Lesson 4

Physics 168

Luis Anchordoqui

Drag forces

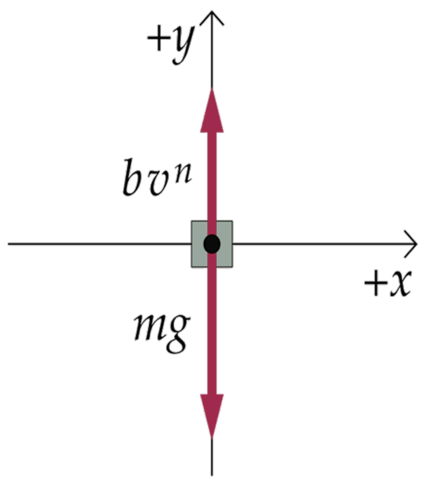
When an object moves through a fluid such as air or water fluid exerts a drag force (or retarding force) that opposites motion of object

Drag force depends on: shape of object
properties of fluid
and speed of object relative to fluid

Drag force on parachute slows this dragster

Drag forces

Drop object from rest and falling under influence of gravity



Magnitude of drag force $\Rightarrow \; F_d = bv^n$

$$mg - bv^n = ma_y$$

solving this equation for acceleration

$$a_y = g - bv^n/m$$

$@ \; t = 0$

speed and drag force are zero but acceleration is g downwards

$@ \; t > 0$

speed of object and drag force increase so acceleration decreases

Eventually speed is great enough

for magnitude of drag force to approach force of gravity

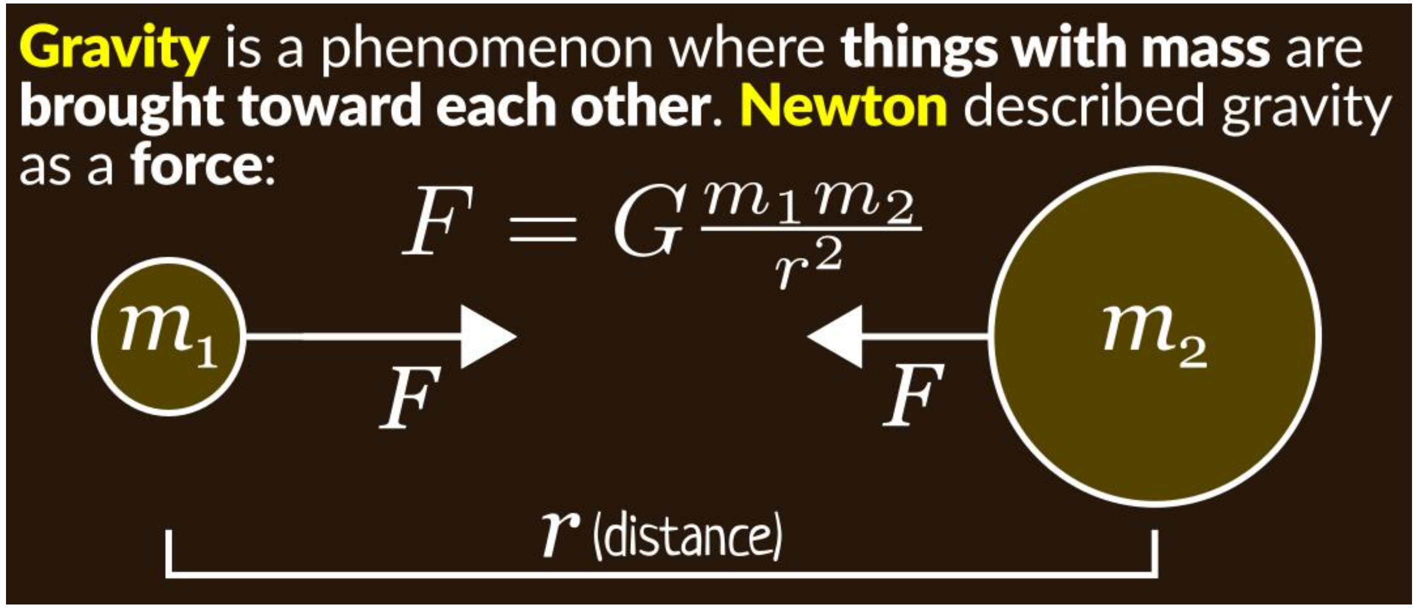
At terminal speed $\quad bv_T^n = mg \;\rightarrow\; v_T = (mg/b)^{\frac{1}{n}}$

Larger constant b smaller terminal speed

Parachute design to maximize b so terminal speed is small


Circular motion
Luis Anchordoqui

In the last lecture we have seen that ...



One could ask why the Moon doesn’t fall on Earth as an apple from the tree
The reason is that the Moon is never still
It constantly moves around us
Without the force of gravity from the Earth, it would just float away into space
This mix of velocity and distance from the Earth allows the Moon to always be in balance between fall and escape
If it was faster, it would escape; any slower and it would fall!
Moon is constantly accelerating towards the Earth
Orbiting is like falling without ever hitting the ground

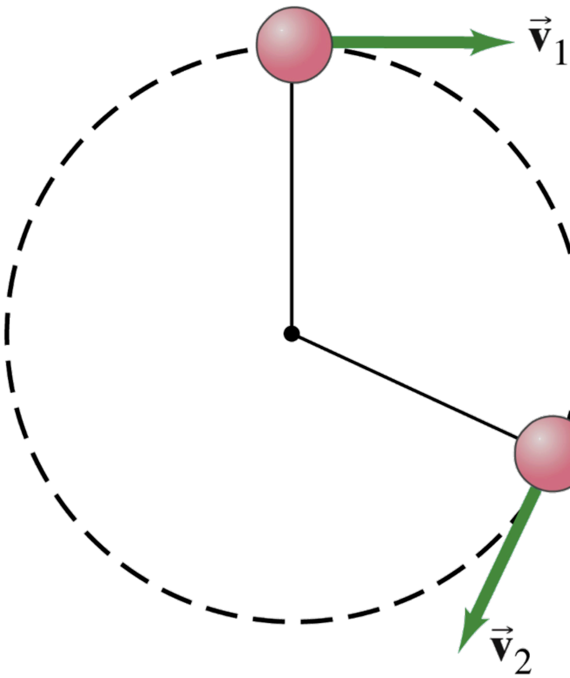
Kinematics of uniform circular motion

Object moving in a circle at constant speed

experience uniform circular motion

Magnitude of velocity remains constant

but velocity direction continuously changes



as object moves around circle

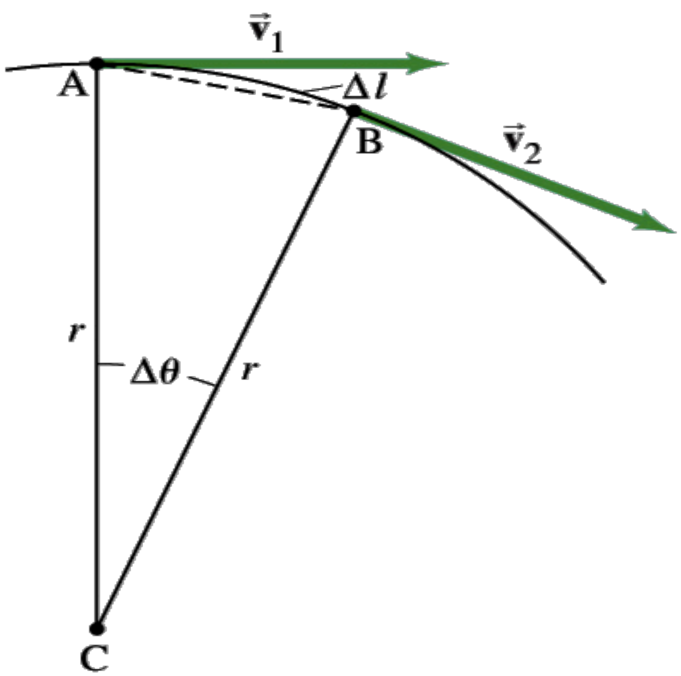
An object revolving in a circle is continuously accelerating

even when speed remains constant

Kinematics of uniform circular motion

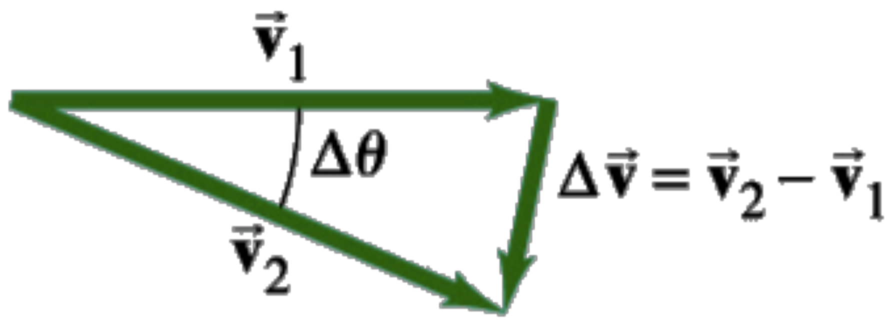
Acceleration is defined as $\vec{a} = \dfrac{\vec{v_2} - \vec{v_1}}{\Delta t}$

To make clear drawing consider a non-zero time interval



$$\Delta\vec{v} \perp \vec{v_1} \wedge \vec{v_2}$$

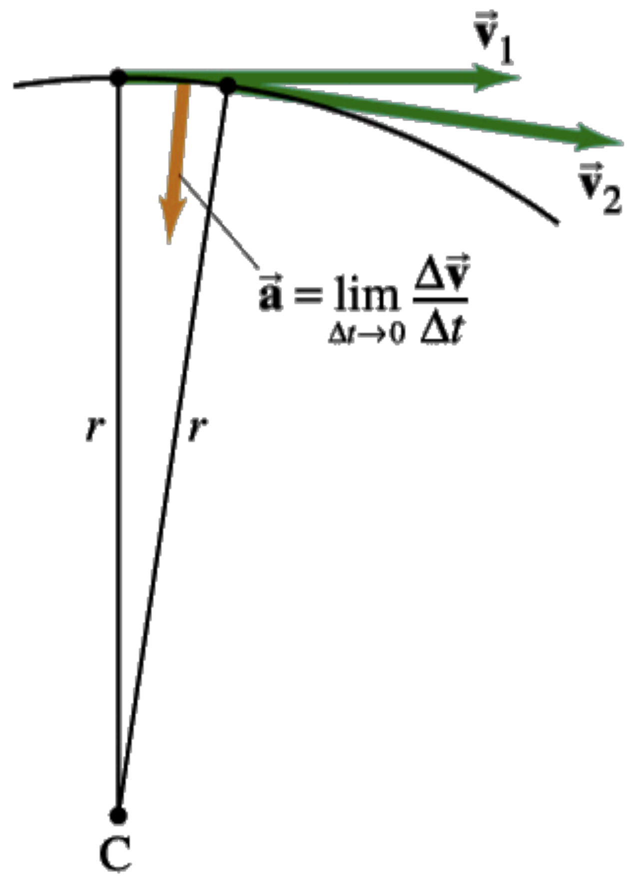
pointing to center of circle



$$\lim_{\Delta t \to 0} \Leftrightarrow \frac{\Delta l}{r} \ll 1 \wedge \Delta\theta \ll 1 \Rightarrow \vec{v_1} \parallel \vec{v_2}$$

Kinematics of uniform circular motion

$\vec{a}$ must too point to the center of the circle



Kinematics of uniform circular motion

Magnitude of velocity is not changing we can write

$$\frac{\Delta v}{v} \approx \frac{\Delta l}{r}$$

This is an exact equality when Δt approaches zero

Let Δt approach zero and solve for Δv

$$\Delta v = \frac{v}{r} \Delta l$$

To get the centripetal acceleration we divide by Δt

$$a_R = \frac{\Delta v}{\Delta t} = \frac{v}{r} \frac{\Delta l}{\Delta t}$$

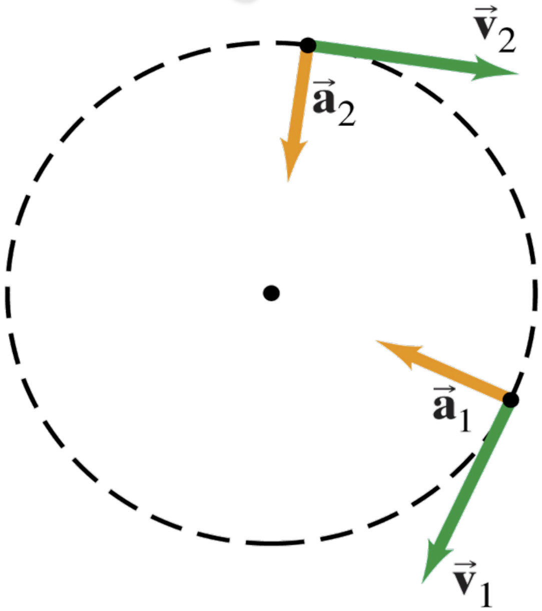
$\frac{\Delta l}{\Delta t}$ is just the linear speed

$$a_R = \frac{v^2}{r}$$

Kinematics of uniform circular motion

Acceleration vector points towards center of circle

Velocity vector always points in direction of motion



Circular motion often described in terms of frequency

↓

number of revolutions per second

Period of object revolving in circle
time required for one complete revolution $\Rightarrow T = 1/f$

For object revolving in circle at constant speed $v = \dfrac{2\pi r}{T}$

A SATELLITE'S MOTION

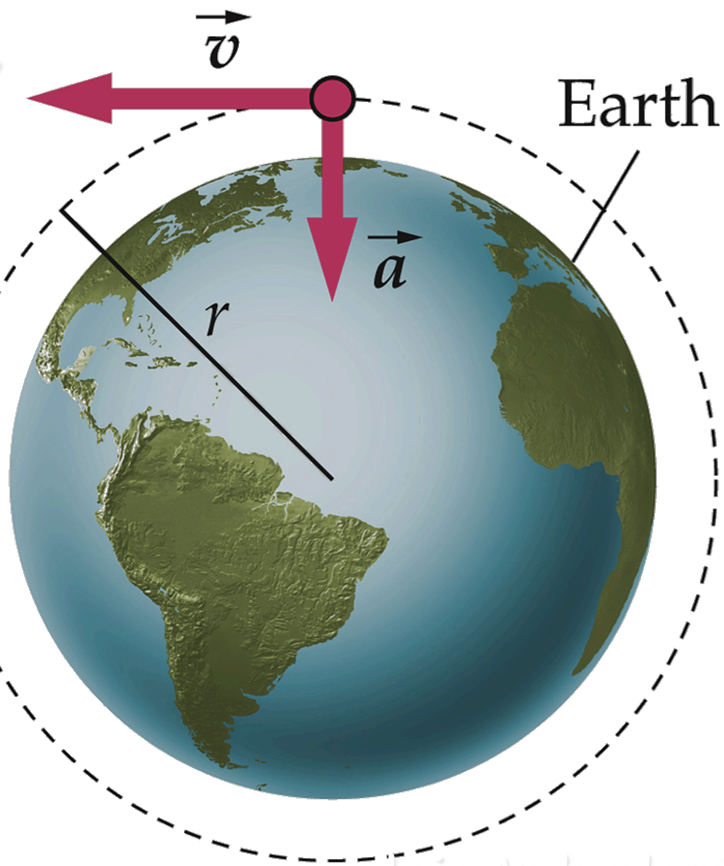
A satellite moves at constant speed in a circular orbit about center of Earth near surface of Earth.

If the magnitude of its acceleration is $g = 9.81\ \mathrm{m/s^2}$ find

(a) its speed and

(b) time for one complete revolution

$$R_{\oplus} = 6,370\ \mathrm{km}$$



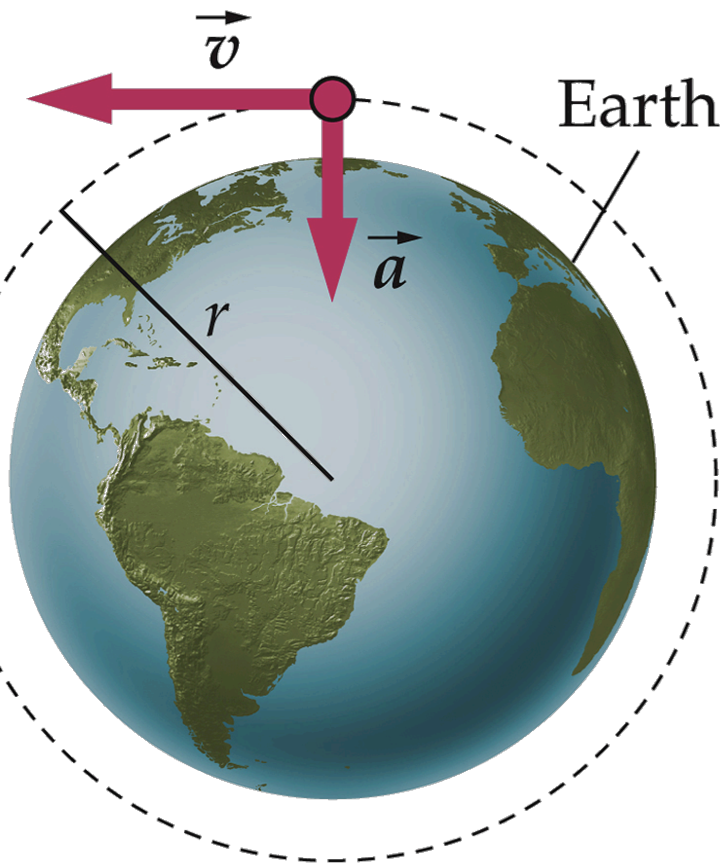
A SATELLITE'S MOTION

$$a_c = \frac{v^2}{R_\oplus} = g$$

$$\Rightarrow v = \sqrt{gR_\oplus} = 7.91 \text{ km/s}$$

$$T = \frac{2\pi R_\oplus}{v} = 5,060 \text{ s}$$

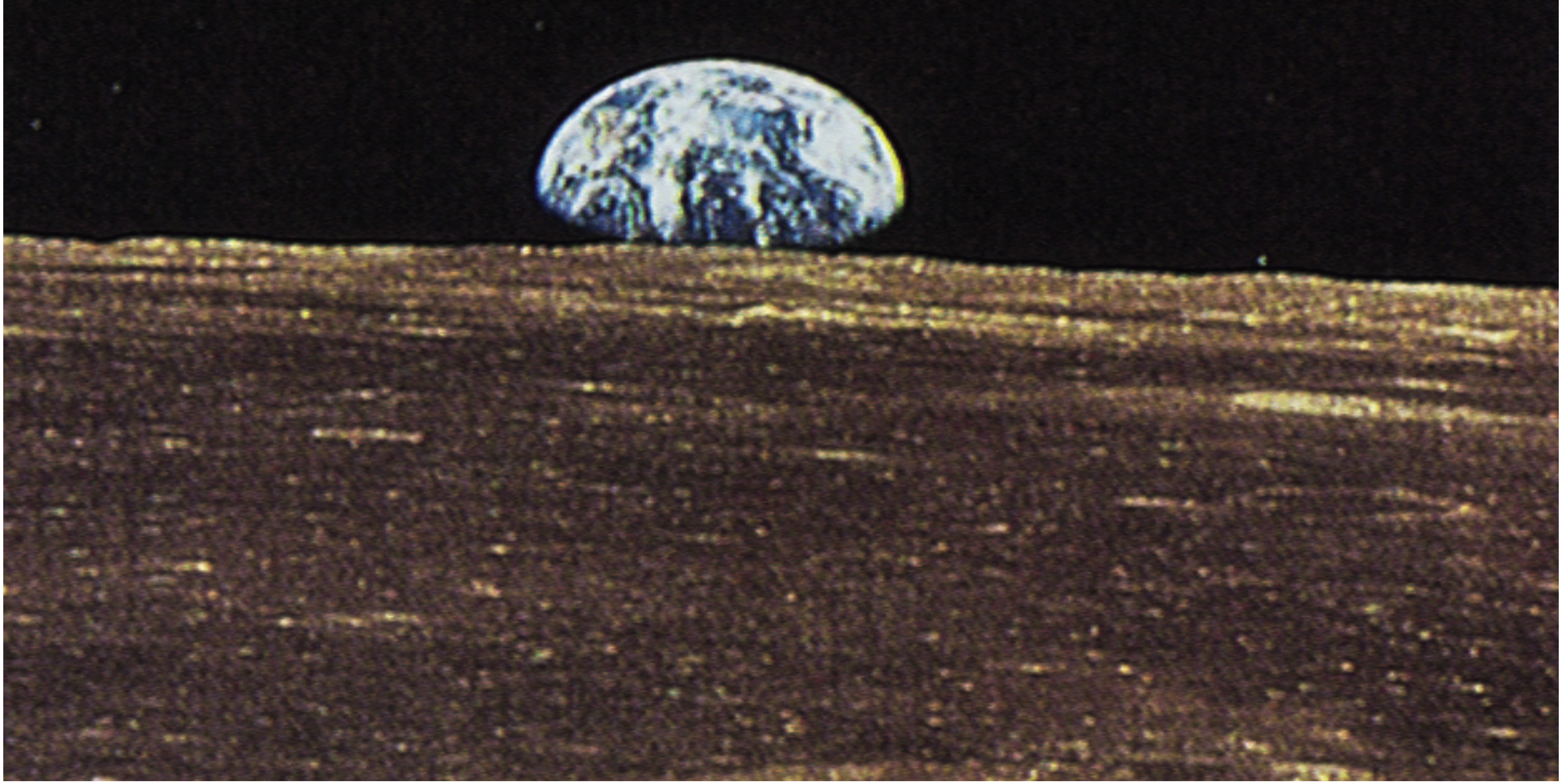
$$= 84.3 \text{ minutes}$$



MOON'S CENTRIPETAL ACCELERATION

Moon's nearly circular orbit about Earth has a radius of about 384,000km and a period T of 27.3 days.

Determine acceleration of Moon towards Earth



MOON'S CENTRIPETAL ACCELERATION

Earth as seen from Apollo 11 orbiting Moon on July 16, 1969 (NASA)

$$a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2} = 2.72 \times 10^{-3}\ \mathrm{m/s^2}$$

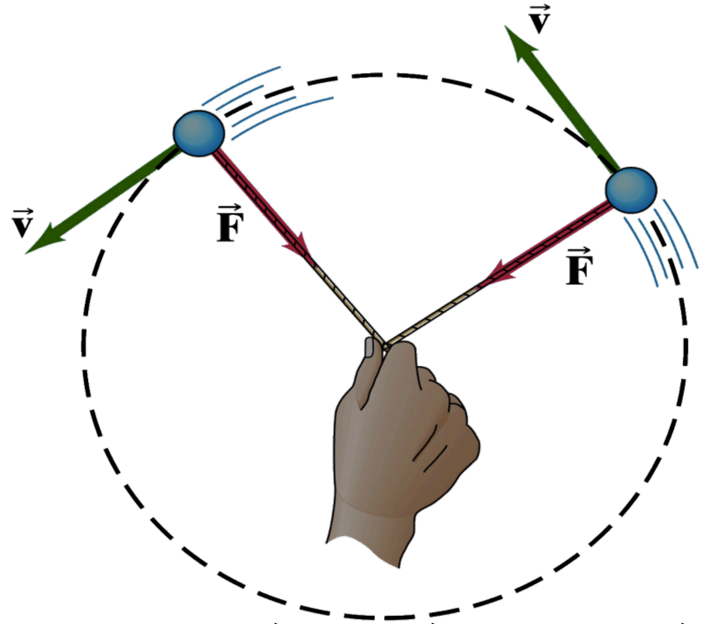
$$a = 2.78 \times 10^{-4}\, g$$

DYNAMICS OF UNIFORM CIRCULAR MOTION

According to Newton's second law

object that is accelerating must have net force acting on it

Object moving in circle such as ball on end of string



must therefore have force applied to it to keep it moving on that circle

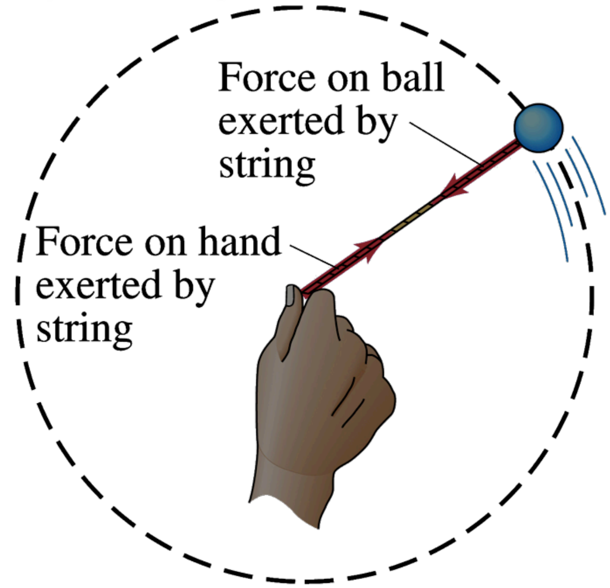
Magnitude of force can be calculated

using Newton's second law for radial component

$$\sum F_R = ma_R = m\frac{v^2}{r}$$

DYNAMICS OF UNIFORM CIRCULAR MOTION

Consider person swinging ball ot end of string around her head



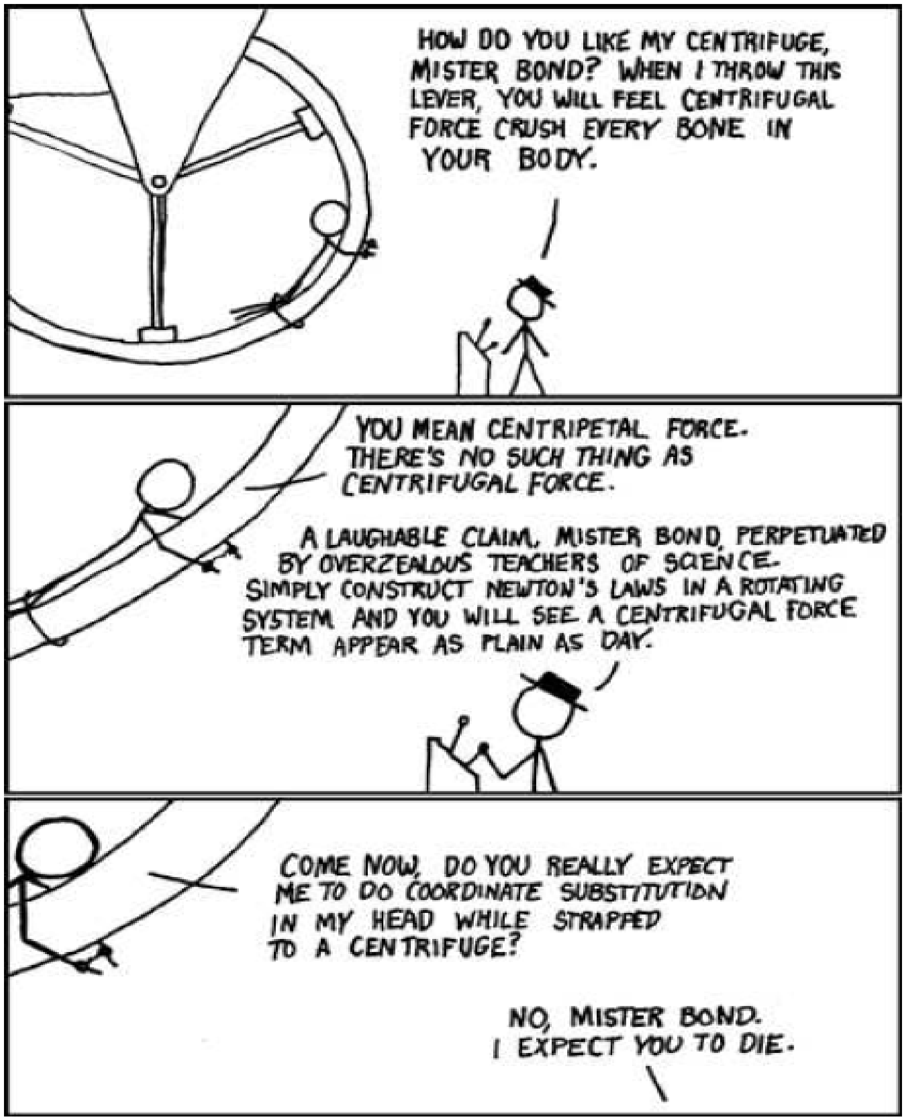
If you ever done this yourself
you know that you feel a force pulling outward on your hand

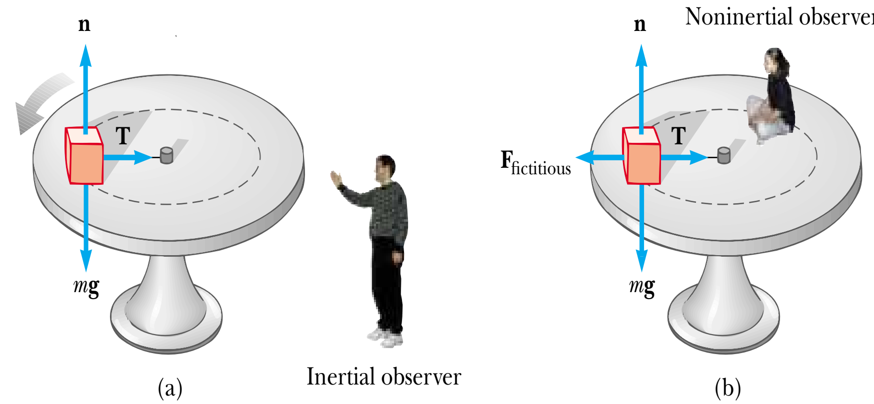
To keep ball moving on circle
you pull inwardly on string and string exerts this force on ball

Ball exerts equal and opposite force on the string (Newton's third law)
and this is outward force your hand feels

DYNAMICS OF UNIFORM CIRCULAR MOTION

There is a common misconception that object moving in a circle has an outward force acting on it: centrifugal ("center feeling") force





Block of mass m connected to a string tied to center of rotating turntable

(a) Inertial observer claims that force causing circular motion is provided by force T exerted by string on block

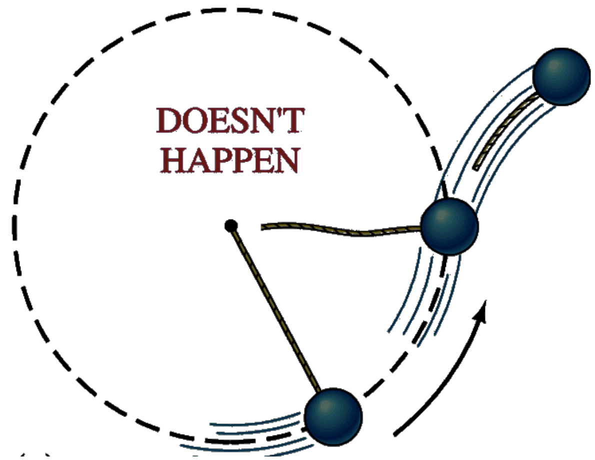
(b) Noninertial observer claims that block is not accelerating and therefore she introduces a fictitious force of magnitude mvv/r that acts outward and balances force T

There is no outward force on revolving object

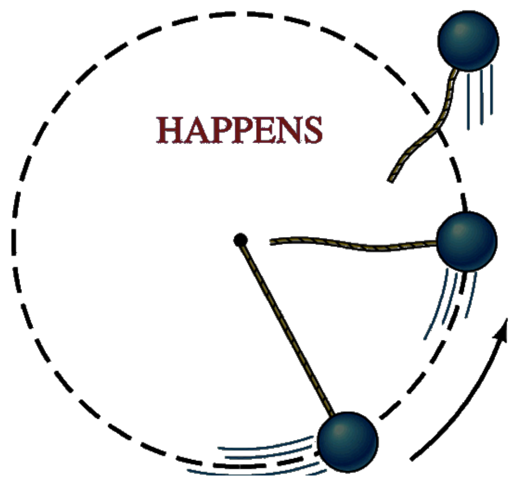
For convincing evidence that "centrifugal force" does not act on ball

➔ consider what happens when you let go of the string

If a centrifugal force were acting the ball would fly outward

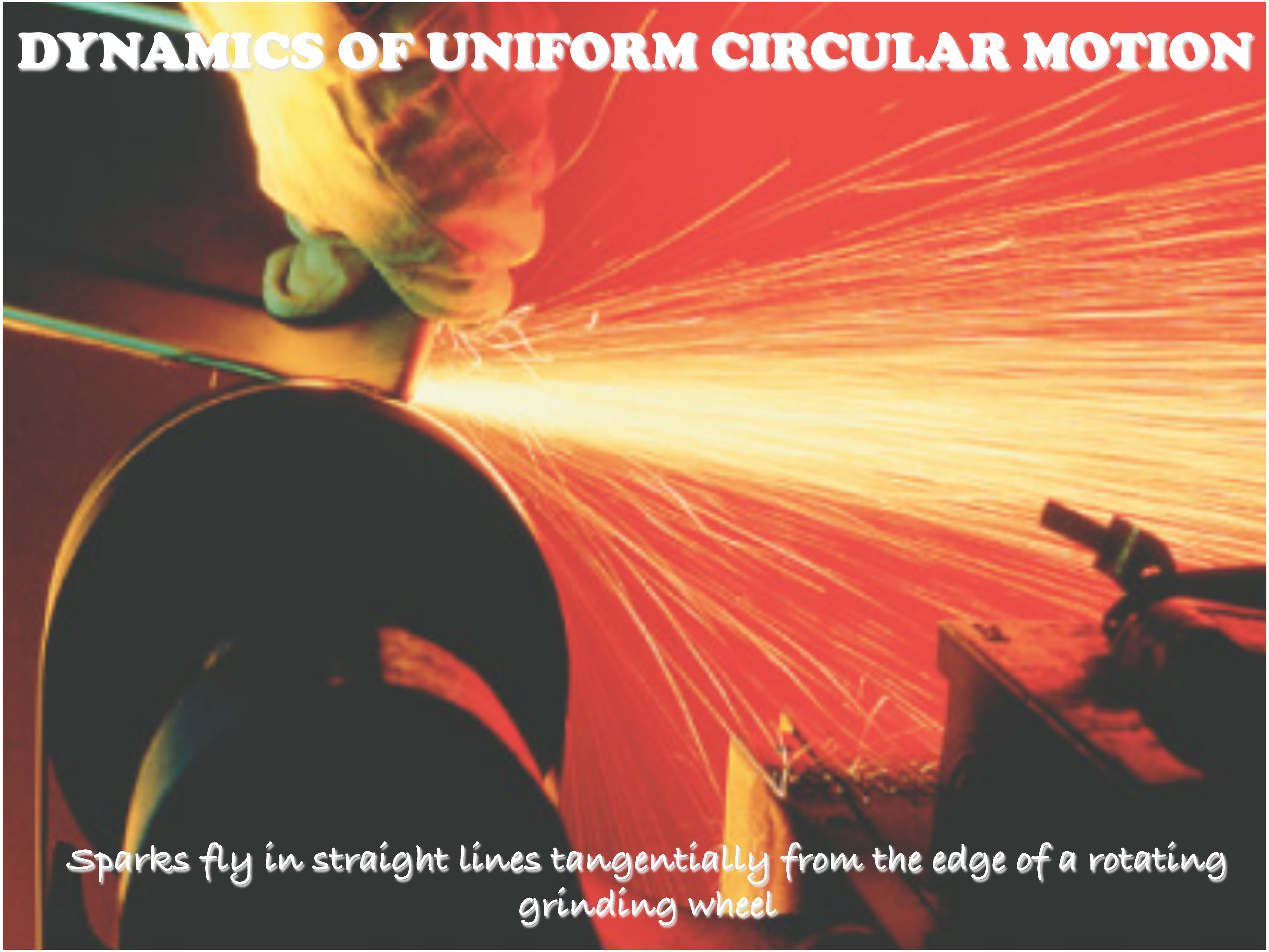


Ball flies off tangentially in the direction of velocity it had at moment it was released because inward force no longer acts



Try it and see!

DYNAMICS OF UNIFORM CIRCULAR MOTION

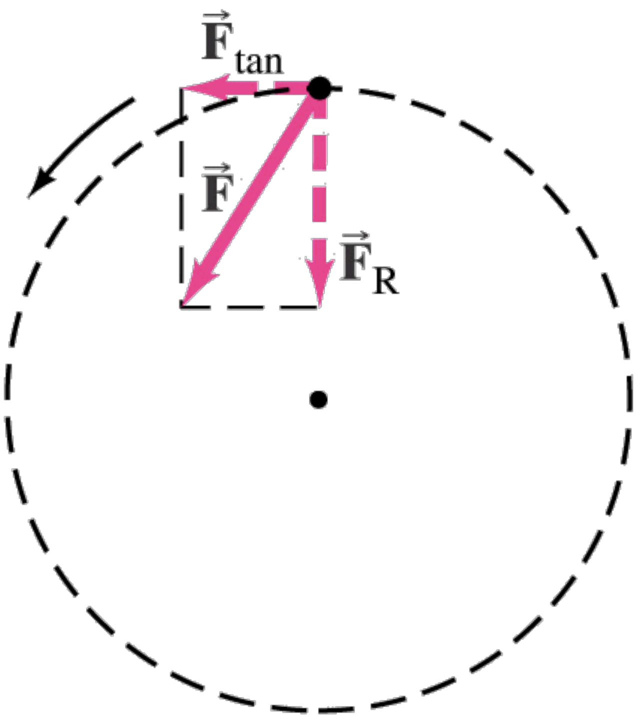


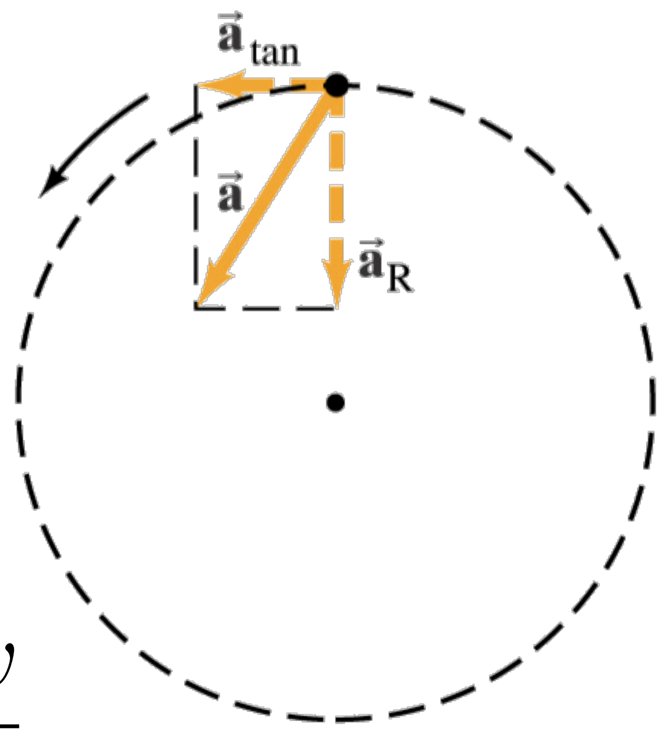
Sparks fly in straight lines tangentially from the edge of a rotating grinding wheel

NONUNIFORM CIRCULAR MOTION

If an object is moving in a circular path but at varying speeds

it must have a tangential component to its acceleration as well as radial one





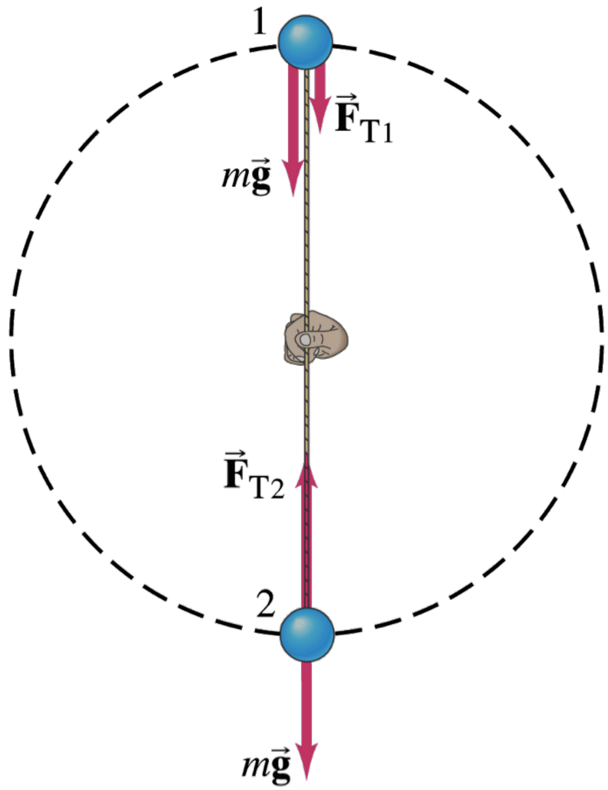
$$a_{\tan} = \frac{dv}{dt}$$

REVOLVING BALL (VERTICAL CIRCLE)

A 0.15 kg ball on the end of a 1 m long cord (of negligible mass) is swung in a vertical circle.

(a) Determine minimum speed ball must have at top of its arc so that ball continues moving in a circle

(b) Calculate tension in cord at bottom of arc assuming ball is moving at twice speed of part (a)



At the top

$$\left(\sum F\right)_R = ma_R \Rightarrow F_{T_1} + mg = m\frac{v^2}{r}$$

The larger the velocity the larger the tension

For minimum speed $\Rightarrow F_{T_1} = 0$

$$v_* = \sqrt{gr} = 3.13 \text{ m/s}$$

At the bottom

$$\left(\sum F\right)_R = ma_r \Rightarrow F_{T_2} - mg = 4m\frac{v_*^2}{r}$$

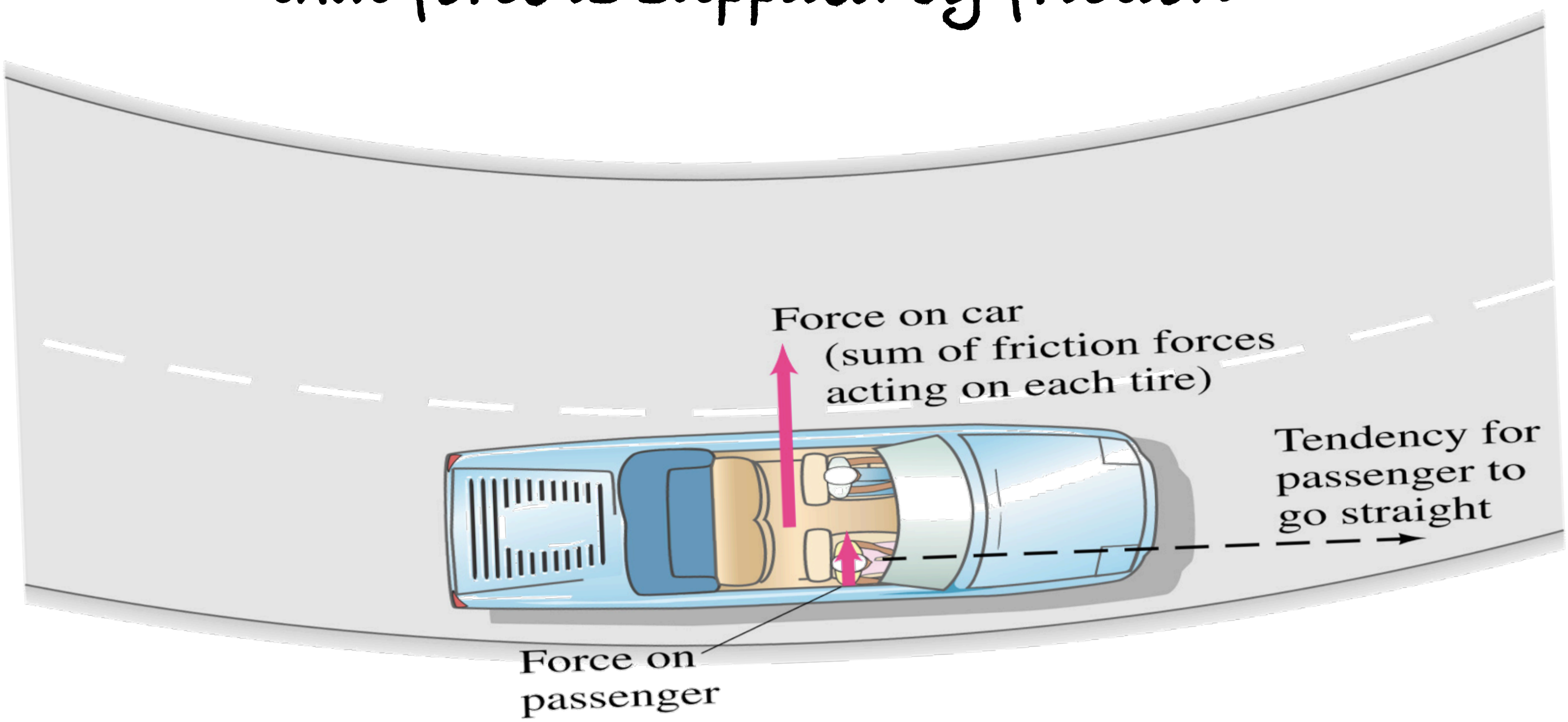
$$F_{T_2} = m\left(4\frac{v_*^2}{r} + g\right) = 7.34 \text{ N}$$

Highway Curves, Banked and Unbanked

When car goes around curve on flat road

must be net force towards center of circle of which curve is arc

that force is supplied by friction



If frictional force is insufficient
car will tend to move more nearly in a straight line

as skid marks show

HIGHWAY CURVES BANKED AND UNBANKED

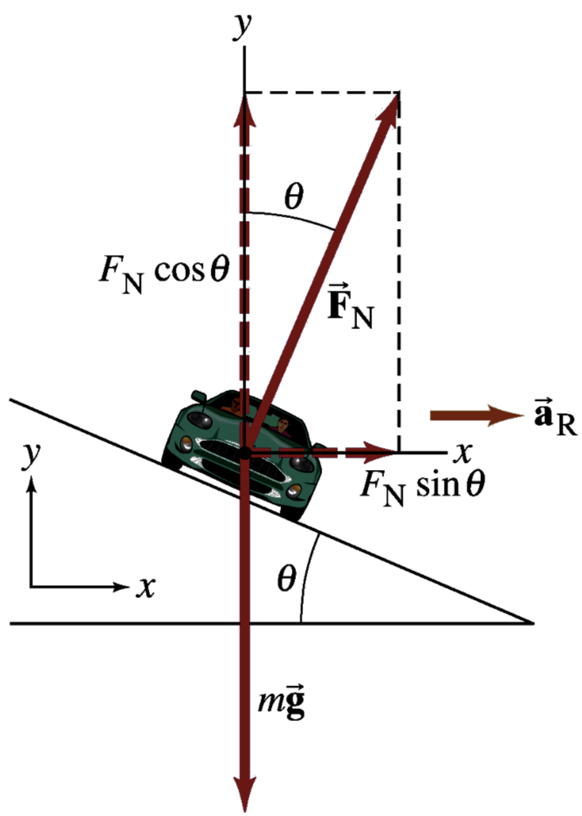
As long as the tires do not slip → friction is static
If the tires do start to slip → friction is kinetic
which is bad in two ways:

1. The kinetic frictional force is smaller than the static

2. Static frictional force points towards center of circle but kinetic frictional force opposes direction of motion making it very difficult to regain control of the car and continue around the curve

Banking the curve can help keep cars from skidding

For every banked curve there is one speed
where the entire centripetal force
is supplied by horizontal component of the normal force
and no friction is required



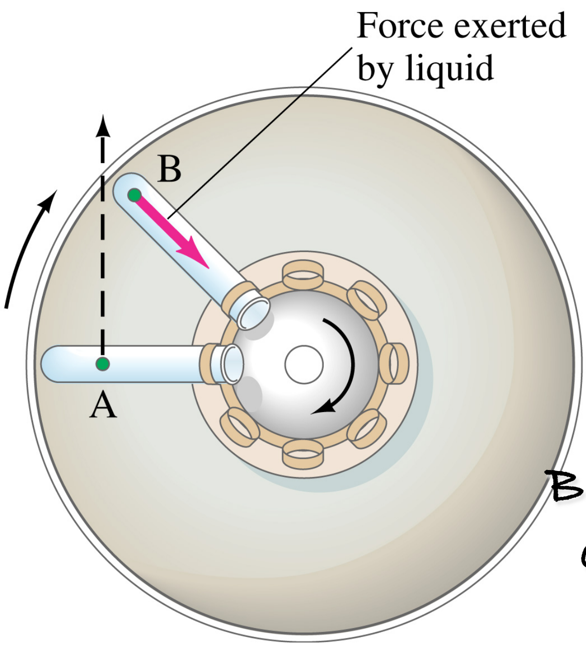
This occurs when:

$$F_N \sin\theta = m\frac{v^2}{r}$$

CENTRIFUGATION

These devices are used to sediment materials quickly or to separate materials

Test tubes are held in centrifugal rotor
which is accelerated to very high rotational speeds



Small green dot represents a small particle (macromolecule) in a fluid filled test tube

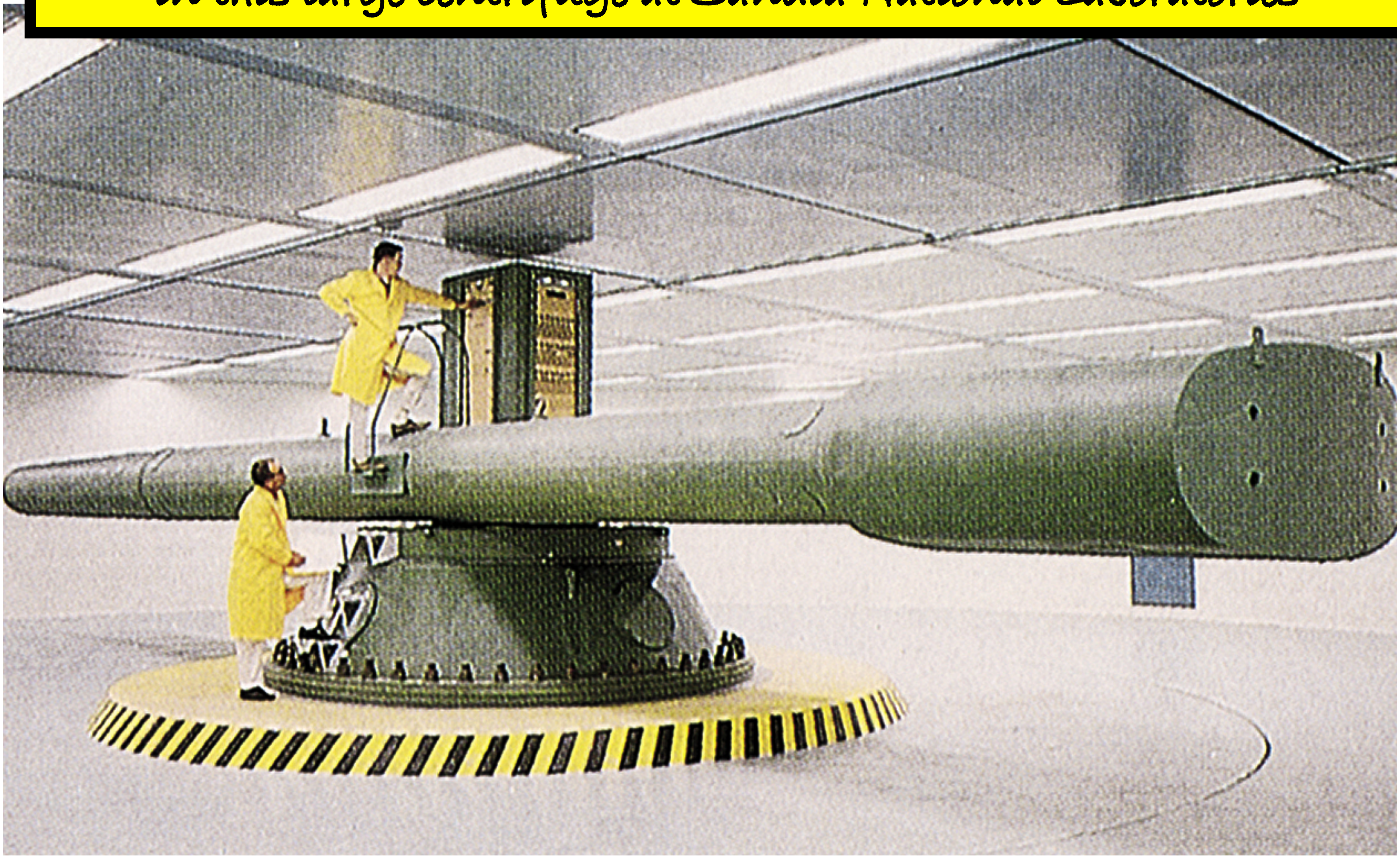
When tube is at position A and rotor is turning particle has a tendency to move in a straight line in direction of dashed arrow

But fluid that resists motion of these particles exerts a centripetal force that keeps particles moving nearly in a circle

Usually resistance of tube does not quite equal mv^2/r and particles eventually reach bottom of tube

Purpose of a centrifuge is to provide and **effective gravity** much larger than normal gravity because of high rotational speeds thus causing more rapid sedimentation

In 1993 a descendent probe containing instruments
went deep into the Jovian atmosphere of Jupiter
Fully assemble probed was tested at accelerations up to 200 g
in this large centrifuge at Sandia National Laboratories



ULTRACENTRIFUGE

The rotor of an ultracentrifuge rotates at 50,000 rpm.

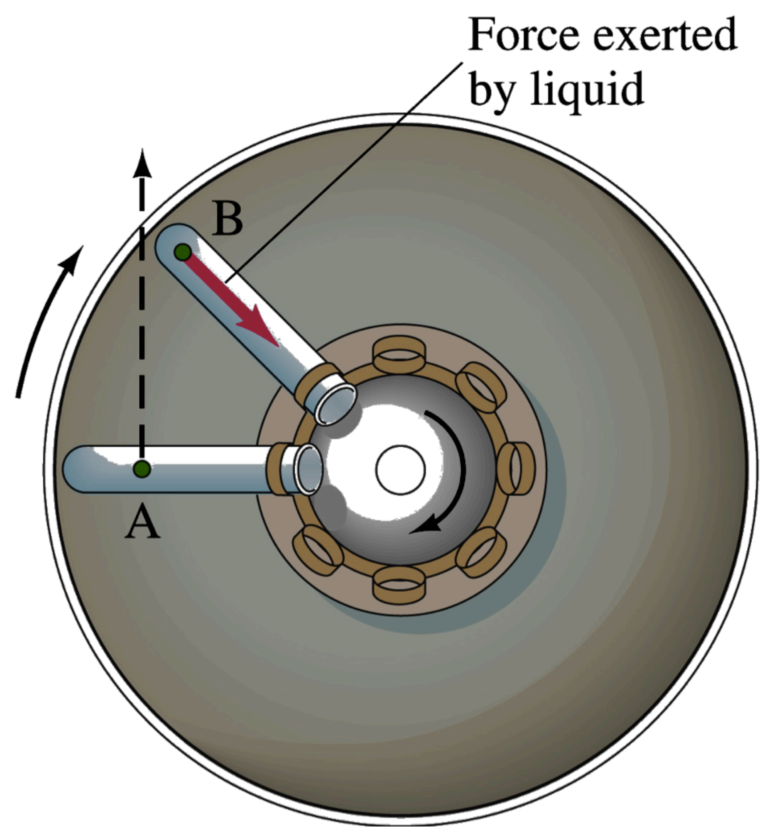
The top of the 4cm long test tube is 6cm from the rotation axis and is perpendicular to it.

The bottom of the tube is 10 cm from the axis of rotation.

Calculate the centripetal acceleration in g at

(a) the top

(b) the bottom of the tube



At the top

$$2\pi r = 2\pi\, 0.06 \text{ m} = 0.377 \text{ m per revolution}$$

It makes 5×10^4 such revolutions per minute

on dividing by $60 \text{ min/s} \Rightarrow 833 \text{ rev/s}$

Time to make 1 revolution $\Rightarrow T = \dfrac{1}{833 \text{ rev/s}}$

$$v = \frac{2\pi r}{T} = 3.14 \times 10^2 \text{ m/s}$$

$$a_R = \frac{v^2}{r} = 1.64 \times 10^6 \text{ m/s}^2 = 1.67 \times 10^5 \ g$$

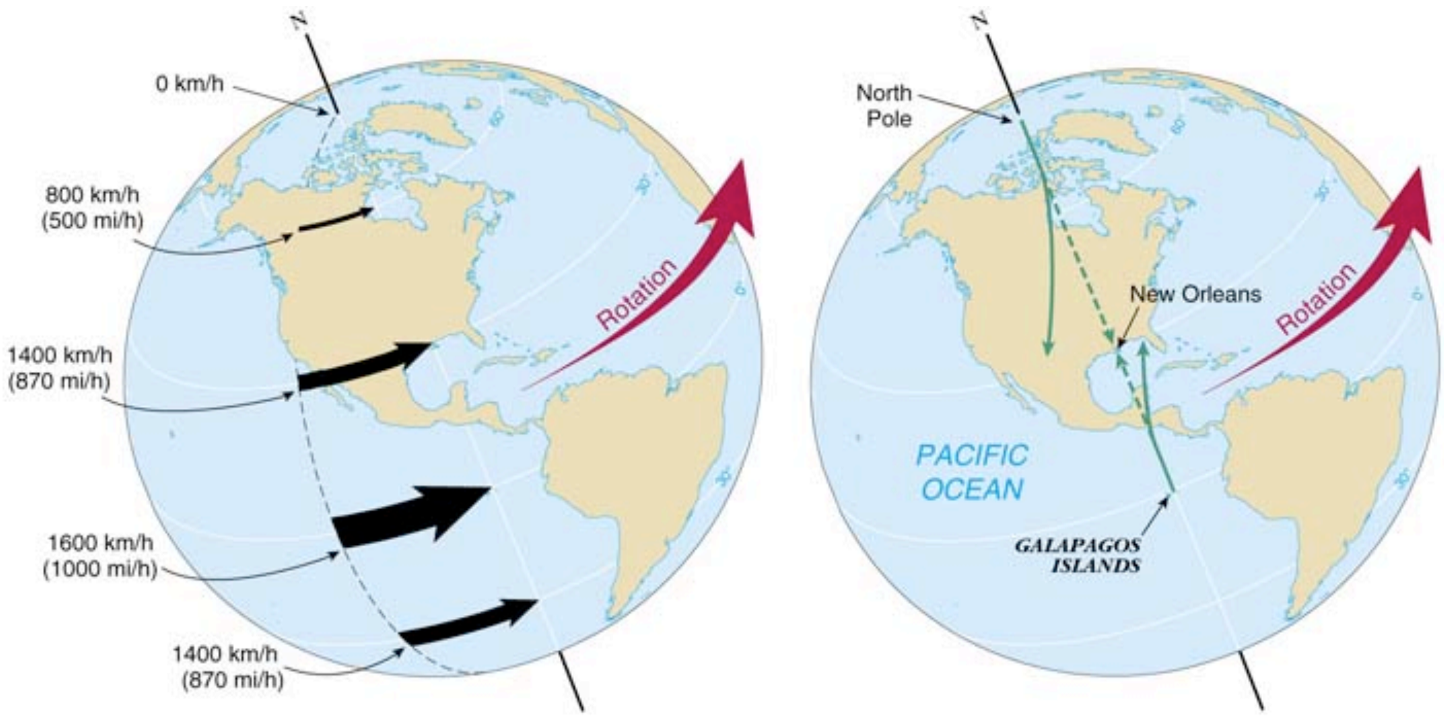
At the bottom

$$r = 0.1 \text{ m}$$

$$v = \frac{2\pi r}{T} = 5.23 \times 10^2 \text{ m/s}$$

$$a_R = \frac{v^2}{r} = 2.74 \times 10^6 \text{ m/s}^2 = 2.8 \times 10^5 \ g$$

How long would a day be if the Earth were rotating so fast that objects at the equator were apparently weightless?



For an object to be apparently weightless
would mean that the object would have a centripetal acceleration equal to g

This is the same as asking what orbital period would be for object orbiting Earth
with orbital radius equal to Earth radius

$$a_c = \frac{v^2}{R_\oplus} = g$$

$$\Rightarrow v = \sqrt{gR_\oplus} = 7.91 \text{ km/s}$$

$$T = \frac{2\pi R_\oplus}{v} = 5,060 \text{ s}$$

$$= 84.3 \text{ minutes}$$