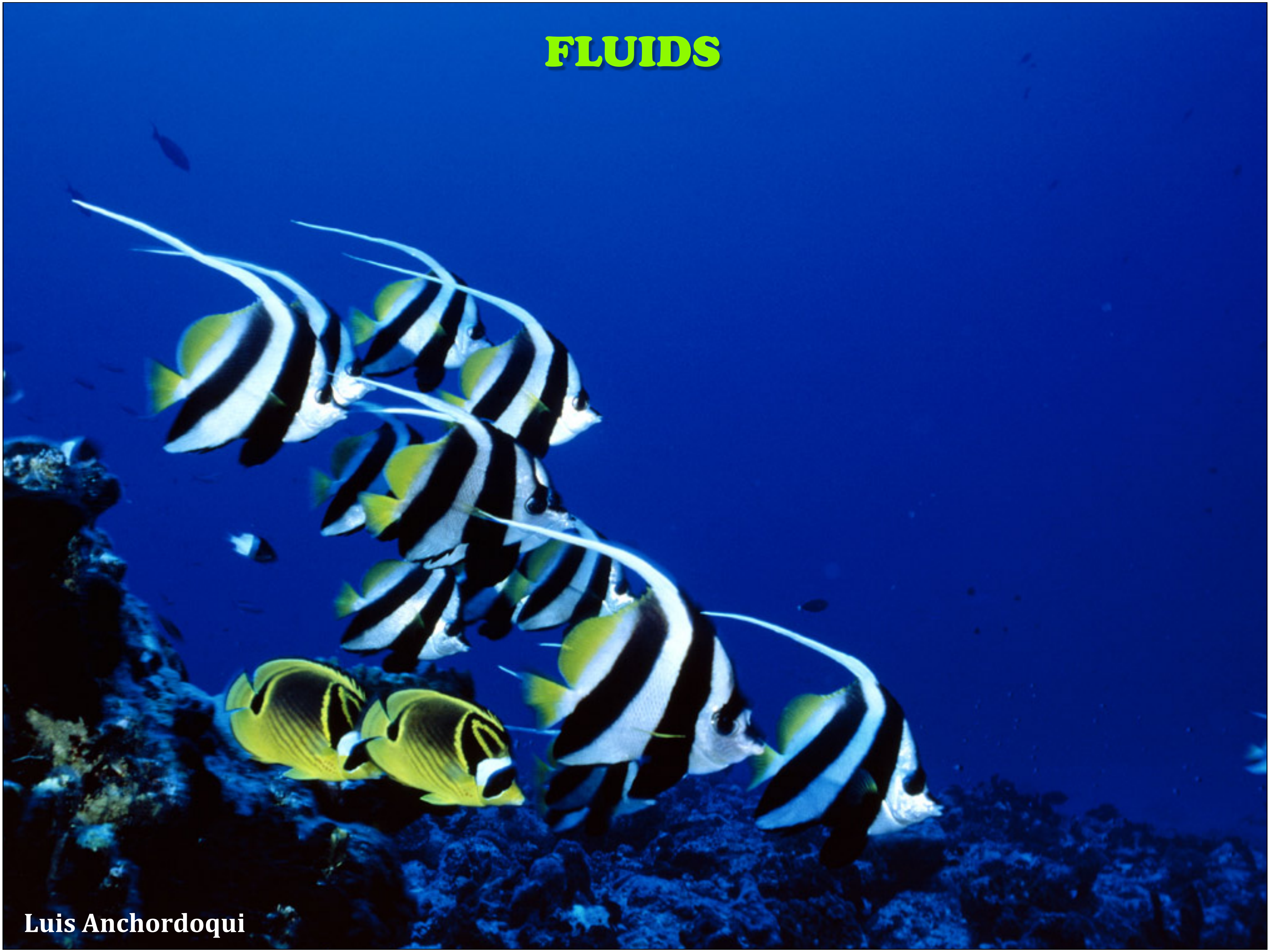
Lesson 10

Physics 168

Luis Anchordoqui


FLUIDS
Luis Anchordoqui

Phases of matter

Three common phases of matter are

- **solid** ☛ has definite shape and size
- **liquid** ☛ has fixed volume but can be any shape
- **gas** ☛ can be any shape and also can be easily compressed

Liquids and gases both flow and are called fluids

Density and Specific Gravity

Density ρ of an object is its mass per unit volume:

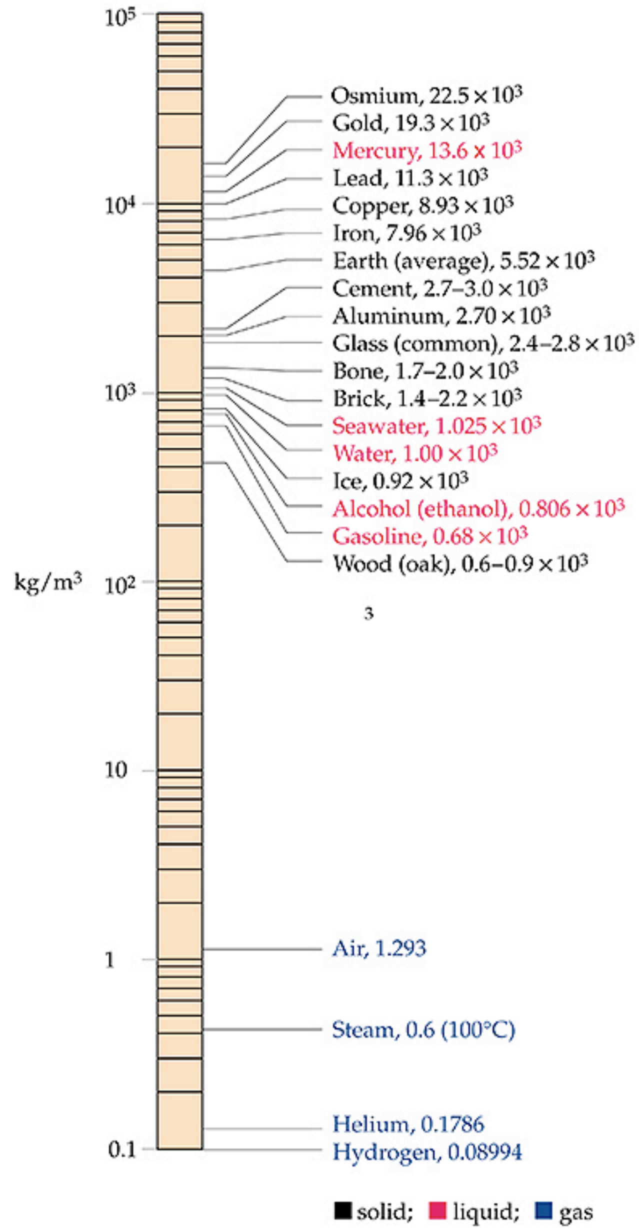
$$\rho = \frac{m}{V}$$

SI unit for density is kg/m^3

Water at $4^\circ\, C$ has a density of $1\, g/cm^3 = 1.000\, kg/m^3$

Specific gravity of a substance ➡ ratio of its density to that of water

Densities of selected substances



Density values exceed five orders of magnitude

Pressure

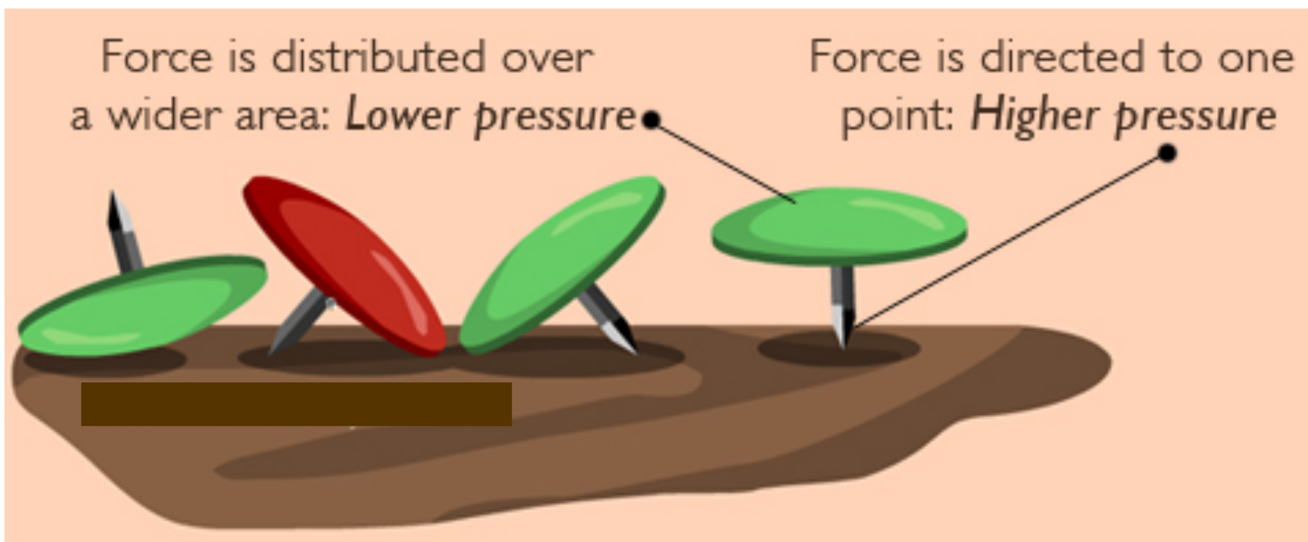
Pressure ☛ force applied perpendicular to surface of an object per unit area over which that force is distributted

$$P = F/A$$

Pressure is a scalar

Units of pressure in SI system are pascals

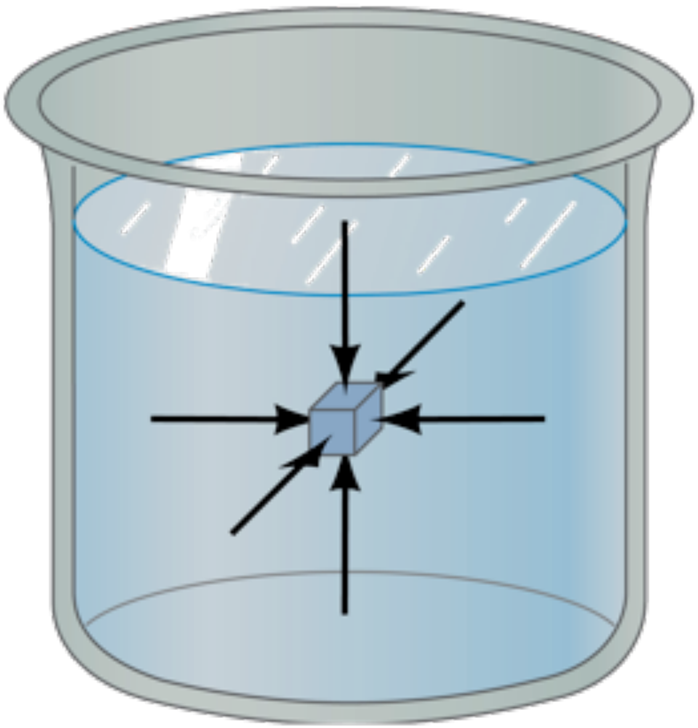
$$1 \text{ Pa} = 1 \text{ N/m}^2$$



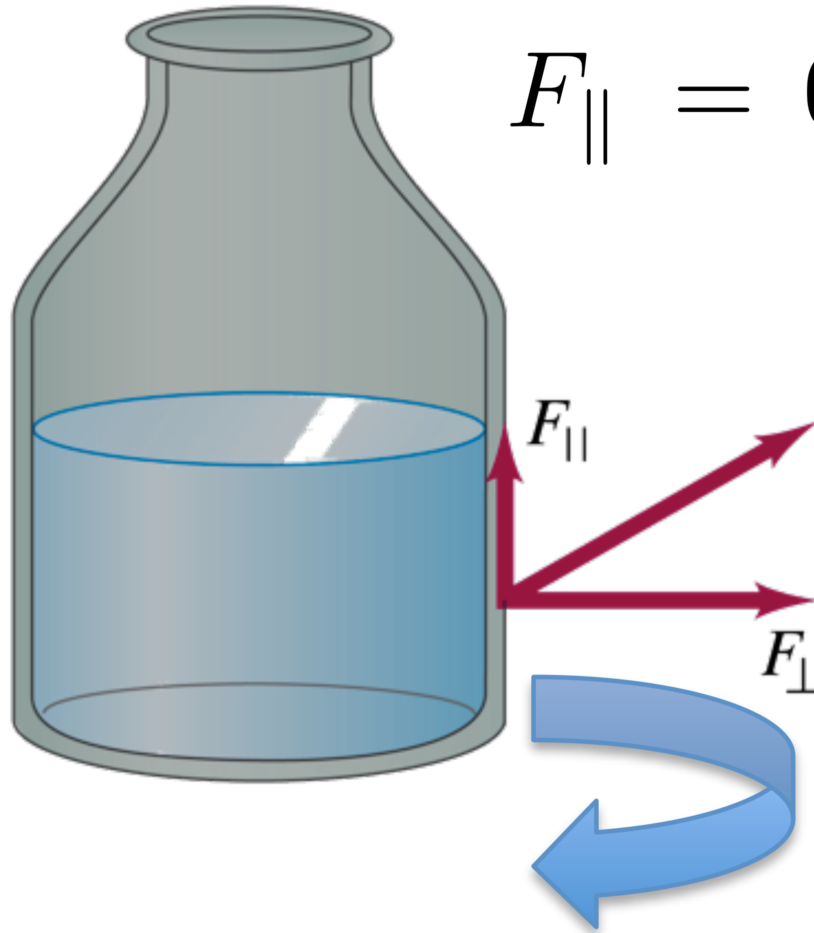
Pressure in Fluids

For fluid at rest

1- Pressure is same in every direction in a fluid at a given depth
if it were not fluid would flow



$$F_{\parallel} = 0$$

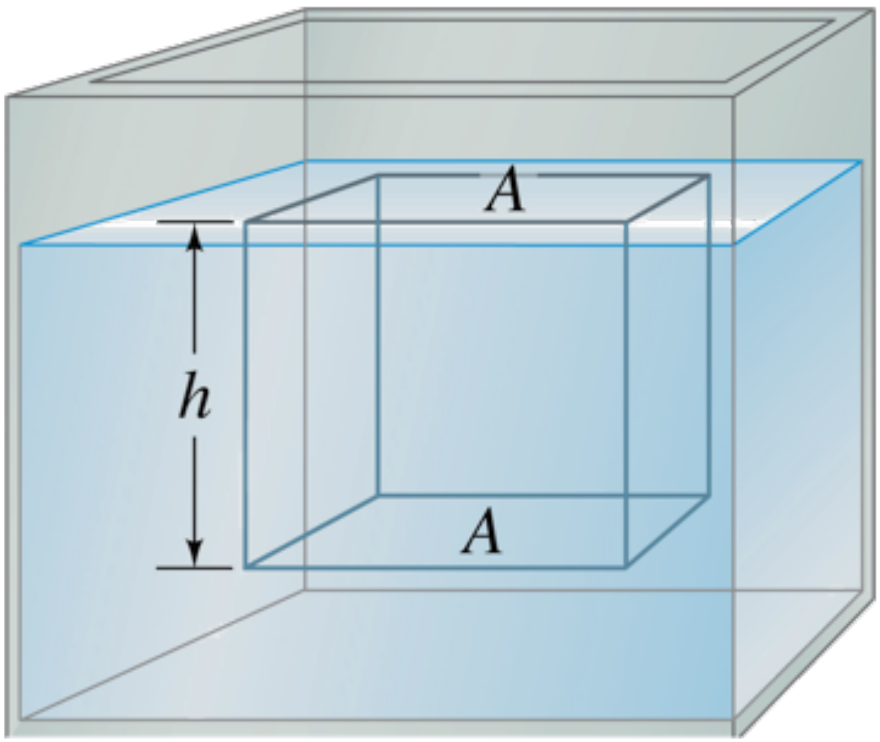


2- There is no component of force parallel to any solid surface
again ☛ if there were fluid would flow

Pressure in Fluids (cont'd)

Pressure at depth h below surface of liquid

is due to weight of liquid above it



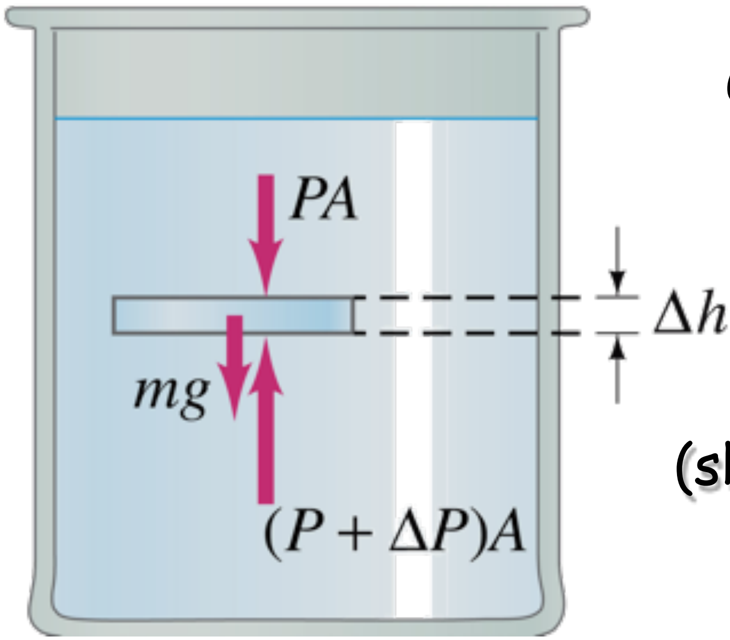
We can quickly calculate :

$$P = mg/A = \rho g h$$

This relation is valid for any liquid whose density does not change with depth

Pressure in Fluids (cont'd)

To a good approximation liquids can be considered incompressible



Gases on other hand are very compressible and density can vary significantly

Forces on a thin slab of fluids (shown as a liquid but it could instead be a gas)

We assume fluid is at rest so net force on slab is zero

$$(P + \Delta P)\,A - PA - \rho A \Delta h g = 0$$

$$\Delta P = \rho g \Delta h$$

$$\rho \approx \text{constant over } \Delta h$$

Atmospheric Pressure and Gauge Pressure

At sea level atmospheric pressure is about

$$P_{\rm atm} = 1.013 \times 10^5 \ {\rm N/m^2}$$

this is called one atmosphere (atm)

Another unit of pressure is bar:

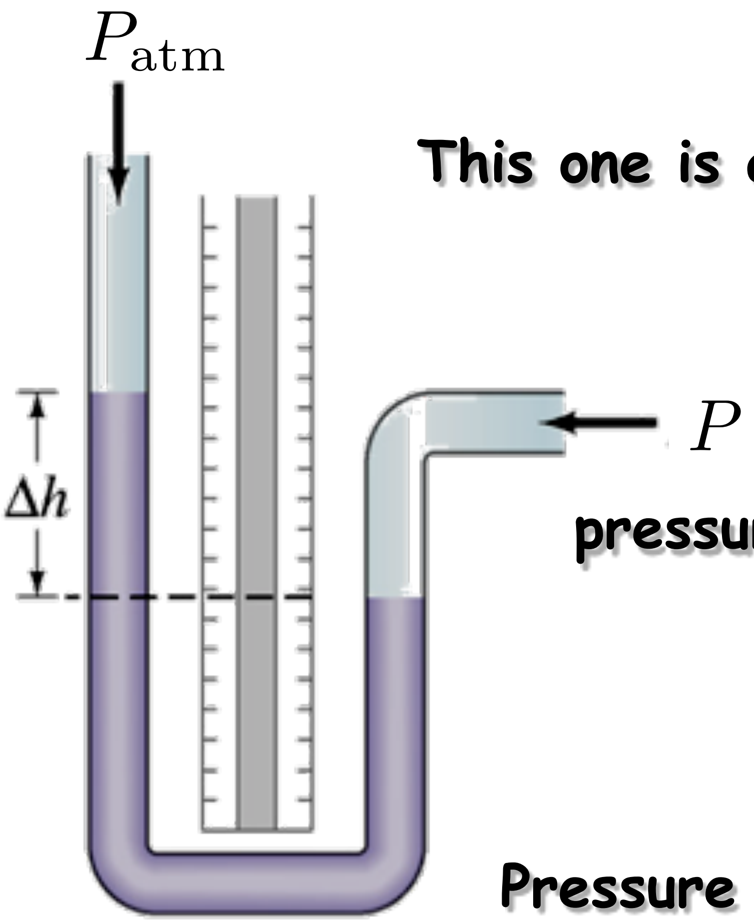
$$1 \ {\rm bar} = 1.00 \times 10^5 \ {\rm N/m^2}$$

Standard atmospheric pressure is just over 1 bar

This pressure does not crush us
because our cells maintain an internal pressure that balances it

Measurement of Pressure manometer

There are a number of different types of pressure gauges



This one is an open-tube manometer

pressure measured

Pressure in open end is atmospheric pressure

Pressure being measured will cause fluid
to rise until pressures on both sides at same height are equal

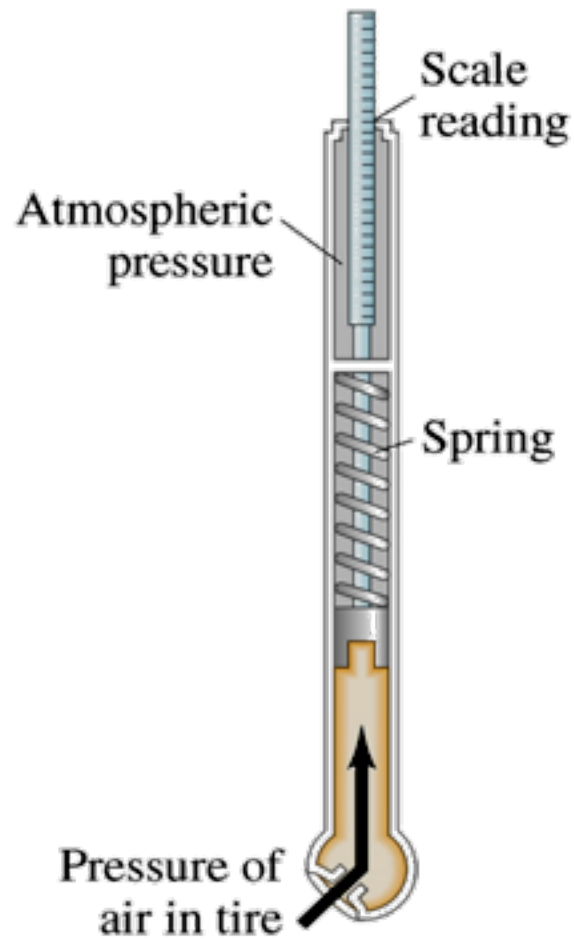
Gauge Pressure

Most pressure gauges measure pressure above atmospheric pressure

this is called **gauge pressure**

Absolute pressure is sum of atmospheric pressure and gauge pressure

$$P = P_{\rm atm} + P_{\rm gauge}$$



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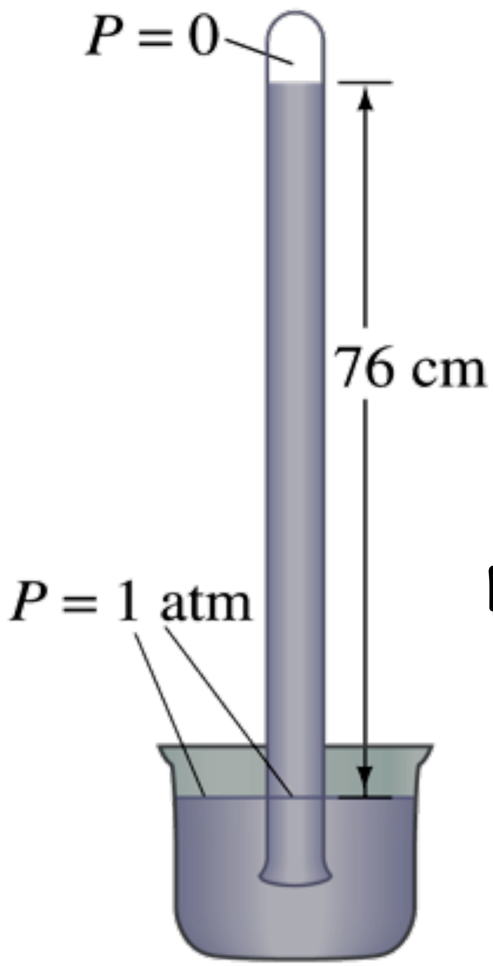
Measurement of Pressure Barometer

This is a mercury barometer

developed by Torricelli to measure atmospheric pressure



Height of column of mercury is such that pressure in tube at surface level is 1 atm

pressure is often quoted in millimeters of mercury

Measurement of Pressure Barometer (Cont'd)

Any liquid can serve in a Torricelli-style barometer

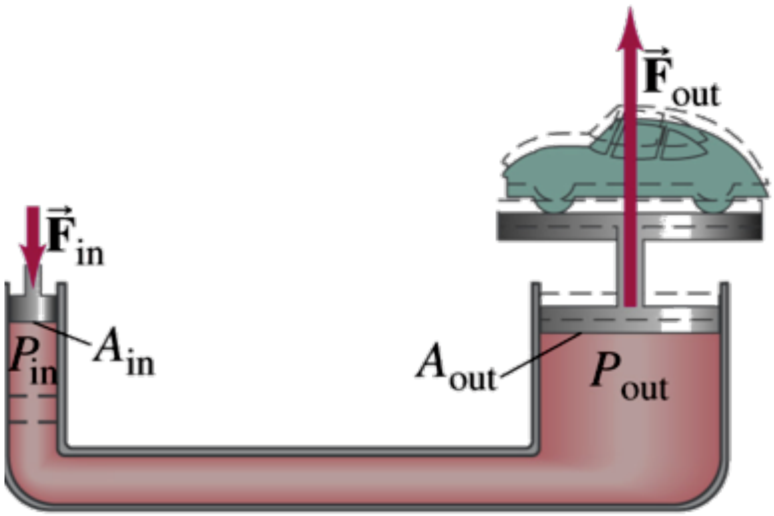
but most dense ones are most convenient

This barometer uses water $10.3\, m$ high!

Pascal's Principle

If an external pressure is applied to a confined fluid
pressure at every point within fluid increases by that amount

This principle is used in hydraulic lifts



$$P_{out} = P_{in}$$

$$\frac{F_{out}}{A_{out}} = \frac{F_{in}}{A_{in}}$$

$$\frac{F_{out}}{F_{in}} = \frac{A_{out}}{A_{in}}$$

Quantity F_{out}/F_{in} is called mechanical advantage of hydraulic lift

For example if area of output piston is 20 times that of input cylinder
force is multiplied by a factor of 20

Force of 200 lb could lift $4,000 \text{ lb}$ car

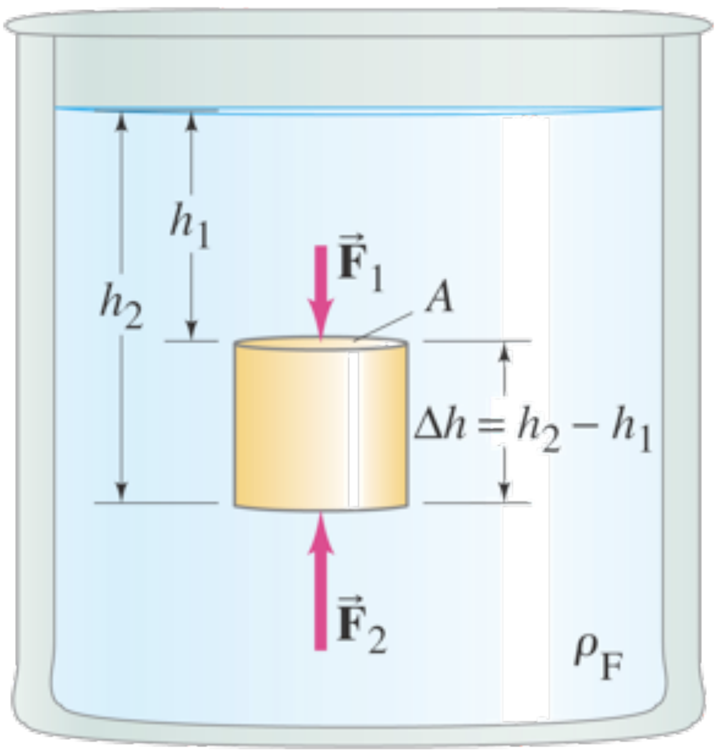
Buoyancy and Archimedes' Principle

Consider an object submerged in a fluid

There is a net force on object

because pressures at top and bottom of it are different

Buoyant force is found to be upward force on same volume of water



$$F_B = F_2 - F_1 = \rho_F \, g \, A (h_2 - h_1)$$

$$= \rho_F \, g \, A \, \Delta h$$

$$= \rho_F \, V \, g$$

$$= m_F \, g$$

Archimedes principle

We can derive Archimedes' principle in general

by following simple but elegant argument

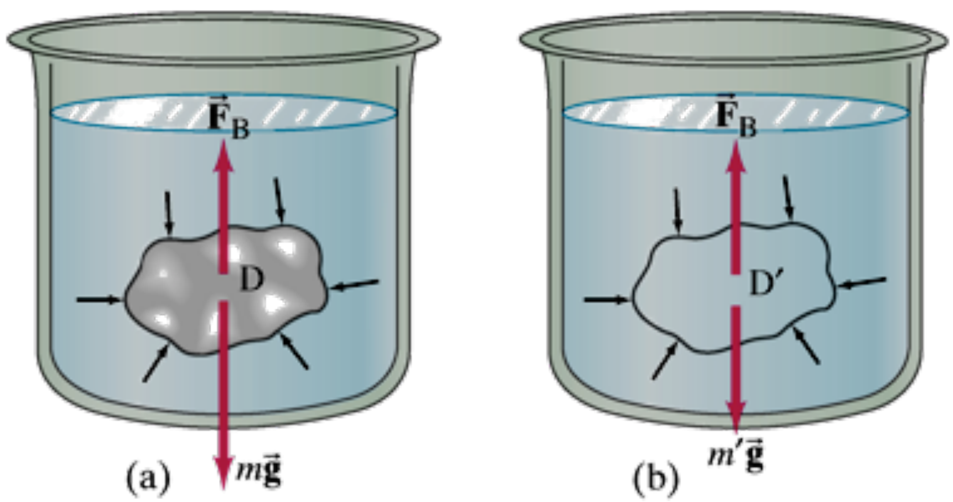
Irregularity shaped object D shown in figure

is acted on by force of gravity and buoyant force

To determine buoyant force we next consider a body D' this time made of fluid itself with shape and size of original object and located at same depth

You might think of this body of fluid as being separated from rest of fluid

by an imaginary membrane

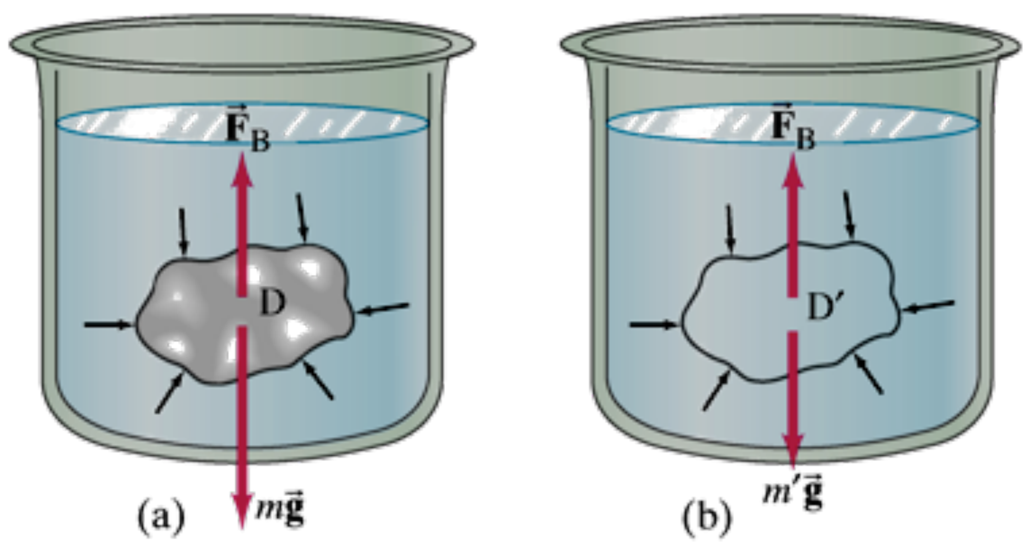


Archimedes principle (Cont'd)

Buoyant force on this body of fluid will be exactly same as that on original object since surrounding fluid which exerts F is in exactly same configuration

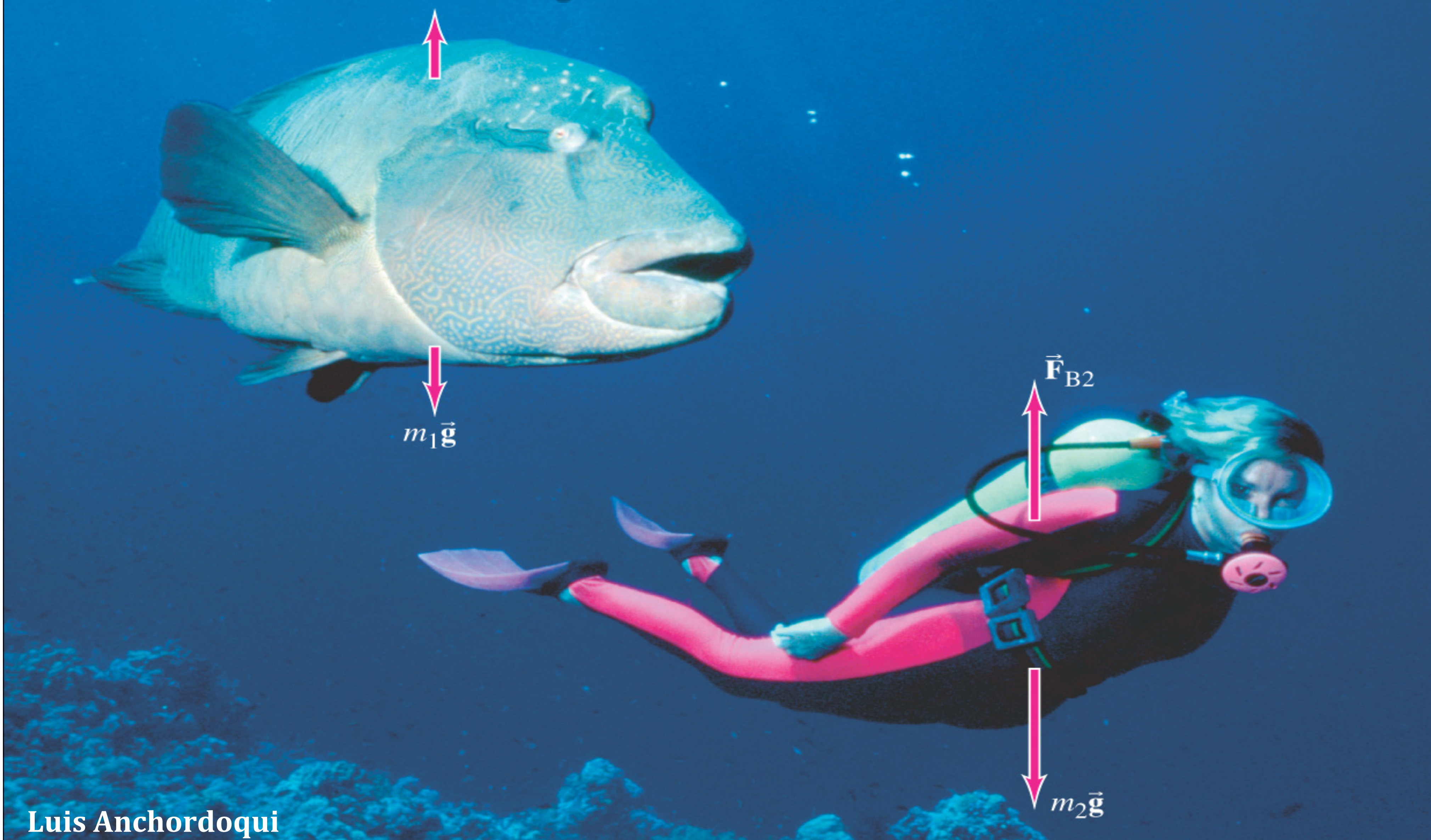
Body of fluid D' is in equilibrium

Buoyant force is equal to weight of body of fluid whose volume equals volume of original submerged object



Buoyancy and Archimedes' principle

Net force on object is then difference between buoyant force and gravitational force



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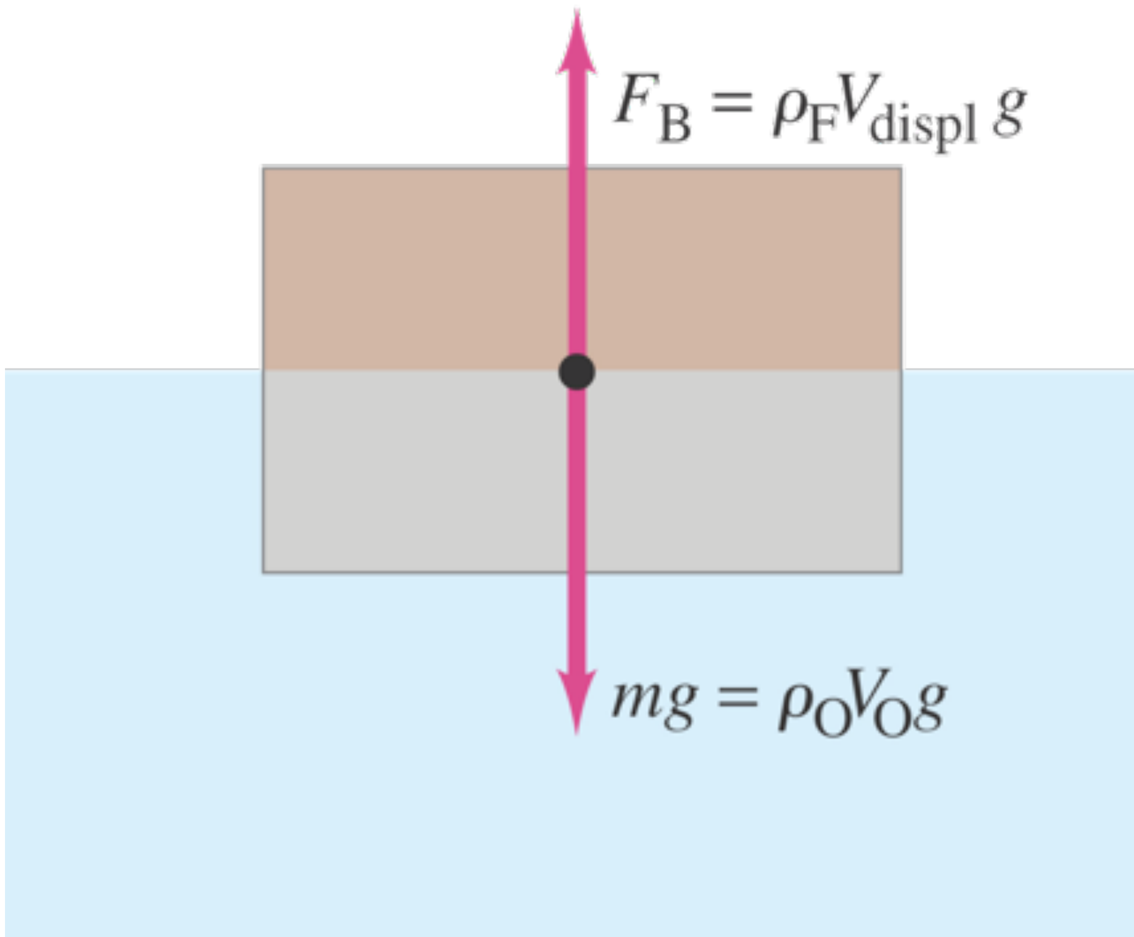
Buoyancy and Archimedes' principle (cont'd)

For a floating object

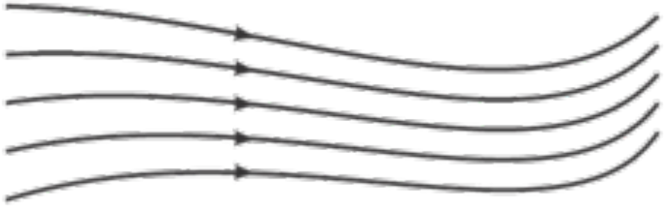
fraction that is submerged

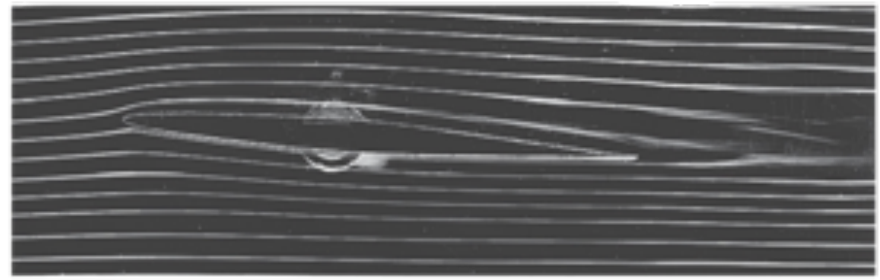
is given by ratio of object's density to that of fluid



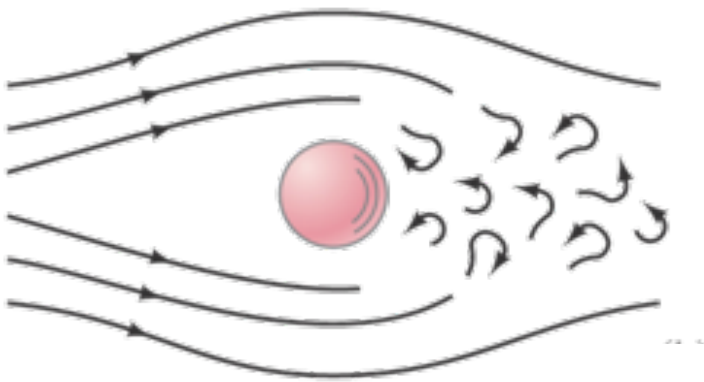
Fluids in Motion
Flow Rate and Equation of Continuity

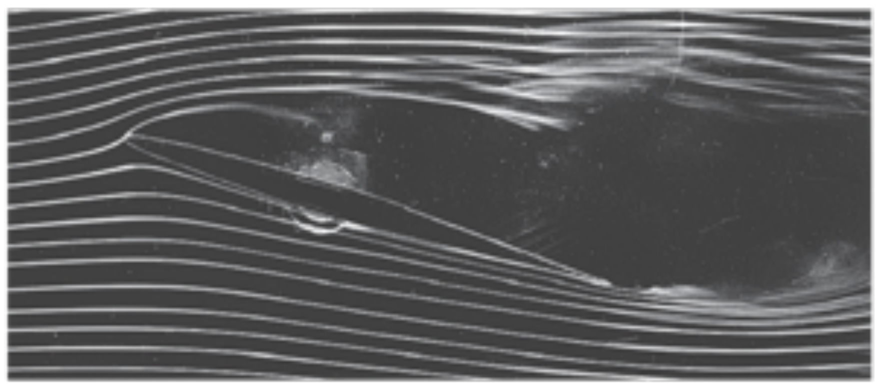
If flow of fluid is smooth ☛ it is called streamline or laminar flow





Above a certain speed ☛ flow becomes turbulent





Fluids in Motion
Flow Rate and Equation of Continuity

We will deal with laminar flow

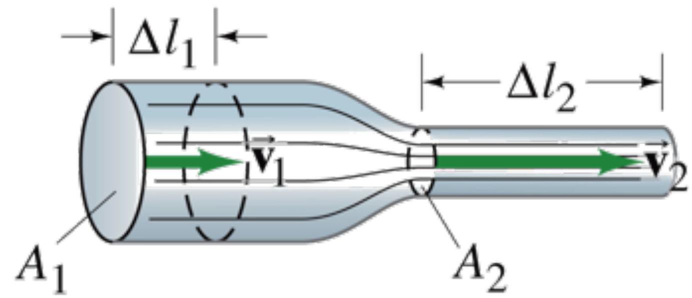
Mass flow rate is mass that passes a given point per unit time

Flow rates at any two points must be equal
as long as no fluid is being added or taken away

This gives us equation of continuity

Equation continuty

Consider a steady laminar flow of a fluid through an enclosed pipe



$$\text{Mass flow rate} = \frac{\Delta m}{\Delta t}$$

Volume of fluid passing point 1 (that is through area A_1) in a time Δt is

$$\Delta V_1 = A_1 \underbrace{\Delta l_1}$$

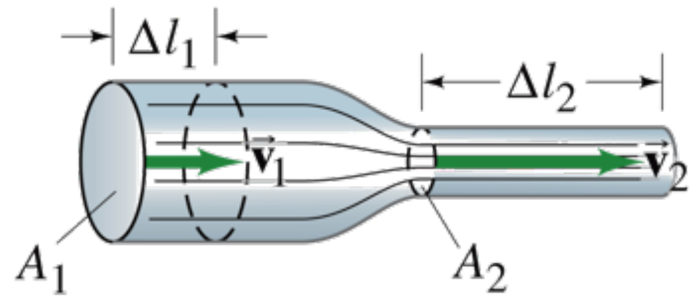
Distance fluid moves in time Δt

Since velocity of fluid passing through point 1 is $v_1 = \Delta l_1 / \Delta t$

$$\frac{\Delta m_1}{\Delta t} = \frac{\rho_1 \Delta V_1}{\Delta t} = \frac{\rho_1 A_1 \Delta l_1}{\Delta t} = \rho_1 A_1 v_1$$

Equation continuty (cont'd)

Similarly at point 2 flow rate is ☛ $$\frac{\Delta m_2}{\Delta t} = \rho_2 A_2 v_2$$



Since no fluid flows in or out sides flow rates through A_1 and A_2 must be equal

$$\frac{\Delta m_1}{\Delta t} = \frac{\Delta m_2}{\Delta t}$$

$$\rho_1 \, A_1 \, v_1 = \rho_2 \, A_2 \, v_2$$

If $\rho =$ constant continuity equation becomes

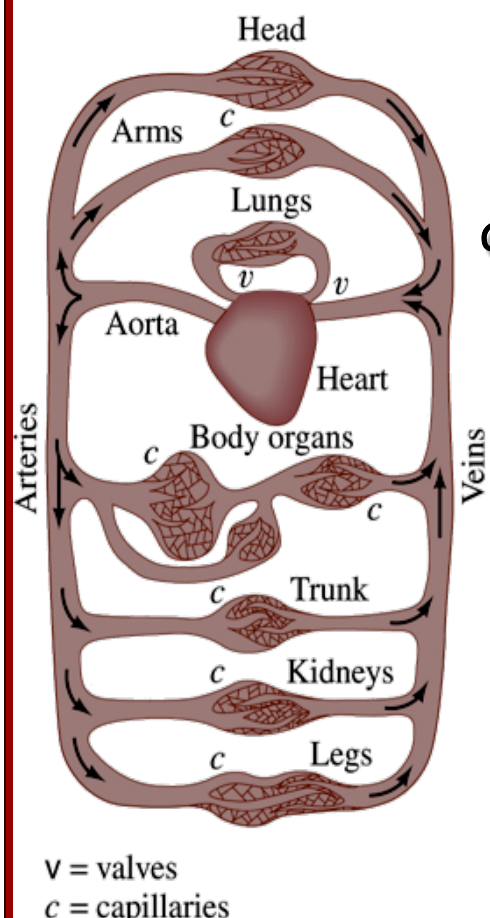
$$A_1 \, v_1 = A_2 \, v_2$$

In humans ☛ blood flows from heart into aorta

from which it passes into major arteries

These branch into small arteries (arterioles)

which in turn branch into myriads of tiny capilars



Blood returns to heart via veins

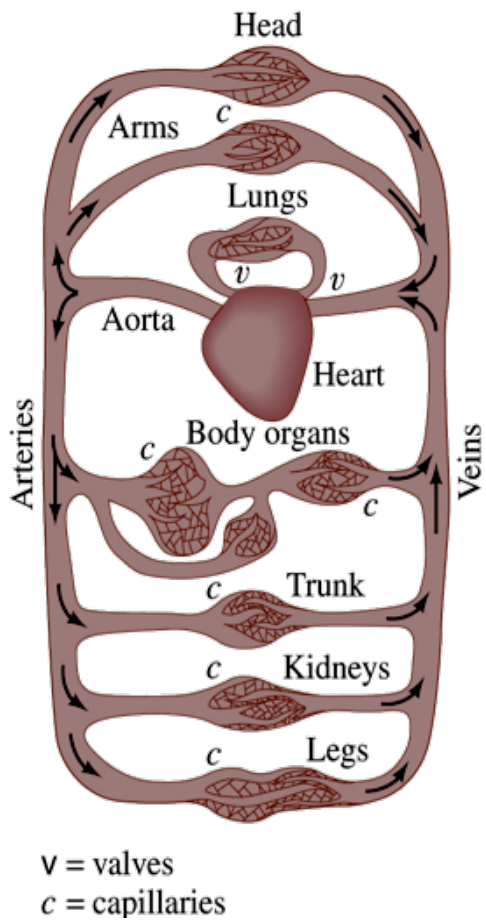
Radius of aorta is about 1.2 cm ,

and blood passing through it has a speed of about 40 cm/s

A typical capilar has a radius of about 0.0004 cm

and blood flows through it at a speed of about 0.0005 m/s

Estimate number of capillars that are in body

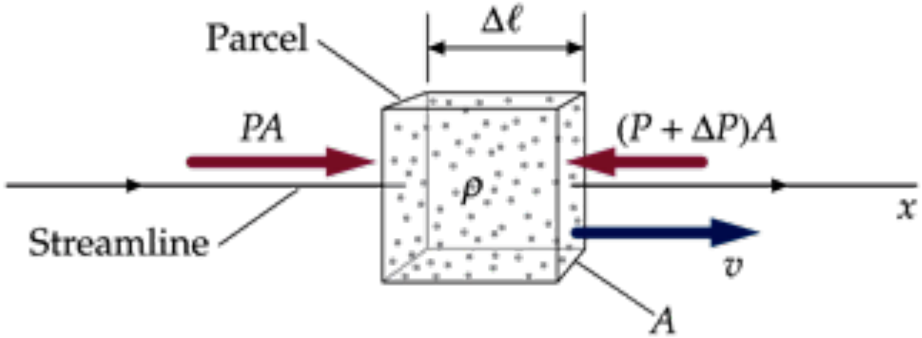


$$v_2 A_2 = v_1 A_1 \Rightarrow v_2 N \pi r_{\rm cap}^2 = v_1 \pi r_{\rm aorta}^2$$

$$N = \frac{v_1 \times r_{\rm aorta}^2}{v_2 \times r_{\rm cap}^2} = 7 \times 10^9$$

Bernoulli's equation

Consider a small parcel of air moving along a streamline



into a region of reduced pressure

$$F = m\,\frac{dv}{dt}$$

$$F = PA - (P + \Delta P)\,A = -A\Delta P$$

Parcel is so small that ΔP can be accurately expressed using differential approximation

$$\frac{\Delta P}{\Delta l} = \frac{dP}{dx} \Rightarrow \Delta P = \frac{dP}{dx}\,\Delta l$$

Substituting

$$-A\,\frac{dP}{dx}\,\Delta l = \rho\,A\,\Delta l\,\frac{dv}{dt}$$

$$dP = -\rho\,\frac{dv}{dt}\,dx = -\rho\,v\,d\,v$$

Bernoulli's equation (Cont'd)

Integrating both sides

$$\int_{P_1}^{P_2} dP = -\rho \int_{v_1}^{v_2} v\, dv$$

We obtain Bernoulli equation

$$P_2 - P_1 = \frac{1}{2}\rho\, v_1^2 - \frac{1}{2}\rho\, v_2^2$$

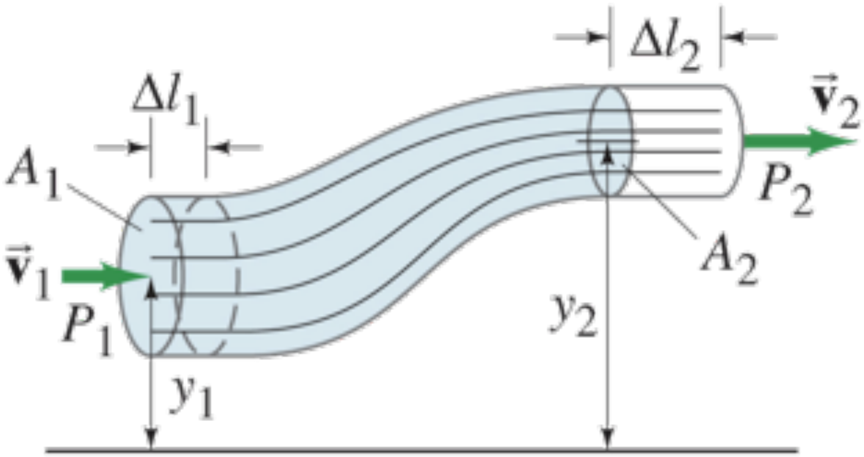
or equivalently

$$P_2 + \frac{1}{2}\rho\, v_2^2 = P_1 + \frac{1}{2}\rho v_1^2$$

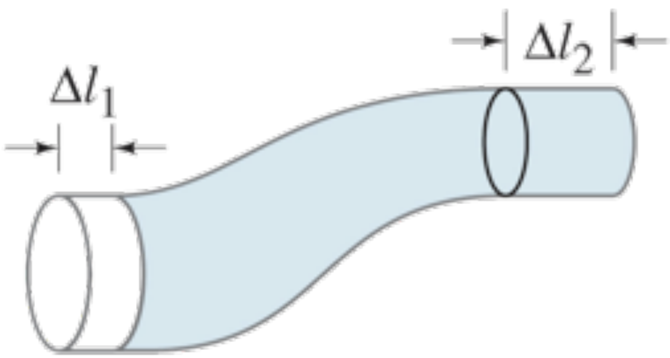
Bernoulli's equation (Cont'd)

A fluid can also change its height

By looking at work done as it moves we find:



$$P + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$$



Bernoulli's equation tells us that

as speed goes up ☛ pressure goes down

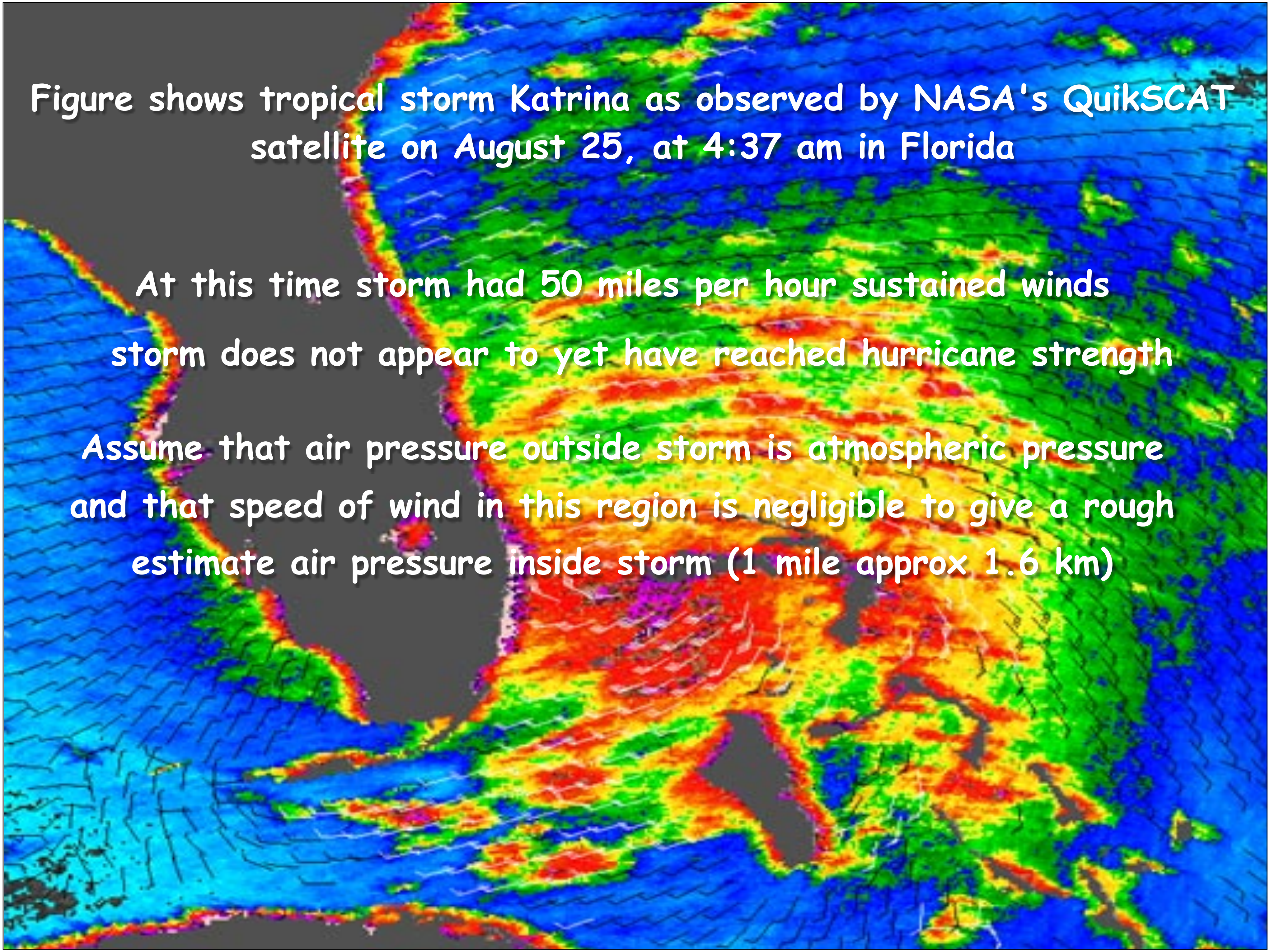
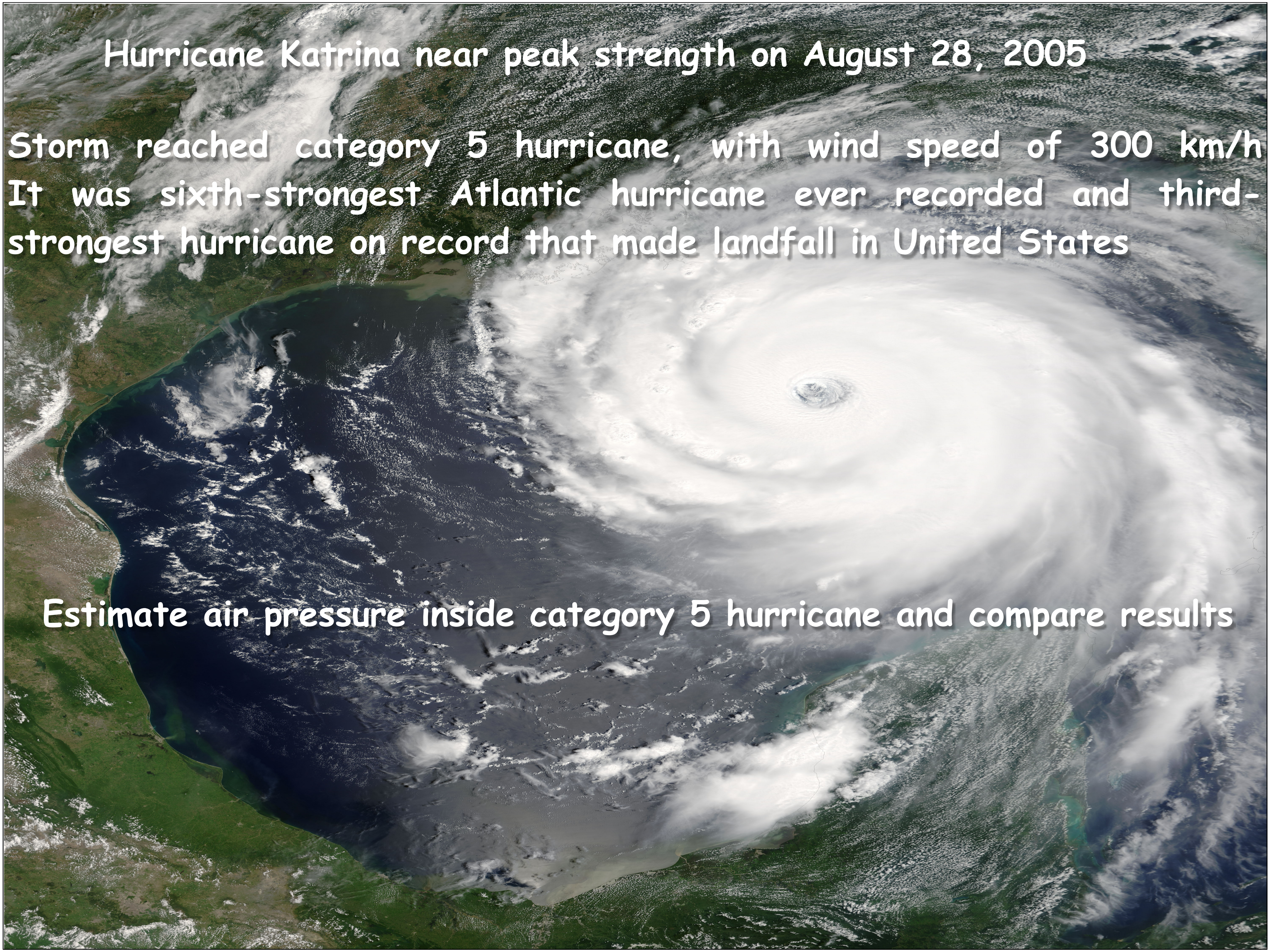


Figure shows tropical storm Katrina as observed by NASA's QuikSCAT satellite on August 25, at 4:37 am in Florida

At this time storm had 50 miles per hour sustained winds

storm does not appear to yet have reached hurricane strength

Assume that air pressure outside storm is atmospheric pressure and that speed of wind in this region is negligible to give a rough estimate air pressure inside storm (1 mile approx 1.6 km)


Hurricane Katrina near peak strength on August 28, 2005
Storm reached category 5 hurricane, with wind speed of 300 km/h
It was sixth-strongest Atlantic hurricane ever recorded and third-strongest hurricane on record that made landfall in United States
Estimate air pressure inside category 5 hurricane and compare results

Applications of Bernoulli's Principle: Hurricanes

$$P_{\text{inside}} + \frac{1}{2}\rho v_{\text{inside}}^2 + \rho g y_{\text{inside}} = P_{\text{outside}} + \frac{1}{2}\rho v_{\text{outside}}^2 + \rho g y_{\text{outside}}$$

measurement taken @ same altitude

$$y_{\text{inside}} = y_{\text{outside}}$$

$$v_{\text{outside}} \approx 0$$

storm $P_{\text{inside}} = 1.013 \times 10^5 \text{ Pa}$

hurricane $P_{\text{inside}} = 9.7 \times 10^4 \text{ Pa} \approx 0.96 \text{ atm}$

Viscosity

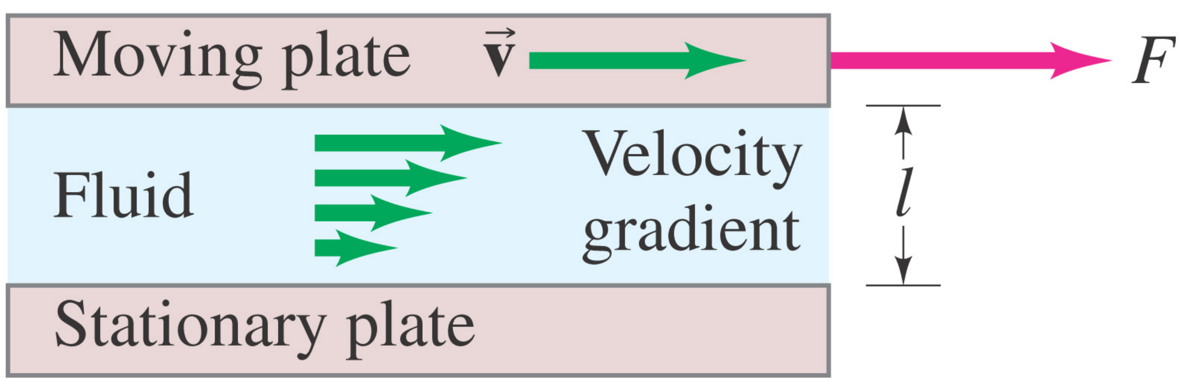
Real fluids have some internal friction called viscosity

Viscosity can be measured

It is found from relation

$$F = \eta A \frac{v}{l}$$

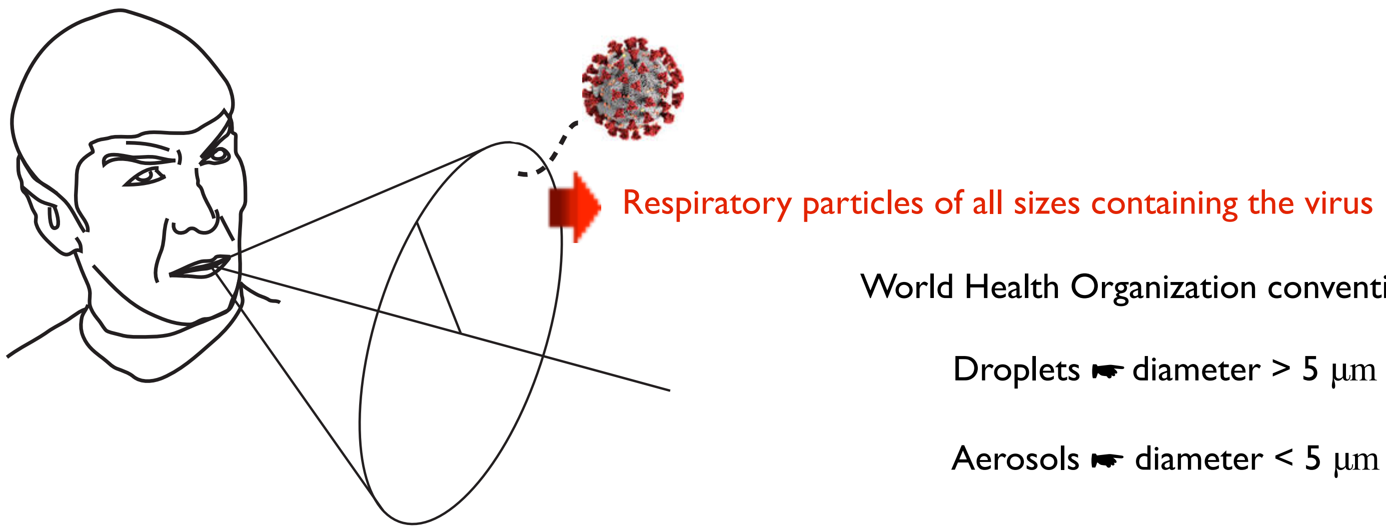
η → coefficient of viscosity



Coefficients of Viscosity for Various Fluids

Fluid	t, °C	η, mPa · s
Water	0	1.8
	20	1.00
	60	0.65
Blood (whole)	37	4.0
Engine oil (SAE 10W)	30	200
Glycerin	0	10,000
	20	1,410
	60	81
Air	20	0.018

Coronavirus Airborne Transmission



World Health Organization convention

Droplets ☛ diameter > 5 μm

Aerosols ☛ diameter < 5 μm

Two major questions of 2020 pandemic have been:

1) How long does it take for virus-containing droplet of given size to fall to ground by gravity to potentially contaminate surface?

2) For given relative humidity ☛ how much time does it take for water evaporation to reduce virus-containing droplet to size that leaves it floating in air for sufficiently long time to allow direct transmission of virus to another person?

Answer to first question is easily obtained by simply equating gravitational and Stokesian viscous forces on falling object to obtain its terminal velocity

Consider spherical particle of radius R moving with velocity v through air

aerodynamic drag force

$$F_{\text{Stokes}} = 6\pi\eta R v$$

air viscosity $@\ 25^\circ\text{C} \Rightarrow \eta \simeq 1.86 \times 10^{-8}\ \text{g} \cdot \mu\text{m}^{-1} \cdot \text{s}^{-1}$

counterbalanced by excess of gravitational attraction over air buoyancy force

$$F_g - F_b = \frac{4}{3}\pi R^3 (\rho_{\text{H}_2\text{O}} - \rho_{\text{air}}) g$$

$$\rho_{\text{H}_2\text{O}} \gg \rho_{\text{air}}$$

$$\rho_{\text{H}_2\text{O}} = 10^{-12}\ \text{g}/\mu\text{m}^3 \qquad g = 9.8 \times 10^6\ \mu\text{m}/\text{s}^2$$

terminal velocity

$$v_{\text{terminal}} = \frac{2}{9}\frac{R^2\ \rho_{\text{H}_2\text{O}}\ g}{\eta}$$

mean time for particle to reach ground from height z_0

$$\tau_{\text{sed}} = \frac{9}{2}\frac{\eta\ z_0}{R^2\ \rho_{\text{H}_2\text{O}}\ g}$$

Evaporation rate of respiratory particles is proportional to exposed surface area

Time it takes for complete evaporation of pure water droplet/aerosol of initial radius R_0

$$\tau_{\rm ev} = \frac{R_0^2}{\xi(1-\zeta_{\rm RH})}$$

$@\ 25^\circ{\rm C} \Rightarrow \xi = 4.2 \times 10^2 \mu{\rm m}^2/{\rm s}$ $\zeta_{\rm RH}$ ☛ relative humidity

Mean time for droplet/aerosol of initial R_0 to shrink to $R_{\rm eq} = R_0/3$ from water evaporation

$$t(R_{\rm eq}) \approx \frac{R_0^2 - R_{\rm eq}^2}{\xi(1-\zeta_{\rm RH})}$$

Critical initial radius for which evaporation and settling times are equal

$$t(R_{\rm eq}) = \tau_{\rm sed} \Rightarrow R_0^{\rm crit} = \left[\frac{81}{16}\frac{\eta z_0 \xi(1-\xi_{\rm RH})}{\rho_{\rm H_2O}\ g}\right]^{1/4}$$

$$\text{For } \zeta_{\rm RH} = 0.5 \text{ and } z_0 = 1.5 \text{ m} \Rightarrow R_0^{\rm crit} \simeq 42\ \mu{\rm m}$$

This means that droplets with radii > 42 μm will fall to the ground before drying out whereas droplets/aerosols with radii < 42 μm will remain floating in the air in a dry state

Coronavirus Airborne Infection

$R_0(\mu m)$	$\varkappa(\text{virions/min})$	$t(R_{eq})(\text{min})$	$\tau_{sed}(R_0)(\text{min})$	$\tau_{sed}(R_{eq})(\text{min})$
1	3	7×10^{-5}	200	2×10^{3}
3	80	6×10^{-4}	20	200
5	400	2×10^{-3}	8	80
10	3×10^{3}	7×10^{-3}	2	20
20	2×10^{4}	3×10^{-2}	0.5	5
40	2×10^{5}	0.1	0.1	1

$$\varkappa = \frac{4}{3}\pi R_0^3 a\, b$$

$$a = 10^5 \text{ particles/min}$$

doi:10.1056/NEJMc2007800

doi:10.1073/pnas.2006874117

$$b = 7 \times 10^{-6} \text{ virions}/\mu\text{m}^3$$

doi:10.1038/s41586-020-2196-x

Number of virions required for infection ☛ unknown

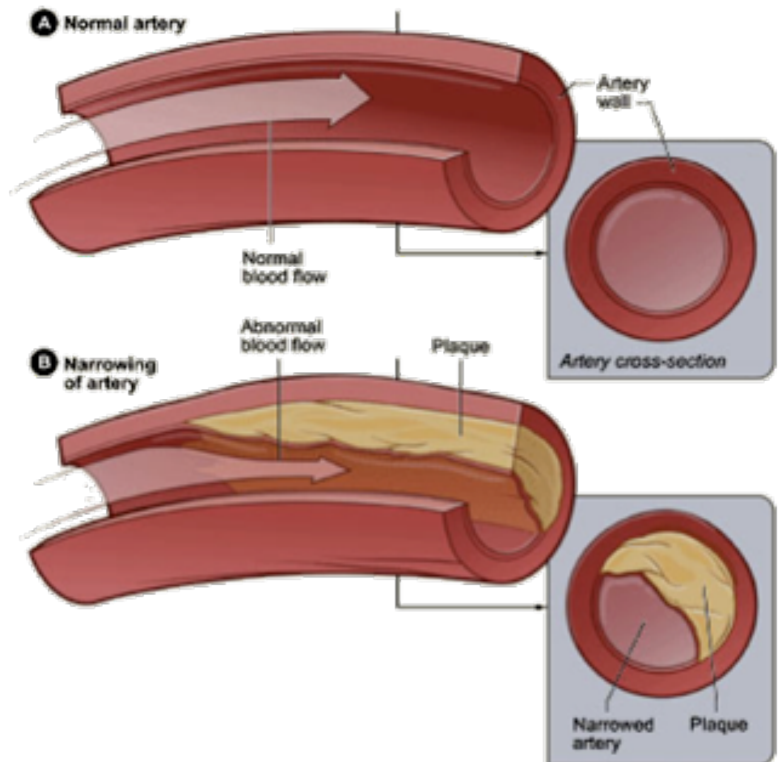
Poiseuille's Equation

Rate of flow in a fluid in a round tube depends on viscosity of fluid pressure difference and dimensions of tube

Volume flow rate is proportional to pressure difference inversely proportional to length of tube L and proportional to fourth power of radius R of tube

$$Q = \frac{\pi R^4 (P_1 - P_2)}{8 \eta L}$$

If cholesterol build-up reduces diameter of an artery by 15% what will be effect on blood flow?



$$\frac{Q_{\text{final}}}{R^4_{\text{final}}} = \frac{Q_{\text{initial}}}{R^4_{\text{initial}}} \Rightarrow \frac{Q_{\text{final}}}{Q_{\text{initial}}} = \frac{R^4_{\text{final}}}{R^4_{\text{initial}}} = 0.85^4 = 0.52$$

Flow rate is 52% of original value

Turbulence: Reynolds Number

When flow speed of a fluid becomes sufficiently great laminar flows breaks down and turbulence flow sets in

Critical speed above which flow through a tube is turbulent depends on density and viscosity of fluid and on radius of tube

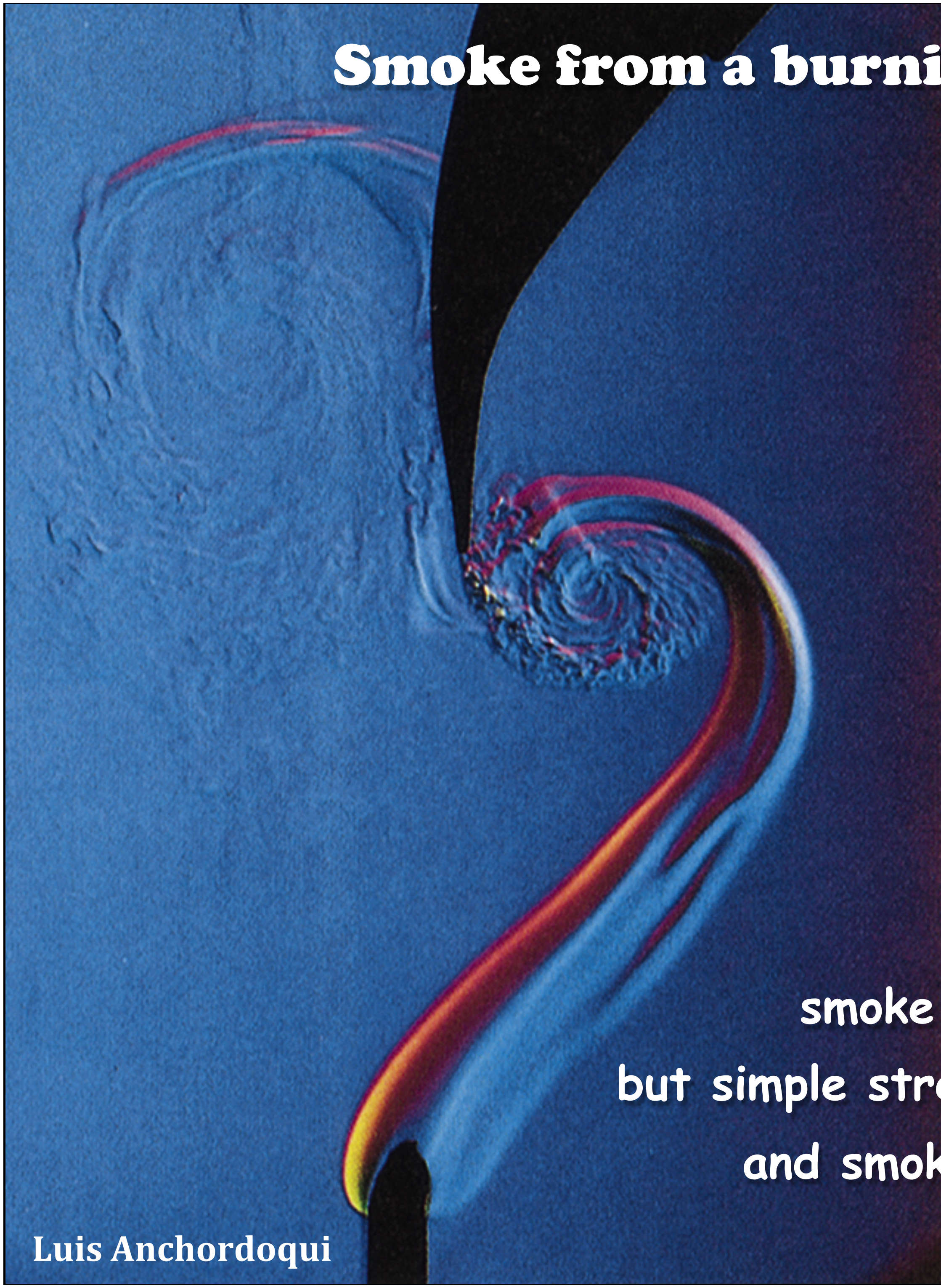
Flow of a fluid can be characterized by a dimensionless number called Reynolds number

$$N_R = \frac{2r\,\rho v}{\eta}$$

Experiments have shown that

flow will remain laminar if Reynolds number is less than about 2000 and turbulent if it is greater than 3000

Smoke from a burning cigarrete



At first

smoke rises in a regular stream

but simple streamline quickly becomes turbulent

and smoke begins to swirl irregularly

Luis Anchordoqui