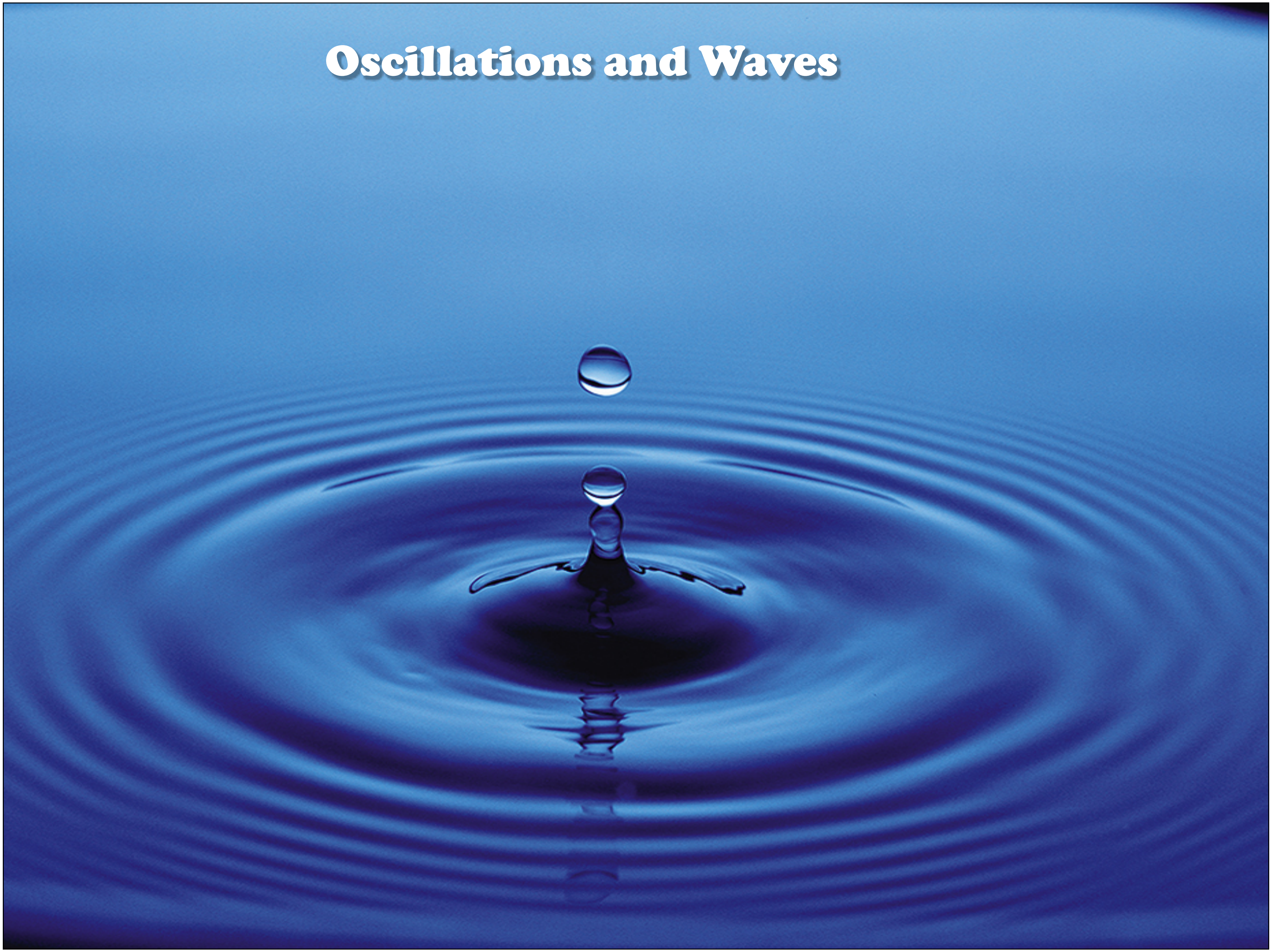
Lesson 11

Physics 168

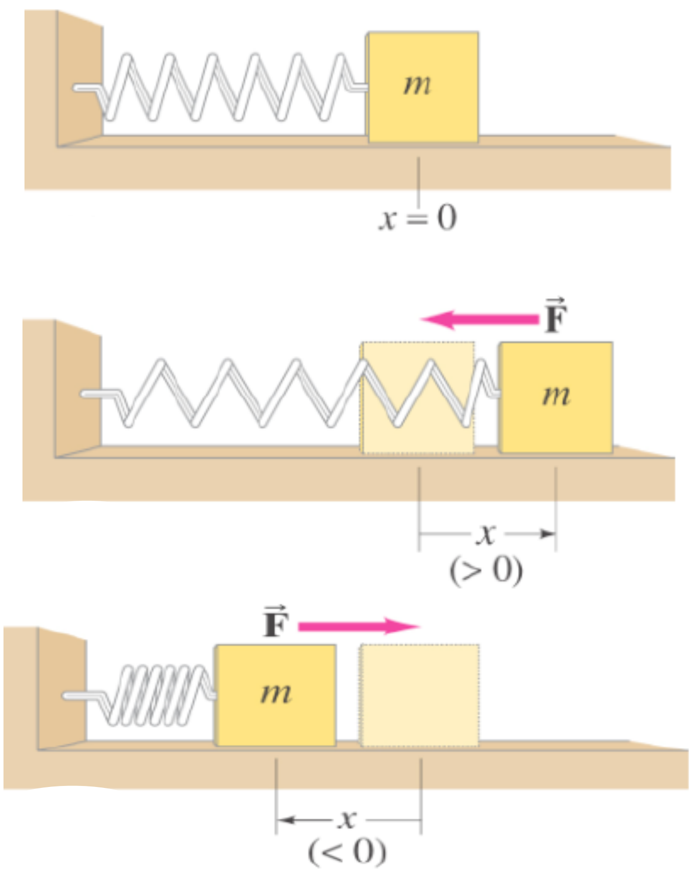
Luis Anchordoqui


Oscillations and Waves

Simple harmonic motion

If object vibrates or oscillates back and forth over same path each cycle taking same amount of time ☛ motion is called periodic

Mass and spring system is useful model of periodic system



Simple harmonic motion (cont'd)

We assume that surface is frictionless

There is point where spring is neither stretched nor compressed

⇩

Equilibrium position

We measure displacement from that point ($x = 0$)

Force exerted by spring depends on displacement

$$F = -kx$$

Simple harmonic motion (cont'd)

Minus sign on force indicates that it is restoring force

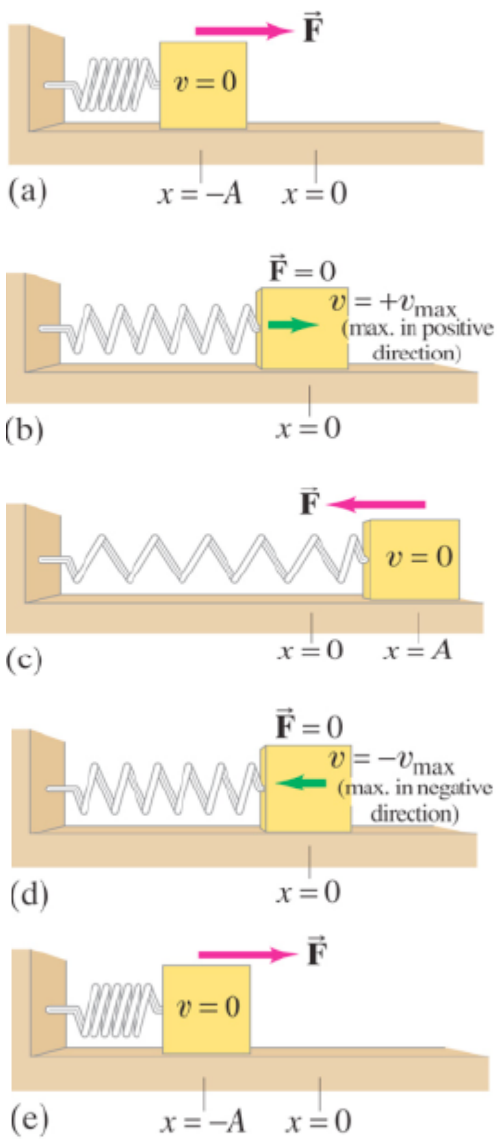
⇩

it is directed to restore mass to its equilibrium position

k **is spring constant**

Force is not constant so acceleration is not constant either

Simple harmonic motion (cont'd)



Displacement ☛ measured from equilibrium point

Amplitude ☛ maximum displacement

Cycle ☛ full to-and-from motion

Period ☛ time required to complete one cycle

Frequency ☛ number of cycles completed per second

Simple harmonic motion (cont'd)

Any vibrating system where restoring force

is proportional to negative of displacement

is in simple harmonic motion (SHM)

and is often called a simple harmonic oscillator

We know that potential energy of a spring is given by

$$PE = \frac{1}{2} k x^2$$

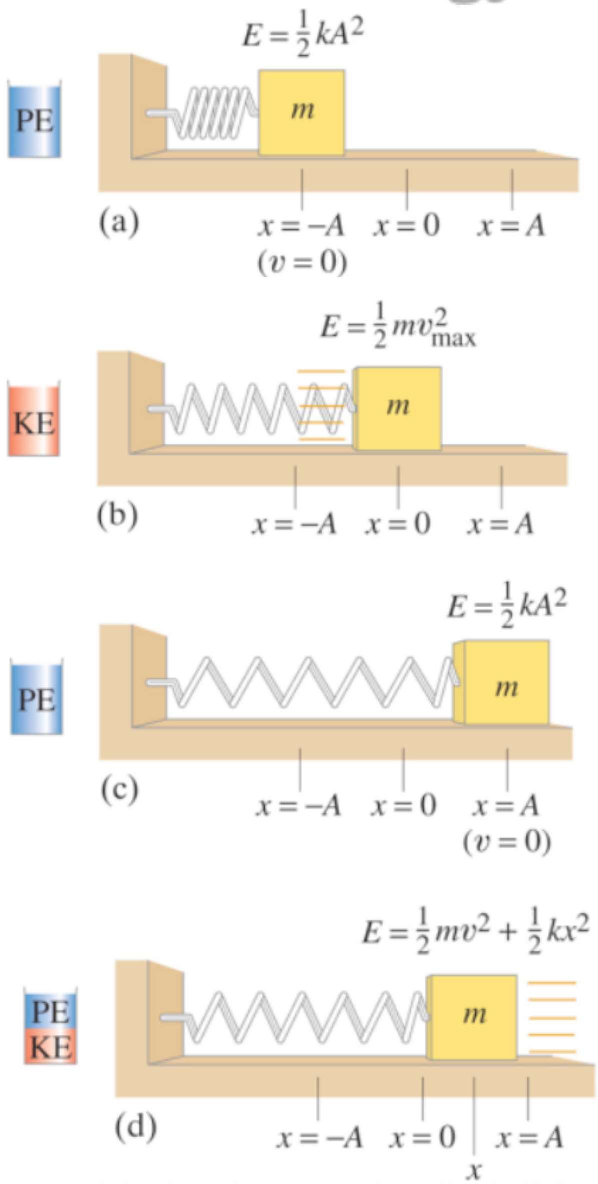
Total mechanical energy is then

$$E = \frac{1}{2} m v^2 + \frac{1}{2} k x^2$$

Total mechanical energy will be conserved

as we are assuming system is frictionless

Energy in simple harmonic oscillator



If mass is at limits of its motion

energy is all potential

If mass is at equilibrium point

energy is all kinetic

We know what potential energy

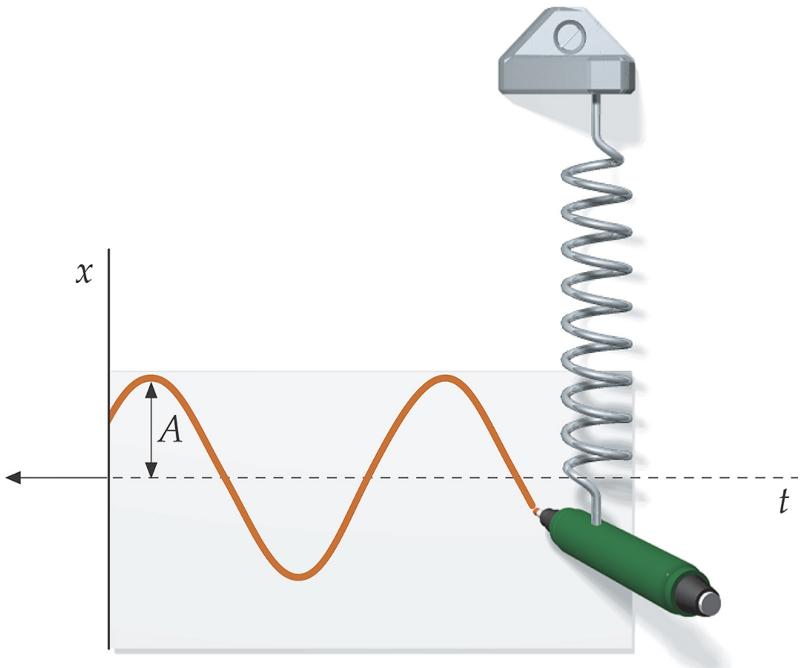
is at turning points

Period and sinusoidal nature of SHM

Figure shows how to get experimentally x versus t for mass on spring

A marking pen is attached to mass on spring and paper is pulled to left

As paper moves with constant speed pen traces out displacement x as function of time



General equation for such curve is

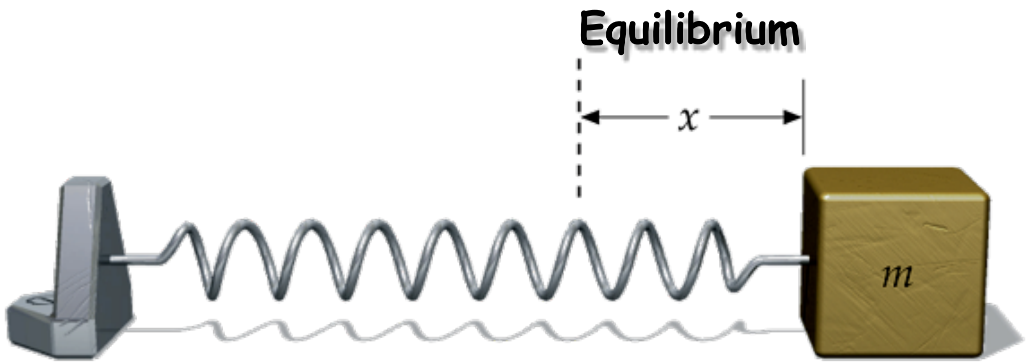
$$x = A\cos(\omega t + \delta)$$

$\omega \rightarrow 2\pi f$

$\delta \rightarrow$ Phase constant

Sinusoidal Nature of SHM

Consider an object on spring on frictionless surface



$$F_x = -kx$$

Using Newton's second law

$$m\frac{d^2x}{dt^2} = -kx$$

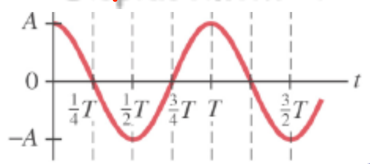
General solution is

$$x = A\cos(\omega t + \delta)$$

Sinusoidal Nature of SHM (cont'd)

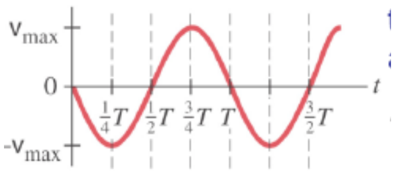
Velocity and acceleration can be calculated as function of time

Displacement x



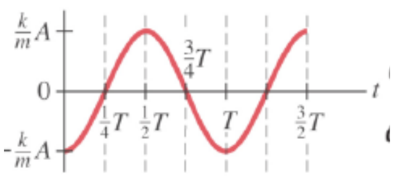
$$v = -v_{\max} \sin \omega t$$

$$v_{\max} = A\,(k/m)^{\frac{1}{2}}$$



Velocity v

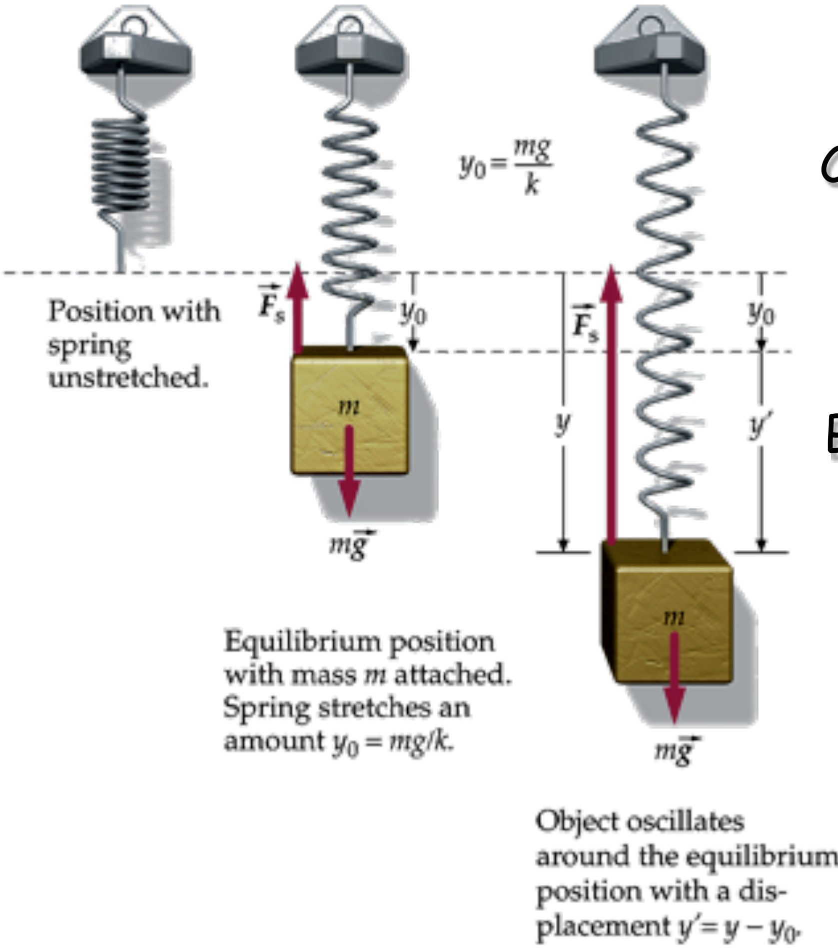
$$a = -a_{\max} \cos\left(2\pi\, t/T\right)$$



$$a_{\max} = k\,A/m$$

Acceleration a

Oscillating systems: object on a vertical spring



$$\sum F_y = -ky + mg$$

Changing variables $y' = y - y_0$

$$\sum F_y = -k(y' + y_0) + mg$$

But $ky_0 = mg \rightarrow \sum F_y = -ky'$

From Newton's second law

$$-ky' = m\frac{d^2y}{dt^2}$$

$$dy = dy'$$

$$\frac{d^2y'}{dt^2} = -\frac{k}{m}y'$$

Solution is ☛ $y' = A\cos(\omega t + \delta)$

$$\omega = (k/m)^{\frac{1}{2}}$$

Spider Web

A spider of mass 0.3 g waits in its web of negligible mass

A slight movement causes the web to vibrate

with a frequency of about 15 Hz

a) Estimate value of spring stifness constant k for web

(b) At what frequency would you expect web to vibrate if an insect of mass 0.1 g were trapped in addition to spider?

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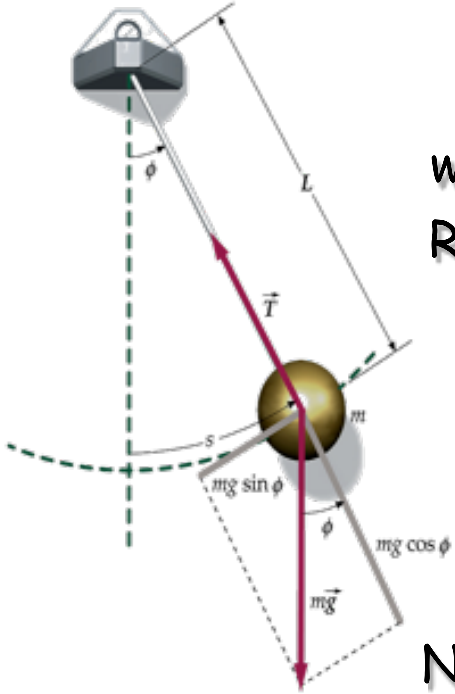
Frequency of SHM is given by

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \Rightarrow k = (2\pi\nu)^2 m = 2.7 \text{ N/m}$$

For $m = 4 \times 10^{-4}$ kg **we have**

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 13 \text{ Hz}$$

Oscillating systems: simple pendulum



$$-mg \sin\phi = m \frac{d^2 s}{dt^2}$$

where arc length $s = L\phi$

Repeatedly differentiating on both sides of s gives

$$\frac{d^2 s}{dt^2} = L \frac{d^2 \phi}{dt^2}$$

Substituting and re-arranging gives

$$\frac{d^2 \phi}{dt^2} = -\frac{g}{L} \sin\phi$$

Note that motion of pendulum does not depend on its mass

For small $\phi \to \sin\phi \approx \phi$

$$\frac{d^2 \phi}{dt^2} \approx -\frac{g}{L}\phi \qquad \phi \ll 1$$

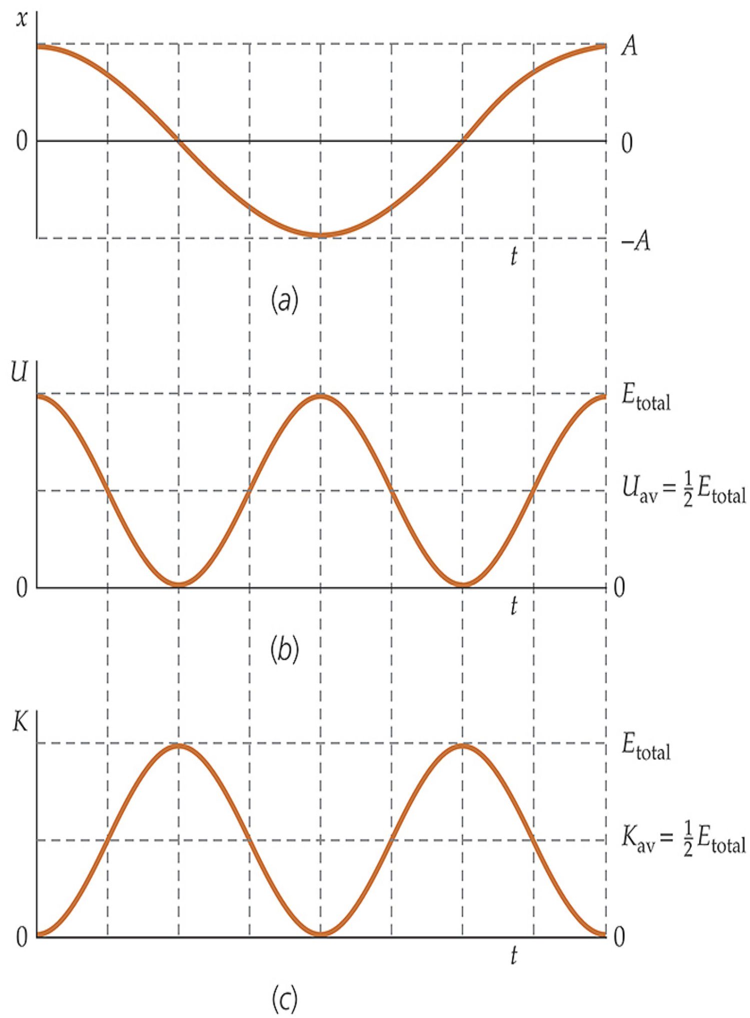
General solution for small oscillation ☛ $\phi = \phi_0 \cos(\omega t + \delta)$

Where $\omega^2 = \frac{g}{L}$ and $T = \frac{2\pi}{\omega} = 2\pi(L/g)^{\frac{1}{2}}$

$\sin\theta$ at small angles

θ (Degrees)	θ (Radians)	$\sin\theta$	% Difference
0	0	0	0
1°	0.01745	0.01745	$0.005\,\%$
5°	0.08727	0.08716	$0.1\,\%$
10°	0.17453	0.17365	$0.5\,\%$
15°	0.26180	0.25882	$1.1\,\%$
20°	0.34907	0.34202	$2.0\,\%$
30°	0.52360	0.50000	$4.7\,\%$

Energy in a simple harmonic motion



$$U = \frac{1}{2}kx^2$$

$$U = \frac{1}{2}kA^2\cos^2(\omega t + \delta)$$

$$K = \frac{1}{2}mv^2$$

$$K = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \delta)$$

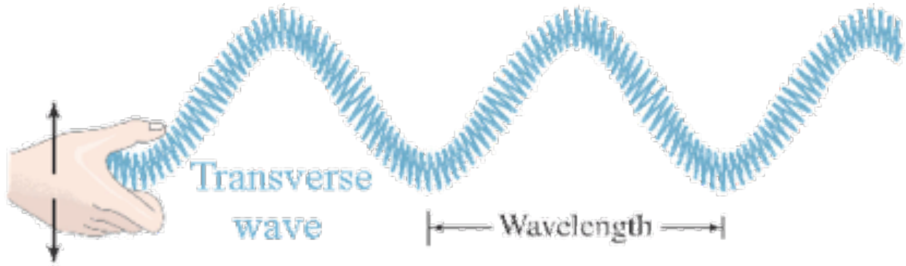
Using $\omega^2 = k/m$

$$E = U + K = \frac{1}{2}kA^2\left[\cos^2(\omega t + \delta) + \sin^2(\omega t + \delta)\right] = \frac{1}{2}kA^2$$

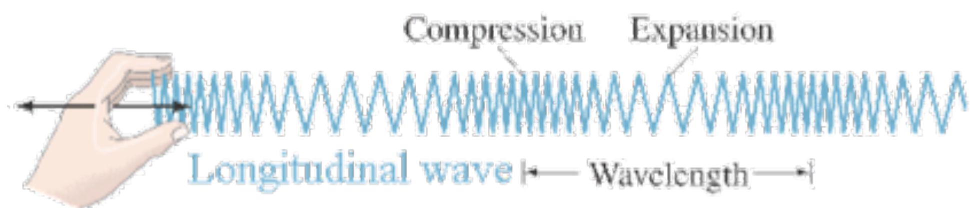
Waves

- Wave ☞ type of energy transmission
  that results from periodic disturbance ☞ vibration
- Waves transfer energy from one place to another
  without transferring matter
- They are composed of series of repeating patterns
- Two classes of waves ☞ { transverse / longitudinal
  - traverse ☞ vibration is perpendicular to direction of motion of wave
  - longitudinal ☞ vibration is in same direction as direction of wave

Types of waves



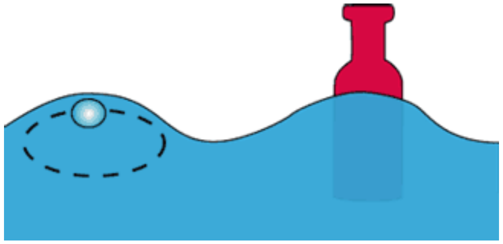
Waves in which motion of medium (molecules of water, particles on string) is perpendicular to direction of propagation are called transverse waves



Waves in which motion of medium is along (parallel to) direction of propagation of disturbance are called longitudinal waves

(Sound waves are examples of longitudinal waves)

- Everyone has seen waves on surface water
- Water wave can travel hundreds of kilometers over ocean but water just moves up and down as waves passes
- Energy is transferred from one water molecule to next by forces that hold molecules together
- In open ocean ☞ water waves are transverse
- Near shore ☞ water waves becomes also longitudinal
- We live surrounded by waves
- Some are visible ☞ others are not
- By observing visible waves (e.g. ☞ in water) we can describe some characteristics that all waves (including invisibles ones) have in common



Wave Motion

Mechanical wave is caused by disturbance in medium

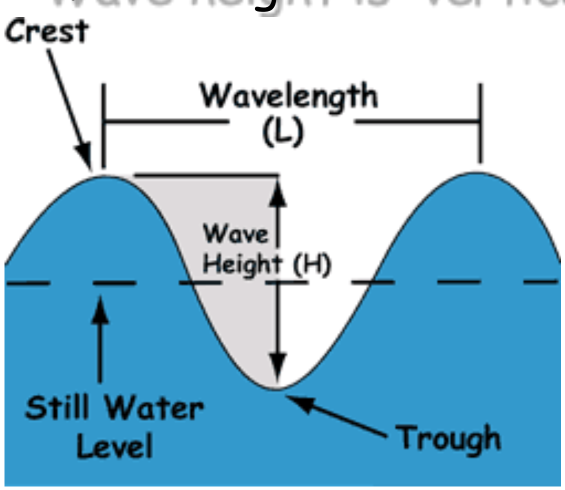
As wind passes over water's surface friction forces it to ripple

Strength of wind, distance wind blows, and duration
determine how big ripples will become

Crest is highest point on wave & Trough is lowest point on wave

Wavelength is horizontal distance
either between crests or troughs of two consecutive waves

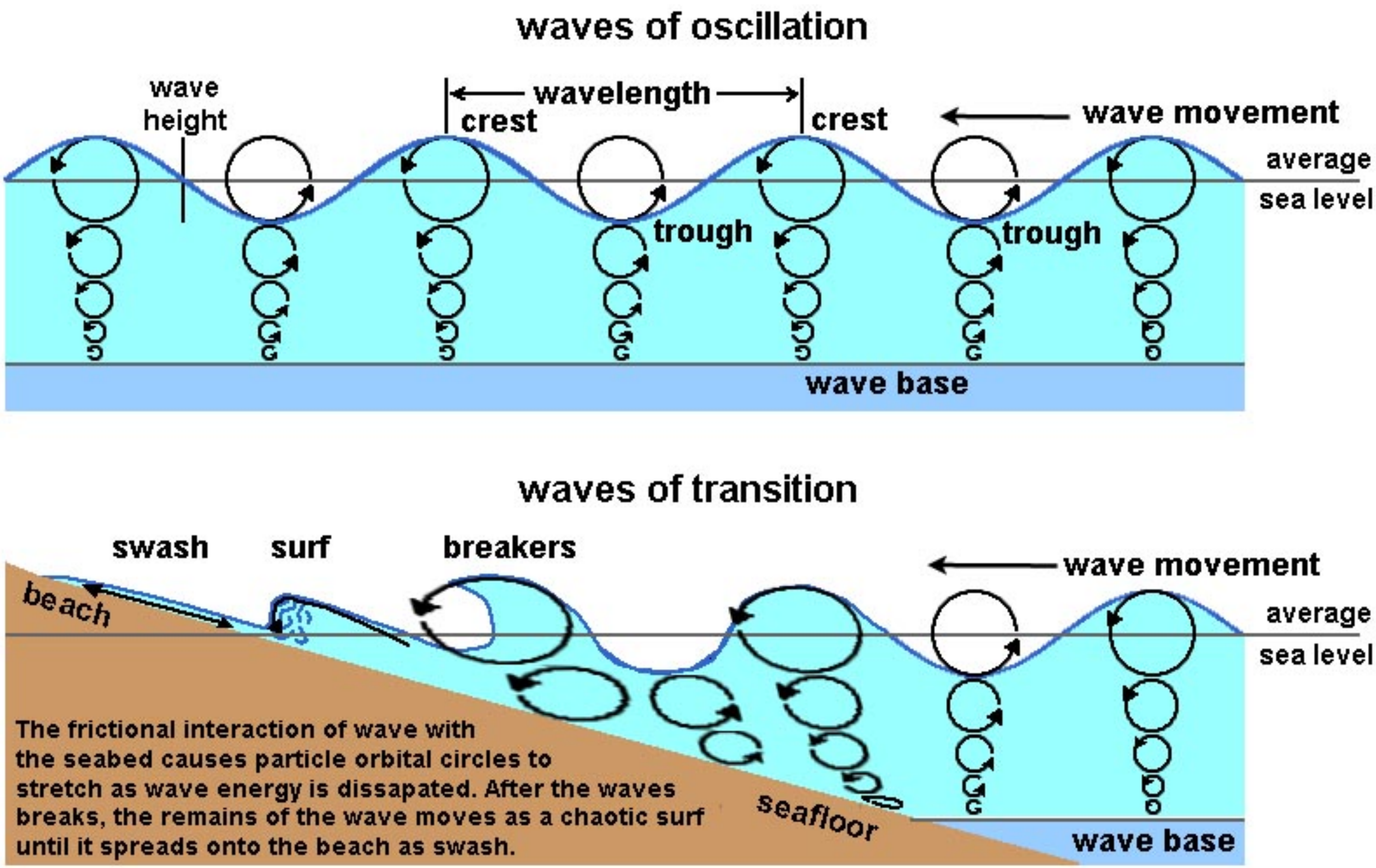
Wave height is vertical distance between wave's crest and next trough



Wave period measures size of wave in time

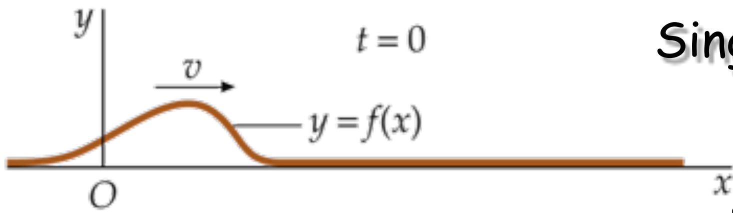
Wave Motion (cont'd)

Circular paths of particles due to oscillations from passing waves



Looking more closely at how a wave forms and how it comes to travel:

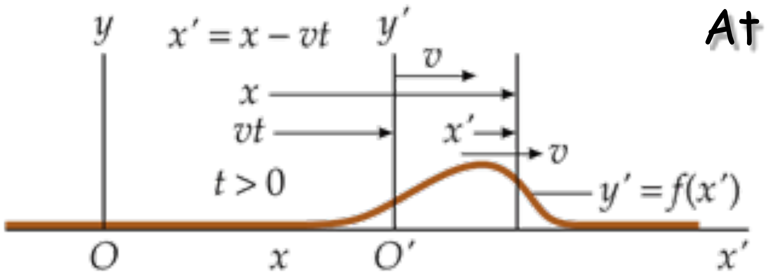
Pulse



Single wave bump or pulse can be formed on rope by quick up-and-down motion of hand

Figure shows pulse on string @ $t = 0$

Shape of string at this instant can be represented by function $y = f(x)$



At some later time pulse is farther down string

In a new coordinate system with origin O' that moves to right with same speed as pulse

☛ pulse is stationary

String is described in this frame by $f(x')$ for all times

x-coordinates of two reference frames are related by $x' = x - vt$

Shape of string in original reference frame is

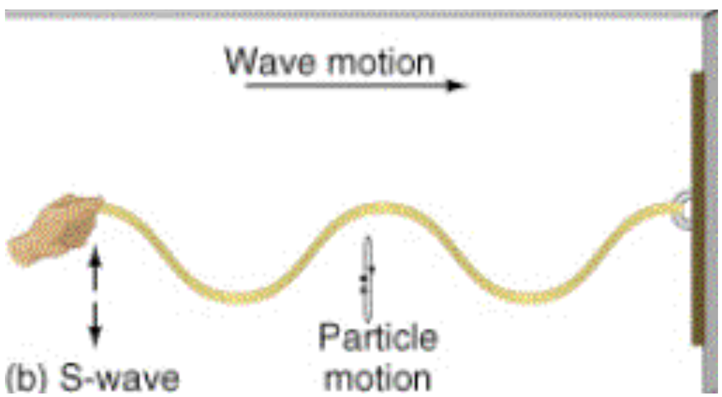
$$f(x') = f(x - vt)$$ wave moving in $+x$ direction

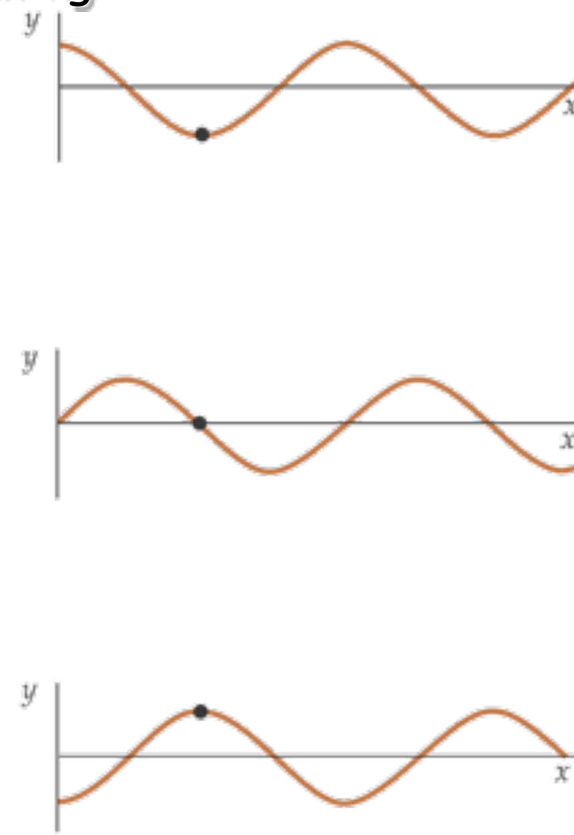
Same line of reasoning for a pulse moving to left leads to $y = f(x + vt)$

Looking more closely at how a wave forms and how it comes to travel:

Periodic wave

A continuous or periodic wave has as its source a disturbance that is continuous and oscillating





disturbance in this case is change in shape of string

Its propagation arises from interaction of each string segment with adjacent segments

Segments of string move in direction perpendicular to string as pulses that propagate back and forth along string

Speed of waves

Speed of waves relative to medium

depends on elastic and inertial properties of medium

but is independent of motion of source of waves

String tension

For a pulse on a rope ☛

$$v = \left(\frac{F_T}{\mu}\right)^{\frac{1}{2}}$$

Linear mass density

Bulk modulus

For sounds waves ☛

$$v = \left(\frac{B}{\rho}\right)^{\frac{1}{2}}$$

Volume mass density

Sound Waves

- A sound wave (as any other wave) can be characterized by its:

  1. amplitude A ☞ distance from midpoint of wave to a crest or trough (maximum displacement from equilibrium)
  2. frequency ν ☞ number of repeating patterns (cycles) per unit time
  3. period $\mathcal{T}$ ☞ time for one cycle
  4. wavelength λ ☞ distance from crest (or trough) to another crest (or trough)
  5. speed

$$v = \lambda \nu = \lambda / \mathcal{T}$$

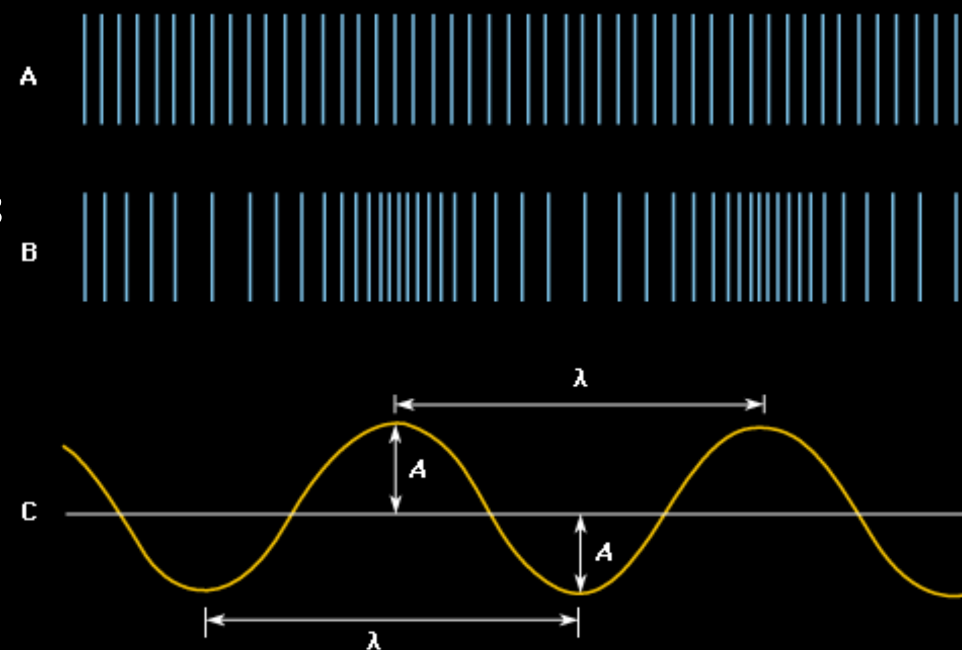
- Speed of sound in dry air

$$v_{\rm sound} = \left[331.5 + 0.6 \left(\frac{T}{^\circ{\rm C}} \right) \right] \ {\rm m/s}$$

- Human ear can hear from 20 to $20,000 \ {\rm Hz}$
- Infrasonic is below this frequency and ultrasonic above

Graphic representations of a sound wave

(A) Air at equilibrium in absence of a sound wave



(B) Compressions and rarefactions that constitute a sound wave

(C) Transverse representation of wave showing amplitude (A) and wavelength (λ)

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- Sound can bounce off of objects

  angle of incidence = angle of *reflection*

- Sound *reflection* gives rise to echoes
- Change of sound speed in different mediums

  can bend wave if it hits different medium at non 90° angle

- This is called *refraction*
- Waves can superimpose

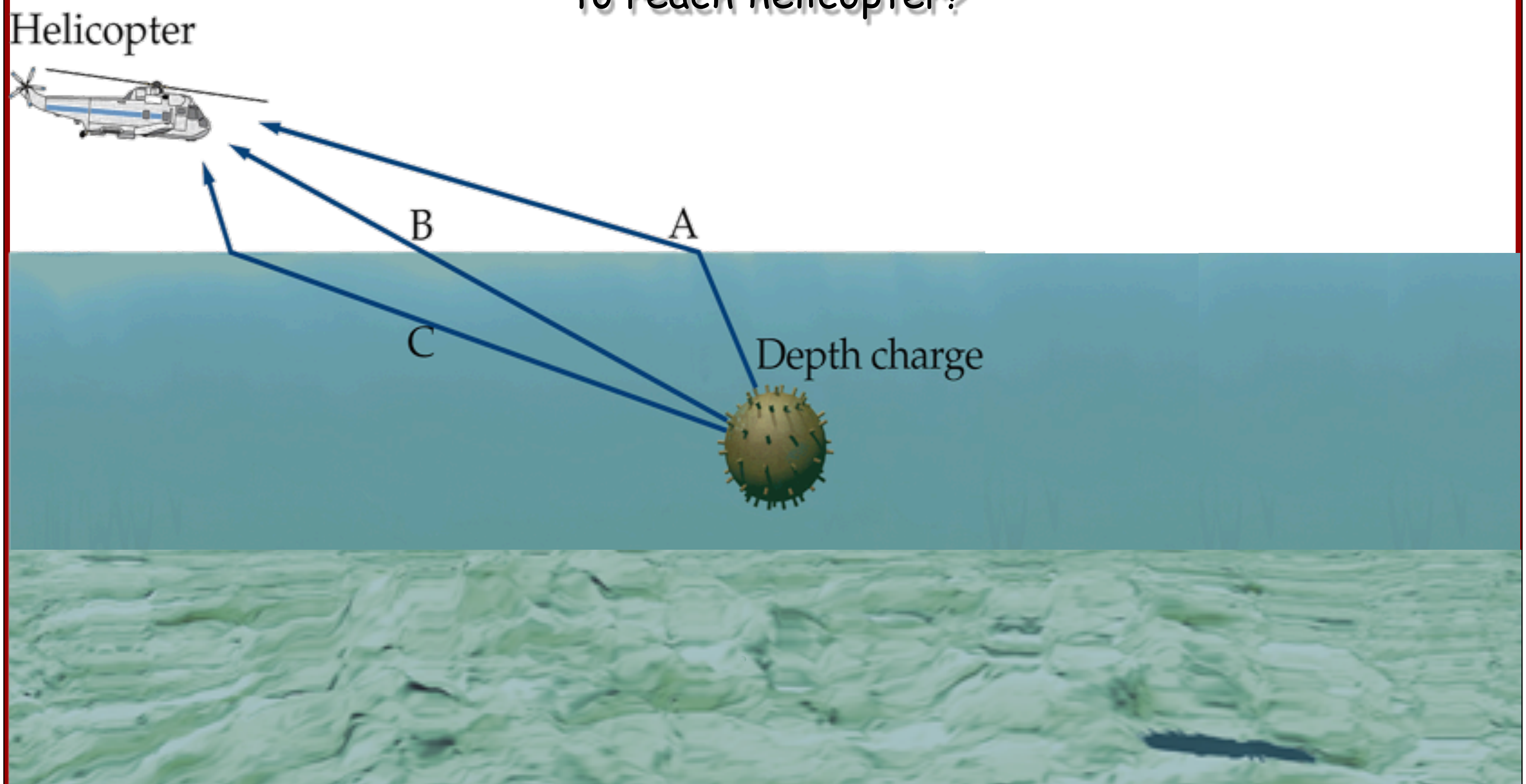
  and constructively and destructively interfere

  increasing each other or destroying each other

- Standing waves are formed when a wave is reflected
  and constructively interferes such that wave appears to stand still

Explosion of a depth charge beneath surface of a body of water is recorded by an helicopter hovering above water's surface as shown in figure

Along which path (A, B, or, C) will sound wave take least time to reach helicopter?



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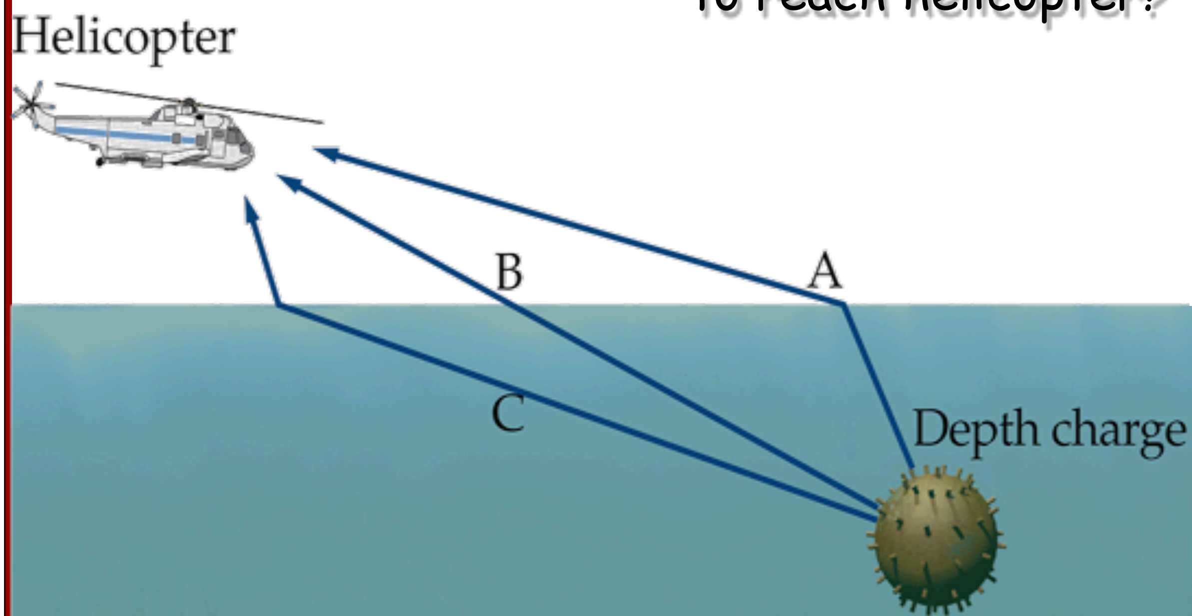


Speed of sound in water is greater than speed of sound in air ☛ path C

fluis are almost incompressible

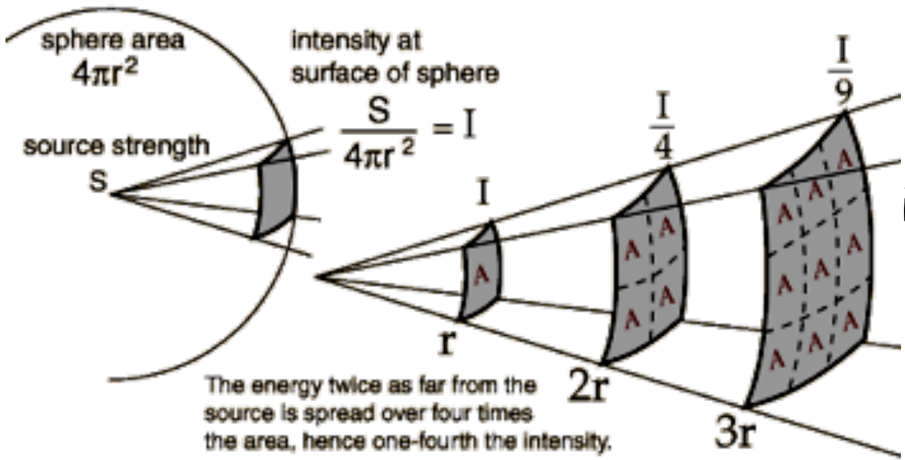
implies disturbance propagates very quickly

Wave intensity

Wave transports energy from one place to another

As waves travel through a medium energy is transferred as vibrational energy from a particle to particle in medium

If a point source emits waves uniformly in all directions then energy at a distance r from source is distributed uniformly on a spherical surface of radius r and area $A = 4\pi r^2$

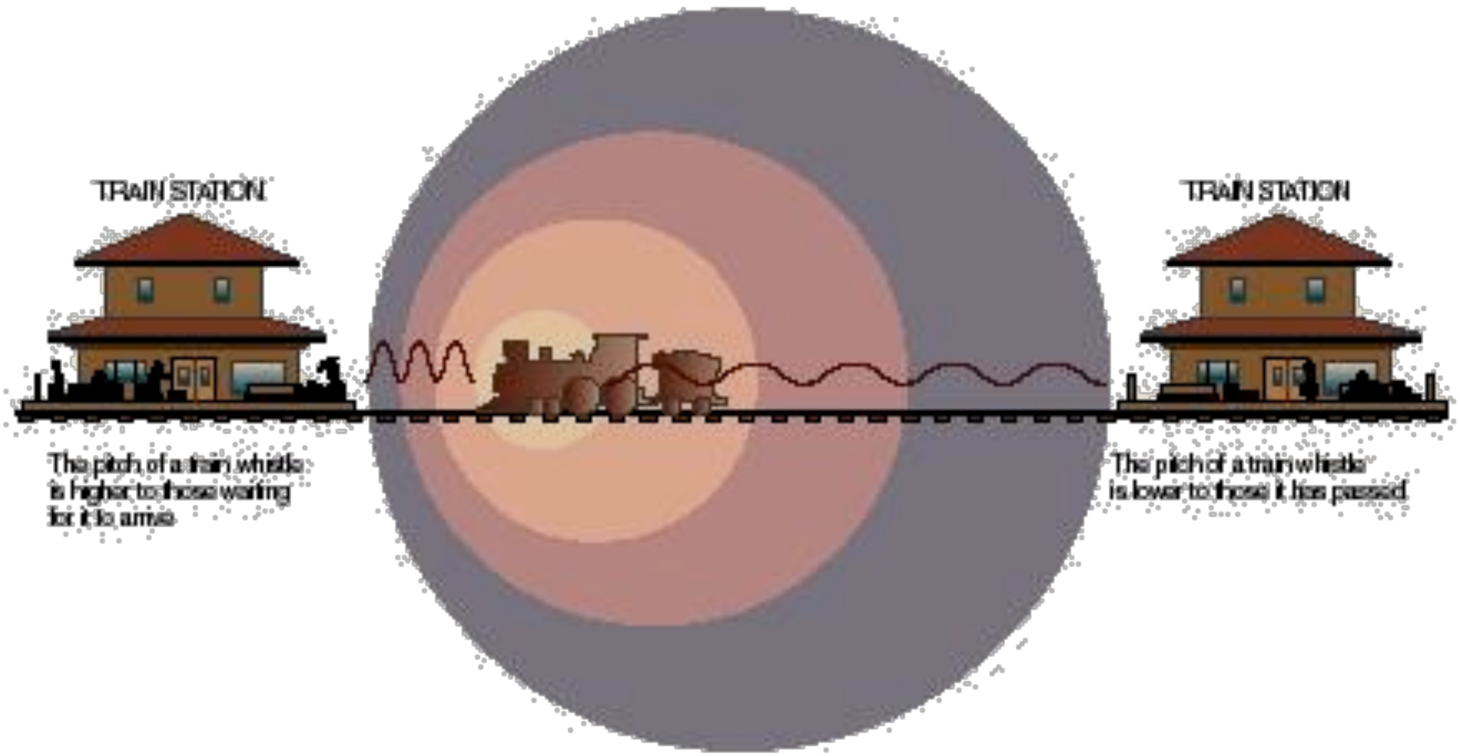


Average power per unit area that is incident perpendicular to direction of propagation is called intensity

$$I = \frac{\langle P \rangle}{A} = \frac{S}{4\pi r^2}$$

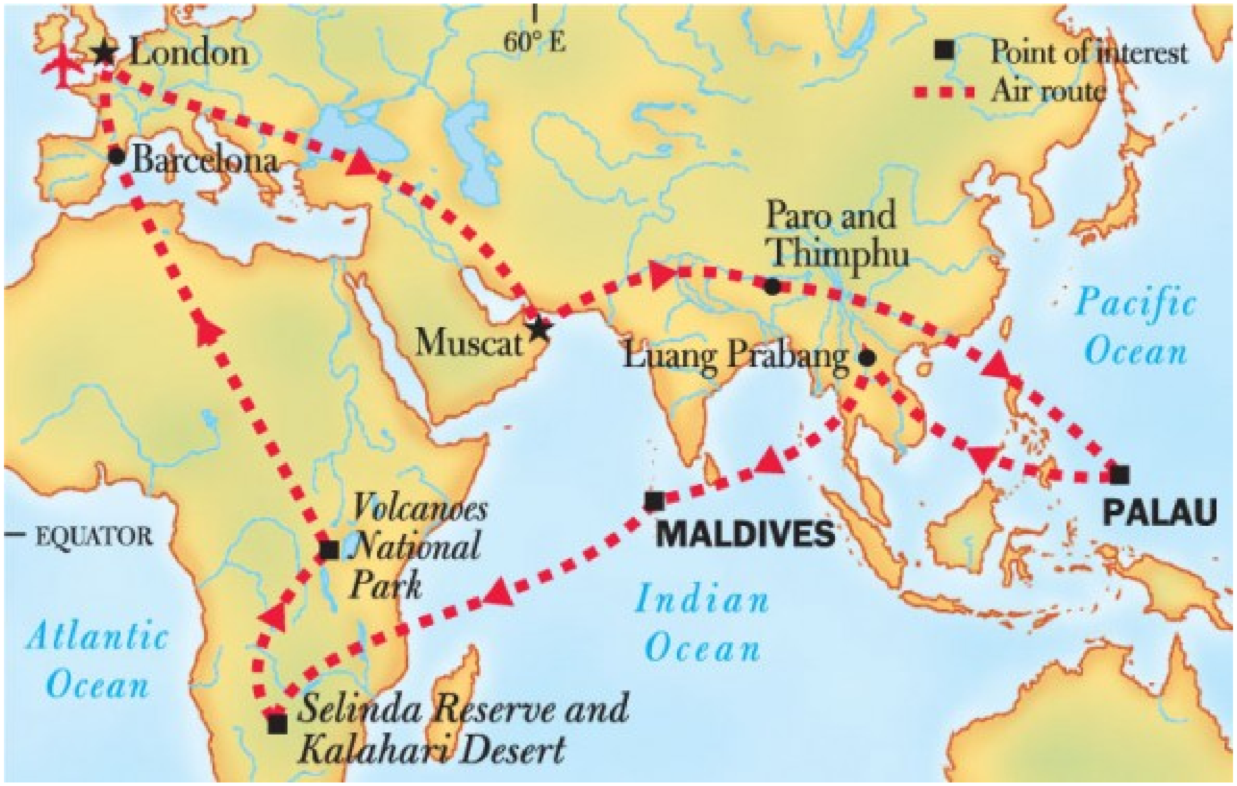
Doppler Effect

You may have noticed that you hear higher pitch of whistle on a speeding train dropped abruptly as it passes you



This phenomenon is known as Doppler effect

- Weinberg's analogy



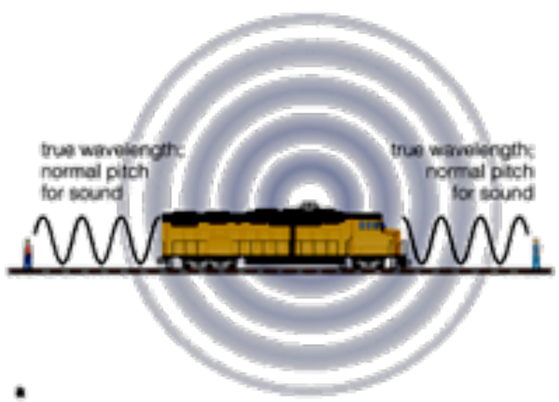
- *Doppler effect* ☞ change in observed frequency of source due to relative motion between source and receiver
- Relative motion that affects observed frequency is only motion in line-of-sight between source and receiver

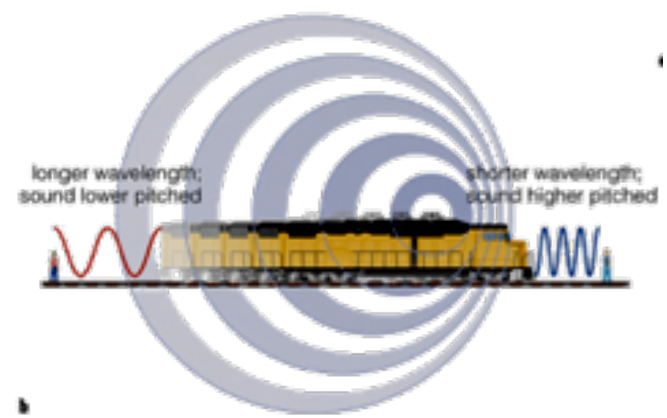
- When we observe sound wave from source at rest
  time between arrival wave crests at our instruments
  is same as time between crests as they leave source

- If source is moving toward us
  time between arrivals of wave crests is decreased
  because each successive crest has shorter distance to go

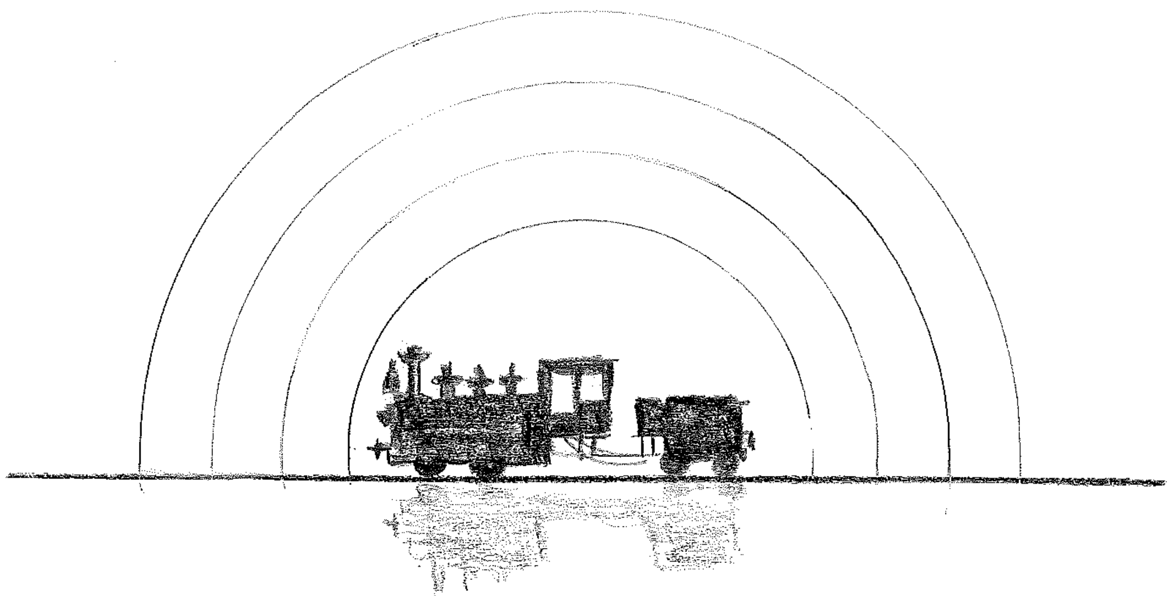
- Time between crests ☞ wavelength divided by speed of wave

- A wave sent out by source moving towards us
  will appear to have shorter wavelength than if source were at rest





- We first consider relative motion of receiver with V_{receiver}
- Stationary source emitting sound waves



- If receiver moves towards the source with velocity V_{receiver} each successive sound wave will be detected earlier than it would have if receiver were stationary due to motion of receiver along line-of-sight

- Detected frequency of each successive wave front will be changed by this relative motion ☞ $\Delta\nu = V_{\rm receiver}/\lambda_{\rm emitted}$

  $\Delta\nu = \nu_{\rm received} - \nu_{\rm emitted}$ ☞ change in the observed frequency

  $\lambda_{\rm emitted}$ ☞ original wavelength of source

- Since $\nu_{\rm emitted} = v_{\rm sound}/\lambda_{\rm emitted}$ and $\nu_{\rm received} = \nu_{\rm emitted} + \Delta\nu$

$$\nu_{\rm received} = \frac{v_{\rm sound} + V_{\rm receiver}}{\lambda_{\rm emitted}} = \nu_{\rm emitted}\left(\frac{v_{\rm sound} + V_{\rm receiver}}{v_{\rm sound}}\right)$$

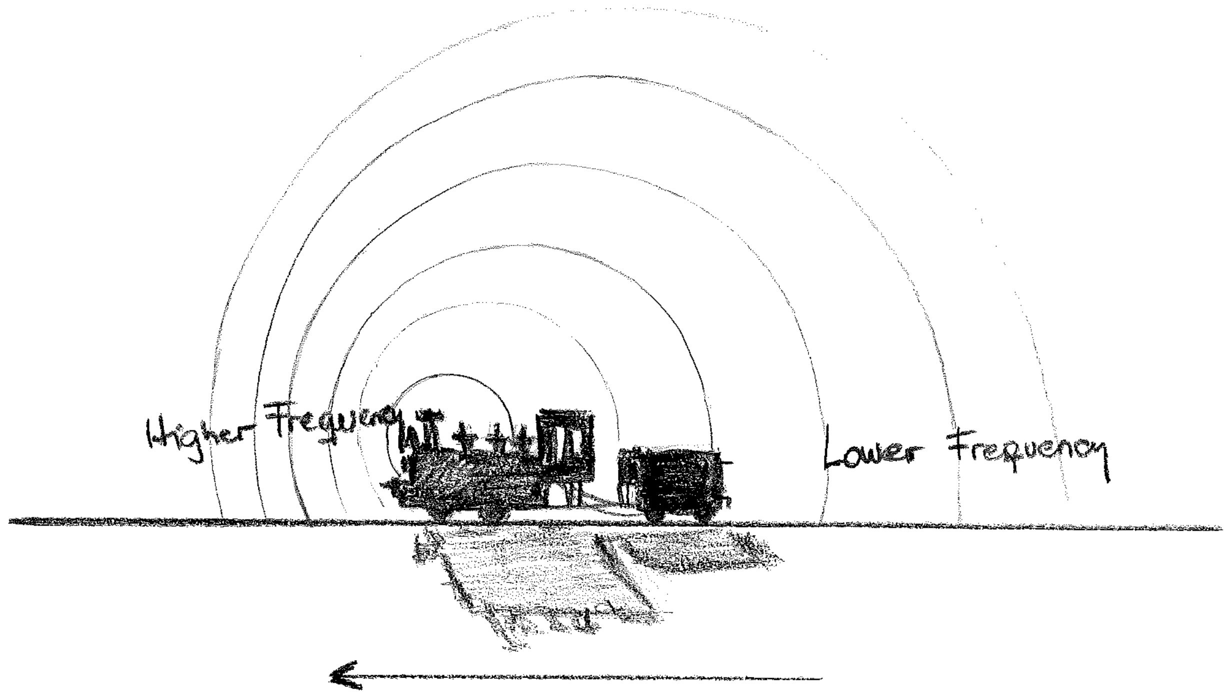
- If motion is away from source relative velocity would be in opposite direction

$$\nu_{\rm received} = \nu_{\rm emitted}\left(\frac{v_{\rm sound} - V_{\rm receiver}}{v_{\rm sound}}\right)$$

- Two equations are usually combined and expressed as

$$\nu_{\rm received} = \nu_{\rm emitted}\left(\frac{v_{\rm sound} \pm V_{\rm receiver}}{v_{\rm sound}}\right)$$

- If source is moving towards receiver with $V_{\rm source}$
  spacing between successive wave fronts would be less



- This would be expressed as ☞ $\Delta\lambda = V_{\rm source}/\nu_{\rm emitted}$
- To calculate the observed frequency

$$\nu_{\rm received} = \frac{v_{\rm sound}}{\lambda_{\rm emitted} + \Delta\lambda} = \nu_{\rm emitted}\left(\frac{v_{\rm sound}}{v_{\rm sound} - V_{\rm source}}\right)$$

- If source is moving away

$$\nu_{\rm received} = \nu_{\rm emitted} \left(\frac{v_{\rm sound}}{v_{\rm sound} + V_{\rm source}} \right)$$

- When combined with previous result

$$\nu_{\rm received} = \nu_{\rm emitted} \left(\frac{v_{\rm sound}}{v_{\rm sound} \mp V_{\rm source}} \right)$$

- By combining all previous results

$$\nu_{\rm received} = \nu_{\rm emitted} \left(\frac{v_{\rm sound} \pm V_{\rm receiver}}{v_{\rm sound} \mp V_{\rm source}} \right)$$

- One interesting application of Doppler effect ☞ active sonar
- We must carefully define *source* and *receiver*
- For outgoing active pulse ☞ reciver is target

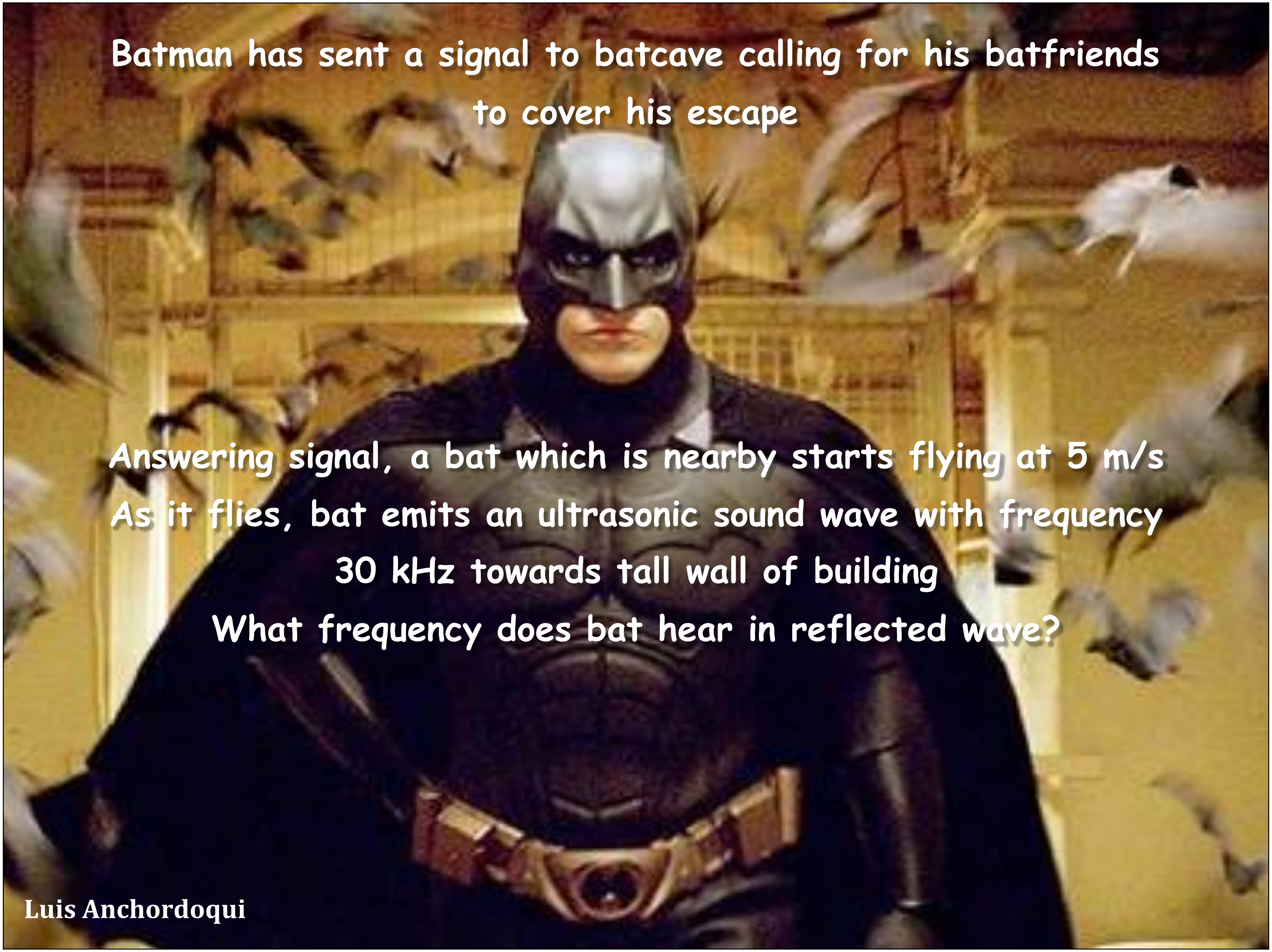
$$\nu_{\rm received}^{\rm target} = \nu_{\rm emitted} \left(\frac{v_{\rm sound} \pm V_{\rm target}}{v_{\rm sound} \mp V_{\rm source}} \right)$$

- For return pulse (echo) ☞ receiver is ship sending original pulse

$$\nu_{\rm echo} = \nu_{\rm received}^{\rm target} \left(\frac{v_{\rm sound} \pm V_{\rm source}}{v_{\rm sound} \mp V_{\rm target}} \right)$$

- Substituting for $\nu_{\rm received}^{\rm target}$

$$\nu_{\rm echo} = \nu_{\rm emitted} \left(\frac{v_{\rm sound} \pm V_{\rm target}}{v_{\rm sound} \mp V_{\rm source}} \right) \left(\frac{v_{\rm sound} \pm V_{\rm source}}{v_{\rm sound} \mp V_{\rm target}} \right)$$


Batman has sent a signal to batcave calling for his batfriends
to cover his escape
Answering signal, a bat which is nearby starts flying at 5 m/s
As it flies, bat emits an ultrasonic sound wave with frequency
30 kHz towards tall wall of building
What frequency does bat hear in reflected wave?
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The wall is treated as stationary observer for calculation frequency it receives

Bat is flying towards wall so

$$f'_{\rm wall} = f_{\rm bat} \frac{1}{1 - v_{\rm bat}/v_{\rm sound}}$$

Wall is treated as stationary source emitting frequency $f'_{\rm wall}$

and bat as moving observer flying towards wall

$$f''_{\rm bat} = f'_{\rm wall} \left(1 + \frac{v_{\rm bat}}{v_{\rm sound}}\right) = f_{\rm bat} \frac{1}{1 - v_{\rm bat}/v_{\rm sound}} \left(1 + \frac{v_{\rm bat}}{v_{\rm sound}}\right)$$

$$= f_{\rm bat} \frac{v_{\rm sound} + v_{\rm bat}}{v_{\rm sound} - v_{\rm bat}} = 3.09 \times 10^4 \ {\rm Hz}$$

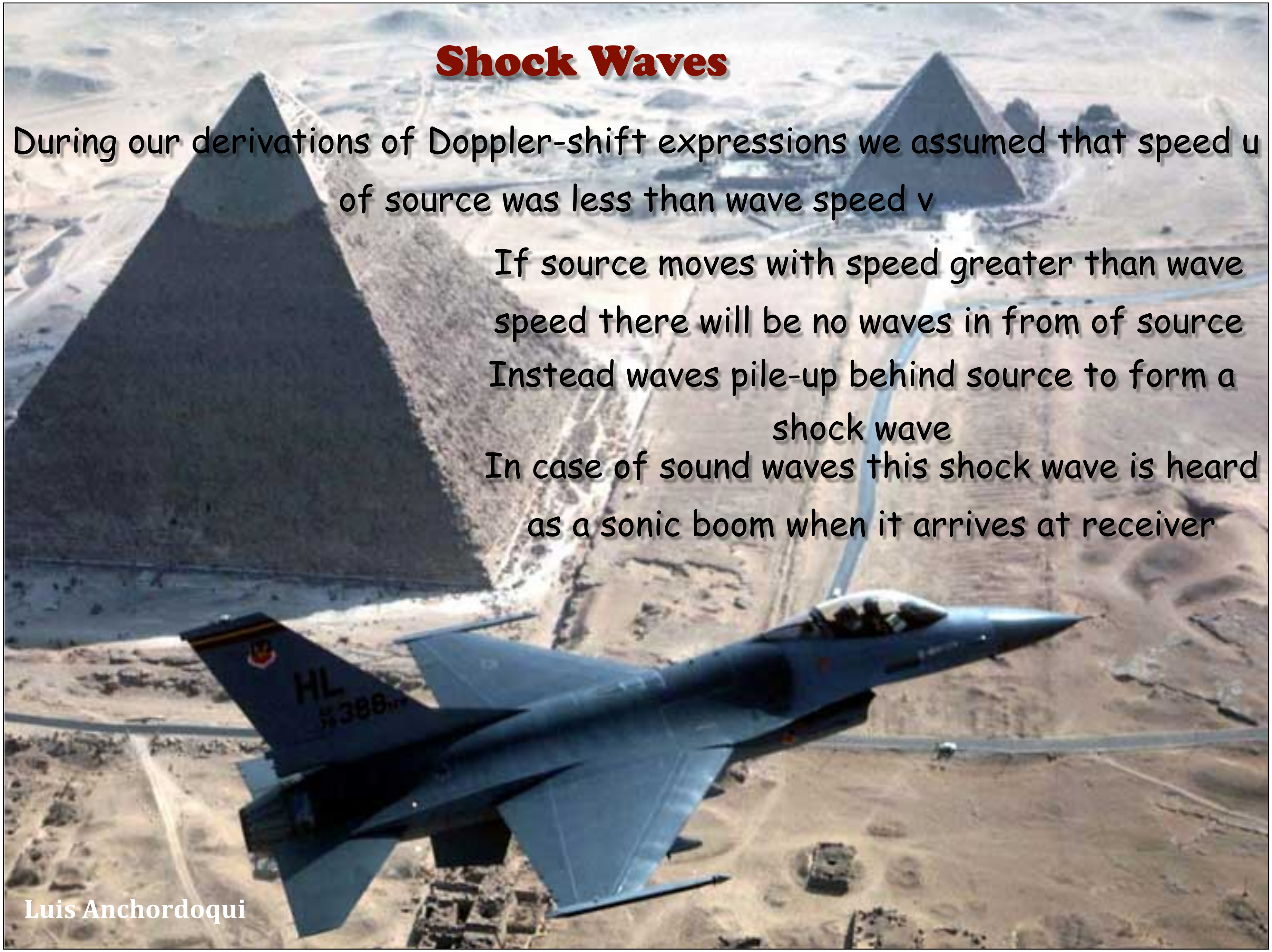
Shock Waves

During our derivations of Doppler-shift expressions we assumed that speed u of source was less than wave speed v

If source moves with speed greater than wave speed there will be no waves in from of source

Instead waves pile-up behind source to form a shock wave

In case of sound waves this shock wave is heard as a sonic boom when it arrives at receiver



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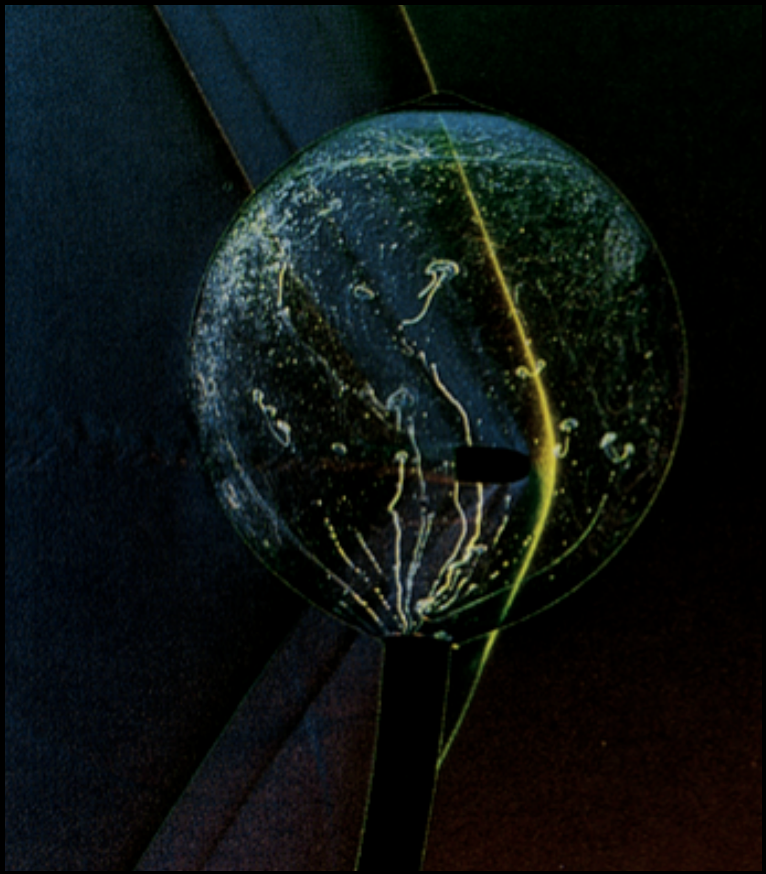
Shock waves produced by a bullet traversing a helium balloon



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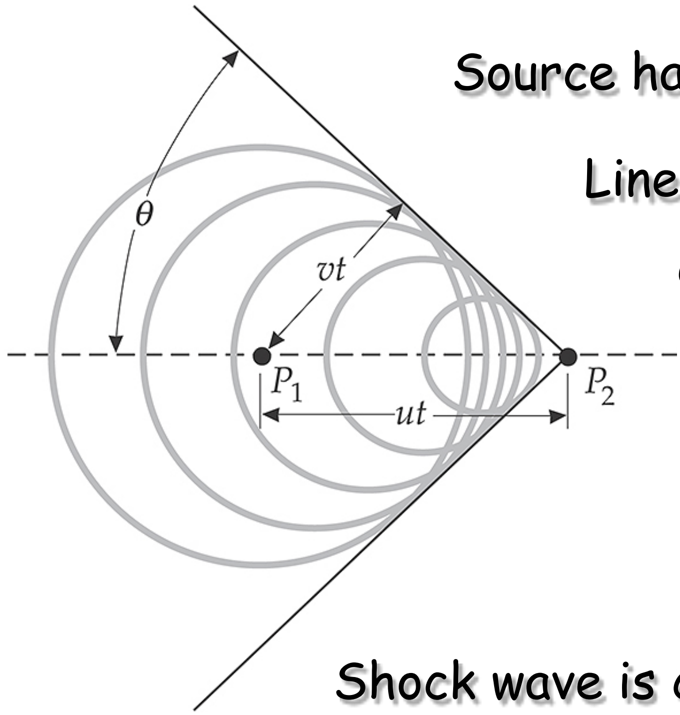
Mach Number

Figure shows a source originally at point P_1 moving to right with velocity u

After some time t wave emitted from point P_1 has traveled a distance vt



Source has traveled a distance ut and will be at point P_2

Line from this new position of source to wavefront emitted when source was at P_1 makes an angle θ with path of source known as Mach angle +

$$\sin\,\theta = \frac{vt}{ut} = \frac{v}{u}$$

Shock wave is confined to a cone that narrows as u increases

Ratio of source speed u to wave speed v is called Mach number

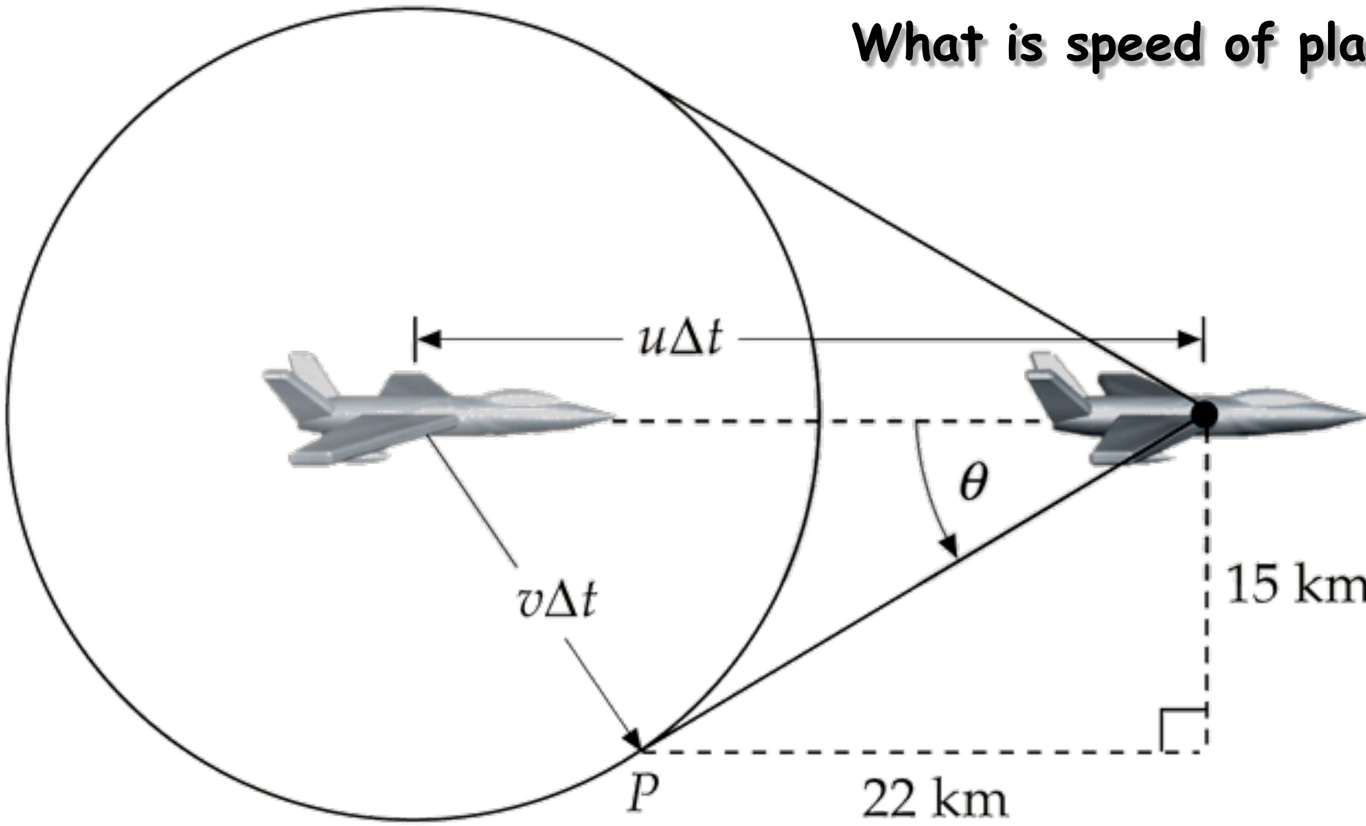
$$\text{Mach number} = \frac{u}{v}$$


Sonic Boom
Shock waves from a supersonic plane
Luis Anchordoqui

A supersonic plane flying due east at an altitude of 15 km passes directly over point P

Sonic boom is heard at point P when plane is 22 km east of point P

What is speed of plane?



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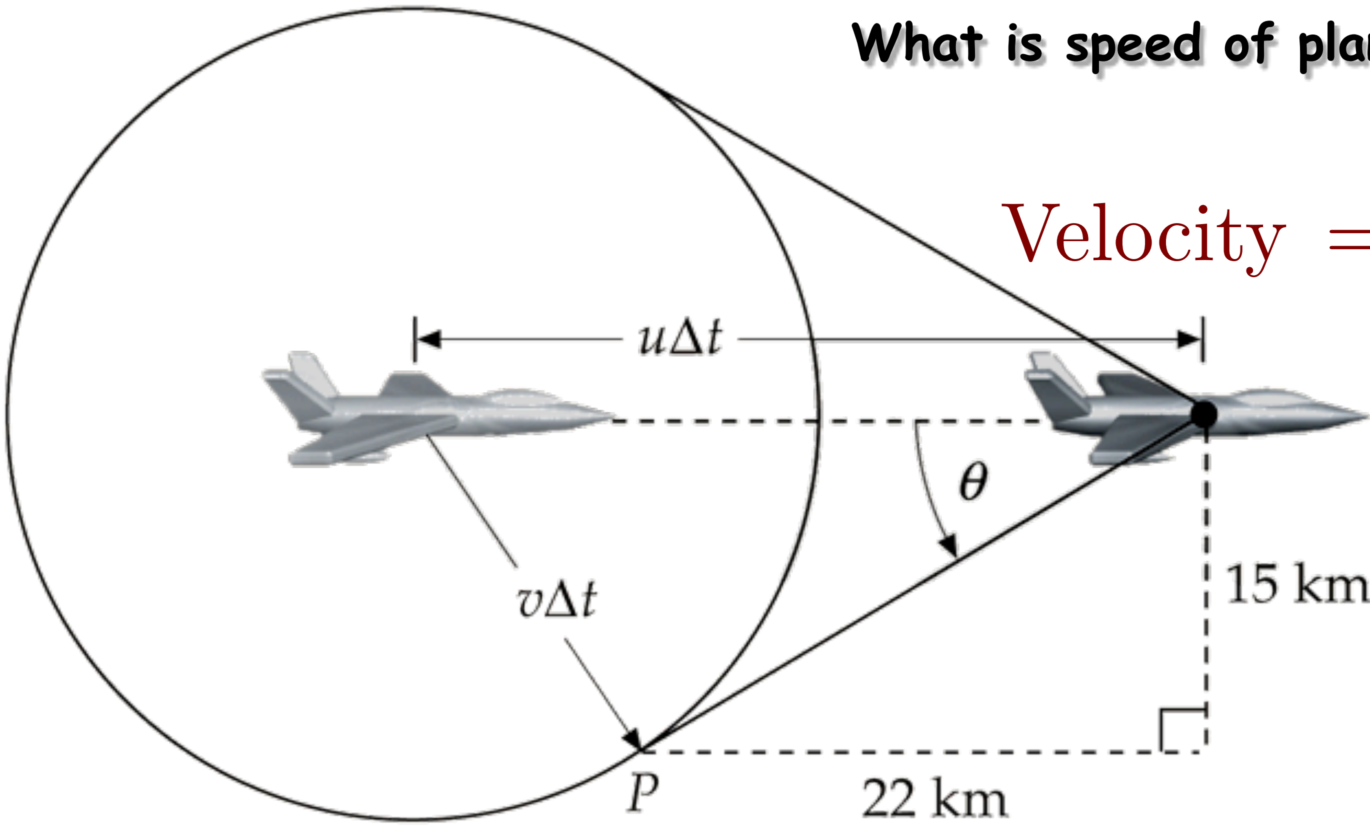

Velocity $= 610\, m/s$

Doppler Effect (summary)

Stationary Sound Source

Doppler Shift

Source moving with $v_{\text{source}} < v_{\text{sound}}$

Breaking Sound Barrier ☛ Sonic Boom

Source moving with $v_{\text{source}} > v_{\text{sound}}$