Lesson 5

Physics 168

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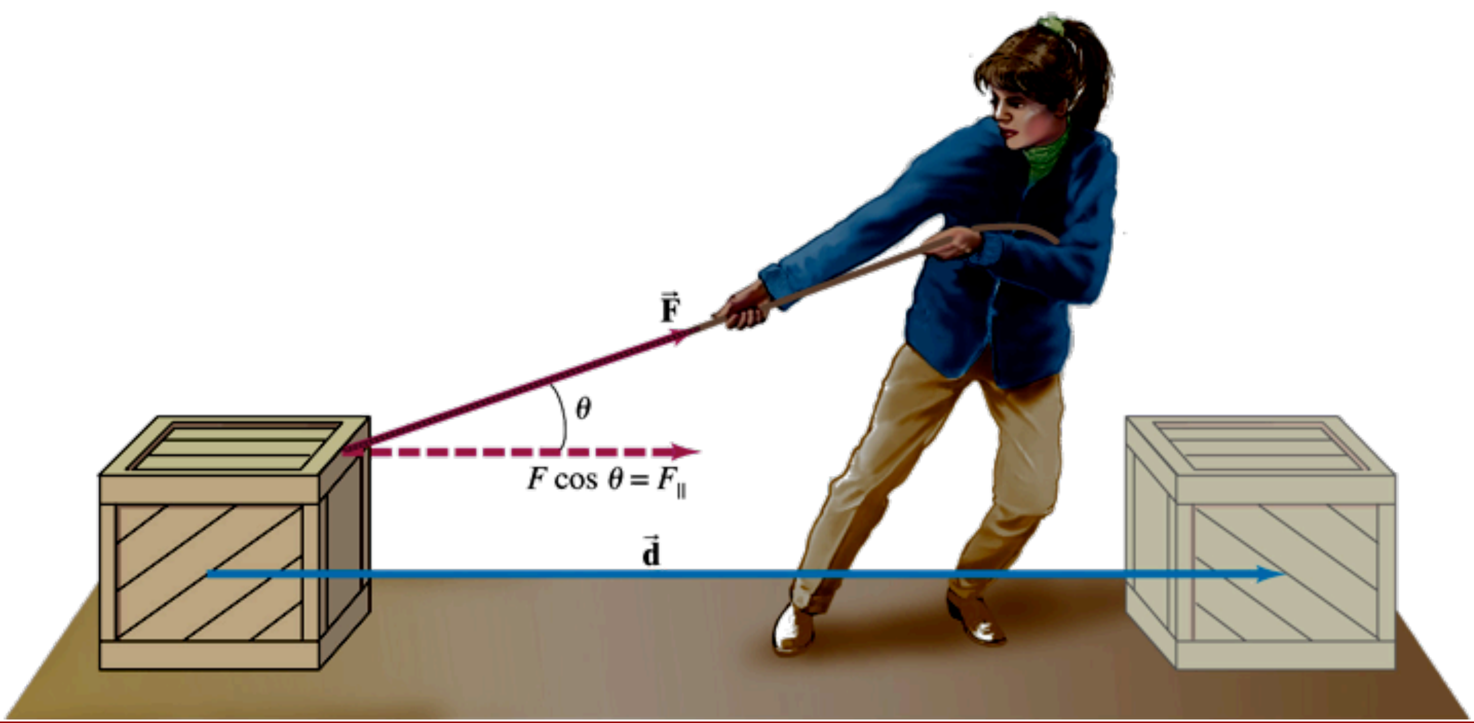
Conservation Theorems: Energy

Work Done by a Constant Force

distance moved times component of force in direction of displacement

$$W = Fd\cos\theta$$

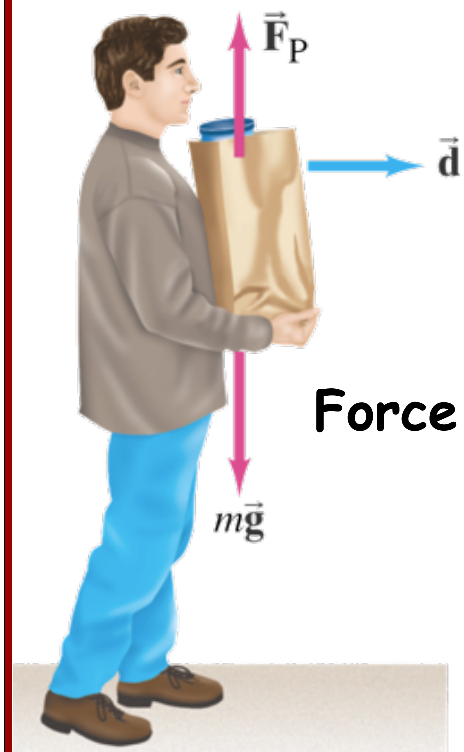


Work Done by a Constant Force (Cont'd)

In SI system ➤ units of work are joules:

$$1\mathrm{J} = 1\,\mathrm{N}\cdot\mathrm{m}$$

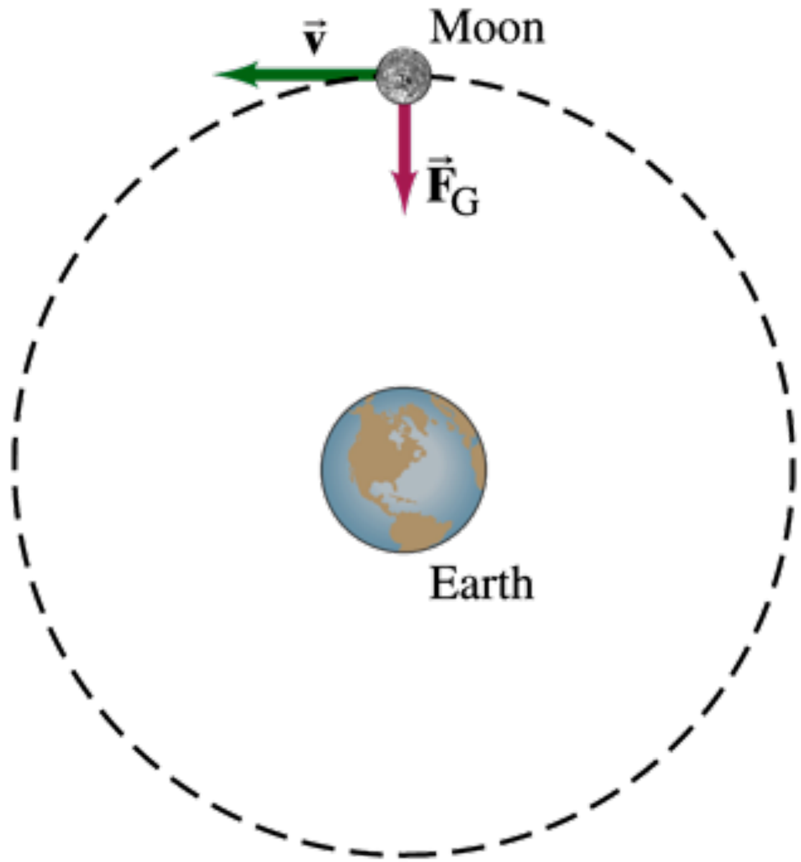
As long as this person does not lift or lower bag of groceries he is doing no work on it



Force he exerts has no component in direction of motion

Work Done by a Constant Force

What about centripetal forces?



Centripetal forces do no work
as they are always perpendicular to direction of motion

Kinetic Energy and Work-Energy Principle

If constant net force acts on particle that moves along x axis

Newton's second law leads to

$$F_{\mathrm{net},x} = m a_x$$

If net force is constant ☛ acceleration is constant

convince yourself that ☛ $v_{\mathrm{f}}^2 = v_{\mathrm{i}}^2 + 2 a_x \Delta x$

Solving for a_x

$$a_x = \frac{1}{2\Delta x}\left(v_{\mathrm{f}}^2 - v_{\mathrm{i}}^2\right)$$

⬇

$$F_{\mathrm{net},x}\Delta x = \frac{1}{2} m v_{\mathrm{f}}^2 - \frac{1}{2} m v_{\mathrm{i}}^2$$

⬇

$$W_{\mathrm{net}} = \frac{1}{2} m v_{\mathrm{f}}^2 - \frac{1}{2} m v_{\mathrm{i}}^2$$

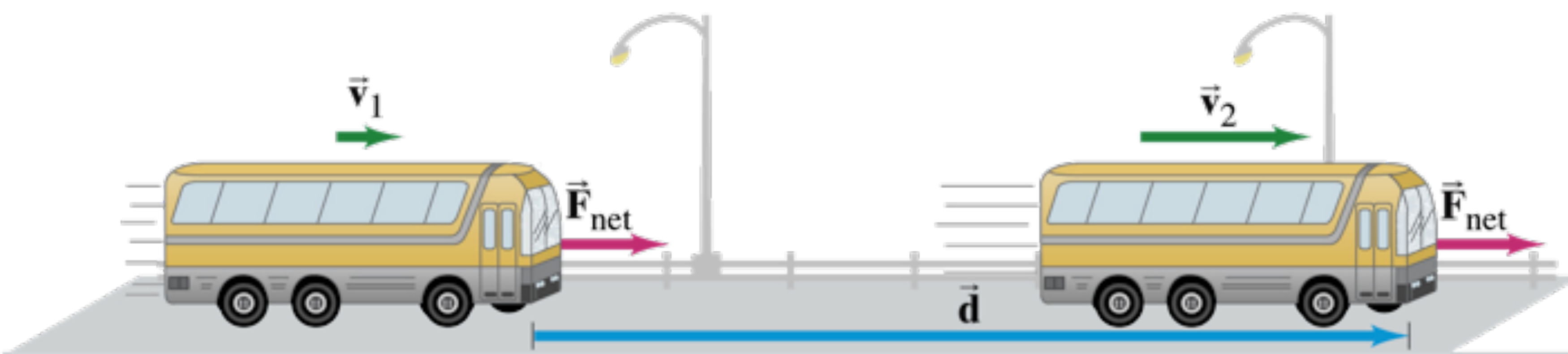
Kinetic Energy and Work-Energy Principle

If we write acceleration in terms of velocity and distance

we find that work done is

$$W_{\rm net} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

We define kinetic energy ☛ $\mathrm{KE} = \frac{1}{2}mv^2$



Kinetic Energy and Work-Energy Principle

Work done is equal to change in kinetic energy

$$W_{\text{net}} = \Delta \text{KE}$$

If net work

Is positive
kinetic energy increases

Is negative
kinetic energy decreases

Work Done by a Varying Force

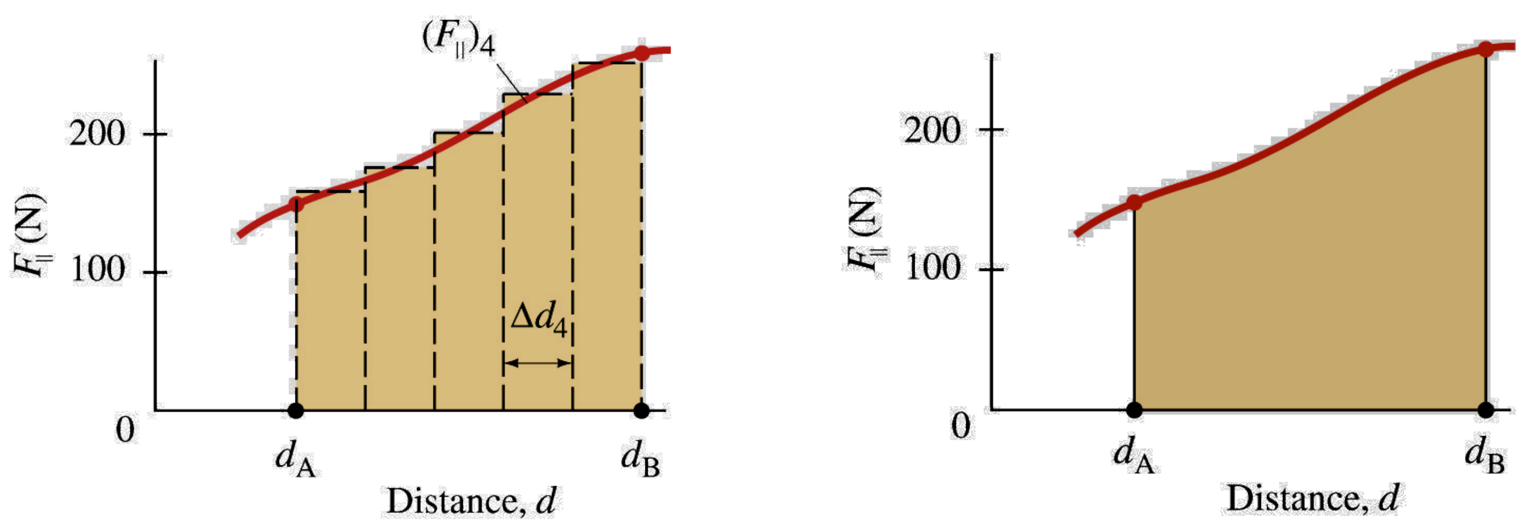
For varying force work can be approximated

by dividing distance up into small pieces

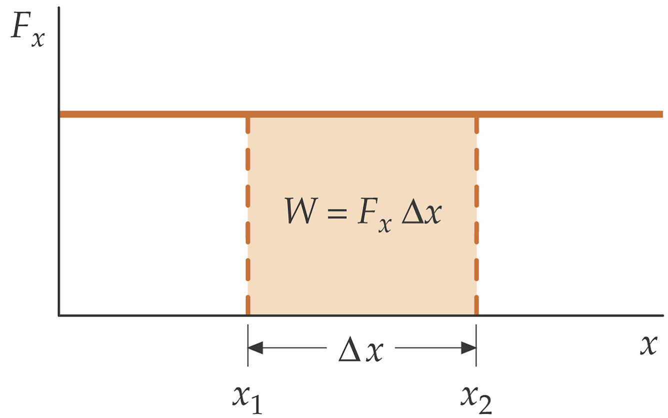
finding work done during each and adding them up

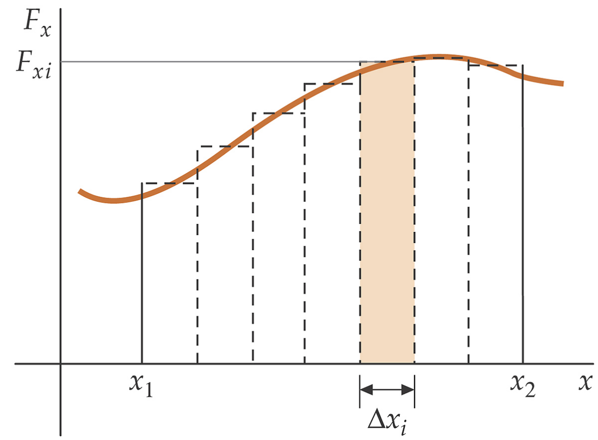
As pieces become very narrow

work done is area under force vs. distance curve



Work done on a variable force – straight line motion





$$W = \lim_{\Delta x_i \to 0} \sum_i F_{x_i} \, \Delta x_i$$

$$W = \int_{x_1}^{x_2} F_x \, dx$$

Replacing for $F_x = m a_x$

$$W = \int_{x_1}^{x_2} m \, a \, dx = \int_{x_1}^{x_2} m \frac{d^2 x}{dt^2} dx = \int_{x_1}^{x_2} m \frac{dv}{dt} dx$$

$$W = \int_{v_1}^{v_2} m \, v \, dv = \frac{1}{2} m \left(v_2^2 - v_1^2 \right)$$

Potential Energy

An object can have potential energy by virtue of its surroundings

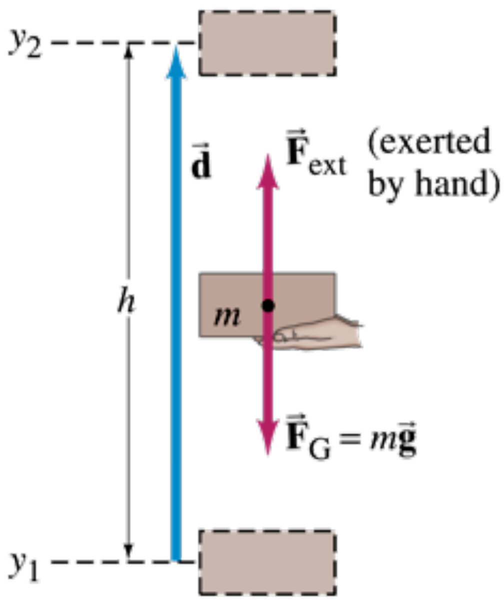
Familiar examples of potential energy:

- A wound-up spring
- A stretched elastic band
- An object at some height above ground

Gravitational Potential Energy

In raising a mass m to a height h work done by external force is

$$W_{\rm ext} = F_{\rm ext}\, d \cos 0^\circ = mgh$$
$$= mg(y_2 - y_1)$$



We therefore define gravitational potential energy

$$PE_{\rm grav} = mgy$$

Gravitational Potential Energy (cont'd)

This potential energy can become kinetic energy if object is dropped

Potential energy is a property of a system as a whole not just of object (because it depends on external forces)

If $PE_{\rm grav} = mgy$ ☚ where do we measure y from?

It turns out not to matter as long as we are consistent about where we choose $y = 0$

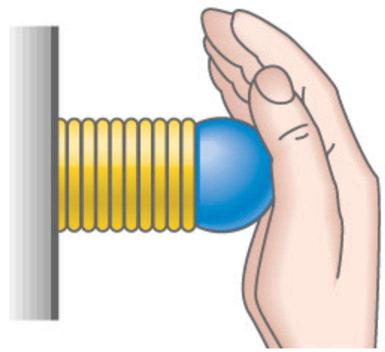
Only changes in potential energy can be measured

Elastic Potential Energy

Potential energy can also be stored in a spring when it is compressed

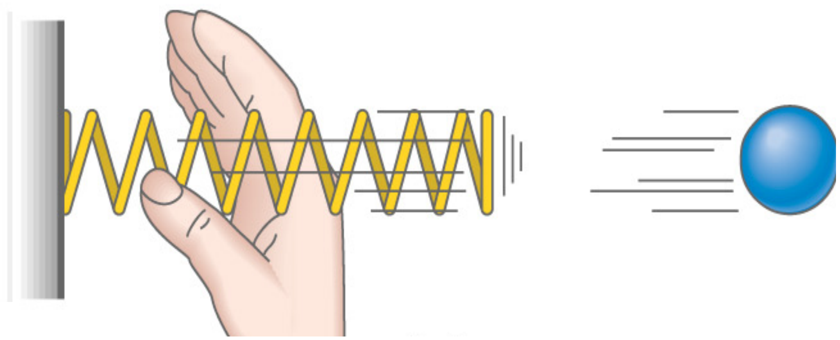
Elastic Potential Energy (cont'd)

Potential energy can also be stored in a spring when it is compressed



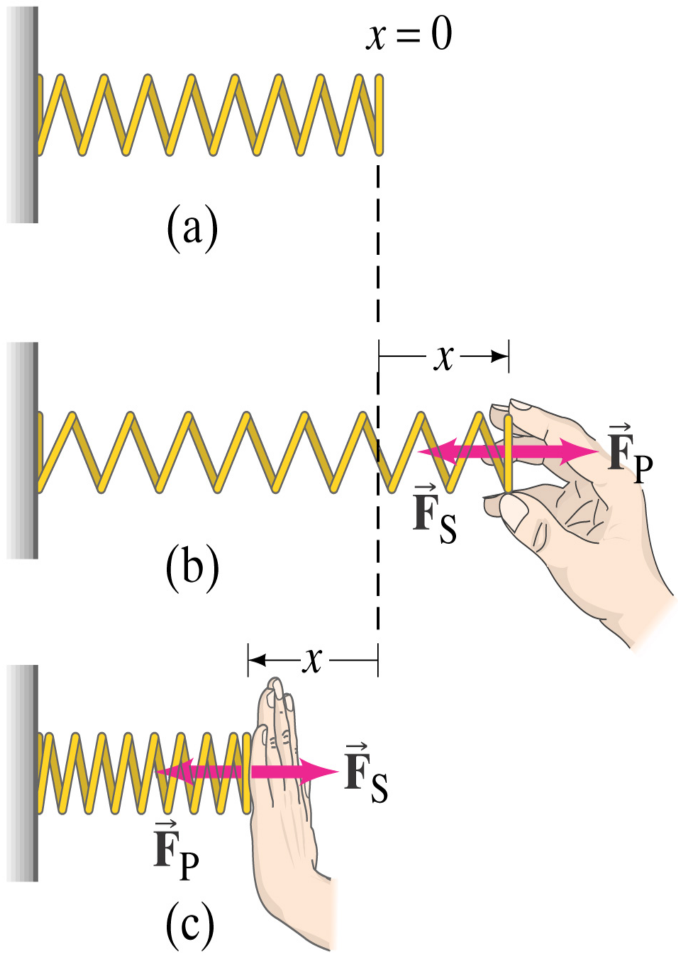
Potential Energy (cont'd)

Potential energy can also be stored in a spring when it is compressed



Elastic Potential Energy (cont'd)

Force required to compress or stretch a spring is



$$F_s = -kx$$

spring constant k

needs to be measured for each spring

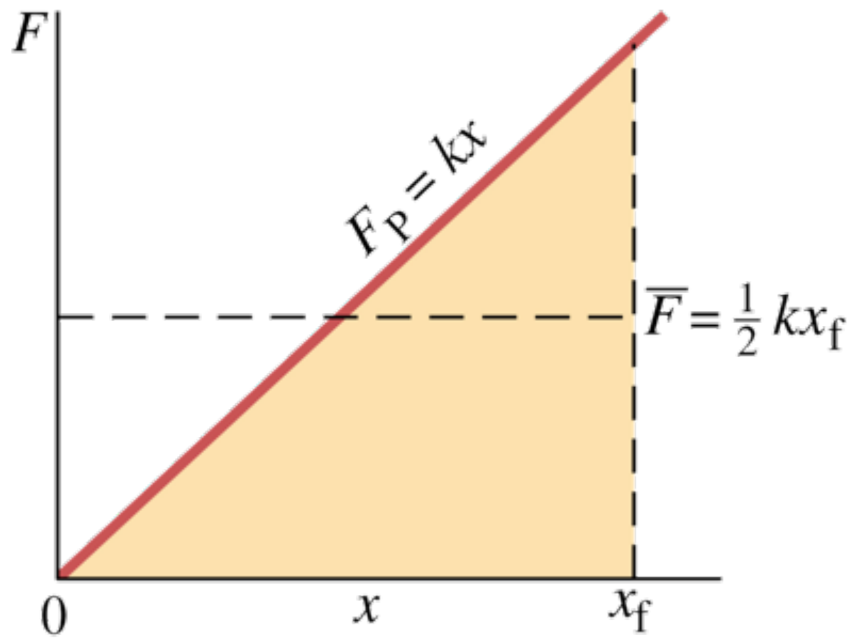
Elastic Potential Energy (cont'd)

Force increases as spring is stretched or compressed further

Potential energy of compressed or stretched spring

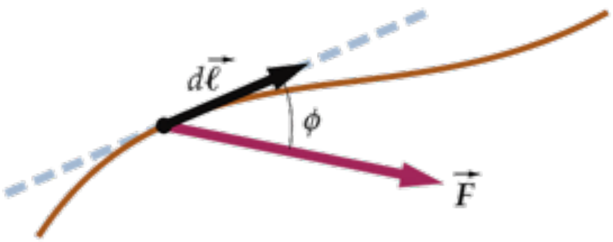
measured from its equilibrium position ☛

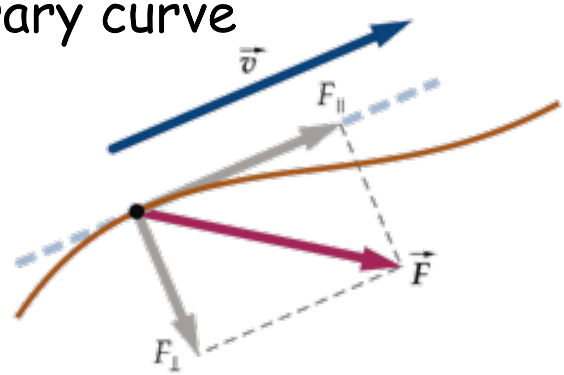
$$W_{\text{by spring}} = \int_{x_i}^{x_f} F_x\, dx = -k \int_{x_i}^{x_f} x\, dx = -k\left(\frac{x_f^2}{2} - \frac{x_i^2}{2}\right)$$



Scalar product

Consider a particle moving along an arbitrary curve





Component $F_{||}$ is related to angle ϕ (between directions of $\vec{F}$ and $\vec{dl}$) by

$$F_{||} = \vec{F} \cos\phi$$

Work done by F for displacement $\vec{dl}$ is

$$dW = F_{||}\, dl = F \cos\phi\, dl$$

This combination of two vectors and cosine of angle between their directions is called scalar product

Scalar product of two general vectors $\vec{A}$ and $\vec{B}$ is

$$\vec{A} \cdot \vec{B} = AB \cos\Phi$$

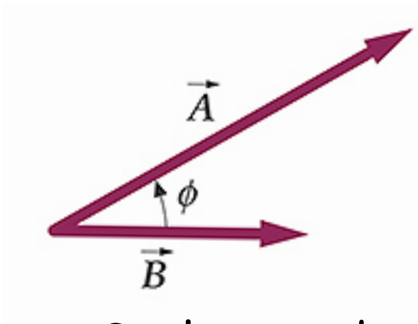
Φ ➤ angle between $\vec{A}$ and $\vec{B}$

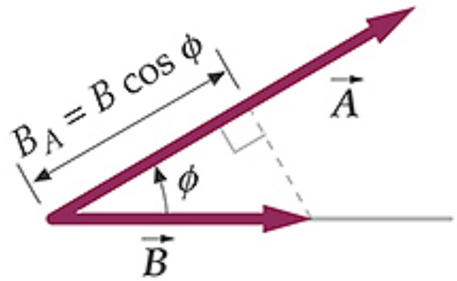
Properties of Scalar Products

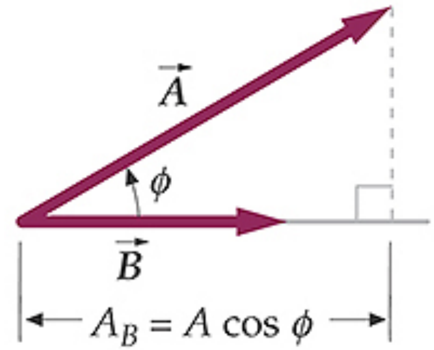
If		Then
$\vec{A}$ and $\vec{B}$ are perpendicular	☛	$\vec{A} \cdot \vec{B} = 0$ (because $\phi = 90^\circ$, $\cos\phi = 0$)
$\vec{A}$ and $\vec{B}$ are parallel	☛	$\vec{A} \cdot \vec{B} = AB$ (because $\phi = 0$, $\cos\phi = 1$)
$\vec{A} \cdot \vec{B} = 0$	☛	$\vec{A} = 0$ or $\vec{B} = 0$ or $\vec{A} \perp \vec{B}$
$\vec{A} \cdot \vec{A} = A^2$	☛	Because $\vec{A}$ is parallel to itself
$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$	☛	Commutative rule of multiplication
$(\vec{A} + \vec{B}) \cdot \vec{C} = \vec{A} \cdot \vec{C} + \vec{B} \cdot \vec{C}$	☛	Distributive rule of multiplication

Scalar product (cont'd)

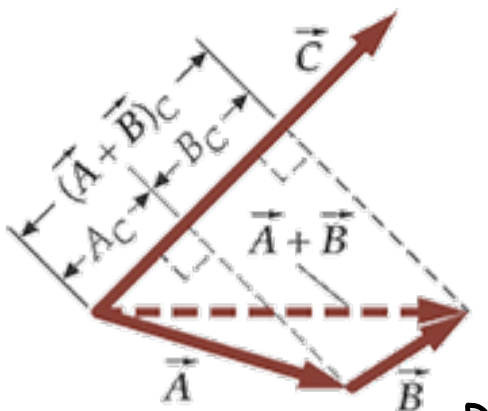
Product of A and projection of $\vec{B}$ on $\vec{A}$ ➤ and vice versa







Scalar product is distributive over addition



$$(\vec{A} + \vec{B}) \cdot \vec{C} = \vec{A} \cdot \vec{C} + \vec{B} \cdot \vec{C}$$

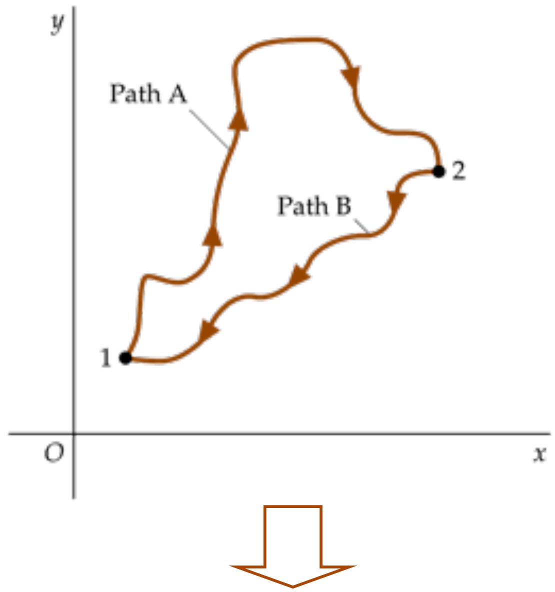
Rule of differentiating a scalar product is

$$\frac{d}{dt}(\vec{A} \cdot \vec{B}) = \frac{d\vec{A}}{dt} \cdot \vec{B} + \vec{A} \cdot \frac{d\vec{B}}{dt}$$

Conservative and Nonconservative Forces

Work done by a conservative force on a particle

is independent of path taken as particles moves from one point from another



A force is conservative if work it does on a particle is zero

when particle moves around any closed path returning to its initial position

A force is said to be non conservative

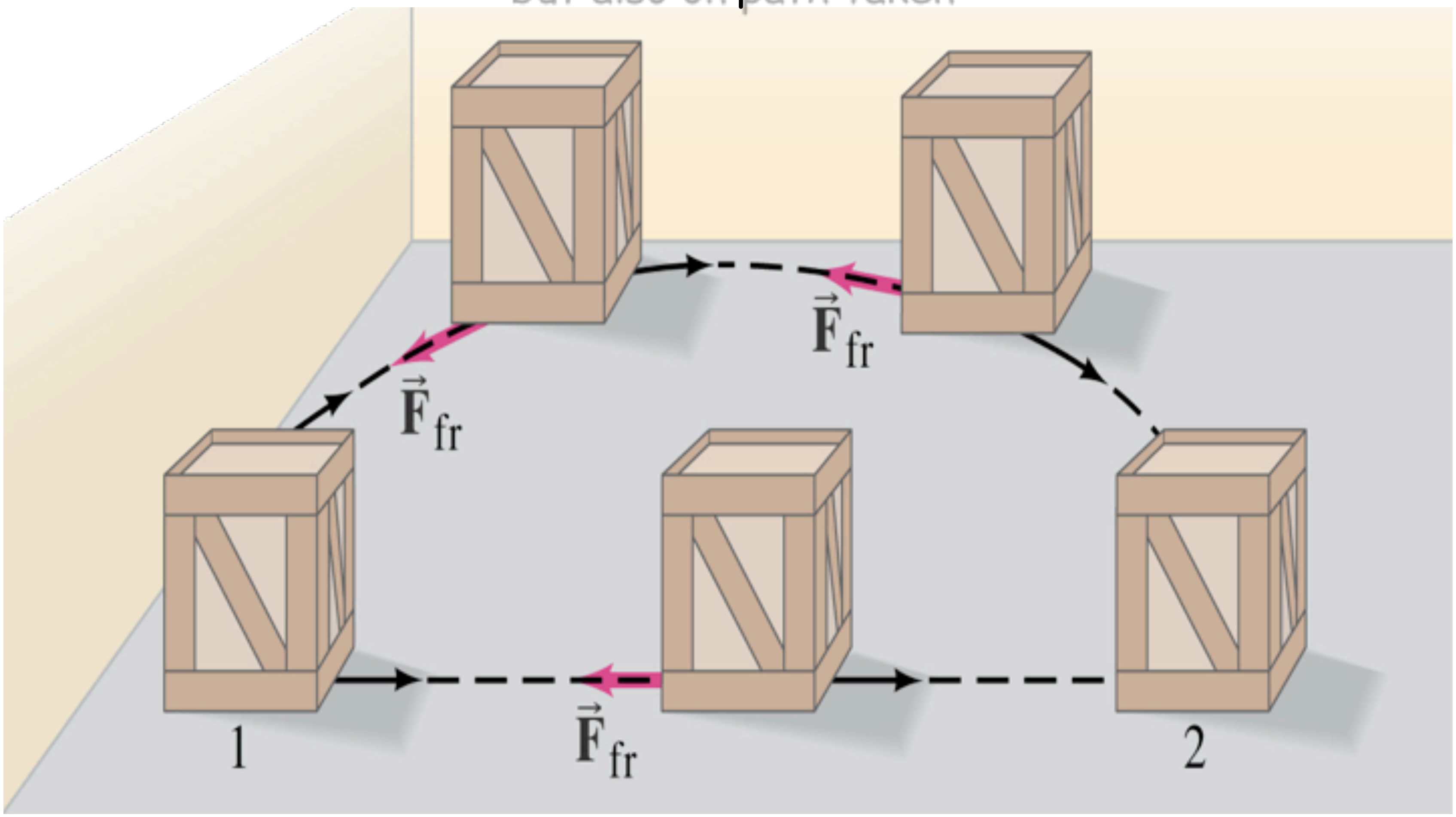
if it does not meet definition of conservative forces

Conservative and Nonconservative Forces

If friction is present

work done depends not only on starting and ending points

but also on path taken



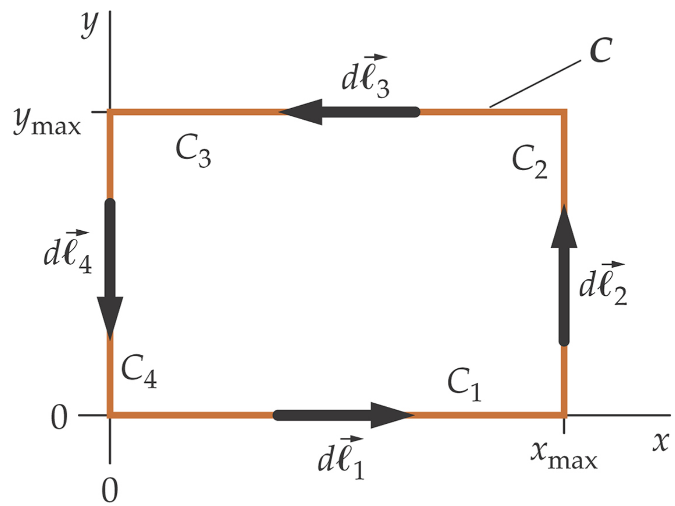
Friction ☛ **nonconservative force**

To calculate work done by a force F around a closed curve (path) C we evaluate

$$\oint_C \vec{F} \cdot d\vec{l}$$

Circle on integral means that integration is evaluated for one complete trip around C

Calculate integral around closed path shown in figure if



$$\vec{F} = Ax\hat{\imath}$$

Force is described by Hooke's law

Integral is zero as force for a spring is conservative

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Conservative and Nonconservative Forces (cont'd)

We distinguish between: work done by conservative forces and work done by nonconservative forces

Work done by nonconservative forces is equal to total change in kinetic and potential energies

$$W_{NC} = \Delta KE + \Delta PE$$

Mechanical Energy and Its Conservation

If there are no nonconservative forces

sum of changes in kinetic energy and in potential energy is zero

Kinetic and potential energy changes are equal but opposite in sign

Define total mechanical energy:

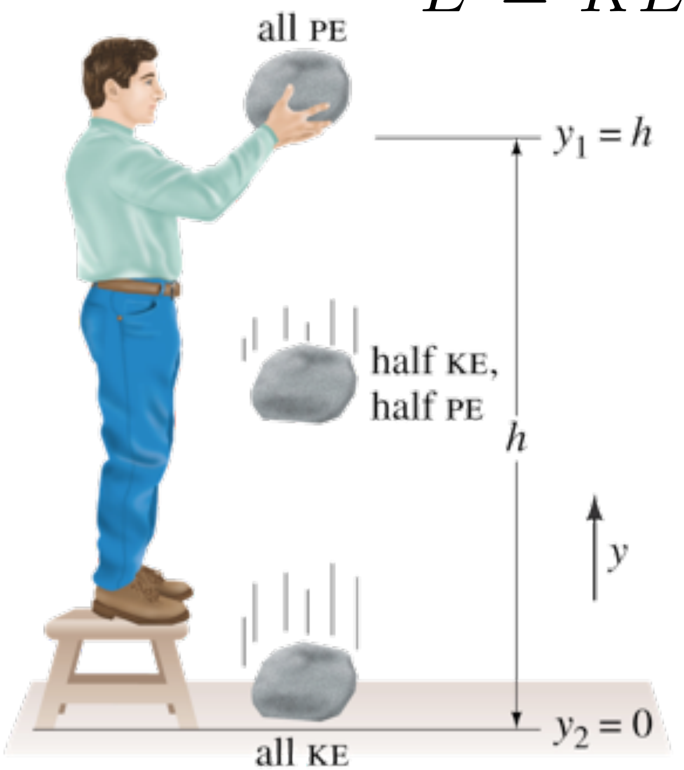
$$E = KE + PE$$

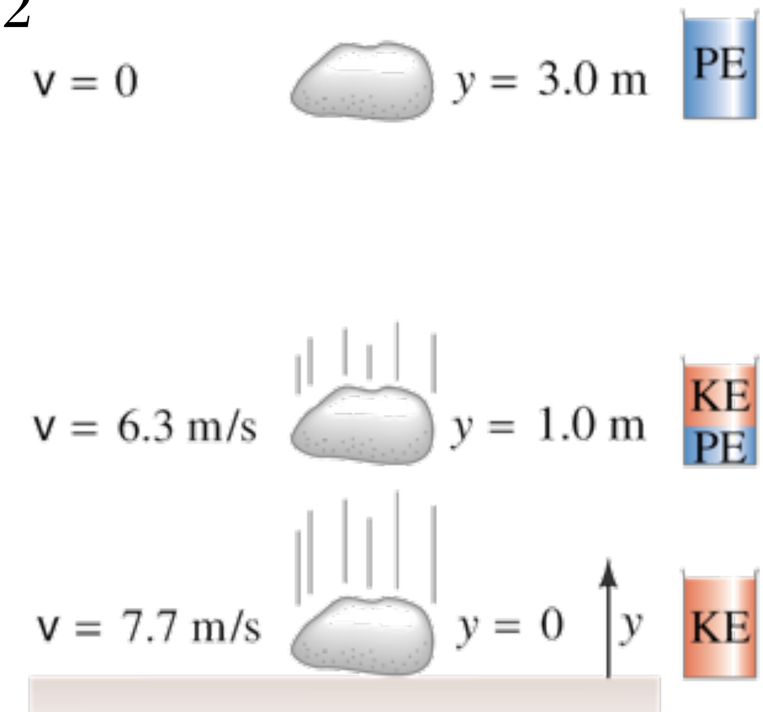
and its conservation: $E_2 = E_1 = \text{Constant}$

Problem Solving Using Conservation of Mechanical Energy

In left image total mechanical energy is:

$$E = KE + PE = \frac{1}{2}mv^2 + mgy$$





Energy buckets (right) show how energy moves from all potential to all kinetic

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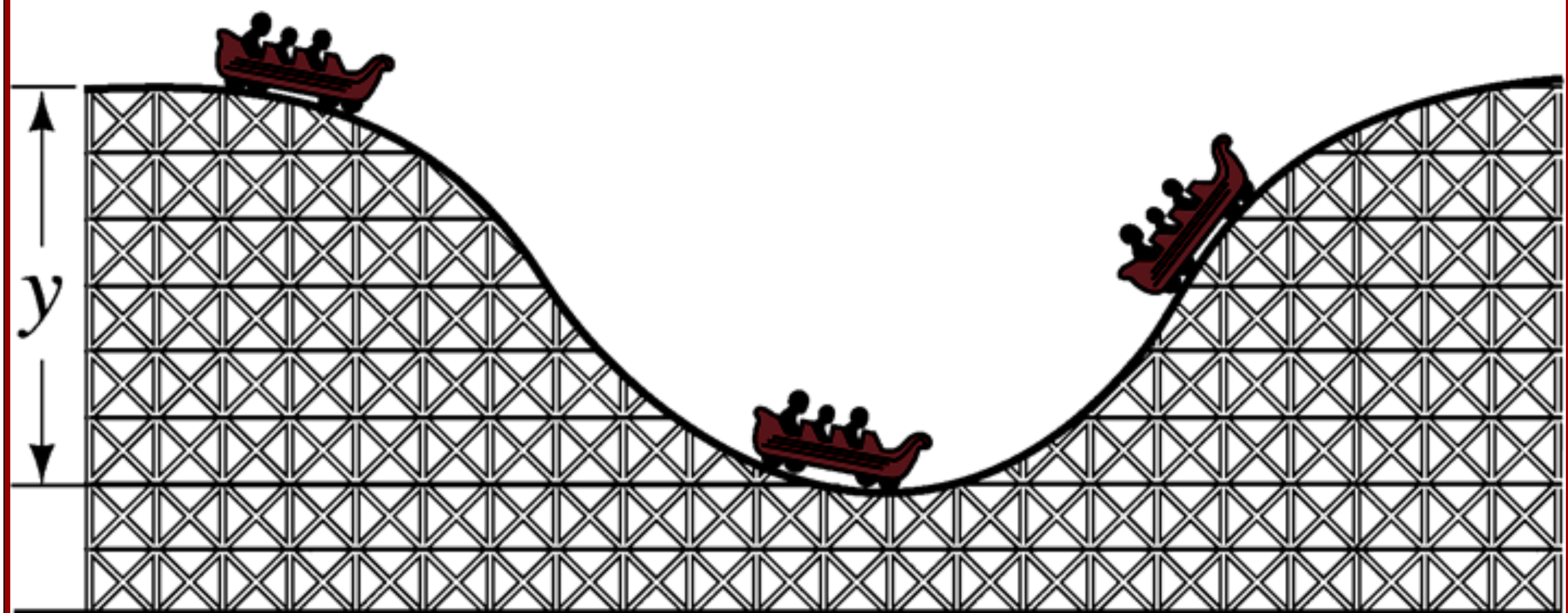
Problem Solving Using Conservation of Mechanical Energy

If there is no friction speed of a roller coaster

will depend only on its height compared to its starting height



Work done on a skier

You and your friend are at a ski resort with two ski runs

a beginner's run and an expert's run

Both runs begin at top of ski lift and end at finish line at bottom of same lift

Let h be vertical descent for both runs

Beginner's run is longer and less steep than expert's run

You and your friend, who is a much better skier than you,

are testing some experimental frictionless skis

To make things interesting you offer a wager

that if she takes expert's run and you take beginner's run

her speed at finish line will not be greater than your speed at finish line

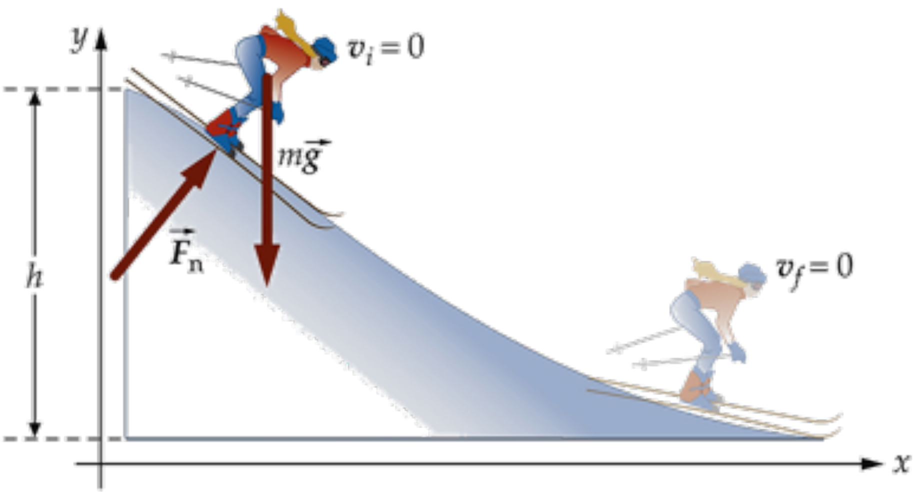
Forgetting that you study physics she accepts bet

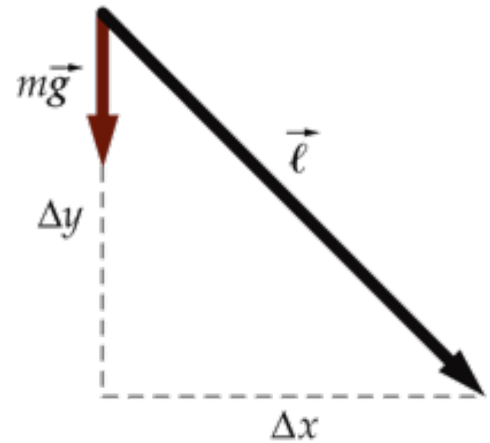
Conditions are that you both start from rest at top of lift

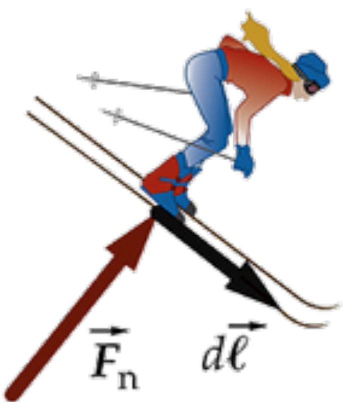
and both of you coast for entire trip

Who wins bet? (Assume air drag is negligible)

Work done on a skier (cont'd)







$$W_{\text{total}} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$W_{\text{total}} = W_{F_n} + W_{F_g}$$

$$F_n \perp d\vec{\ell} \Rightarrow W_{F_n} = 0$$

$$W_{F_g} = mg\Delta y$$

$$\Delta \text{KE} = mg\Delta y \Rightarrow v_f = \sqrt{2gh}$$

Final speed depends only on h which is same for both runs

Both of you will have same final speed

YOU WIN!!

Bungee Jumping

A $62\,\mathrm{kg}$ bungee jumper jumps from a bridge

He is tied to a bungee cord whose un-stretched length is $L_1 = 12\,\mathrm{m}$

and falls a total of $L_1 + L_2 = 31\,\mathrm{m}$

Calculate

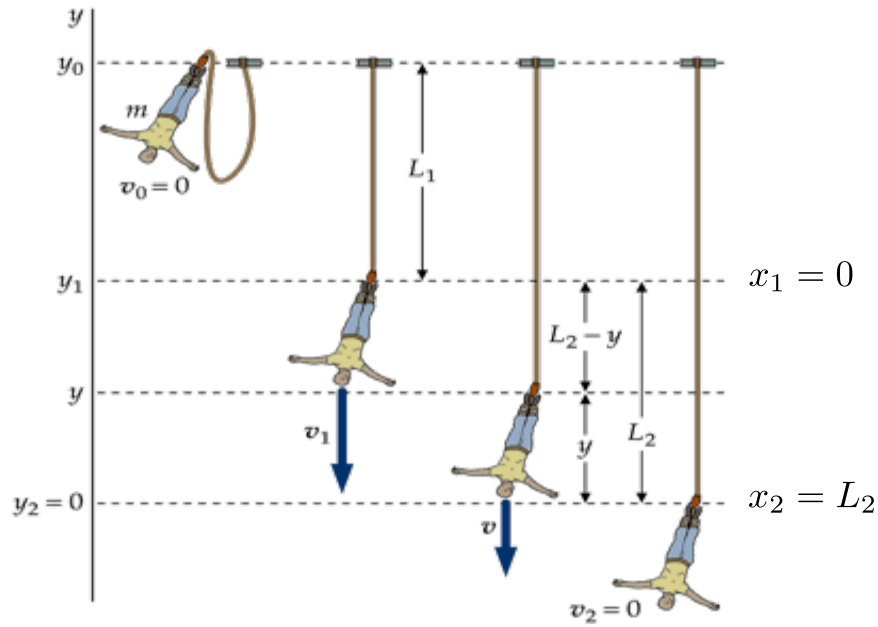
ⓐ spring stiffness constant k of bungee cord, assuming Hooke's law applies

ⓑ Calculate maximum acceleration he experiences

ⓒ Calculate velocity just before cord is starting to stretch

ⓓ Calculate the position of maximum velocity

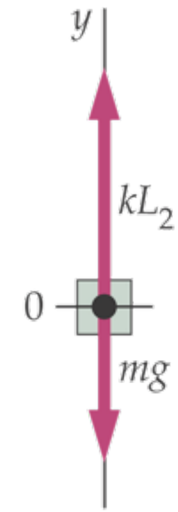
ⓐ Conservation of energy



$$\frac{1}{2}mv_0^2 + mgy_0 + \frac{1}{2}kx_0^2 = \frac{1}{2}mv_2^2 + mgy_2 + \frac{1}{2}kx_2^2$$

$$mgy_0 = \frac{1}{2}kx_2^2 \Rightarrow k = \frac{2mgy_0}{x_2^2} = 104.4 \text{ N/m}$$

ⓑ Maximum acceleration occurs when bungee cord has maximum stretch



$$F_{\text{net}} = F_{\text{cord}} - mg = kx_2 - mg = ma$$

$$a = \frac{kx_2}{m} - g = 22\ \text{m/s}^2 = 2.2g$$

ⓒ Just before cord is starting to stretch $x = 0$

$$mgy_0 = \frac{1}{2}mv_1^2 + mgy_1 \Rightarrow \frac{1}{2}mv_1^2 = mgL_1$$

$$v_1 = \sqrt{2gL_1} = 15.3\ \text{m/s}$$

ⓓ Maximum velocity @ $a = 0$

$$v_{\text{max}}\ @\ kx - mg = 0 \Rightarrow x = mg/k = 5.8\ \text{m}$$

Energy Conservation with Dissipative Processes

If there is a nonconservative force such as friction where do kinetic and potential energies go?

Others Forms of Energy

Some other forms of energy

Electric energy | nuclear energy | thermal energy | chemical energy

Work is done when energy is transferred from one object to another

Accounting for all forms of energy

we find that total energy neither increases nor decreases

Energy as a whole is conserved

Conservative and Nonconservative Forces

Potential energy can only be defined for conservative forces

Conservative forces	Nonconservative forces
Gravitional	Friction
Elastic	Air resistance
	Tension in cord
	Motor or rocket propulsion
	Push or pull by a person