milky Way halo
C. The velocity of the Milky Way toward the Virgo cluster
D. The velocity of stars deep within the Galactic center
E. The velocity of Andromeda toward the Milky Way

Cosmological Parameters

3. The dark energy density parameter

We can express a similar density parameter for lambda again by using the Friedmann equation and setting rh = 0

We then get

$$\Omega_\Lambda = \frac{\Lambda c^2}{3H^2}$$

Cosmological model: ΛCDM?

The Cosmic Inflation

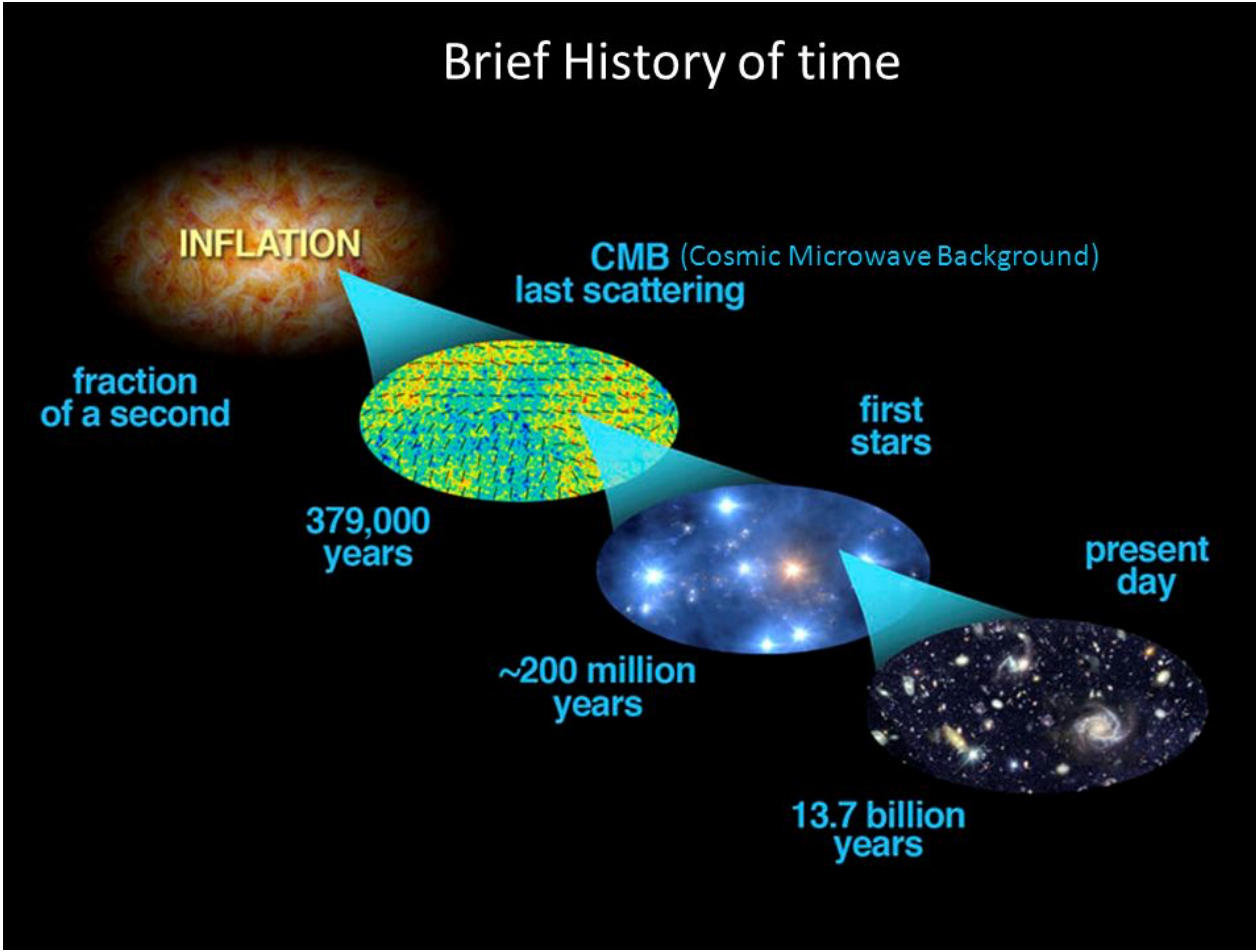
Inflation

* Physical models invoke a large reservoir of potential energy in the very early universe that decays to produce the sudden expansion

* No consensus on the nature of the potential energy (scalar field with a slow-roll downhill or something else?)

* The acceleration of the universe seen currently is a much milder version of inflation

* An over-riding question is why now and why so little acceleration?


Brief History of time
INFLATION
CMB (Cosmic Microwave Background)
last scattering
fraction
of a second
first
stars
379,000
years
present
day
~200 million
years
13.7 billion
years

QUERY 28

The age of the universe (that is, the time since the Big Bang) is 14 billion years

The age of the Solar System is 4.56 billion years

Thus, the Solar System has existed for 32.6% of the age of the universe

For what percentage of the total age of the universe have existed: (i) helium nuclei, (ii) neutral atoms, (iii) galaxies, and (iv) the U.S.

QUERY 28

Helium nuclei have been around since the time of primordial nucleosynthesis, at a time

$$t_{BBN} \approx 7 \text{ min} \approx 420 \text{ s}$$

after the Big Bang

The age of the universe, expressed in seconds, is

$$t_U = 1.4 \times 10^{10} \text{ yr} \left(\frac{3.16 \times 10^7 \text{ s}}{1 \text{ yr}} \right) = 4.4 \times 10^{17} \text{ s}$$

The fraction of the age of the universe during which helium nuclei have been around is

$$F = \frac{t_U - t_{BBN}}{t_U} = 1 - \frac{t_{BBN}}{t_U} = 1 - \frac{420 \text{ s}}{4.4 \times 10^{17} \text{ s}} = 1 - 10^{-15} = 0.999999999999999$$

Expressed as a percentage, this is 99.9999999999999% of the age of the universe

(There should be 13 nines after the decimal point I hope I counted them correctly!)

QUERY 28

(ii) Neutral atoms have been around since the universe became transparent, at a time t_{trans} = 350, 000 yr after the Big Bang

The fraction of the age of the universe during which neutral atoms have been around is

$$F = 1 - \frac{t_{\text{trans}}}{t_{\text{U}}} = 1 - \frac{3.5 \times 10^5 \text{ yr}}{1.4 \times 10^{10} \text{ yr}} = 1 - 0.000025 = 0.999975$$

Expressed as a percentage, this is 99.9975% of the age of the universe

(iii) Galaxies have been around since the universe had an age t_{gal} = 5×10^8 yr

The fraction of the age of the universe during which galaxies have been present is

QUERY 28

$$F = 1 - \frac{t_{\rm gal}}{t_{\rm U}} = 1 - \frac{5 \times 10^8 \text{ yr}}{1.4 \times 10^{10} \text{ yr}} = 1 - 0.0357 = 0.964$$

Expressed as a percentage, this is 96.4% of the age of the universe

(iv) Dating from the Declaration of Independence, the U.S.A. has existed for 243 yr.

Written as a fraction of the age of the universe, the age of the U.S.A. is

$$F = \frac{243 \; yr}{1.4 \times 10^{10}} = 1.7 \times 10^{-8}$$

Expressed as a percentage, this is 0.0000017% of the age of the universe

QUERY 29

Hydrogen has an absorption line at a wavelength λ_0 = 656.3 nm (as long as the hydrogen is at rest). You observe a distant galaxy for which the same hydrogen absorption line has a wavelength λ = 715.4 nm. (i) What is the redshift, $z = (\lambda - \lambda_0)/\lambda_0$, of the galaxy? (ii) What is the radial velocity of the galaxy, in kilometers per second? (iii) From Hubble's law, what is the distance to the galaxy? [Hint: assume $H_0 \approx 71$ km/s/Mpc.]

QUERY 29

(a) This first part is straight plug and chug

From the question above, we know that λ_0 = 656.3 nm

Plugging these in to the formula provided, find:

$$z = \frac{\lambda - \lambda_0}{\lambda_0} = \frac{715.4\,\text{nm} - 656.3\,\text{nm}}{656.3\,\text{nm}} = \frac{59.1\,\cancel{\text{nm}}}{656.3\,\cancel{\text{nm}}} = 0.09005 \approx 0.09$$

Please note that the units (nm) have cancelled!

Redshift is a so-called dimensionless number, meaning it does not have any units

Since the wavelengths we started with each had 4 digits, it's OK to keep all 4 digits in your answewr here (0.09005)

However, rounding to 0.09 will make the math a bit easier in the coming parts

QUERY 29

(b) Using your answer to part (a), the radial velocity can be easily computed using the formula v = cz

Using c = 3.0×10^5 km/s, I find:

$$v = c\,z = (3.0 \times 10^5 \text{ km/s})(0.09005) = 27,015 \text{ km/s} \approx 27,020 \text{ km/s}$$

As in part (a), I rounded this answer off after 4 digits

Using z = 0.09 instead, you would find

v = 27,000 km/s = 2.7×10^4 km/s

which you can easily do without a calculator (since 9 x 3 = 27)

(c) This last part provides one final plug and chug opportunity. Hubble's Law, written v = H_0 d, relates the distance of galaxies to their recession (radial) velocity

Using H_0 = 71 km/s/Mpc and the answer from part (b), I find:

$$v = H_0\,d \;\rightarrow\; d = \frac{v}{H_0} = \frac{27,015 \text{ km/s}}{71 \text{ km/s/Mpc}}$$

QUERY 29

It helps to be extremely careful with units to avoid mistakes, particularly with velocities!

To be sure to get this correct, I rewrite my units as conventional fractions

The Hubble Constant 71 km/s/Mpc thus becomes 71 $\frac{\text{km}}{\text{s. Mpc}}$

Plugging this back in, I have:

$$d = \frac{v}{H_0} = \left(27,015 \cdot \frac{\cancel{\text{km}}}{\cancel{\text{s}}}\right)\left(\frac{\cancel{\text{s}} \cdot \text{Mpc}}{71\ \cancel{\text{km}}}\right) = \frac{27,015}{71}\,\text{Mpc} = 380.5\,\text{Mpc}$$

Please notice that the km and s units cancel completely, leaving behind Mpc

It's always important to check whether or not the units make sense. Fortunately, Mpc is a measure of distance, which is what we're looking for

QUERY 30

As we have seen in the lectures, if the Hubble constant is H_0 = 71 km/s/Mpc, then the Hubble time is $1/H_0$ = 14 billionyears. Sir Edwin Hubble himself, because he grossly underestimated the distance to galaxies, believed that the Hubble constant was H_0 = 500 km/s/Mpc.

For H_0 = 500 km/s/Mpc, what is $1/H_0$, in billions of years?

QUERY 30

You can either calculate the new Hubble time directly with unit conversion, or you can solve for it by comparison to the actual Hubble time using ratios. Both methods give the same answer (i.e., they both work)

If you opt to do the unit conversion directly, take great care to make sure that you cancel out your units properly!

As in the previous question, I'll rewrite the units in a more standard fraction form to avoid mistakes:

$$H_0 = 500 \text{ km/s/Mpc} = \frac{500 \text{ km}}{\text{s} \cdot \text{Mpc}}$$

Using conversion factors 1 Mpc = 10^6 pc and 1pc = 3.08 x 10^{13} km, I find:

$$H_0 = \left(\frac{500 \text{ km}}{\text{s} \cdot \text{Mpc}}\right)\left(\frac{1 \text{ Mpc}}{10^6 \text{ pc}}\right)\left(\frac{1 \text{ pc}}{3.08 \times 10^{13} \text{ km}}\right) = \frac{500}{3.08 \times 10^{19} \text{ s}} = 1.62 \times 10^{-17} \cdot \frac{1}{\text{s}}$$

QUERY 30

Please note two things: firstly, nearly all of the units cancelled. Secondly, H_0 has units of inverse seconds (not seconds)! To compute the Hubble time, I take the reciprocal of the above expression

I find (taking care not to drop powers of 10!):

$$\frac{1}{H_0} = \frac{\text{s}}{1.62 \times 10^{-17}} = \frac{10^{17}\ \text{s}}{1.62} = 6.173 \times 10^{16}\ \text{s}$$

Converting the above value to years, I get:

$$\frac{1}{H_0} = (6.173 \times 10^{16}\ \cancel{\text{s}}) \left(\frac{1\,\text{yr}}{3.15 \times 10^{7}\ \cancel{\text{s}}} \right) = \frac{6.173 \times 10^{16}}{3.15 \times 10^{7}}\ \text{yr} = 1.96 \times 10^{9}\ \text{yr} = 1.96\ \text{billion years}$$

QUERY 30

Alternatively, you could have note that the new $1/H_0$ must be smaller than the true value by the ratio of H0 values (if H_0 gets bigger, then $1/H_0$ must get smaller) Doing this you would find

$$\frac{1}{H_0} = \left(\frac{71\ \text{km/s/Mpc}}{500\ \text{km/s/Mpc}}\right) \cdot 14\ \text{billion years} = 0.142 \cdot 14\ \text{billion years} = 1.99\ \text{billion years}$$

Note that all the units in the H_0 ratio above cancelled immediately. Setting up the problem to intentionally cancel units is a useful way to avoid needless conversion mistakes

QUERY 31

There are 411,000,000 cosmic microwave photons per cubic meter of the universe. The average energy of a cosmic microwave photon is very small: only $E = 1.02 \times 10^{-22}$ joules. What is the energy density of the Cosmic Microwave Background, in joules per cubic meter? Using Einstein's relation, $E = mc^2$, what is the equivalent mass density, in kilograms per cubic meter? What fraction of the critical density, $\rho_{crit} = 10^{-26}$ kg/m^3, does this density represent?

QUERY 31

First, compute the energy density of the CMB.

We know the number of photons per unit volume (n_{CMB} = 4.11 × 10^8 photons / m^3) and the average energy that each carries E = 1.02 × 10^{-22} J / photon)

Multiplying these together (note that the units work out) gives us the CMB energy density

$$\text{CMB energy density} = n_{CMB} \cdot E$$

$$\text{CMB energy density} = (4.11 \times 10^8 \text{ ~~photons~~ } / \text{m}^3) \cdot (1.02 \times 10^{-22} \text{ J } / \text{ ~~photon~~})$$

$$\text{CMB energy density} = 4.19 \times 10^{-14} \text{ J} / \text{m}^3$$

Next we'll calculate the equivalent mass density ρ_m using E = mc^2

QUERY 31

To simplify this calculation, recall that:

$$c^2 = (3.0 \times 10^8 \text{ m}/\text{s})^2 = 9.0 \times 10^{16} \text{ J}/\text{kg}$$

CAUTIONARY NOTE: Choosing the correct conversion factor is extremely important

When using the relation $E = mc^2$, take care with your units!!

Since 1 Joule = 1 kg m^2 / s^2, you Must use c = 3.0×10^8 m/s!!

If you have E in Joules, m in kg, but use c = 3.0×10^5 km/s, the units DO NOT WORK as expected, because km/s does not properly cancel the m/s built into Joules! This is a very common mistake

QUERY 31

Converting the previously-found CMB energy density with the c2 value above, find:

$$\text{CMB mass density} = \rho_m = \frac{\text{CMB energy density}}{c^2} = \frac{4.19 \times 10^{-14}\ \cancel{J} / \text{m}^3}{9.0 \times 10^{16}\ \cancel{J} / \text{kg}} = \frac{4.19}{9.0} \times 10^{-30}\ \text{kg} / \text{m}^3$$

$$\rho_m = 4.66 \times 10^{-31}\ \text{kg} / \text{m}^3$$

Lastly, we determine what fracción of the critical density (ρ_{crit}) this value represent

Recall from the question that $\rho_{crit} = 10^{-26}$ kg/m³

Using this value, find:

$$\frac{\rho_m}{\rho_{crit}} = \frac{4.66 \times 10^{-31}\ \cancel{\text{kg} / \text{m}^3}}{10^{-26}\ \cancel{\text{kg} / \text{m}^3}} \rightarrow \boxed{\frac{\rho_m}{\rho_{crit}} = 4.66 \times 10^{-5}}$$