are the same, apart from the additional repulsive Coulomb force for the proton–proton interaction.

Evidence for the limited range of nuclear forces comes from scattering experiments and from studies of nuclear binding energies. The short range of the nuclear force is shown in the neutron–proton (n–p) potential energy plot of Figure 44.3a obtained by scattering neutrons from a target containing hydrogen. The depth of the n–p potential energy well is 40 to 50 MeV, and there is a strong repulsive component that prevents the nucleons from approaching much closer than 0.4 fm.

The nuclear force does not affect electrons, enabling energetic electrons to serve as point-like probes of nuclei. The charge independence of the nuclear force also means that the main difference between the n–p and p–p interactions is that the p–p potential energy consists of a *superposition* of nuclear and Coulomb interactions as shown in Figure 44.3b. At distances less than 2 fm, both p–p and n–p potential energies are nearly identical, but for distances of 2 fm or greater, the p–p potential has a positive energy barrier with a maximum at 4 fm.

The existence of the nuclear force results in approximately 270 stable nuclei; hundreds of other nuclei have been observed, but they are unstable. A plot of neutron number N versus atomic number Z for a number of stable nuclei is given in Figure 44.4. The stable nuclei are represented by the black dots, which lie in a narrow range called the *line of stability*. Notice that the light stable nuclei contain an equal number of protons and neutrons; that is, $N = Z$. Also notice that in heavy stable nuclei, the number of neutrons exceeds the number of protons: above $Z = 20$, the line of stability deviates upward from the line representing $N = Z$. This deviation can be understood by recognizing that as the number of protons increases, the strength of the Coulomb force increases, which tends to break the nucleus apart. As a result, more neutrons are needed to keep the nucleus stable because neutrons experience only the attractive nuclear force. Eventually, the repulsive Coulomb forces between protons cannot be compensated by the addition of more neutrons. This point occurs at $Z = 83$, meaning that elements that contain more than 83 protons do not have stable nuclei.

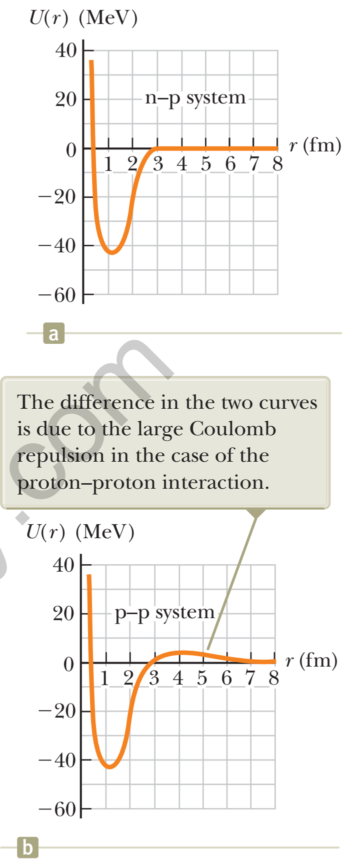


Figure 44.3 (a) Potential energy versus separation distance for a neutron–proton system. (b) Potential energy versus separation distance for a proton–proton system. To display the difference in the curves on this scale, the height of the peak for the proton–proton curve has been exaggerated by a factor of 10.

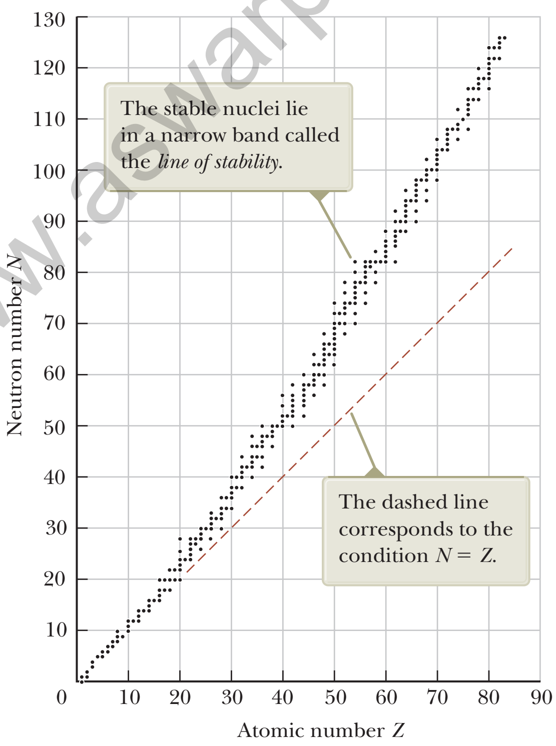


Figure 44.4 Neutron number N versus atomic number Z for stable nuclei (black dots).

44.2 Nuclear Binding Energy

As mentioned in the discussion of ^{12}C in Section 44.1, the total mass of a nucleus is less than the sum of the masses of its individual nucleons. Therefore, the rest energy of the bound system (the nucleus) is less than the combined rest energy of the separated nucleons. This difference in energy is called the **binding energy** of the nucleus and can be interpreted as the energy that must be added to a nucleus to break it apart into its components. Therefore, to separate a nucleus into protons and neutrons, energy must be delivered to the system.

Conservation of energy and the Einstein mass–energy equivalence relationship show that the binding energy E_b in MeV of any nucleus is

Binding energy of a nucleus

$$E_b = [ZM(\mathrm{H}) + Nm_n - M(^A_Z\mathrm{X})] \times 931.494 \text{ MeV/u} \quad (44.2)$$

where $M(\mathrm{H})$ is the atomic mass of the neutral hydrogen atom, m_n is the mass of the neutron, $M(^A_Z\mathrm{X})$ represents the atomic mass of an atom of the isotope $^A_Z\mathrm{X}$, and the masses are all in atomic mass units. The mass of the Z electrons included in $M(\mathrm{H})$ cancels with the mass of the Z electrons included in the term $M(^A_Z\mathrm{X})$ within a small difference associated with the atomic binding energy of the electrons. Because atomic binding energies are typically several electron volts and nuclear binding energies are several million electron volts, this difference is negligible.

A plot of binding energy per nucleon E_b/A as a function of mass number A for various stable nuclei is shown in Figure 44.5. Notice that the binding energy in Figure 44.5 peaks in the vicinity of $A = 60$. That is, nuclei having mass numbers either greater or less than 60 are not as strongly bound as those near the middle of the periodic table. The decrease in binding energy per nucleon for $A > 60$ implies that energy is released when a heavy nucleus splits, or *fissions,* into two lighter nuclei. Energy is released in fission because the nucleons in each product nucleus are more tightly bound to one another than are the nucleons in the original nucleus. The important process of fission and a second important process of *fusion,* in which energy is released as light nuclei combine, shall be considered in detail in Chapter 45.

Pitfall Prevention 44.2

Binding Energy When separate nucleons are combined to form a nucleus, the energy of the system is reduced. Therefore, the change in energy is negative. The absolute value of this change is called the binding energy. This difference in sign may be confusing. For example, an *increase* in binding energy corresponds to a *decrease* in the energy of the system.

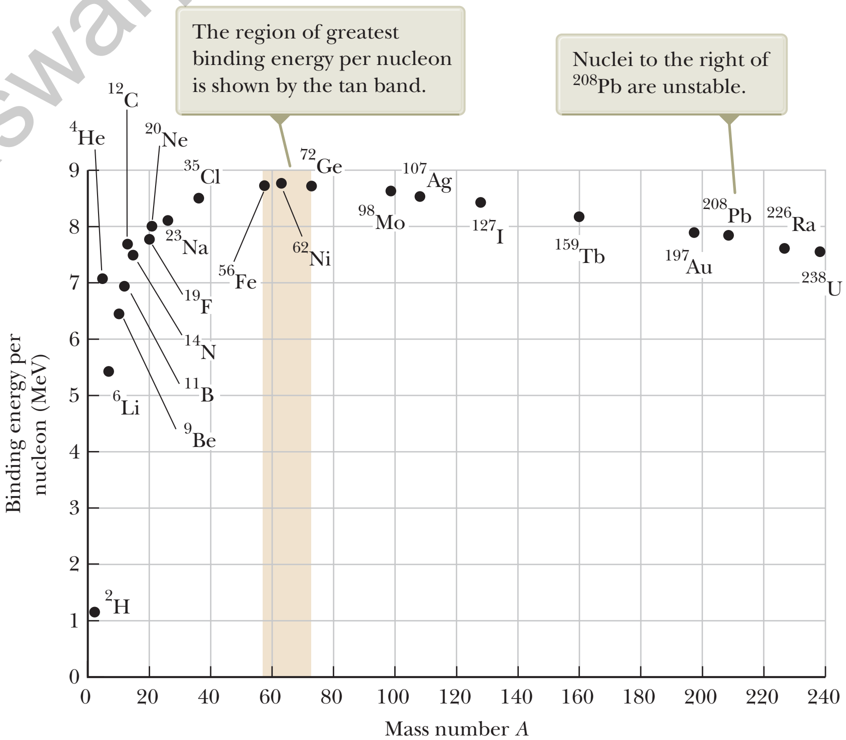


Figure 44.5 Binding energy per nucleon versus mass number for nuclides that lie along the line of stability in Figure 44.4. Some representative nuclides appear as black dots with labels.

Another important feature of Figure 44.5 is that the binding energy per nucleon is approximately constant at around 8 MeV per nucleon for all nuclei with $A > 50$. For these nuclei, the nuclear forces are said to be *saturated,* meaning that in the closely packed structure shown in Figure 44.2, a particular nucleon can form attractive bonds with only a limited number of other nucleons.

Figure 44.5 provides insight into fundamental questions about the origin of the chemical elements. In the early life of the Universe, the only elements that existed were hydrogen and helium. Clouds of cosmic gas coalesced under gravitational forces to form stars. As a star ages, it produces heavier elements from the lighter elements contained within it, beginning by fusing hydrogen atoms to form helium. This process continues as the star becomes older, generating atoms having larger and larger atomic numbers, up to the tan band shown in Figure 44.5.

The nucleus $^{63}_{28}Ni$ has the largest binding energy per nucleon of 8.794 5 MeV. It takes additional energy to create elements with mass numbers larger than 63 because of their lower binding energies per nucleon. This energy comes from the supernova explosion that occurs at the end of some large stars' lives. Therefore, all the heavy atoms in your body were produced from the explosions of ancient stars. You are literally made of stardust!

44.3 Nuclear Models

The details of the nuclear force are still an area of active research. Several nuclear models have been proposed that are useful in understanding general features of nuclear experimental data and the mechanisms responsible for binding energy. Two such models, the liquid-drop model and the shell model, are discussed below.

The Liquid-Drop Model

In 1936, Bohr proposed treating nucleons like molecules in a drop of liquid. In this **liquid-drop model,** the nucleons interact strongly with one another and undergo frequent collisions as they jiggle around within the nucleus. This jiggling motion is analogous to the thermally agitated motion of molecules in a drop of liquid.

Four major effects influence the binding energy of the nucleus in the liquid-drop model:

- **The volume effect.** Figure 44.5 shows that for $A > 50$, the binding energy per nucleon is approximately constant, which indicates that the nuclear force on a given nucleon is due only to a few nearest neighbors and not to all the other nucleons in the nucleus. On average, then, the binding energy associated with the nuclear force for each nucleon is the same in all nuclei: that associated with an interaction with a few neighbors. This property indicates that the total binding energy of the nucleus is proportional to A and therefore proportional to the nuclear volume. The contribution to the binding energy of the entire nucleus is C_1A, where C_1 is an adjustable constant that can be determined by fitting the prediction of the model to experimental results.
- **The surface effect.** Because nucleons on the surface of the drop have fewer neighbors than those in the interior, surface nucleons reduce the binding energy by an amount proportional to their number. Because the number of surface nucleons is proportional to the surface area $4\pi r^2$ of the nucleus (modeled as a sphere) and because $r^2 \propto A^{2/3}$ (Eq. 44.1), the surface term can be expressed as $-C_2A^{2/3}$, where C_2 is a second adjustable constant.
- **The Coulomb repulsion effect.** Each proton repels every other proton in the nucleus. The corresponding potential energy per pair of interacting protons is $k_e e^2/r$, where k_e is the Coulomb constant. The total electric potential energy is equivalent to the work required to assemble Z protons, initially infinitely far apart, into a sphere of volume V. This energy is proportional to the number

of proton pairs $Z(Z-1)/2$ and inversely proportional to the nuclear radius. Consequently, the reduction in binding energy that results from the Coulomb effect is $-C_3Z(Z-1)/A^{1/3}$, where C_3 is yet another adjustable constant.

- **The symmetry effect.** Another effect that lowers the binding energy is related to the symmetry of the nucleus in terms of values of N and Z. For small values of A, stable nuclei tend to have $N \approx Z$. Any large asymmetry between N and Z for light nuclei reduces the binding energy and makes the nucleus less stable. For larger A, the value of N for stable nuclei is naturally larger than Z. This effect can be described by a binding-energy term of the form $-C_4(N-Z)^2/A$, where C_4 is another adjustable constant.[1] For small A, any large asymmetry between values of N and Z makes this term relatively large and reduces the binding energy. For large A, this term is small and has little effect on the overall binding energy.

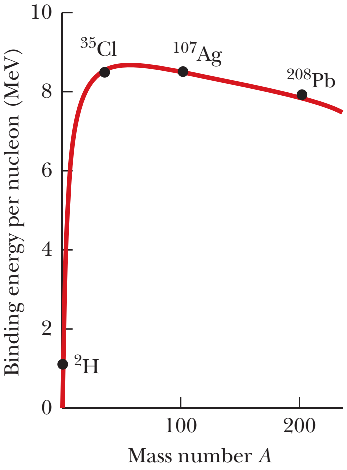


Figure 44.6 The binding-energy curve plotted by using the semiempirical binding-energy formula (red-brown). For comparison to the theoretical curve, experimental values for four sample nuclei are shown.

Adding these contributions gives the following expression for the total binding energy:

$$E_b = C_1A - C_2A^{2/3} - C_3\frac{Z(Z-1)}{A^{1/3}} - C_4\frac{(N-Z)^2}{A} \qquad \textbf{(44.3)}$$

This equation, often referred to as the **semiempirical binding-energy formula,** contains four constants that are adjusted to fit the theoretical expression to experimental data. For nuclei having $A \geq 15$, the constants have the values

$$C_1 = 15.7 \text{ MeV} \qquad C_2 = 17.8 \text{ MeV}$$
$$C_3 = 0.71 \text{ MeV} \qquad C_4 = 23.6 \text{ MeV}$$

Equation 44.3, together with these constants, fits the known nuclear mass values very well as shown by the theoretical curve and sample experimental values in Figure 44.6. The liquid-drop model does not, however, account for some finer details of nuclear structure, such as stability rules and angular momentum. Equation 44.3 is a *theoretical* equation for the binding energy, based on the liquid-drop model, whereas binding energies calculated from Equation 44.2 are *experimental* values based on mass measurements.

Example 44.3 Applying the Semiempirical Binding-Energy Formula

The nucleus ^{64}Zn has a tabulated binding energy of 559.09 MeV. Use the semiempirical binding-energy formula to generate a theoretical estimate of the binding energy for this nucleus.

SOLUTION

Conceptualize Imagine bringing the separate protons and neutrons together to form a ^{64}Zn nucleus. The rest energy of the nucleus is smaller than the rest energy of the individual particles. The difference in rest energy is the binding energy.

Categorize From the text of the problem, we know to apply the liquid-drop model. This example is a substitution problem.

For the ^{64}Zn nucleus, $Z = 30$, $N = 34$, and $A = 64$. Evaluate the four terms of the semiempirical binding-energy formula:

$$C_1A = (15.7 \text{ MeV})(64) = 1\,005 \text{ MeV}$$

$$C_2A^{2/3} = (17.8 \text{ MeV})(64)^{2/3} = 285 \text{ MeV}$$

$$C_3\frac{Z(Z-1)}{A^{1/3}} = (0.71 \text{ MeV})\frac{(30)(29)}{(64)^{1/3}} = 154 \text{ MeV}$$

$$C_4\frac{(N-Z)^2}{A} = (23.6 \text{ MeV})\frac{(34-30)^2}{64} = 5.90 \text{ MeV}$$

[1]The liquid-drop model *describes* that heavy nuclei have $N > Z$. The shell model, as we shall see shortly, *explains* why that is true with a physical argument.

44.3 continued

Substitute these values into Equation 44.3: $E_b = 1\,005 \text{ MeV} - 285 \text{ MeV} - 154 \text{ MeV} - 5.90 \text{ MeV} = 560 \text{ MeV}$

This value differs from the tabulated value by less than 0.2%. Notice how the sizes of the terms decrease from the first to the fourth term. The fourth term is particularly small for this nucleus, which does not have an excessive number of neutrons.

The Shell Model

The liquid-drop model describes the general behavior of nuclear binding energies relatively well. When the binding energies are studied more closely, however, we find the following features:

- Most stable nuclei have an even value of A. Furthermore, only eight stable nuclei have odd values for both Z and N.
- Figure 44.7 shows a graph of the difference between the binding energy per nucleon calculated by Equation 44.3 and the measured binding energy. There is evidence for regularly spaced peaks in the data that are not described by the semiempirical binding-energy formula. The peaks occur at values of N or Z that have become known as **magic numbers:**

$$Z \text{ or } N = 2, 8, 20, 28, 50, 82 \tag{44.4}$$

◀ **Magic numbers**

- High-precision studies of nuclear radii show deviations from the simple expression in Equation 44.1. Graphs of experimental data show peaks in the curve of radius versus N at values of N equal to the magic numbers.
- A group of *isotones* is a collection of nuclei having the same value of N and varying values of Z. When the number of stable isotones is graphed as function of N, there are peaks in the graph, again at the magic numbers in Equation 44.4.
- Several other nuclear measurements show anomalous behavior at the magic numbers.[2]

These peaks in graphs of experimental data are reminiscent of the peaks in Figure 42.20 for the ionization energy of atoms, which arose because of the shell structure of the atom. The **shell model** of the nucleus, also called the **independent-particle model,** was developed independently by two German scientists: Maria Goeppert-Mayer in 1949 and Hans Jensen (1907–1973) in 1950. Goeppert-Mayer and Jensen

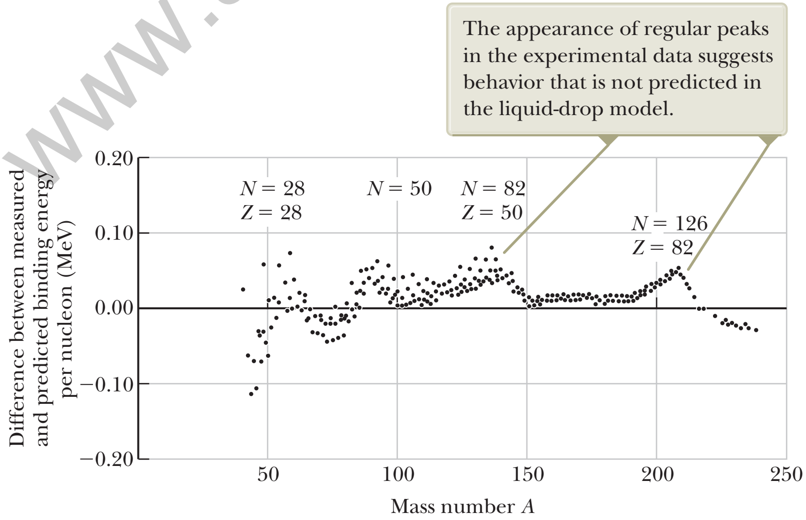


Figure 44.7 The difference between measured binding energies and those calculated from the liquid-drop model as a function of A. (Adapted from R. A. Dunlap, *The Physics of Nuclei and Particles,* Brooks/Cole, Belmont, CA, 2004.)

[2]For further details, see chapter 5 of R. A. Dunlap, *The Physics of Nuclei and Particles,* Brooks/Cole, Belmont, CA, 2004.


Science Source

Maria Goeppert-Mayer
German Scientist (1906–1972)
Goeppert-Mayer was born and educated in Germany. She is best known for her development of the shell model (independent-particle model) of the nucleus, published in 1950. A similar model was simultaneously developed by Hans Jensen, another German scientist. Goeppert-Mayer and Jensen were awarded the Nobel Prize in Physics in 1963 for their extraordinary work in understanding the structure of the nucleus.

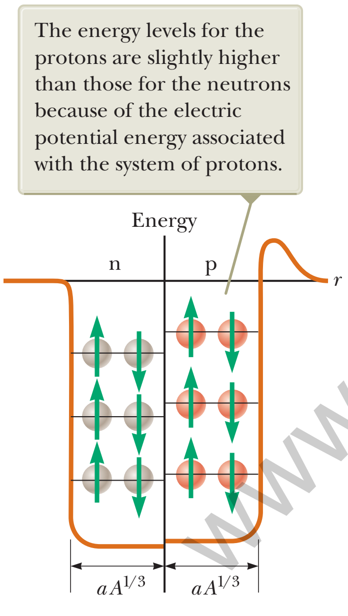


Figure 44.8 A square potential well containing 12 nucleons. The red spheres represent protons, and the gray spheres represent neutrons.

shared the 1963 Nobel Prize in Physics for their work. In this model, each nucleon is assumed to exist in a shell, similar to an atomic shell for an electron. The nucleons exist in quantized energy states, and there are few collisions between nucleons. Obviously, the assumptions of this model differ greatly from those made in the liquid-drop model.

The quantized states occupied by the nucleons can be described by a set of quantum numbers. Because both the proton and the neutron have spin $\frac{1}{2}$, the exclusion principle can be applied to describe the allowed states (as it was for electrons in Chapter 42). That is, each state can contain only two protons (or two neutrons) having *opposite* spins (Fig. 44.8). The proton states differ from those of the neutrons because the two species move in different potential wells. The proton energy levels are farther apart than the neutron levels because the protons experience a superposition of the Coulomb force and the nuclear force, whereas the neutrons experience only the nuclear force.

One factor influencing the observed characteristics of nuclear ground states is *nuclear spin–orbit* effects. The atomic spin–orbit interaction between the spin of an electron and its orbital motion in an atom gives rise to the sodium doublet discussed in Section 42.6 and is magnetic in origin. In contrast, the nuclear spin–orbit effect for nucleons is due to the nuclear force. It is much stronger than in the atomic case, and it has opposite sign. When these effects are taken into account, the shell model is able to account for the observed magic numbers.

The shell model helps us understand why nuclei containing an even number of protons and neutrons are more stable than other nuclei. (There are 160 stable even–even isotopes.) Any particular state is filled when it contains two protons (or two neutrons) having opposite spins. An extra proton or neutron can be added to the nucleus only at the expense of increasing the energy of the nucleus. This increase in energy leads to a nucleus that is less stable than the original nucleus. A careful inspection of the stable nuclei shows that the majority have a special stability when their nucleons combine in pairs, which results in a total angular momentum of zero.

The shell model also helps us understand why nuclei tend to have more neutrons than protons. As in Figure 44.8, the proton energy levels are higher than those for neutrons due to the extra energy associated with Coulomb repulsion. This effect becomes more pronounced as Z increases. Consequently, as Z increases and higher states are filled, a proton level for a given quantum number will be much higher in energy than the neutron level for the same quantum number. In fact, it will be even higher in energy than neutron levels for higher quantum numbers. Hence, it is more energetically favorable for the nucleus to form with neutrons in the lower energy levels rather than protons in the higher energy levels, so the number of neutrons is greater than the number of protons.

More sophisticated models of the nucleus have been and continue to be developed. For example, the *collective model* combines features of the liquid-drop and shell models. The development of theoretical models of the nucleus continues to be an active area of research.

44.4 Radioactivity

In 1896, Becquerel accidentally discovered that uranyl potassium sulfate crystals emit an invisible radiation that can darken a photographic plate even though the plate is covered to exclude light. After a series of experiments, he concluded that the radiation emitted by the crystals was of a new type, one that requires no external stimulation and was so penetrating that it could darken protected photographic plates and ionize gases. This process of spontaneous emission of radiation by uranium was soon to be called **radioactivity.**

Subsequent experiments by other scientists showed that other substances were more powerfully radioactive. The most significant early investigations of this type were conducted by Marie and Pierre Curie (1859–1906). After several years of careful and laborious chemical separation processes on tons of pitchblende, a radioactive ore, the Curies reported the discovery of two previously unknown elements, both radioactive, named polonium and radium. Additional experiments, including Rutherford's famous work on alpha-particle scattering, suggested that radioactivity is the result of the *decay,* or disintegration, of unstable nuclei.

Three types of radioactive decay occur in radioactive substances: alpha (α) decay, in which the emitted particles are ^{4}He nuclei; beta (β) decay, in which the emitted particles are either electrons or positrons; and gamma (γ) decay, in which the emitted particles are high-energy photons. A **positron** is a particle like the electron in all respects except that the positron has a charge of $+e$. (The positron is the *antiparticle* of the electron; see Section 46.2.) The symbol e^- is used to designate an electron, and e^+ designates a positron.

We can distinguish among these three forms of radiation by using the scheme described in Figure 44.9. The radiation from radioactive samples that emit all three types of particles is directed into a region in which there is a magnetic field. Following the particle in a field (magnetic) analysis model, the radiation beam splits into three components, two bending in opposite directions and the third experiencing no change in direction. This simple observation shows that the radiation of the undeflected beam carries no charge (the gamma ray), the component deflected upward corresponds to positively charged particles (alpha particles), and the component deflected downward corresponds to negatively charged particles (e^-). If the beam includes a positron (e^+), it is deflected upward like the alpha particle, but it follows a different trajectory due to its smaller mass.

The three types of radiation have quite different penetrating powers. Alpha particles barely penetrate a sheet of paper, beta particles can penetrate a few millimeters of aluminum, and gamma rays can penetrate several centimeters of lead.

The decay process is probabilistic in nature and can be described with statistical calculations for a radioactive substance of macroscopic size containing a large number of radioactive nuclei. For such large numbers, the rate at which a particular decay process occurs in a sample is proportional to the number of radioactive nuclei present (that is, the number of nuclei that have not yet decayed). If N is the number of undecayed radioactive nuclei present at some instant, the rate of change of N with time is

$$\frac{dN}{dt} = -\lambda N \quad \textbf{(44.5)}$$

where λ, called the **decay constant,** is the probability of decay per nucleus per second. The negative sign indicates that dN/dt is negative; that is, N decreases in time.

Equation 44.5 can be written in the form

$$\frac{dN}{N} = -\lambda\, dt$$

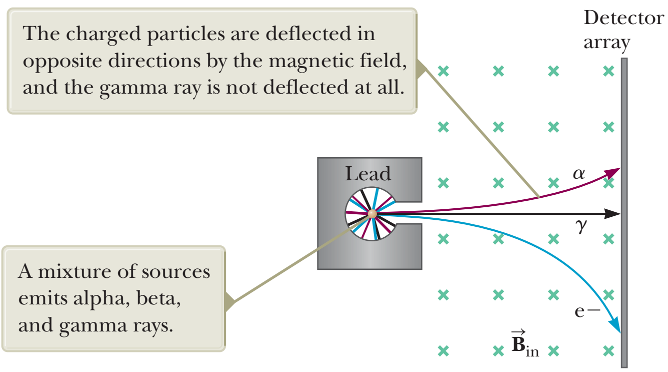


Figure 44.9 The radiation from radioactive sources can be separated into three components by using a magnetic field to deflect the charged particles. The detector array at the right records the events.

Time & Life Pictures/Getty Images

Marie Curie
Polish Scientist (1867–1934)
In 1903, Marie Curie shared the Nobel Prize in Physics with her husband, Pierre, and with Becquerel for their studies of radioactive substances. In 1911, she was awarded a Nobel Prize in Chemistry for the discovery of radium and polonium.

Pitfall Prevention 44.3

Rays or Particles? Early in the history of nuclear physics, the term *radiation* was used to describe the emanations from radioactive nuclei. We now know that alpha radiation and beta radiation involve the emission of particles with nonzero rest energy. Even though they are not examples of electromagnetic radiation, the use of the term *radiation* for all three types of emission is deeply entrenched in our language and in the physics community.

Pitfall Prevention 44.4

Notation Warning In Section 44.1, we introduced the symbol N as an integer representing the number of neutrons in a nucleus. In this discussion, the symbol N represents the number of undecayed nuclei in a radioactive sample remaining after some time interval. As you read further, be sure to consider the context to determine the appropriate meaning for the symbol N.

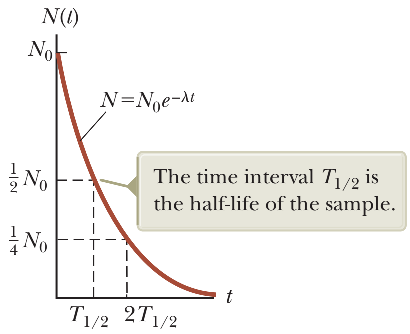


Figure 44.10 Plot of the exponential decay of radioactive nuclei. The vertical axis represents the number of undecayed radioactive nuclei present at any time t, and the horizontal axis is time.

which, upon integration, gives

Exponential behavior of the number of undecayed nuclei

$$N = N_0 e^{-\lambda t} \tag{44.6}$$

where the constant N_0 represents the number of undecayed radioactive nuclei at $t = 0$. Equation 44.6 shows that the number of undecayed radioactive nuclei in a sample decreases exponentially with time. The plot of N versus t shown in Figure 44.10 illustrates the exponential nature of the decay. The curve is similar to that for the time variation of electric charge on a discharging capacitor in an RC circuit, as studied in Section 28.4.

The **decay rate** R, which is the number of decays per second, can be obtained by combining Equations 44.5 and 44.6:

Exponential behavior of the decay rate

$$R = \left|\frac{dN}{dt}\right| = \lambda N = \lambda N_0 e^{-\lambda t} = R_0 e^{-\lambda t} \tag{44.7}$$

where $R_0 = \lambda N_0$ is the decay rate at $t = 0$. The decay rate R of a sample is often referred to as its **activity.** Note that both N and R decrease exponentially with time.

Another parameter useful in characterizing nuclear decay is the **half-life** $T_{1/2}$:

Pitfall Prevention 44.5

Half-life It is *not* true that all the original nuclei have decayed after two half-lives! In one half-life, half of the original nuclei will decay. In the second half-life, half of those remaining will decay, leaving $\frac{1}{4}$ of the original number.

The **half-life** of a radioactive substance is the time interval during which half of a given number of radioactive nuclei decay.

To find an expression for the half-life, we first set $N = N_0/2$ and $t = T_{1/2}$ in Equation 44.6 to give

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$$

Canceling the N_0 factors and then taking the reciprocal of both sides, we obtain $e^{\lambda T_{1/2}} = 2$. Taking the natural logarithm of both sides gives

Half-life

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda} \tag{44.8}$$

After a time interval equal to one half-life, there are $N_0/2$ radioactive nuclei remaining (by definition); after two half-lives, half of these remaining nuclei have decayed and $N_0/4$ radioactive nuclei are left; after three half-lives, $N_0/8$ are left; and so on. In general, after n half-lives, the number of undecayed radioactive nuclei remaining is

$$N = N_0\left(\tfrac{1}{2}\right)^n \tag{44.9}$$

where n can be an integer or a noninteger.

A frequently used unit of activity is the **curie** (Ci), defined as

The curie

$$1 \text{ Ci} \equiv 3.7 \times 10^{10} \text{ decays/s}$$

This value was originally selected because it is the approximate activity of 1 g of radium. The SI unit of activity is the **becquerel** (Bq):

The becquerel

$$1 \text{ Bq} \equiv 1 \text{ decay/s}$$

Therefore, $1 \text{ Ci} = 3.7 \times 10^{10}$ Bq. The curie is a rather large unit, and the more frequently used activity units are the millicurie and the microcurie.

Quick Quiz 44.2 On your birthday, you measure the activity of a sample of ^{210}Bi, which has a half-life of 5.01 days. The activity you measure is 1.000 μCi. What is the activity of this sample on your next birthday? **(a)** 1.000 μCi **(b)** 0 **(c)** $\sim$ 0.2 μCi **(d)** $\sim$ 0.01 μCi **(e)** $\sim 10^{-22}$ μCi

Example 44.4 How Many Nuclei Are Left?

The isotope carbon-14, $^{14}_{6}$C, is radioactive and has a half-life of 5 730 years. If you start with a sample of 1 000 carbon-14 nuclei, how many nuclei will still be undecayed in 25 000 years?

SOLUTION

Conceptualize The time interval of 25 000 years is much longer than the half-life, so only a small fraction of the originally undecayed nuclei will remain.

Categorize The text of the problem allows us to categorize this example as a substitution problem involving radioactive decay.

Analyze Divide the time interval by the half-life to determine the number of half-lives:

$$n = \frac{25\,000 \text{ yr}}{5\,730 \text{ yr}} = 4.363$$

Determine how many undecayed nuclei are left after this many half-lives using Equation 44.9:

$$N = N_0\left(\tfrac{1}{2}\right)^n = 1\,000\left(\tfrac{1}{2}\right)^{4.363} = \boxed{49}$$

Finalize As we have mentioned, radioactive decay is a probabilistic process and accurate statistical predictions are possible only with a very large number of atoms. The original sample in this example contains only 1 000 nuclei, which is certainly not a very large number. Therefore, if you counted the number of undecayed nuclei remaining after 25 000 years, it might not be exactly 49.

Example 44.5 The Activity of Carbon

At time $t = 0$, a radioactive sample contains 3.50 μg of pure $^{11}_{6}$C, which has a half-life of 20.4 min.

(A) Determine the number N_0 of nuclei in the sample at $t = 0$.

SOLUTION

Conceptualize The half-life is relatively short, so the number of undecayed nuclei drops rapidly. The molar mass of $^{11}_{6}$C is approximately 11.0 g/mol.

Categorize We evaluate results using equations developed in this section, so we categorize this example as a substitution problem.

Find the number of moles in 3.50 μg of pure $^{11}_{6}$C:

$$n = \frac{3.50 \times 10^{-6} \text{ g}}{11.0 \text{ g/mol}} = 3.18 \times 10^{-7} \text{ mol}$$

Find the number of undecayed nuclei in this amount of pure $^{11}_{6}$C:

$$N_0 = (3.18 \times 10^{-7} \text{ mol})(6.02 \times 10^{23} \text{ nuclei/mol}) = \boxed{1.92 \times 10^{17} \text{ nuclei}}$$

(B) What is the activity of the sample initially and after 8.00 h?

SOLUTION

Find the initial activity of the sample using Equations 44.7 and 44.8:

$$R_0 = \lambda N_0 = \frac{0.693}{T_{1/2}} N_0 = \frac{0.693}{20.4 \text{ min}}\left(\frac{1 \text{ min}}{60 \text{ s}}\right)(1.92 \times 10^{17})$$

$$= (5.66 \times 10^{-4} \text{ s}^{-1})(1.92 \times 10^{17}) = \boxed{1.09 \times 10^{14} \text{ Bq}}$$

continued

44.5 continued

Use Equation 44.7 to find the activity at $t = 8.00$ h $= 2.88 \times 10^4$ s:

$$R = R_0 e^{-\lambda t} = (1.09 \times 10^{14}\text{ Bq})e^{-(5.66\times10^{-4}\text{ s}^{-1})(2.88\times10^4\text{ s})} = 8.96 \times 10^6\text{ Bq}$$

Example 44.6 A Radioactive Isotope of Iodine

A sample of the isotope ^{131}I, which has a half-life of 8.04 days, has an activity of 5.0 mCi at the time of shipment. Upon receipt of the sample at a medical laboratory, the activity is 2.1 mCi. How much time has elapsed between the two measurements?

SOLUTION

Conceptualize The sample is continuously decaying as it is in transit. The decrease in the activity is 58% during the time interval between shipment and receipt, so we expect the elapsed time to be greater than the half-life of 8.04 d.

Categorize The stated activity corresponds to many decays per second, so N is large and we can categorize this problem as one in which we can use our statistical analysis of radioactivity.

Analyze Solve Equation 44.7 for the ratio of the final activity to the initial activity:

$$\frac{R}{R_0} = e^{-\lambda t}$$

Take the natural logarithm of both sides:

$$\ln\left(\frac{R}{R_0}\right) = -\lambda t$$

Solve for the time t:

$$(1)\quad t = -\frac{1}{\lambda}\ln\left(\frac{R}{R_0}\right)$$

Use Equation 44.8 to substitute for λ:

$$t = -\frac{T_{1/2}}{\ln 2}\ln\left(\frac{R}{R_0}\right)$$

Substitute numerical values:

$$t = -\frac{8.04\text{ d}}{0.693}\ln\left(\frac{2.1\text{ mCi}}{5.0\text{ mCi}}\right) = 10\text{ d}$$

Finalize This result is indeed greater than the half-life, as expected. This example demonstrates the difficulty in shipping radioactive samples with short half-lives. If the shipment is delayed by several days, only a small fraction of the sample might remain upon receipt. This difficulty can be addressed by shipping a combination of isotopes in which the desired isotope is the product of a decay occurring within the sample. It is possible for the desired isotope to be in *equilibrium,* in which case it is created at the same rate as it decays. Therefore, the amount of the desired isotope remains constant during the shipping process and subsequent storage. When needed, the desired isotope can be separated from the rest of the sample; its decay from the initial activity begins at this point rather than upon shipment.

44.5 The Decay Processes

As we stated in Section 44.4, a radioactive nucleus spontaneously decays by one of three processes: alpha decay, beta decay, or gamma decay. Figure 44.11 shows a close-up view of a portion of Figure 44.4 from $Z = 65$ to $Z = 80$. The black circles are the stable nuclei seen in Figure 44.4. In addition, unstable nuclei above and below the line of stability for each value of Z are shown. Above the line of stability, the blue circles show unstable nuclei that are neutron-rich and undergo a beta decay process in which an electron is emitted. Below the black circles are red circles corresponding to proton-rich unstable nuclei that primarily undergo a beta-decay process in which a positron is emitted or a competing process called electron capture. Beta decay and electron capture are described in more detail below. Further below the line of stability (with a few exceptions) are tan circles that represent very proton-rich nuclei for which the primary decay mechanism is alpha decay, which we discuss first.

Alpha Decay

A nucleus emitting an alpha particle ($^4_2$He) loses two protons and two neutrons. Therefore, the atomic number Z decreases by 2, the mass number A decreases by 4, and the neutron number decreases by 2. The decay can be written

$$^A_Z\mathrm{X} \rightarrow \, ^{A-4}_{Z-2}\mathrm{Y} + \, ^4_2\mathrm{He} \quad \textbf{(44.10)}$$

where X is called the **parent nucleus** and Y the **daughter nucleus.** As a general rule in any decay expression such as this one, (1) the sum of the mass numbers A must be the same on both sides of the decay and (2) the sum of the atomic numbers Z must be the same on both sides of the decay. As examples, ^{238}U and ^{226}Ra are both alpha emitters and decay according to the schemes

$$^{238}_{92}\mathrm{U} \rightarrow \, ^{234}_{90}\mathrm{Th} + \, ^4_2\mathrm{He} \quad \textbf{(44.11)}$$

$$^{226}_{88}\mathrm{Ra} \rightarrow \, ^{222}_{86}\mathrm{Rn} + \, ^4_2\mathrm{He} \quad \textbf{(44.12)}$$

The decay of ^{226}Ra is shown in Figure 44.12.

When the nucleus of one element changes into the nucleus of another as happens in alpha decay, the process is called **spontaneous decay.** In any spontaneous decay, relativistic energy and momentum of the parent nucleus as an isolated system must be conserved. The final components of the system are the daughter nucleus and the alpha particle. If we call M_X the mass of the parent nucleus, M_Y the mass of the daughter nucleus, and M_α the mass of the alpha particle, we can define the **disintegration energy** Q of the system as

$$Q = (M_X - M_Y - M_\alpha)c^2 \quad \textbf{(44.13)}$$

The energy Q is in joules when the masses are in kilograms and c is the speed of light, 3.00×10^8 m/s. When the masses are expressed in atomic mass units u, however, Q can be calculated in MeV using the expression

$$Q = (M_X - M_Y - M_\alpha) \times 931.494 \text{ MeV/u} \quad \textbf{(44.14)}$$

Table 44.2 (page 1396) contains information on selected isotopes, including masses of neutral atoms that can be used in Equation 44.14 and similar equations.

The disintegration energy Q is the amount of rest energy transformed and appears in the form of kinetic energy in the daughter nucleus and the alpha particle and is sometimes referred to as the Q value of the nuclear decay. Consider the case of the ^{226}Ra decay described in Figure 44.12. If the parent nucleus is at rest before the decay, the total kinetic energy of the products is 4.87 MeV. (See Example 44.7.) Most of this kinetic energy is associated with the alpha particle because this particle is much less massive than the daughter nucleus ^{222}Rn. That is, because the system is also isolated in terms of momentum, the lighter alpha particle recoils with a much higher speed than does the daughter nucleus. Generally, less massive particles carry off most of the energy in nuclear decays.

Experimental observations of alpha-particle energies show a number of discrete energies rather than a single energy because the daughter nucleus may be left in an

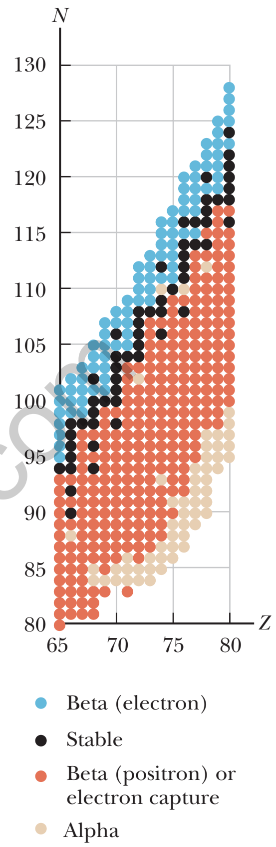


Figure 44.11 A close-up view of the line of stability in Figure 44.4 from $Z = 65$ to $Z = 80$. The black dots represent stable nuclei as in Figure 44.4. The other colored dots represent unstable isotopes above and below the line of stability, with the color of the dot indicating the primary means of decay.

Pitfall Prevention 44.6

Another *Q* We have seen the symbol Q before, but this use is a brand-new meaning for this symbol: the disintegration energy. In this context, it is not heat, charge, or quality factor for a resonance, for which we have used Q before.

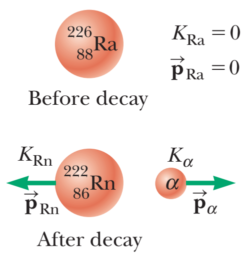


Figure 44.12 The alpha decay of radium-226. The radium nucleus is initially at rest. After the decay, the radon nucleus has kinetic energy K_{Rn} and momentum $\vec{p}_{Rn}$ and the alpha particle has kinetic energy K_α and momentum $\vec{p}_\alpha$.

Table 44.2 Chemical and Nuclear Information for Selected Isotopes

Atomic Number Z	Element	Chemical Symbol	Mass Number A (* means radioactive)	Mass of Neutral Atom (u)	Percent Abundance	Half-life, if Radioactive $T_{1/2}$
−1	electron	e^-	0	0.000 549		
0	neutron	n	1*	1.008 665		614 s
1	hydrogen	$^1H = p$	1	1.007 825	99.988 5	
	[deuterium	$^2H = D$]	2	2.014 102	0.011 5	
	[tritium	$^3H = T$]	3*	3.016 049		12.33 yr
2	helium	He	3	3.016 029	0.000 137	
	[alpha particle	$\alpha = {}^4He$]	4	4.002 603	99.999 863	
			6*	6.018 889		0.81 s
3	lithium	Li	6	6.015 123	7.5	
			7	7.016 005	92.5	
4	beryllium	Be	7*	7.016 930		53.3 d
			8*	8.005 305		10^{-17} s
			9	9.012 182	100	
5	boron	B	10	10.012 937	19.9	
			11	11.009 305	80.1	
6	carbon	C	11*	11.011 434		20.4 min
			12	12.000 000	98.93	
			13	13.003 355	1.07	
			14*	14.003 242		5 730 yr
7	nitrogen	N	13*	13.005 739		9.96 min
			14	14.003 074	99.632	
			15	15.000 109	0.368	
8	oxygen	O	14*	14.008 596		70.6 s
			15*	15.003 066		122 s
			16	15.994 915	99.757	
			17	16.999 132	0.038	
			18	17.999 161	0.205	
9	fluorine	F	18*	18.000 938		109.8 min
			19	18.998 403	100	
10	neon	Ne	20	19.992 440	90.48	
11	sodium	Na	23	22.989 769	100	
12	magnesium	Mg	23*	22.994 124		11.3 s
			24	23.985 042	78.99	
13	aluminum	Al	27	26.981 539	100	
14	silicon	Si	27*	26.986 705		4.2 s
15	phosphorus	P	30*	29.978 314		2.50 min
			31	30.973 762	100	
			32*	31.973 907		14.26 d
16	sulfur	S	32	31.972 071	94.93	
19	potassium	K	39	38.963 707	93.258 1	
			40*	39.963 998	0.011 7	1.28×10^9 yr
20	calcium	Ca	40	39.962 591	96.941	
			42	41.958 618	0.647	
			43	42.958 767	0.135	
25	manganese	Mn	55	54.938 045	100	
26	iron	Fe	56	55.934 938	91.754	
			57	56.935 394	2.119	

continued

Table 44.2 Chemical and Nuclear Information for Selected Isotopes *(continued)*

Atomic Number Z	Element	Chemical Symbol	Mass Number A (* means radioactive)	Mass of Neutral Atom (u)	Percent Abundance	Half-life, if Radioactive $T_{1/2}$
27	cobalt	Co	57*	56.936 291		272 d
			59	58.933 195	100	
			60*	59.933 817		5.27 yr
28	nickel	Ni	58	57.935 343	68.076 9	
			60	59.930 786	26.223 1	
29	copper	Cu	63	62.929 598	69.17	
			64*	63.929 764		12.7 h
			65	64.927 789	30.83	
30	zinc	Zn	64	63.929 142	48.63	
37	rubidium	Rb	87*	86.909 181	27.83	
38	strontium	Sr	87	86.908 877	7.00	
			88	87.905 612	82.58	
			90*	89.907 738		29.1 yr
41	niobium	Nb	93	92.906 378	100	
42	molybdenum	Mo	94	93.905 088	9.25	
44	ruthenium	Ru	98	97.905 287	1.87	
54	xenon	Xe	136*	135.907 219		2.4×10^{21} yr
55	cesium	Cs	137*	136.907 090		30 yr
56	barium	Ba	137	136.905 827	11.232	
58	cerium	Ce	140	139.905 439	88.450	
59	praseodymium	Pr	141	140.907 653	100	
60	neodymium	Nd	144*	143.910 087	23.8	2.3×10^{15} yr
61	promethium	Pm	145*	144.912 749		17.7 yr
79	gold	Au	197	196.966 569	100	
80	mercury	Hg	198	197.966 769	9.97	
			202	201.970 643	29.86	
82	lead	Pb	206	205.974 465	24.1	
			207	206.975 897	22.1	
			208	207.976 652	52.4	
			214*	213.999 805		26.8 min
83	bismuth	Bi	209	208.980 399	100	
84	polonium	Po	210*	209.982 874		138.38 d
			216*	216.001 915		0.145 s
			218*	218.008 973		3.10 min
86	radon	Rn	220*	220.011 394		55.6 s
			222*	222.017 578		3.823 d
88	radium	Ra	226*	226.025 410		1 600 yr
90	thorium	Th	232*	232.038 055	100	1.40×10^{10} yr
			234*	234.043 601		24.1 d
92	uranium	U	234*	234.040 952		2.45×10^{5} yr
			235*	235.043 930	0.720 0	7.04×10^{8} yr
			236*	236.045 568		2.34×10^{7} yr
			238*	238.050 788	99.274 5	4.47×10^{9} yr
93	neptunium	Np	236*	236.046 570		1.15×10^{5} yr
			237*	237.048 173		2.14×10^{6} yr
94	plutonium	Pu	239*	239.052 163		24 120 yr

Source: G. Audi, A. H. Wapstra, and C. Thibault, "The AME2003 Atomic Mass Evaluation," *Nuclear Physics A* **729**:337–676, 2003.

excited quantum state after the decay. As a result, not all the disintegration energy is available as kinetic energy of the alpha particle and daughter nucleus. The emission of an alpha particle is followed by one or more gamma-ray photons (discussed shortly) as the excited nucleus decays to the ground state. The observed discrete alpha-particle energies represent evidence of the quantized nature of the nucleus and allow a determination of the energies of the quantum states.

If one assumes ^{238}U (or any other alpha emitter) decays by emitting either a proton or a neutron, the mass of the decay products would exceed that of the parent nucleus, corresponding to a negative Q value. A negative Q value indicates that such a proposed decay does not occur spontaneously.

Quick Quiz 44.3 Which of the following is the correct daughter nucleus associated with the alpha decay of $^{157}_{72}$Hf? **(a)** $^{153}_{72}$Hf **(b)** $^{153}_{70}$Yb **(c)** $^{157}_{70}$Yb

Example 44.7 The Energy Liberated When Radium Decays AM

The ^{226}Ra nucleus undergoes alpha decay according to Equation 44.12.

(A) Calculate the Q value for this process. From Table 44.2, the masses are 226.025 410 u for ^{226}Ra, 222.017 578 u for ^{222}Rn, and 4.002 603 u for $^4_2$He.

SOLUTION

Conceptualize Study Figure 44.12 to understand the process of alpha decay in this nucleus.

Categorize The parent nucleus is an *isolated system* that decays into an alpha particle and a daughter nucleus. The system is isolated in terms of both *energy* and *momentum*.

Analyze Evaluate Q using Equation 44.14:

$$Q = (M_X - M_Y - M_\alpha) \times 931.494 \text{ MeV/u}$$

$$= (226.025\,410 \text{ u} - 222.017\,578 \text{ u} - 4.002\,603 \text{ u}) \times 931.494 \text{ MeV/u}$$

$$= (0.005\,229 \text{ u}) \times 931.494 \text{ MeV/u} = \boxed{4.87 \text{ MeV}}$$

(B) What is the kinetic energy of the alpha particle after the decay?

Analyze The value of 4.87 MeV is the disintegration energy for the decay. It includes the kinetic energy of both the alpha particle and the daughter nucleus after the decay. Therefore, the kinetic energy of the alpha particle would be *less* than 4.87 MeV.

Set up a conservation of momentum equation, noting that the initial momentum of the system is zero:

$$(1) \quad 0 = M_Y v_Y - M_\alpha v_\alpha$$

Set the disintegration energy equal to the sum of the kinetic energies of the alpha particle and the daughter nucleus (assuming the daughter nucleus is left in the ground state):

$$(2) \quad Q = \tfrac{1}{2}M_\alpha {v_\alpha}^2 + \tfrac{1}{2}M_Y {v_Y}^2$$

Solve Equation (1) for v_Y and substitute into Equation (2):

$$Q = \tfrac{1}{2}M_\alpha {v_\alpha}^2 + \tfrac{1}{2}M_Y\left(\frac{M_\alpha v_\alpha}{M_Y}\right)^2 = \tfrac{1}{2}M_\alpha {v_\alpha}^2\left(1 + \frac{M_\alpha}{M_Y}\right)$$

$$Q = K_\alpha\left(\frac{M_Y + M_\alpha}{M_Y}\right)$$

Solve for the kinetic energy of the alpha particle:

$$K_\alpha = Q\left(\frac{M_Y}{M_Y + M_\alpha}\right)$$

Evaluate this kinetic energy for the specific decay of ^{226}Ra that we are exploring in this example:

$$K_\alpha = (4.87 \text{ MeV})\left(\frac{222}{222 + 4}\right) = \boxed{4.78 \text{ MeV}}$$

Finalize The kinetic energy of the alpha particle is indeed less than the disintegration energy, but notice that the alpha particle carries away *most* of the energy available in the decay.

To understand the mechanism of alpha decay, let's model the parent nucleus as a system consisting of (1) the alpha particle, already formed as an entity within the nucleus, and (2) the daughter nucleus that will result when the alpha particle is emitted. Figure 44.13 shows a plot of potential energy versus separation distance r between the alpha particle and the daughter nucleus, where the distance marked R is the range of the nuclear force. The curve represents the combined effects of (1) the repulsive Coulomb force, which gives the positive part of the curve for $r > R$, and (2) the attractive nuclear force, which causes the curve to be negative for $r < R$. As shown in Example 44.7, a typical disintegration energy Q is approximately 5 MeV, which is the approximate kinetic energy of the alpha particle, represented by the lower dashed line in Figure 44.13.

According to classical physics, the alpha particle is trapped in a potential well. How, then, does it ever escape from the nucleus? The answer to this question was first provided by George Gamow (1904–1968) in 1928 and independently by R. W. Gurney (1898–1953) and E. U. Condon (1902–1974) in 1929, using quantum mechanics. In the view of quantum mechanics, there is always some probability that a particle can tunnel through a barrier (Section 41.5). That is exactly how we can describe alpha decay: the alpha particle tunnels through the barrier in Figure 44.13, escaping the nucleus. Furthermore, this model agrees with the observation that higher-energy alpha particles come from nuclei with shorter half-lives. For higher-energy alpha particles in Figure 44.13, the barrier is narrower and the probability is higher that tunneling occurs. The higher probability translates to a shorter half-life.

As an example, consider the decays of ^{238}U and ^{226}Ra in Equations 44.11 and 44.12, along with the corresponding half-lives and alpha-particle energies:

$$^{238}\text{U:} \quad T_{1/2} = 4.47 \times 10^9 \text{ yr} \qquad K_\alpha = 4.20 \text{ MeV}$$

$$^{226}\text{Ra:} \quad T_{1/2} = 1.60 \times 10^3 \text{ yr} \qquad K_\alpha = 4.78 \text{ MeV}$$

Notice that a relatively small difference in alpha-particle energy is associated with a tremendous difference of six orders of magnitude in the half-life. The origin of this effect can be understood as follows. Figure 44.13 shows that the curve below an alpha-particle energy of 5 MeV has a slope with a relatively small magnitude. Therefore, a small difference in energy on the vertical axis has a relatively large effect on the width of the potential barrier. Second, recall Equation 41.22, which describes the exponential dependence of the probability of transmission on the barrier width. These two factors combine to give the very sensitive relationship between half-life and alpha-particle energy that the data above suggest.

A life-saving application of alpha decay is the household smoke detector, shown in Figure 44.14. The detector consists of an ionization chamber, a sensitive current detector, and an alarm. A weak radioactive source (usually $^{241}_{95}$Am) ionizes the air in the chamber of the detector, creating charged particles. A voltage is maintained between the plates inside the chamber, setting up a small but detectable current in the external circuit due to the ions acting as charge carriers between the plates. As long as the current is maintained, the alarm is deactivated. If smoke drifts into the chamber, however, the ions become attached to the smoke particles. These heavier particles do not drift as readily as do the lighter ions, which causes a decrease in the detector current. The external circuit senses this decrease in current and sets off the alarm.

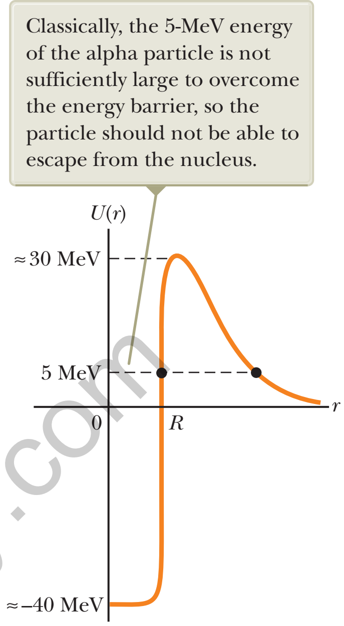


Figure 44.13 Potential energy versus separation distance for a system consisting of an alpha particle and a daughter nucleus. The alpha particle escapes by tunneling through the barrier.

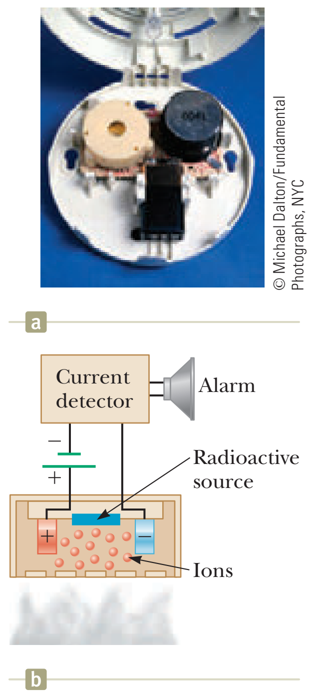


Figure 44.14 (a) A smoke detector uses alpha decay to determine whether smoke is in the air. The alpha source is in the black cylinder at the right. (b) Smoke entering the chamber reduces the detected current, causing the alarm to sound.

Beta Decay

When a radioactive nucleus undergoes beta decay, the daughter nucleus contains the same number of nucleons as the parent nucleus but the atomic number is changed by 1, which means that the number of protons changes:

$$^{A}_{Z}\text{X} \rightarrow {}^{A}_{Z+1}\text{Y} + e^- \quad \text{(incomplete expression)} \tag{44.15}$$

$$^{A}_{Z}\text{X} \rightarrow {}^{A}_{Z-1}\text{Y} + e^+ \quad \text{(incomplete expression)} \tag{44.16}$$

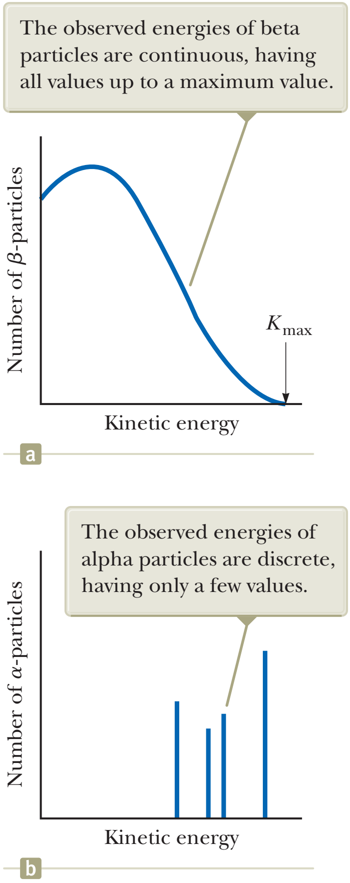


Figure 44.15 (a) Distribution of beta-particle energies in a typical beta decay. (b) Distribution of alpha-particle energies in a typical alpha decay.

where, as mentioned in Section 44.4, e^- designates an electron and e^+ designates a positron, with *beta particle* being the general term referring to either. *Beta decay is not described completely by these expressions.* We shall give reasons for this statement shortly.

As with alpha decay, the nucleon number and total charge are both conserved in beta decays. Because A does not change but Z does, we conclude that in beta decay, either a neutron changes to a proton (Eq. 44.15) or a proton changes to a neutron (Eq. 44.16). Note that the electron or positron emitted in these decays is not present beforehand in the nucleus; it is created in the process of the decay from the rest energy of the decaying nucleus. Two typical beta-decay processes are

$$^{14}_{6}\text{C} \rightarrow {}^{14}_{7}\text{N} + e^- \quad \text{(incomplete expression)} \qquad \textbf{(44.17)}$$

$$^{12}_{7}\text{N} \rightarrow {}^{12}_{6}\text{C} + e^+ \quad \text{(incomplete expression)} \qquad \textbf{(44.18)}$$

Let's consider the energy of the system undergoing beta decay before and after the decay. As with alpha decay, energy of the isolated system must be conserved. Experimentally, it is found that beta particles from a single type of nucleus are emitted over a continuous range of energies (Fig. 44.15a), as opposed to alpha decay, in which the alpha particles are emitted with discrete energies (Fig. 44.15b). The kinetic energy of the system after the decay is equal to the decrease in rest energy of the system, that is, the Q value. Because all decaying nuclei in the sample have the same initial mass, however, *the Q value must be the same for each decay.* So, why do the emitted particles have the range of kinetic energies shown in Figure 44.15a? The isolated system model and the law of conservation of energy seem to be violated! It becomes worse: further analysis of the decay processes described by Equations 44.15 and 44.16 shows that the laws of conservation of angular momentum (spin) and linear momentum are also violated!

After a great deal of experimental and theoretical study, Pauli in 1930 proposed that a third particle must be present in the decay products to carry away the "missing" energy and momentum. Fermi later named this particle the **neutrino** (little neutral one) because it had to be electrically neutral and have little or no mass. Although it eluded detection for many years, the neutrino (symbol ν, Greek nu) was finally detected experimentally in 1956 by Frederick Reines (1918–1998), who received the Nobel Prize in Physics for this work in 1995. The neutrino has the following properties:

Properties of the neutrino ▶

- It has zero electric charge.
- Its mass is either zero (in which case it travels at the speed of light) or very small; much recent persuasive experimental evidence suggests that the neutrino mass is not zero. Current experiments place the upper bound of the mass of the neutrino at approximately 7 eV/c^2.
- It has a spin of $\frac{1}{2}$, which allows the law of conservation of angular momentum to be satisfied in beta decay.
- It interacts very weakly with matter and is therefore very difficult to detect.

We can now write the beta-decay processes (Eqs. 44.15 and 44.16) in their correct and complete form:

Beta decay processes ▶

$$^{A}_{Z}\text{X} \rightarrow {}_{Z+1}^{\;\;A}\text{Y} + e^- + \bar{\nu} \quad \text{(complete expression)} \qquad \textbf{(44.19)}$$

$$^{A}_{Z}\text{X} \rightarrow {}_{Z-1}^{\;\;A}\text{Y} + e^+ + \nu \quad \text{(complete expression)} \qquad \textbf{(44.20)}$$

as well as those for carbon-14 and nitrogen-12 (Eqs. 44.17 and 44.18):

$$^{14}_{6}\text{C} \rightarrow {}^{14}_{7}\text{N} + e^- + \bar{\nu} \quad \text{(complete expression)} \qquad \textbf{(44.21)}$$

$$^{12}_{7}\text{N} \rightarrow {}^{12}_{6}\text{C} + e^+ + \nu \quad \text{(complete expression)} \qquad \textbf{(44.22)}$$

where the symbol $\bar{\nu}$ represents the **antineutrino,** the antiparticle to the neutrino. We shall discuss antiparticles further in Chapter 46. For now, it suffices to say that a neutrino is emitted in positron decay and an antineutrino is emitted in electron

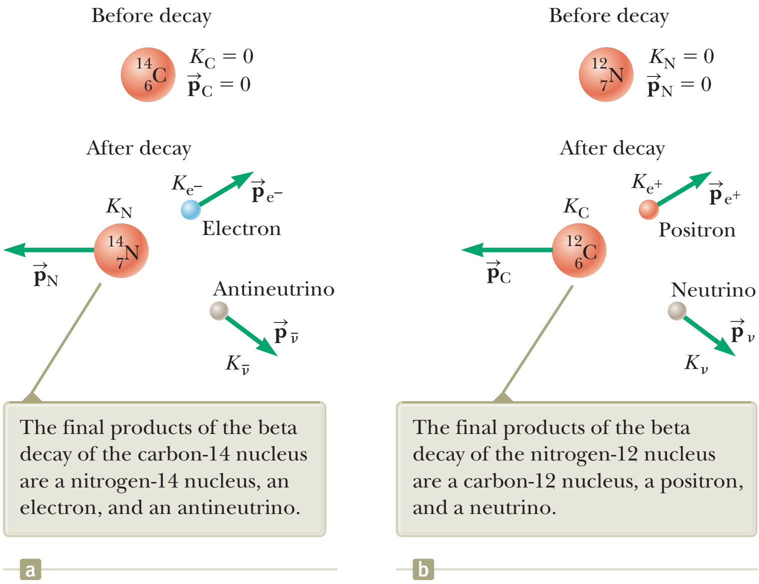


Figure 44.16 (a) The beta decay of carbon-14. (b) The beta decay of nitrogen-12.

decay. As with alpha decay, the decays listed above are analyzed by applying conservation laws, but relativistic expressions must be used for beta particles because their kinetic energy is large (typically 1 MeV) compared with their rest energy of 0.511 MeV. Figure 44.16 shows a pictorial representation of the decays described by Equations 44.21 and 44.22.

In Equation 44.19, the number of protons has increased by one and the number of neutrons has decreased by one. We can write the fundamental process of e^- decay in terms of a neutron changing into a proton as follows:

$$n \rightarrow p + e^- + \bar{\nu} \tag{44.23}$$

The electron and the antineutrino are ejected from the nucleus, with the net result that there is one more proton and one fewer neutron, consistent with the changes in Z and $A - Z$. A similar process occurs in e^+ decay, with a proton changing into a neutron, a positron, and a neutrino. This latter process can only occur within the nucleus, with the result that the nuclear mass decreases. It cannot occur for an isolated proton because its mass is less than that of the neutron.

A process that competes with e^+ decay is **electron capture,** which occurs when a parent nucleus captures one of its own orbital electrons and emits a neutrino. The final product after decay is a nucleus whose charge is $Z - 1$:

$$^{A}_{Z}\text{X} + {}^{0}_{-1}\text{e} \rightarrow {}_{Z-1}^{\;\;A}\text{Y} + \nu \tag{44.24}$$

◀ **Electron capture**

In most cases, it is a K-shell electron that is captured and the process is therefore referred to as **K capture.** One example is the capture of an electron by $^{7}_{4}\text{Be}$:

$$^{7}_{4}\text{Be} + {}^{0}_{-1}\text{e} \rightarrow {}^{7}_{3}\text{Li} + \nu$$

Because the neutrino is very difficult to detect, electron capture is usually observed by the x-rays given off as higher-shell electrons cascade downward to fill the vacancy created in the K shell.

Finally, we specify Q values for the beta-decay processes. The Q values for e^- decay and electron capture are given by $Q = (M_X - M_Y)c^2$, where M_X and M_Y are the masses of neutral atoms. In e^- decay, the parent nucleus experiences an increase in atomic number and, for the atom to become neutral, an electron must be absorbed by the atom. If the neutral parent atom and an electron (which will eventually combine with the daughter to form a neutral atom) is the initial system and the final system is the neutral daughter atom and the beta-ejected electron, the system contains a free electron both before and after the decay. Therefore, in subtracting the initial and final masses of the system, this electron mass cancels.

Pitfall Prevention 44.7

Mass Number of the Electron An alternative notation for an electron, as we see in Equation 44.24, is the symbol ${}^{0}_{-1}\text{e}$, which does not imply that the electron has zero rest energy. The mass of the electron is so much smaller than that of the lightest nucleon, however, that we approximate it as zero in the context of nuclear decays and reactions.

The Q values for e^+ decay are given by $Q = (M_X - M_Y - 2m_e)c^2$. The extra term $-2m_ec^2$ in this expression is necessary because the atomic number of the parent decreases by one when the daughter is formed. After it is formed by the decay, the daughter atom sheds one electron to form a neutral atom. Therefore, the final products are the daughter atom, the shed electron, and the ejected positron.

These relationships are useful in determining whether or not a process is energetically possible. For example, the Q value for proposed e^+ decay for a particular parent nucleus may turn out to be negative. In that case, this decay does not occur. The Q value for electron capture for this parent nucleus, however, may be a positive number, so electron capture can occur even though e^+ decay is not possible. Such is the case for the decay of ${}^{7}_{4}Be$ shown above.

Quick Quiz 44.4 Which of the following is the correct daughter nucleus associated with the beta decay of ${}^{184}_{72}Hf$? **(a)** ${}^{183}_{72}Hf$ **(b)** ${}^{183}_{73}Ta$ **(c)** ${}^{184}_{73}Ta$

Carbon Dating

The beta decay of ${}^{14}C$ (Eq. 44.21) is commonly used to date organic samples. Cosmic rays in the upper atmosphere cause nuclear reactions (Section 44.7) that create ${}^{14}C$. The ratio of ${}^{14}C$ to ${}^{12}C$ in the carbon dioxide molecules of our atmosphere has a constant value of approximately $r_0 = 1.3 \times 10^{-12}$. The carbon atoms in all living organisms have this same ${}^{14}C/{}^{12}C$ ratio r_0 because the organisms continuously exchange carbon dioxide with their surroundings. When an organism dies, however, it no longer absorbs ${}^{14}C$ from the atmosphere, and so the ${}^{14}C/{}^{12}C$ ratio decreases as the ${}^{14}C$ decays with a half-life of 5 730 yr. It is therefore possible to measure the age of a material by measuring its ${}^{14}C$ activity. Using this technique, scientists have been able to identify samples of wood, charcoal, bone, and shell as having lived from 1 000 to 25 000 years ago. This knowledge has helped us reconstruct the history of living organisms—including humans—during this time span.

A particularly interesting example is the dating of the Dead Sea Scrolls. This group of manuscripts was discovered by a shepherd in 1947. Translation showed them to be religious documents, including most of the books of the Old Testament. Because of their historical and religious significance, scholars wanted to know their age. Carbon dating applied to the material in which they were wrapped established their age at approximately 1 950 yr.

Conceptual Example 44.8 The Age of Iceman

In 1991, German tourists discovered the well-preserved remains of a man, now called "Ötzi the Iceman," trapped in a glacier in the Italian Alps. (See the photograph at the opening of this chapter.) Radioactive dating with ${}^{14}C$ revealed that this person was alive approximately 5 300 years ago. Why did scientists date a sample of Ötzi using ${}^{14}C$ rather than ${}^{11}C$, which is a beta emitter having a half-life of 20.4 min?

SOLUTION

Because ${}^{14}C$ has a half-life of 5 730 yr, the fraction of ${}^{14}C$ nuclei remaining after thousands of years is high enough to allow accurate measurements of changes in the sample's activity. Because ${}^{11}C$ has a very short half-life, it is not useful; its activity decreases to a vanishingly small value over the age of the sample, making it impossible to detect.

An isotope used to date a sample must be present in a known amount in the sample when it is formed. As a general rule, the isotope chosen to date a sample should also have a half-life that is on the same order of magnitude as the age of the sample. If the half-life is much less than the age of the sample, there won't be enough activity left to measure because almost all the original radioactive nuclei will have decayed. If the half-life is much greater than the age of the sample, the amount of decay that has taken place since the sample died will be too small to measure. For example, if you have a specimen estimated

44.8 continued

to have died 50 years ago, neither ^{14}C (5 730 yr) nor ^{11}C (20 min) is suitable. If you know your sample contains hydrogen, however, you can measure the activity of ^{3}H (tritium), a beta emitter that has a half-life of 12.3 yr.

Example 44.9 Radioactive Dating

A piece of charcoal containing 25.0 g of carbon is found in some ruins of an ancient city. The sample shows a ^{14}C activity R of 250 decays/min. How long has the tree from which this charcoal came been dead?

SOLUTION

Conceptualize Because the charcoal was found in ancient ruins, we expect the current activity to be smaller than the initial activity. If we can determine the initial activity, we can find out how long the wood has been dead.

Categorize The text of the question helps us categorize this example as a carbon dating problem.

Analyze Solve Equation 44.7 for t:

$$(1)\quad t = -\frac{1}{\lambda}\ln\left(\frac{R}{R_0}\right)$$

Evaluate the ratio R/R_0 using Equation 44.7, the initial value of the $^{14}C/^{12}C$ ratio r_0, the number of moles n of carbon, and Avogadro's number N_A:

$$\frac{R}{R_0} = \frac{R}{\lambda N_0(^{14}\text{C})} = \frac{R}{\lambda r_0 N_0(^{12}\text{C})} = \frac{R}{\lambda r_0 n N_A}$$

Replace the number of moles in terms of the molar mass M of carbon and the mass m of the sample and substitute for the decay constant λ:

$$\frac{R}{R_0} = \frac{R}{(\ln 2/T_{1/2})r_0(m/M)N_A} = \frac{RMT_{1/2}}{r_0 m N_A \ln 2}$$

Substitute numerical values:

$$\frac{R}{R_0} = \frac{(250\text{ min}^{-1})(12.0\text{ g/mol})(5\,730\text{ yr})}{(1.3\times10^{-12})(25.0\text{ g})(6.022\times10^{23}\text{ mol}^{-1})\ln 2}\left(\frac{3.156\times10^{7}\text{ s}}{1\text{ yr}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)$$

$$= 0.667$$

Substitute this ratio into Equation (1) and substitute for the decay constant λ:

$$t = -\frac{1}{\lambda}\ln\left(\frac{R}{R_0}\right) = -\frac{T_{1/2}}{\ln 2}\ln\left(\frac{R}{R_0}\right)$$

$$= -\frac{5\,730\text{ yr}}{\ln 2}\ln(0.667) = \boxed{3.4\times10^{3}\text{ yr}}$$

Finalize Note that the time interval found here is on the same order of magnitude as the half-life, so ^{14}C is a valid isotope to use for this sample, as discussed in Conceptual Example 44.8.

Gamma Decay

Very often, a nucleus that undergoes radioactive decay is left in an excited energy state. The nucleus can then undergo a second decay to a lower-energy state, perhaps to the ground state, by emitting a high-energy photon:

$$^{A}_{Z}\text{X}^{*} \rightarrow {}^{A}_{Z}\text{X} + \gamma \qquad (44.25)$$

◄ **Gamma decay**

where X* indicates a nucleus in an excited state. The typical half-life of an excited nuclear state is 10^{-10} s. Photons emitted in such a de-excitation process are called gamma rays. Such photons have very high energy (1 MeV to 1 GeV) relative to the energy of visible light (approximately 1 eV). Recall from Section 42.3 that the energy of a photon emitted or absorbed by an atom equals the difference in energy

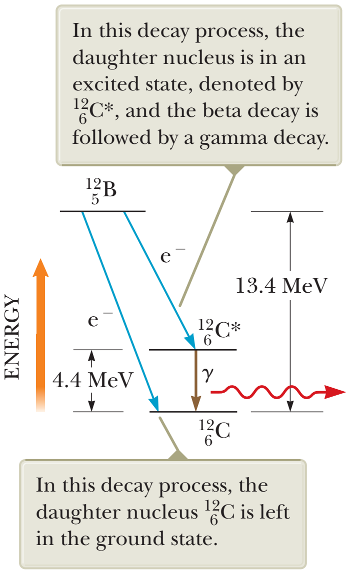


Figure 44.17 An energy-level diagram showing the initial nuclear state of a ^{12}B nucleus and two possible lower-energy states of the ^{12}C nucleus.

between the two electronic states involved in the transition. Similarly, a gamma-ray photon has an energy hf that equals the energy difference ΔE between two nuclear energy levels. When a nucleus decays by emitting a gamma ray, the only change in the nucleus is that it ends up in a lower-energy state. There are no changes in Z, N, or A.

A nucleus may reach an excited state as the result of a violent collision with another particle. More common, however, is for a nucleus to be in an excited state after it has undergone alpha or beta decay. The following sequence of events represents a typical situation in which gamma decay occurs:

$$^{12}_{5}B \rightarrow {}^{12}_{6}C^* + e^- + \bar{\nu} \quad (44.26)$$

$$^{12}_{6}C^* \rightarrow {}^{12}_{6}C + \gamma \quad (44.27)$$

Figure 44.17 shows the decay scheme for ^{12}B, which undergoes beta decay to either of two levels of ^{12}C. It can either (1) decay directly to the ground state of ^{12}C by emitting a 13.4-MeV electron or (2) undergo beta decay to an excited state of $^{12}C^*$ followed by gamma decay to the ground state. The latter process results in the emission of a 9.0-MeV electron and a 4.4-MeV photon.

The various pathways by which a radioactive nucleus can undergo decay are summarized in Table 44.3.

44.6 Natural Radioactivity

Radioactive nuclei are generally classified into two groups: (1) unstable nuclei found in nature, which give rise to **natural radioactivity,** and (2) unstable nuclei produced in the laboratory through nuclear reactions, which exhibit **artificial radioactivity.**

As Table 44.4 shows, there are three series of naturally occurring radioactive nuclei. Each series starts with a specific long-lived radioactive isotope whose half-life exceeds that of any of its unstable descendants. The three natural series begin with the isotopes ^{238}U, ^{235}U, and ^{232}Th, and the corresponding stable end products are three isotopes of lead: ^{206}Pb, ^{207}Pb, and ^{208}Pb. The fourth series in Table 44.4 begins with ^{237}Np and has as its stable end product ^{209}Bi. The element ^{237}Np is a *transuranic* element (one having an atomic number greater than that of uranium) not found in nature. This element has a half-life of "only" 2.14×10^6 years.

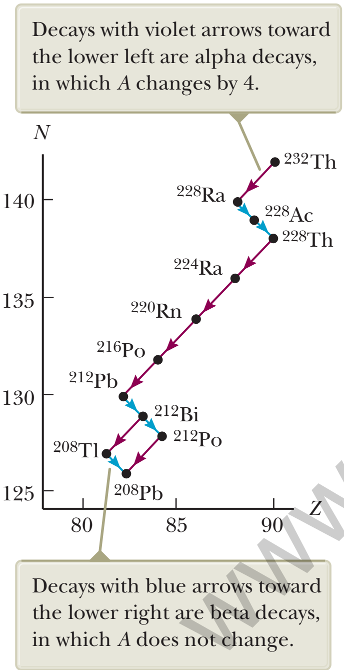


Figure 44.18 Successive decays for the ^{232}Th series.

Figure 44.18 shows the successive decays for the ^{232}Th series. First, ^{232}Th undergoes alpha decay to ^{228}Ra. Next, ^{228}Ra undergoes two successive beta decays to ^{228}Th. The series continues and finally branches when it reaches ^{212}Bi. At this point, there are two decay possibilities. The sequence shown in Figure 44.18 is characterized by a mass-number decrease of either 4 (for alpha decays) or 0 (for beta or gamma decays). The two uranium series are more complex than the ^{232}Th series. In addition, several naturally occurring radioactive isotopes, such as ^{14}C and ^{40}K, are not part of any decay series.

Because of these radioactive series, our environment is constantly replenished with radioactive elements that would otherwise have disappeared long ago. For example, because our solar system is approximately 5×10^9 years old, the supply of

Table 44.3 Various Decay Pathways

Alpha decay	$^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$
Beta decay (e^-)	$^{A}_{Z}X \rightarrow {}_{Z+1}^{A}Y + e^- + \bar{\nu}$
Beta decay (e^+)	$^{A}_{Z}X \rightarrow {}_{Z-1}^{A}Y + e^+ + \nu$
Electron capture	$^{A}_{Z}X + e^- \rightarrow {}_{Z-1}^{A}Y + \nu$
Gamma decay	$^{A}_{Z}X^* \rightarrow {}^{A}_{Z}X + \gamma$

Table 44.4 The Four Radioactive Series

Series		Starting Isotope	Half-life (years)	Stable End Product
Uranium	Natural	$^{238}_{92}U$	4.47×10^9	$^{206}_{82}Pb$
Actinium	Natural	$^{235}_{92}U$	7.04×10^8	$^{207}_{82}Pb$
Thorium	Natural	$^{232}_{90}Th$	1.41×10^{10}	$^{208}_{82}Pb$
Neptunium		$^{237}_{93}Np$	2.14×10^6	$^{209}_{83}Bi$

^{226}Ra (whose half-life is only 1 600 years) would have been depleted by radioactive decay long ago if it were not for the radioactive series starting with ^{238}U.

44.7 Nuclear Reactions

We have studied radioactivity, which is a spontaneous process in which the structure of a nucleus changes. It is also possible to stimulate changes in the structure of nuclei by bombarding them with energetic particles. Such collisions, which change the identity of the target nuclei, are called **nuclear reactions.** Rutherford was the first to observe them, in 1919, using naturally occurring radioactive sources for the bombarding particles. Since then, a wide variety of nuclear reactions has been observed following the development of charged-particle accelerators in the 1930s. With today's advanced technology in particle accelerators and particle detectors, the Large Hadron Collider (see Section 46.10) in Europe can achieve particle energies of 14 000 GeV = 14 TeV. These high-energy particles are used to create new particles whose properties are helping to solve the mysteries of the nucleus.

Consider a reaction in which a target nucleus X is bombarded by a particle a, resulting in a daughter nucleus Y and an outgoing particle b:

$$\mathrm{a} + \mathrm{X} \rightarrow \mathrm{Y} + \mathrm{b} \qquad (44.28)$$

◀ **Nuclear reaction**

Sometimes this reaction is written in the more compact form

$$\mathrm{X(a,\,b)Y}$$

In Section 44.5, the Q value, or disintegration energy, of a radioactive decay was defined as the rest energy transformed to kinetic energy as a result of the decay process. Likewise, we define the **reaction energy** Q associated with a nuclear reaction as *the difference between the initial and final rest energies resulting from the reaction:*

$$Q = (M_\mathrm{a} + M_\mathrm{X} - M_\mathrm{Y} - M_\mathrm{b})c^2 \qquad (44.29)$$

◀ **Reaction energy Q**

As an example, consider the reaction $^7\mathrm{Li(p,}\ \alpha)^4\mathrm{He}$. The notation p indicates a proton, which is a hydrogen nucleus. Therefore, we can write this reaction in the expanded form

$$^1_1\mathrm{H} + {}^7_3\mathrm{Li} \rightarrow {}^4_2\mathrm{He} + {}^4_2\mathrm{He}$$

The Q value for this reaction is 17.3 MeV. A reaction such as this one, for which Q is positive, is called **exothermic.** A reaction for which Q is negative is called **endothermic.** To satisfy conservation of momentum for the isolated system, an endothermic reaction does not occur unless the bombarding particle has a kinetic energy greater than Q. (See Problem 74.) The minimum energy necessary for such a reaction to occur is called the **threshold energy.**

If particles a and b in a nuclear reaction are identical so that X and Y are also necessarily identical, the reaction is called a **scattering event.** If the kinetic energy of the system (a and X) before the event is the same as that of the system (b and Y) after the event, it is classified as *elastic scattering.* If the kinetic energy of the system after the event is less than that before the event, the reaction is described as *inelastic scattering.* In this case, the target nucleus has been raised to an excited state by the event, which accounts for the difference in energy. The final system now consists of b and an excited nucleus Y*, and eventually it will become b, Y, and γ, where γ is the gamma-ray photon that is emitted when the system returns to the ground state. This elastic and inelastic terminology is identical to that used in describing collisions between macroscopic objects as discussed in Section 9.4.

In addition to energy and momentum, the total charge and total number of nucleons must be conserved in any nuclear reaction. For example, consider the reaction ^{19}F(p, α)^{16}O, which has a Q value of 8.11 MeV. We can show this reaction more completely as

$$^{1}_{1}\text{H} + ^{19}_{9}\text{F} \rightarrow ^{16}_{8}\text{O} + ^{4}_{2}\text{He} \qquad \textbf{(44.30)}$$

The total number of nucleons before the reaction (1 + 19 = 20) is equal to the total number after the reaction (16 + 4 = 20). Furthermore, the total charge is the same before (1 + 9) and after (8 + 2) the reaction.

44.8 Nuclear Magnetic Resonance and Magnetic Resonance Imaging

In this section, we describe an important application of nuclear physics in medicine called *magnetic resonance imaging*. To understand this application, we first discuss the spin angular momentum of the nucleus. This discussion has parallels with the discussion of spin for atomic electrons.

In Chapter 42, we discussed that the electron has an intrinsic angular momentum, called spin. Nuclei also have spin because their component particles—neutrons and protons—each have spin $\frac{1}{2}$ as well as orbital angular momentum within the nucleus. All types of angular momentum obey the quantum rules that were outlined for orbital and spin angular momentum in Chapter 42. In particular, two quantum numbers associated with the angular momentum determine the allowed values of the magnitude of the angular momentum vector and its direction in space. The magnitude of the nuclear angular momentum is $\sqrt{I(I+1)}\,\hbar$, where I is called the **nuclear spin quantum number** and may be an integer or a half-integer, depending on how the individual proton and neutron spins combine. The quantum number I is the analog to ℓ for the electron in an atom as discussed in Section 42.6. Furthermore, there is a quantum number m_I that is the analog to m_ℓ, in that the allowed projections of the nuclear spin angular momentum vector on the z axis are $m_I\hbar$. The values of m_I range from $-I$ to $+I$ in steps of 1. (In fact, for *any* type of spin with a quantum number S, there is a quantum number m_S that ranges in value from $-S$ to $+S$ in steps of 1.) Therefore, the maximum value of the z component of the spin angular momentum vector is $I\hbar$. Figure 44.19 is a vector model (see Section 42.6) illustrating the possible orientations of the nuclear spin vector and its projections along the z axis for the case in which $I = \frac{3}{2}$.

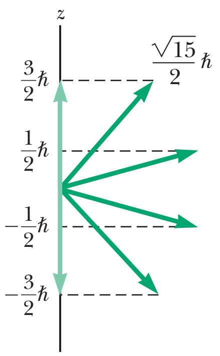


Figure 44.19 A vector model showing possible orientations of the nuclear spin angular momentum vector and its projections along the z axis for the case $I = \frac{3}{2}$.

Nuclear spin has an associated nuclear magnetic moment, similar to that of the electron. The spin magnetic moment of a nucleus is measured in terms of the **nuclear magneton** μ_n, a unit of moment defined as

Nuclear magneton ▶

$$\mu_n \equiv \frac{e\hbar}{2m_p} = 5.05 \times 10^{-27}\ \text{J/T} \qquad \textbf{(44.31)}$$

where m_p is the mass of the proton. This definition is analogous to that of the Bohr magneton μ_B, which corresponds to the spin magnetic moment of a free electron (see Section 42.6). Note that μ_n is smaller than μ_B (= 9.274 × 10^{-24} J/T) by a factor of 1 836 because of the large difference between the proton mass and the electron mass.

The magnetic moment of a free proton is 2.792 8μ_n. Unfortunately, there is no general theory of nuclear magnetism that explains this value. The neutron also has a magnetic moment, which has a value of −1.913 5μ_n. The negative sign indicates that this moment is opposite the spin angular momentum of the neutron. The existence of a magnetic moment for the neutron is surprising in view of the neutron being uncharged. That suggests that the neutron is not a fundamental particle but rather has an underlying structure consisting of charged constituents. We shall explore this structure in Chapter 46.

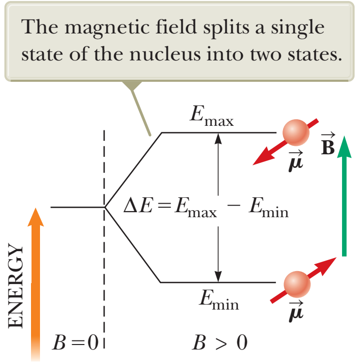


Figure 44.20 A nucleus with spin $\frac{1}{2}$ is placed in a magnetic field.

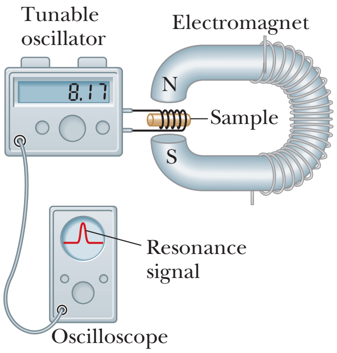


Figure 44.21 Experimental arrangement for nuclear magnetic resonance. The radio-frequency magnetic field created by the coil surrounding the sample and provided by the variable-frequency oscillator is perpendicular to the constant magnetic field created by the electromagnet. When the nuclei in the sample meet the resonance condition, the nuclei absorb energy from the radio-frequency field of the coil; this absorption changes the characteristics of the circuit in which the coil is included. Most modern NMR spectrometers use superconducting magnets at fixed field strengths and operate at frequencies of approximately 200 MHz.

The potential energy associated with a magnetic dipole moment $\vec{\mu}$ in an external magnetic field $\vec{B}$ is given by $-\vec{\mu} \cdot \vec{B}$ (Eq. 29.18). When the magnetic moment $\vec{\mu}$ is lined up with the field as closely as quantum physics allows, the potential energy of the dipole–field system has its minimum value E_{min}. When $\vec{\mu}$ is as antiparallel to the field as possible, the potential energy has its maximum value E_{max}. In general, there are other energy states between these values corresponding to the quantized directions of the magnetic moment with respect to the field. For a nucleus with spin $\frac{1}{2}$, there are only two allowed states, with energies E_{min} and E_{max}. These two energy states are shown in Figure 44.20.

It is possible to observe transitions between these two spin states using a technique called **NMR,** for **nuclear magnetic resonance.** A constant magnetic field ($\vec{B}$ in Fig. 44.20) is introduced to define a z axis and split the energies of the spin states. A second, weaker, oscillating magnetic field is then applied perpendicular to $\vec{B}$, creating a cloud of radio-frequency photons around the sample. When the frequency of the oscillating field is adjusted so that the photon energy matches the energy difference between the spin states, there is a net absorption of photons by the nuclei that can be detected electronically.

Figure 44.21 is a simplified diagram of the apparatus used in nuclear magnetic resonance. The energy absorbed by the nuclei is supplied by the tunable oscillator producing the oscillating magnetic field. Nuclear magnetic resonance and a related technique called *electron spin resonance* are extremely important methods for studying nuclear and atomic systems and the ways in which these systems interact with their surroundings.

A widely used medical diagnostic technique called **MRI,** for **magnetic resonance imaging,** is based on nuclear magnetic resonance. Because nearly two-thirds of the atoms in the human body are hydrogen (which gives a strong NMR signal), MRI works exceptionally well for viewing internal tissues. The patient is placed inside a large solenoid that supplies a magnetic field that is constant in time but whose magnitude varies spatially across the body. Because of the variation in the field, hydrogen atoms in different parts of the body have different energy splittings between spin states, so the resonance signal can be used to provide information about the positions of the protons. A computer is used to analyze the position information to provide data for constructing a final image. Contrast in the final image among different types of tissues is created by computer analysis of the time intervals for the nuclei to return to the lower-energy spin state between pulses of radio-frequency photons. Contrast can be enhanced with the use of contrast agents such as gadolinium compounds or iron oxide nanoparticles taken orally or injected intravenously. An MRI scan showing incredible detail in internal body structure is shown in Figure 44.22.

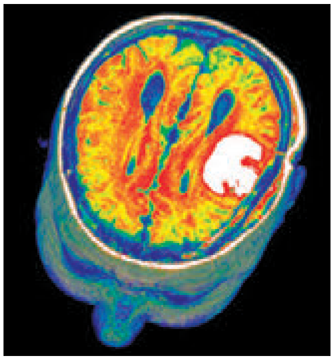


Figure 44.22 A color-enhanced MRI scan of a human brain, showing a tumor in white.

The main advantage of MRI over other imaging techniques is that it causes minimal cellular damage. The photons associated with the radio-frequency signals used in MRI have energies of only about 10^{-7} eV. Because molecular bond strengths are much larger (approximately 1 eV), the radio-frequency radiation causes little cellular damage. In comparison, x-rays have energies ranging from 10^4 to 10^6 eV and can cause considerable cellular damage. Therefore, despite some individuals' fears of the word *nuclear* associated with MRI, the radio-frequency radiation involved is overwhelmingly safer than the x-rays that these individuals might accept more readily. A disadvantage of MRI is that the equipment required to conduct the procedure is very expensive, so MRI images are costly.

The magnetic field produced by the solenoid is sufficient to lift a car, and the radio signal is about the same magnitude as that from a small commercial broadcasting station. Although MRI is inherently safe in normal use, the strong magnetic field of the solenoid requires diligent care to ensure that no ferromagnetic materials are located in the room near the MRI apparatus. Several accidents have occurred, such as a 2000 incident in which a gun pulled from a police officer's hand discharged upon striking the machine.

Summary

Definitions

A nucleus is represented by the symbol ^A_ZX, where A is the **mass number** (the total number of nucleons) and Z is the **atomic number** (the total number of protons). The total number of neutrons in a nucleus is the **neutron number** N, where $A = N + Z$. Nuclei having the same Z value but different A and N values are **isotopes** of each other.

The magnetic moment of a nucleus is measured in terms of the **nuclear magneton** μ_n, where

$$\mu_n \equiv \frac{e\hbar}{2m_p} = 5.05 \times 10^{-27}\ \text{J/T} \qquad \textbf{(44.31)}$$

Concepts and Principles

Assuming nuclei are spherical, their radius is given by

$$r = aA^{1/3} \qquad \textbf{(44.1)}$$

where $a = 1.2$ fm.

Nuclei are stable because of the **nuclear force** between nucleons. This short-range force dominates the Coulomb repulsive force at distances of less than about 2 fm and is independent of charge. Light stable nuclei have equal numbers of protons and neutrons. Heavy stable nuclei have more neutrons than protons. The most stable nuclei have Z and N values that are both even.

The difference between the sum of the masses of a group of separate nucleons and the mass of the compound nucleus containing these nucleons, when multiplied by c^2, gives the **binding energy** E_b of the nucleus. The binding energy of a nucleus can be calculated in MeV using the expression

$$E_b = [ZM(\text{H}) + Nm_n - M(^A_Z\text{X})] \times 931.494\ \text{MeV/u} \qquad \textbf{(44.2)}$$

where $M(\text{H})$ is the atomic mass of the neutral hydrogen atom, $M(^A_Z\text{X})$ represents the atomic mass of an atom of the isotope ^A_ZX, and m_n is the mass of the neutron.

The **liquid-drop model** of nuclear structure treats the nucleons as molecules in a drop of liquid. The four main contributions influencing binding energy are the volume effect, the surface effect, the Coulomb repulsion effect, and the symmetry effect. Summing such contributions results in the **semiempirical binding-energy formula:**

$$E_b = C_1A - C_2A^{2/3} - C_3\frac{Z(Z-1)}{A^{1/3}} - C_4\frac{(N-Z)^2}{A} \qquad \textbf{(44.3)}$$

The **shell model, or independent-particle model,** assumes each nucleon exists in a shell and can only have discrete energy values. The stability of certain nuclei can be explained with this model.

A radioactive substance decays by **alpha decay, beta decay,** or **gamma decay.** An alpha particle is the ^{4}He nucleus, a beta particle is either an electron (e^-) or a positron (e^+), and a gamma particle is a high-energy photon.

If a radioactive material contains N_0 radioactive nuclei at $t = 0$, the number N of nuclei remaining after a time t has elapsed is

$$N = N_0 e^{-\lambda t} \quad \textbf{(44.6)}$$

where λ is the **decay constant,** a number equal to the probability per second that a nucleus will decay. The **decay rate,** or **activity,** of a radioactive substance is

$$R = \left|\frac{dN}{dt}\right| = R_0 e^{-\lambda t} \quad \textbf{(44.7)}$$

where $R_0 = \lambda N_0$ is the activity at $t = 0$. The **half-life** $T_{1/2}$ is the time interval required for half of a given number of radioactive nuclei to decay, where

$$T_{1/2} = \frac{0.693}{\lambda} \quad \textbf{(44.8)}$$

In alpha decay, a helium nucleus is ejected from the parent nucleus with a discrete set of kinetic energies. A nucleus undergoing beta decay emits either an electron (e^-) and an antineutrino ($\bar{\nu}$) or a positron (e^+) and a neutrino (ν). The electron or positron is ejected with a continuous range of energies. In **electron capture,** the nucleus of an atom absorbs one of its own electrons and emits a neutrino. In gamma decay, a nucleus in an excited state decays to its ground state and emits a gamma ray.

Nuclear reactions can occur when a target nucleus X is bombarded by a particle a, resulting in a daughter nucleus Y and an outgoing particle b:

$$a + X \rightarrow Y + b \quad \textbf{(44.28)}$$

The mass–energy conversion in such a reaction, called the **reaction energy** Q, is

$$Q = (M_a + M_X - M_Y - M_b)c^2 \quad \textbf{(44.29)}$$

Objective Questions

[1.] denotes answer available in *Student Solutions Manual/Study Guide*

1. In nuclear magnetic resonance, suppose we increase the value of the constant magnetic field. As a result, the frequency of the photons that are absorbed in a particular transition changes. How is the frequency of the photons absorbed related to the magnetic field? (a) The frequency is proportional to the square of the magnetic field. (b) The frequency is directly proportional to the magnetic field. (c) The frequency is independent of the magnetic field. (d) The frequency is inversely proportional to the magnetic field. (e) The frequency is proportional to the reciprocal of the square of the magnetic field.

2. When the $^{95}_{36}$Kr nucleus undergoes beta decay by emitting an electron and an antineutrino, does the daughter nucleus (Rb) contain (a) 58 neutrons and 37 protons, (b) 58 protons and 37 neutrons, (c) 54 neutrons and 41 protons, or (d) 55 neutrons and 40 protons?

3. When $^{32}_{15}$P decays to $^{32}_{16}$S, which of the following particles is emitted? (a) a proton (b) an alpha particle (c) an electron (d) a gamma ray (e) an antineutrino

4. The half-life of radium-224 is about 3.6 days. What approximate fraction of a sample remains undecayed after two weeks? (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{8}$ (d) $\frac{1}{16}$ (e) $\frac{1}{32}$

[5.] Two samples of the same radioactive nuclide are prepared. Sample G has twice the initial activity of sample H. **(i)** How does the half-life of G compare with the half-life of H? (a) It is two times larger. (b) It is the same. (c) It is half as large. **(ii)** After each has passed through five half-lives, how do their activities compare? (a) G has more than twice the activity of H. (b) G has twice the activity of H. (c) G and H have the same activity. (d) G has lower activity than H.

6. If a radioactive nuclide ^A_ZX decays by emitting a gamma ray, what happens? (a) The resulting nuclide has a different Z value. (b) The resulting nuclide has the same A and Z values. (c) The resulting nuclide has a different A value. (d) Both A and Z decrease by one. (e) None of those statements is correct.

7. Does a nucleus designated as $^{40}_{18}$X contain (a) 20 neutrons and 20 protons, (b) 22 protons and 18 neutrons, (c) 18 protons and 22 neutrons, (d) 18 protons and 40 neutrons, or (e) 40 protons and 18 neutrons?

8. When $^{144}_{60}$Nd decays to $^{140}_{58}$Ce, identify the particle that is released. (a) a proton (b) an alpha particle (c) an electron (d) a neutron (e) a neutrino

9. What is the Q value for the reaction ^{9}Be $+ \alpha \rightarrow {}^{12}$C + n? (a) 8.4 MeV (b) 7.3 MeV (c) 6.2 MeV (d) 5.7 MeV (e) 4.2 MeV

10. **(i)** To predict the behavior of a nucleus in a fission reaction, which model would be more appropriate,

(a) the liquid-drop model or (b) the shell model? **(ii)** Which model would be more successful in predicting the magnetic moment of a given nucleus? Choose from the same answers as in part (i). **(iii)** Which could better explain the gamma-ray spectrum of an excited nucleus? Choose from the same answers as in part (i).

11. A free neutron has a half-life of 614 s. It undergoes beta decay by emitting an electron. Can a free proton undergo a similar decay? (a) yes, the same decay (b) yes, but by emitting a positron (c) yes, but with a very different half-life (d) no

12. Which of the following quantities represents the reaction energy of a nuclear reaction? (a) (final mass − initial mass)/c^2 (b) (initial mass − final mass)/c^2 (c) (final mass − initial mass)c^2 (d) (initial mass − final mass)c^2 (e) none of those quantities

13. In the decay ${}^{234}_{90}\text{Th} \rightarrow {}^{A}_{Z}\text{Ra} + {}^{4}_{2}\text{He}$, identify the mass number and the atomic number of the Ra nucleus: (a) $A = 230$, $Z = 92$ (b) $A = 238$, $Z = 88$ (c) $A = 230$, $Z = 88$ (d) $A = 234$, $Z = 88$ (e) $A = 238$, $Z = 86$

Conceptual Questions

1. denotes answer available in *Student Solutions Manual/Study Guide*

1. If a nucleus such as ^{226}Ra initially at rest undergoes alpha decay, which has more kinetic energy after the decay, the alpha particle or the daughter nucleus? Explain your answer.

2. "If no more people were to be born, the law of population growth would strongly resemble the radioactive decay law.'' Discuss this statement.

3. A student claims that a heavy form of hydrogen decays by alpha emission. How do you respond?

4. In beta decay, the energy of the electron or positron emitted from the nucleus lies somewhere in a relatively large range of possibilities. In alpha decay, however, the alpha-particle energy can only have discrete values. Explain this difference.

5. Can carbon-14 dating be used to measure the age of a rock? Explain.

6. In positron decay, a proton in the nucleus becomes a neutron and its positive charge is carried away by the positron. A neutron, though, has a larger rest energy than a proton. How is that possible?

7. (a) How many values of I_z are possible for $I = \frac{5}{2}$? (b) For $I = 3$?

8. Why do nearly all the naturally occurring isotopes lie above the $N = Z$ line in Figure 44.4?

9. Why are very heavy nuclei unstable?

10. Explain why nuclei that are well off the line of stability in Figure 44.4 tend to be unstable.

11. Consider two heavy nuclei X and Y having similar mass numbers. If X has the higher binding energy, which nucleus tends to be more unstable? Explain your answer.

12. What fraction of a radioactive sample has decayed after two half-lives have elapsed?

13. Figure CQ44.13 shows a watch from the early 20th century. The numbers and the hands of the watch are painted with a paint that contains a small amount of natural radium ${}^{226}_{88}$Ra mixed with a phosphorescent material. The decay of the radium causes the phosphorescent material to glow continuously. The radioactive nuclide ${}^{226}_{88}$Ra has a half-life of approximately 1.60×10^3 years. Being that the solar system is approximately 5 billion years old, why was this isotope still available in the 20th century for use on this watch?

Figure CQ44.13

14. Can a nucleus emit alpha particles that have different energies? Explain.

15. In Rutherford's experiment, assume an alpha particle is headed directly toward the nucleus of an atom. Why doesn't the alpha particle make physical contact with the nucleus?

16. Suppose it could be shown that the cosmic-ray intensity at the Earth's surface was much greater 10 000 years ago. How would this difference affect what we accept as valid carbon-dated values of the age of ancient samples of once-living matter? Explain your answer.

17. Compare and contrast the properties of a photon and a neutrino.

Problems

The problems found in this chapter may be assigned online in Enhanced WebAssign

1. straightforward; **2.** intermediate; **3.** challenging

1. full solution available in the *Student Solutions Manual/Study Guide*

AMT Analysis Model tutorial available in Enhanced WebAssign

GP Guided Problem

M Master It tutorial available in Enhanced WebAssign

W Watch It video solution available in Enhanced WebAssign

Section 44.1 Some Properties of Nuclei

1. Find the nuclear radii of (a) $^{2}_{1}$H, (b) $^{60}_{27}$Co, (c) $^{197}_{79}$Au, and (d) $^{239}_{94}$Pu.

2. (a) Determine the mass number of a nucleus whose radius is approximately equal to two-thirds the radius of $^{230}_{88}$Ra. (b) Identify the element. (c) Are any other answers possible? Explain.

3. M (a) Use energy methods to calculate the distance of closest approach for a head-on collision between an alpha particle having an initial energy of 0.500 MeV and a gold nucleus (^{197}Au) at rest. Assume the gold nucleus remains at rest during the collision. (b) What minimum initial speed must the alpha particle have to approach as close as 300 fm to the gold nucleus?

4. (a) What is the order of magnitude of the number of protons in your body? (b) Of the number of neutrons? (c) Of the number of electrons?

5. Consider the $^{65}_{29}$Cu nucleus. Find approximate values for its (a) radius, (b) volume, and (c) density.

6. Using 2.30×10^{17} kg/m^3 as the density of nuclear matter, find the radius of a sphere of such matter that would have a mass equal to that of a baseball, 0.145 kg.

7. A star ending its life with a mass of four to eight times the Sun's mass is expected to collapse and then undergo a supernova event. In the remnant that is not carried away by the supernova explosion, protons and electrons combine to form a neutron star with approximately twice the mass of the Sun. Such a star can be thought of as a gigantic atomic nucleus. Assume $r = aA^{1/3}$ (Eq. 44.1). If a star of mass 3.98×10^{30} kg is composed entirely of neutrons ($m_n = 1.67 \times 10^{-27}$ kg), what would its radius be?

8. Figure P44.8 shows the potential energy for two protons as a function of separation distance. In the text, it was claimed that, to be visible on such a graph, the peak in the curve is exaggerated by a factor of ten. (a) Find the electric potential energy of a pair of protons separated by 4.00 fm. (b) Verify that the peak in Figure P44.8 is exaggerated by a factor of ten.

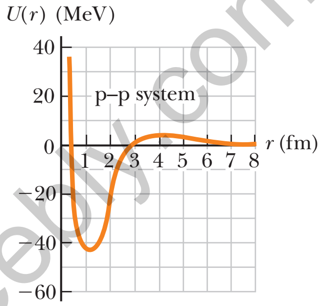


Figure P44.8

9. AMT **Review.** Singly ionized carbon is accelerated through 1 000 V and passed into a mass spectrometer to determine the isotopes present (see Chapter 29). The magnitude of the magnetic field in the spectrometer is 0.200 T. The orbit radius for a ^{12}C isotope as it passes through the field is $r = 7.89$ cm. Find the radius of the orbit of a ^{13}C isotope.

10. **Review.** Singly ionized carbon is accelerated through a potential difference ΔV and passed into a mass spectrometer to determine the isotopes present (see Chapter 29). The magnitude of the magnetic field in the spectrometer is B. The orbit radius for an isotope of mass m_1 as it passes through the field is r_1. Find the radius of the orbit of an isotope of mass m_2.

11. M An alpha particle ($Z = 2$, mass $= 6.64 \times 10^{-27}$ kg) approaches to within 1.00×10^{-14} m of a carbon nucleus ($Z = 6$). What are (a) the magnitude of the maximum Coulomb force on the alpha particle, (b) the magnitude of the acceleration of the alpha particle at the time of the maximum force, and (c) the potential energy of the system of the alpha particle and the carbon nucleus at this time?

12. In a Rutherford scattering experiment, alpha particles having kinetic energy of 7.70 MeV are fired toward a gold nucleus that remains at rest during the collision. The alpha particles come as close as 29.5 fm to the gold nucleus before turning around. (a) Calculate the de Broglie wavelength for the 7.70-MeV alpha particle and compare it with the distance of closest approach, 29.5 fm. (b) Based on this comparison, why is it proper to treat the alpha particle as a particle and not as a wave in the Rutherford scattering experiment?

13. **Review.** Two golf balls each have a 4.30-cm diameter and are 1.00 m apart. What would be the gravitational force exerted by each ball on the other if the balls were made of nuclear matter?

14. Assume a hydrogen atom is a sphere with diameter 0.100 nm and a hydrogen molecule consists of two such spheres in contact. (a) What fraction of the space in a tank of hydrogen gas at 0°C and 1.00 atm is occupied by the hydrogen molecules themselves? (b) What fraction of the space within one hydrogen atom is occupied by its nucleus, of radius 1.20 fm?

Section 44.2 Nuclear Binding Energy

15. Calculate the binding energy per nucleon for (a) ^{2}H, (b) ^{4}He, (c) ^{56}Fe, and (d) ^{238}U.

16. (a) Calculate the difference in binding energy per nucleon for the nuclei $^{23}_{11}$Na and $^{23}_{12}$Mg. (b) How do you account for the difference?

17. A pair of nuclei for which $Z_1 = N_2$ and $Z_2 = N_1$ are called *mirror isobars* (the atomic and neutron numbers are interchanged). Binding-energy measurements on these nuclei can be used to obtain evidence of the charge independence of nuclear forces (that is, proton–proton, proton–neutron, and neutron–neutron nuclear forces are equal). Calculate the difference in binding energy for the two mirror isobars $^{15}_{8}$O and $^{15}_{7}$N. The electric repulsion among eight protons rather than seven accounts for the difference.

18. The peak of the graph of nuclear binding energy per nucleon occurs near ^{56}Fe, which is why iron is prominent in the spectrum of the Sun and stars. Show that ^{56}Fe has a higher binding energy per nucleon than its neighbors ^{55}Mn and ^{59}Co.

19. Nuclei having the same mass numbers are called *isobars.* The isotope $^{139}_{57}$La is stable. A radioactive isobar, $^{139}_{59}$Pr, is located below the line of stable nuclei as shown in Figure P44.19 and decays by e$^+$ emission. Another radioactive isobar of $^{139}_{57}$La, $^{139}_{55}$Cs, decays by e$^-$ emission and is located above the line of stable nuclei in Figure P44.19. (a) Which of these three isobars has the highest neutron-to-proton ratio? (b) Which has the greatest binding energy per nucleon? (c) Which do you expect to be heavier, $^{139}_{59}$Pr or $^{139}_{55}$Cs?

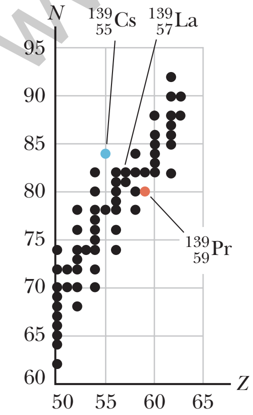


Figure P44.19

20. The energy required to construct a uniformly charged sphere of total charge Q and radius R is $U = 3k_eQ^2/5R$, where k_e is the Coulomb constant (see Problem 77). Assume a ^{40}Ca nucleus contains 20 protons uniformly distributed in a spherical volume. (a) How much energy is required to counter their electrical repulsion according to the above equation? (b) Calculate the binding energy of ^{40}Ca. (c) Explain what you can conclude from comparing the result of part (b) with that of part (a).

21. Calculate the minimum energy required to remove a neutron from the $^{43}_{20}$Ca nucleus.

Section 44.3 Nuclear Models

22. Using the graph in Figure 44.5, estimate how much energy is released when a nucleus of mass number 200 fissions into two nuclei each of mass number 100.

23. (a) Use the semiempirical binding-energy formula (Eq. 44.3) to compute the binding energy for $^{56}_{26}$Fe. (b) What percentage is contributed to the binding energy by each of the four terms?

24. (a) In the liquid-drop model of nuclear structure, why does the surface-effect term $-C_2A^{2/3}$ have a negative sign? (b) **What If?** The binding energy of the nucleus increases as the volume-to-surface area ratio increases. Calculate this ratio for both spherical and cubical shapes and explain which is more plausible for nuclei.

Section 44.4 Radioactivity

25. What time interval is required for the activity of a sample of the radioactive isotope $^{72}_{33}$As to decrease by 90.0% from its original value? The half-life of $^{72}_{33}$As is 26 h.

26. M A freshly prepared sample of a certain radioactive isotope has an activity of 10.0 mCi. After 4.00 h, its activity is 8.00 mCi. Find (a) the decay constant and (b) the half-life. (c) How many atoms of the isotope were contained in the freshly prepared sample? (d) What is the sample's activity 30.0 h after it is prepared?

27. A sample of radioactive material contains 1.00×10^{15} atoms and has an activity of 6.00×10^{11} Bq. What is its half-life?

28. From the equation expressing the law of radioactive decay, derive the following useful expressions for the decay constant and the half-life, in terms of the time interval Δt during which the decay rate decreases from R_0 to R:

$$\lambda = \frac{1}{\Delta t} \ln\left(\frac{R_0}{R}\right) \qquad T_{1/2} = \frac{(\ln 2)\,\Delta t}{\ln (R_0/R)}$$

29. The radioactive isotope ^{198}Au has a half-life of 64.8 h. A sample containing this isotope has an initial activity ($t = 0$) of 40.0 μCi. Calculate the number of nuclei that decay in the time interval between $t_1 = 10.0$ h and $t_2 = 12.0$ h.

30. A radioactive nucleus has half-life $T_{1/2}$. A sample containing these nuclei has initial activity R_0 at $t = 0$. Calculate the number of nuclei that decay during the interval between the later times t_1 and t_2.

31. The half-life of ^{131}I is 8.04 days. (a) Calculate the decay constant for this nuclide. (b) Find the number of ^{131}I nuclei necessary to produce a sample with an activity of 6.40 mCi. (c) A sample of ^{131}I with this initial activity decays for 40.2 d. What is the activity at the end of that period?

32. Tritium has a half-life of 12.33 years. What fraction of the nuclei in a tritium sample will remain (a) after 5.00 yr? (b) After 10.0 yr? (c) After 123.3 yr? (d) According to Equation 44.6, an infinite amount of time is required for the entire sample to decay. Discuss whether that is realistic.

33. Consider a radioactive sample. Determine the ratio of the number of nuclei decaying during the first half of its half-life to the number of nuclei decaying during the second half of its half-life.

34. (a) The daughter nucleus formed in radioactive decay is often radioactive. Let N_{10} represent the number of parent nuclei at time $t = 0$, $N_1(t)$ the number of parent nuclei at time t, and λ_1 the decay constant of the parent. Suppose the number of daughter nuclei at time $t = 0$ is zero. Let $N_2(t)$ be the number of daughter nuclei at time t and let λ_2 be the decay constant of the daughter. Show that $N_2(t)$ satisfies the differential equation

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2$$

(b) Verify by substitution that this differential equation has the solution

$$N_2(t) = \frac{N_{10}\lambda_1}{\lambda_1 - \lambda_2}\left(e^{-\lambda_2 t} - e^{-\lambda_1 t}\right)$$

This equation is the law of successive radioactive decays. (c) ^{218}Po decays into ^{214}Pb with a half-life of 3.10 min, and ^{214}Pb decays into ^{214}Bi with a half-life of 26.8 min. On the same axes, plot graphs of $N_1(t)$ for ^{218}Po and $N_2(t)$ for ^{214}Pb. Let $N_{10} = 1\,000$ nuclei and choose values of t from 0 to 36 min in 2-min intervals. (d) The curve for ^{214}Pb obtained in part (c) at first rises to a maximum and then starts to decay. At what instant t_m is the number of ^{214}Pb nuclei a maximum? (e) By applying the condition for a maximum $dN_2/dt = 0$, derive a symbolic equation for t_m in terms of λ_1 and λ_2. (f) Explain whether the value obtained in part (c) agrees with this equation.

Section 44.5 The Decay Processes

35. Determine which decays can occur spontaneously. (a) $^{40}_{20}\text{Ca} \rightarrow e^+ + ^{40}_{19}\text{K}$ (b) $^{98}_{44}\text{Ru} \rightarrow ^{4}_{2}\text{He} + ^{94}_{42}\text{Mo}$ (c) $^{144}_{60}\text{Nd} \rightarrow ^{4}_{2}\text{He} + ^{140}_{58}\text{Ce}$

36. A ^{3}H nucleus beta decays into ^{3}He by creating an electron and an antineutrino according to the reaction

$$^{3}_{1}\text{H} \rightarrow ^{3}_{2}\text{He} + e^- + \bar{\nu}$$

Determine the total energy released in this decay.

37. The ^{14}C isotope undergoes beta decay according to the process given by Equation 44.21. Find the Q value for this process.

38. Identify the unknown nuclide or particle (X). (a) $X \rightarrow ^{65}_{28}\text{Ni} + \gamma$ (b) $^{215}_{84}\text{Po} \rightarrow X + \alpha$ (c) $X \rightarrow ^{55}_{26}\text{Fe} + e^+ + \nu$

39. Find the energy released in the alpha decay

$$^{238}_{92}\text{U} \rightarrow ^{234}_{90}\text{Th} + ^{4}_{2}\text{He}$$

40. A sample consists of 1.00×10^6 radioactive nuclei with a half-life of 10.0 h. No other nuclei are present at time $t = 0$. The stable daughter nuclei accumulate in the sample as time goes on. (a) Derive an equation giving the number of daughter nuclei N_d as a function of time. (b) Sketch or describe a graph of the number of daughter nuclei as a function of time. (c) What are the maximum and minimum numbers of daughter nuclei, and when do they occur? (d) What are the maximum and minimum rates of change in the number of daughter nuclei, and when do they occur?

41. The nucleus $^{15}_{8}$O decays by electron capture. The nuclear reaction is written

$$^{15}_{8}\text{O} + e^- \rightarrow ^{15}_{7}\text{N} + \nu$$

(a) Write the process going on for a single particle within the nucleus. (b) Disregarding the daughter's recoil, determine the energy of the neutrino.

42. A living specimen in equilibrium with the atmosphere contains one atom of ^{14}C (half-life = 5 730 yr) for every 7.70×10^{11} stable carbon atoms. An archeological sample of wood (cellulose, $C_{12}H_{22}O_{11}$) contains 21.0 mg of carbon. When the sample is placed inside a shielded beta counter with 88.0% counting efficiency, 837 counts are accumulated in one week. We wish to find the age of the sample. (a) Find the number of carbon atoms in the sample. (b) Find the number of carbon-14 atoms in the sample. (c) Find the decay constant for carbon-14 in inverse seconds. (d) Find the initial number of decays per week just after the specimen died. (e) Find the corrected number of decays per week from the current sample. (f) From the answers to parts (d) and (e), find the time interval in years since the specimen died.

Section 44.6 Natural Radioactivity

43. Uranium is naturally present in rock and soil. At one step in its series of radioactive decays, ^{238}U produces the chemically inert gas radon-222, with a half-life of 3.82 days. The radon seeps out of the ground to mix into the atmosphere, typically making open air radioactive with activity 0.3 pCi/L. In homes, ^{222}Rn can be a serious pollutant, accumulating to reach much higher

activities in enclosed spaces, sometimes reaching 4.00 pCi/L. If the radon radioactivity exceeds 4.00 pCi/L, the U.S. Environmental Protection Agency suggests taking action to reduce it such as by reducing infiltration of air from the ground. (a) Convert the activity 4.00 pCi/L to units of becquerels per cubic meter. (b) How many ^{222}Rn atoms are in 1 m^3 of air displaying this activity? (c) What fraction of the mass of the air does the radon constitute?

44. The most common isotope of radon is ^{222}Rn, which has half-life 3.82 days. (a) What fraction of the nuclei that were on the Earth one week ago are now undecayed? (b) Of those that existed one year ago? (c) In view of these results, explain why radon remains a problem, contributing significantly to our background radiation exposure.

45. Enter the correct nuclide symbol in each open tan rectangle in Figure P44.45, which shows the sequences of decays in the natural radioactive series starting with the long-lived isotope uranium-235 and ending with the stable nucleus lead-207.

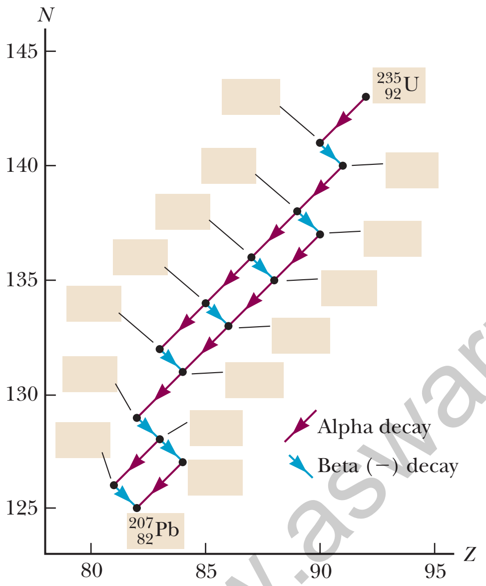


Figure P44.45

46. W ^{238}U, ^{235}U, ^{232}Th, ^{208}Pb, A rock sample contains traces of ^{207}Pb, and ^{206}Pb. Analysis shows that the ratio of the amount of ^{238}U to ^{206}Pb is 1.164. (a) Assuming the rock originally contained no lead, determine the age of the rock. (b) What should be the ratios of ^{235}U to ^{207}Pb and of ^{232}Th to ^{208}Pb so that they would yield the same age for the rock? Ignore the minute amounts of the intermediate decay products in the decay chains. *Note:* This form of multiple dating gives reliable geological dates.

Section 44.7 Nuclear Reactions

47. W A beam of 6.61-MeV protons is incident on a target of $^{27}_{13}$Al. Those that collide produce the reaction

$$\mathrm{p} + {}^{27}_{13}\mathrm{Al} \rightarrow {}^{27}_{14}\mathrm{Si} + \mathrm{n}$$

Ignoring any recoil of the product nucleus, determine the kinetic energy of the emerging neutrons.

48. (a) One method of producing neutrons for experimental use is bombardment of light nuclei with alpha particles. In the method used by James Chadwick in 1932, alpha particles emitted by polonium are incident on beryllium nuclei:

$${}^{4}_{2}\mathrm{He} + {}^{9}_{4}\mathrm{Be} \rightarrow {}^{12}_{6}\mathrm{C} + {}^{1}_{0}\mathrm{n}$$

What is the Q value of this reaction? (b) Neutrons are also often produced by small-particle accelerators. In one design, deuterons accelerated in a Van de Graaff generator bombard other deuterium nuclei and cause the reaction

$${}^{2}_{1}\mathrm{H} + {}^{2}_{1}\mathrm{H} \rightarrow {}^{3}_{2}\mathrm{He} + {}^{1}_{0}\mathrm{n}$$

Calculate the Q value of the reaction. (c) Is the reaction in part (b) exothermic or endothermic?

49. Identify the unknown nuclides and particles X and X′ in the nuclear reactions (a) $\mathrm{X} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{24}_{12}\mathrm{Mg} + {}^{1}_{0}\mathrm{n}$, (b) ${}^{235}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{90}_{38}\mathrm{Sr} + \mathrm{X} + 2({}^{1}_{0}\mathrm{n})$, and (c) $2({}^{1}_{1}\mathrm{H}) \rightarrow {}^{2}_{1}\mathrm{H} + \mathrm{X} + \mathrm{X}'$.

50. Natural gold has only one isotope, $^{197}_{79}$Au. If natural gold is irradiated by a flux of slow neutrons, electrons are emitted. (a) Write the reaction equation. (b) Calculate the maximum energy of the emitted electrons.

51. The following reactions are observed:

$${}^{9}_{4}\mathrm{Be} + \mathrm{n} \rightarrow {}^{10}_{4}\mathrm{Be} + \gamma \qquad Q = 6.812\ \mathrm{MeV}$$

$${}^{9}_{4}\mathrm{Be} + \gamma \rightarrow {}^{8}_{4}\mathrm{Be} + \mathrm{n} \qquad Q = -1.665\ \mathrm{MeV}$$

Calculate the masses of ^{8}Be and ^{10}Be in unified mass units to four decimal places from these data.

Section 44.8 Nuclear Magnetic Resonance and Magnetic Resonance Imaging

52. Construct a diagram like that of Figure 44.19 for the cases when I equals (a) $\frac{5}{2}$ and (b) 4.

53. The radio frequency at which a nucleus having a magnetic moment of magnitude μ displays resonance absorption between spin states is called the Larmor frequency and is given by

$$f = \frac{\Delta E}{h} = \frac{2\mu B}{h}$$

Calculate the Larmor frequency for (a) free neutrons in a magnetic field of 1.00 T, (b) free protons in a magnetic field of 1.00 T, and (c) free protons in the Earth's magnetic field at a location where the magnitude of the field is 50.0 μT.

Additional Problems

54. M A wooden artifact is found in an ancient tomb. Its carbon-14 ($^{14}_{6}$C) activity is measured to be 60.0% of that in a fresh sample of wood from the same region.

Assuming the same amount of ^{14}C was initially present in the artifact as is now contained in the fresh sample, determine the age of the artifact.

55. A 200.0-mCi sample of a radioactive isotope is purchased by a medical supply house. If the sample has a half-life of 14.0 days, how long will it be before its activity is reduced to 20.0 mCi?

56. *Why is the following situation impossible?* A ^{10}B nucleus is struck by an incoming alpha particle. As a result, a proton and a ^{12}C nucleus leave the site after the reaction.

57. (a) Find the radius of the $^{12}_{6}$C nucleus. (b) Find the force of repulsion between a proton at the surface of a $^{12}_{6}$C nucleus and the remaining five protons. (c) How much work (in MeV) has to be done to overcome this electric repulsion in transporting the last proton from a large distance up to the surface of the nucleus? (d) Repeat parts (a), (b), and (c) for $^{238}_{92}$U.

58. (a) Why is the beta decay $p \rightarrow n + e^+ + \nu$ forbidden for a free proton? (b) **What If?** Why is the same reaction possible if the proton is bound in a nucleus? For example, the following reaction occurs:

$$^{13}_{7}\text{N} \rightarrow {}^{13}_{6}\text{C} + e^+ + \nu$$

(c) How much energy is released in the reaction given in part (b)?

59. Review. Consider the Bohr model of the hydrogen atom, with the electron in the ground state. The magnetic field at the nucleus produced by the orbiting electron has a value of 12.5 T. (See Problem 6 in Chapter 30.) The proton can have its magnetic moment aligned in either of two directions perpendicular to the plane of the electron's orbit. The interaction of the proton's magnetic moment with the electron's magnetic field causes a difference in energy between the states with the two different orientations of the proton's magnetic moment. Find that energy difference in electron volts.

60. Show that the ^{238}U isotope cannot spontaneously emit a proton by analyzing the hypothetical process

$$^{238}_{92}\text{U} \rightarrow {}^{237}_{91}\text{Pa} + {}^{1}_{1}\text{H}$$

Note: The ^{237}Pa isotope has a mass of 237.051 144 u.

61. Review. (a) Is the mass of a hydrogen atom in its ground state larger or smaller than the sum of the masses of a proton and an electron? (b) What is the mass difference? (c) How large is the difference as a percentage of the total mass? (d) Is it large enough to affect the value of the atomic mass listed to six decimal places in Table 44.2?

62. *Why is the following situation impossible?* In an effort to study positronium, a scientist places ^{57}Co and ^{14}C in proximity. The ^{57}Co nuclei decay by e^+ emission, and the ^{14}C nuclei decay by e^- emission. Some of the positrons and electrons from these decays combine to form sufficient amounts of positronium for the scientist to gather data.

63. A by-product of some fission reactors is the isotope $^{239}_{94}$Pu, an alpha emitter having a half-life of 24 120 yr:

$$^{239}_{94}\text{Pu} \rightarrow {}^{235}_{92}\text{U} + \alpha$$

Consider a sample of 1.00 kg of pure $^{239}_{94}$Pu at $t = 0$. Calculate (a) the number of $^{239}_{94}$Pu nuclei present at $t = 0$ and (b) the initial activity in the sample. (c) **What If?** For what time interval does the sample have to be stored if a "safe" activity level is 0.100 Bq?

64. After the sudden release of radioactivity from the Chernobyl nuclear reactor accident in 1986, the radioactivity of milk in Poland rose to 2 000 Bq/L due to iodine-131 present in the grass eaten by dairy cattle. Radioactive iodine, with half-life 8.04 days, is particularly hazardous because the thyroid gland concentrates iodine. The Chernobyl accident caused a measurable increase in thyroid cancers among children in Poland and many other Eastern European countries. (a) For comparison, find the activity of milk due to potassium. Assume 1.00 liter of milk contains 2.00 g of potassium, of which 0.011 7% is the isotope ^{40}K with half-life 1.28×10^9 yr. (b) After what elapsed time would the activity due to iodine fall below that due to potassium?

65. A theory of nuclear astrophysics proposes that all the elements heavier than iron are formed in supernova explosions ending the lives of massive stars. Assume equal amounts of ^{235}U and ^{238}U were created at the time of the explosion and the present ^{235}U/^{238}U ratio on the Earth is 0.007 25. The half-lives of ^{235}U and ^{238}U are 0.704×10^9 yr and 4.47×10^9 yr, respectively. How long ago did the star(s) explode that released the elements that formed the Earth?

66. The activity of a radioactive sample was measured over 12 h, with the net count rates shown in the accompanying table. (a) Plot the logarithm of the counting rate as a function of time. (b) Determine the decay constant and half-life of the radioactive nuclei in the sample. (c) What counting rate would you expect for the sample at $t = 0$? (d) Assuming the efficiency of the counting instrument is 10.0%, calculate the number of radioactive atoms in the sample at $t = 0$.

Time (h)	Counting Rate (counts/min)
1.00	3 100
2.00	2 450
4.00	1 480
6.00	910
8.00	545
10.0	330
12.0	200

67. When, after a reaction or disturbance of any kind, a nucleus is left in an excited state, it can return to its normal (ground) state by emission of a gamma-ray photon (or several photons). This process is illustrated by Equation 44.25. The emitting nucleus must recoil to conserve both energy and momentum. (a) Show that the recoil energy of the nucleus is

$$E_r = \frac{(\Delta E)^2}{2Mc^2}$$

where ΔE is the difference in energy between the excited and ground states of a nucleus of mass M. (b) Calculate the recoil energy of the ^{57}Fe nucleus when it decays by gamma emission from the 14.4-keV excited state. For this calculation, take the mass to be 57 u. *Suggestion:* Assume $hf << Mc^2$.

68. In a piece of rock from the Moon, the ^{87}Rb content is assayed to be 1.82×10^{10} atoms per gram of material and the ^{87}Sr content is found to be 1.07×10^9 atoms per gram. The relevant decay relating these nuclides is $^{87}\text{Rb} \rightarrow {}^{87}\text{Sr} + e^- + \bar{\nu}$. The half-life of the decay is 4.75×10^{10} yr. (a) Calculate the age of the rock. (b) **What If?** Could the material in the rock actually be much older? What assumption is implicit in using the radioactive dating method?

69. AMT M Free neutrons have a characteristic half-life of 10.4 min. What fraction of a group of free neutrons with kinetic energy 0.040 0 eV decays before traveling a distance of 10.0 km?

70. On July 4, 1054, a brilliant light appeared in the constellation Taurus the Bull. The supernova, which could be seen in daylight for some days, was recorded by Arab and Chinese astronomers. As it faded, it remained visible for years, dimming for a time with the 77.1-day half-life of the radioactive cobalt-56 that had been created in the explosion. (a) The remains of the star now form the Crab nebula (see the photograph opening Chapter 34). In it, the cobalt-56 has now decreased to what fraction of its original activity? (b) Suppose that an American, of the people called the Anasazi, made a charcoal drawing of the supernova. The carbon-14 in the charcoal has now decayed to what fraction of its original activity?

71. When a nucleus decays, it can leave the daughter nucleus in an excited state. The $^{93}_{43}$Tc nucleus (molar mass 92.910 2 g/mol) in the ground state decays by electron capture and e^+ emission to energy levels of the daughter (molar mass 92.906 8 g/mol in the ground state) at 2.44 MeV, 2.03 MeV, 1.48 MeV, and 1.35 MeV. (a) Identify the daughter nuclide. (b) To which of the listed levels of the daughter are electron capture and e^+ decay of $^{93}_{43}$Tc allowed?

72. The radioactive isotope ^{137}Ba has a relatively short half-life and can be easily extracted from a solution containing its parent ^{137}Cs. This barium isotope is commonly used in an undergraduate laboratory exercise for demonstrating the radioactive decay law. Undergraduate students using modest experimental equipment took the data presented in Figure P44.72. Determine the half-life for the decay of ^{137}Ba using their data.

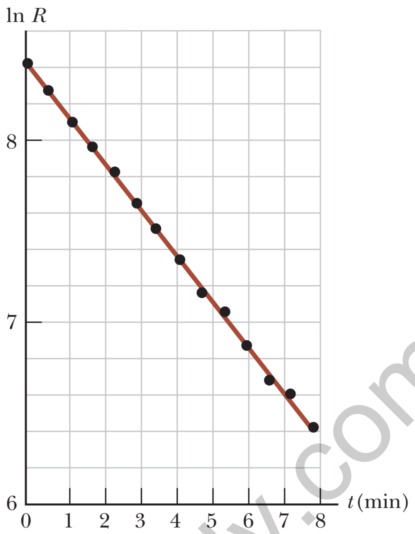


Figure P44.72

73. As part of his discovery of the neutron in 1932, James Chadwick determined the mass of the newly identified particle by firing a beam of fast neutrons, all having the same speed, at two different targets and measuring the maximum recoil speeds of the target nuclei. The maximum speeds arise when an elastic head-on collision occurs between a neutron and a stationary target nucleus. (a) Represent the masses and final speeds of the two target nuclei as m_1, v_1, m_2, and v_2 and assume Newtonian mechanics applies. Show that the neutron mass can be calculated from the equation

$$m_n = \frac{m_1 v_1 - m_2 v_2}{v_2 - v_1}$$

(b) Chadwick directed a beam of neutrons (produced from a nuclear reaction) on paraffin, which contains hydrogen. The maximum speed of the protons ejected was found to be 3.30×10^7 m/s. Because the velocity of the neutrons could not be determined directly, a second experiment was performed using neutrons from the same source and nitrogen nuclei as the target. The maximum recoil speed of the nitrogen nuclei was found to be 4.70×10^6 m/s. The masses of a proton and a nitrogen nucleus were taken as 1.00 u and 14.0 u, respectively. What was Chadwick's value for the neutron mass?

74. When the nuclear reaction represented by Equation 44.28 is endothermic, the reaction energy Q is negative. For the reaction to proceed, the incoming particle must have a minimum energy called the threshold energy, E_{th}. Some fraction of the energy of the incident particle is transferred to the compound nucleus to conserve momentum. Therefore, E_{th} must be greater than Q. (a) Show that

$$E_{th} = -Q\left(1 + \frac{M_a}{M_X}\right)$$

(b) Calculate the threshold energy of the incident alpha particle in the reaction

$$^{4}_{2}\text{He} + ^{14}_{7}\text{N} \rightarrow ^{17}_{8}\text{O} + ^{1}_{1}\text{H}$$

75. In an experiment on the transport of nutrients in a plant's root structure, two radioactive nuclides X and Y are used. Initially, 2.50 times more nuclei of type X are present than of type Y. At a time 3.00 d later, there are 4.20 times more nuclei of type X than of type Y. Isotope Y has a half-life of 1.60 d. What is the half-life of isotope X?

76. In an experiment on the transport of nutrients in a plant's root structure, two radioactive nuclides X and Y are used. Initially, the ratio of the number of nuclei of type X present to that of type Y is r_1. After a time interval Δt, the ratio of the number of nuclei of type X present to that of type Y is r_2. Isotope Y has a half-life of T_Y. What is the half-life of isotope X?

Challenge Problems

77. **Review.** Consider a model of the nucleus in which the positive charge (Ze) is uniformly distributed throughout a sphere of radius R. By integrating the energy density $\frac{1}{2}\epsilon_0 E^2$ over all space, show that the electric potential energy may be written

$$U = \frac{3Z^2e^2}{20\pi\epsilon_0 R} = \frac{3k_e Z^2 e^2}{5R}$$

Problem 72 in Chapter 25 derived the same result by a different method.

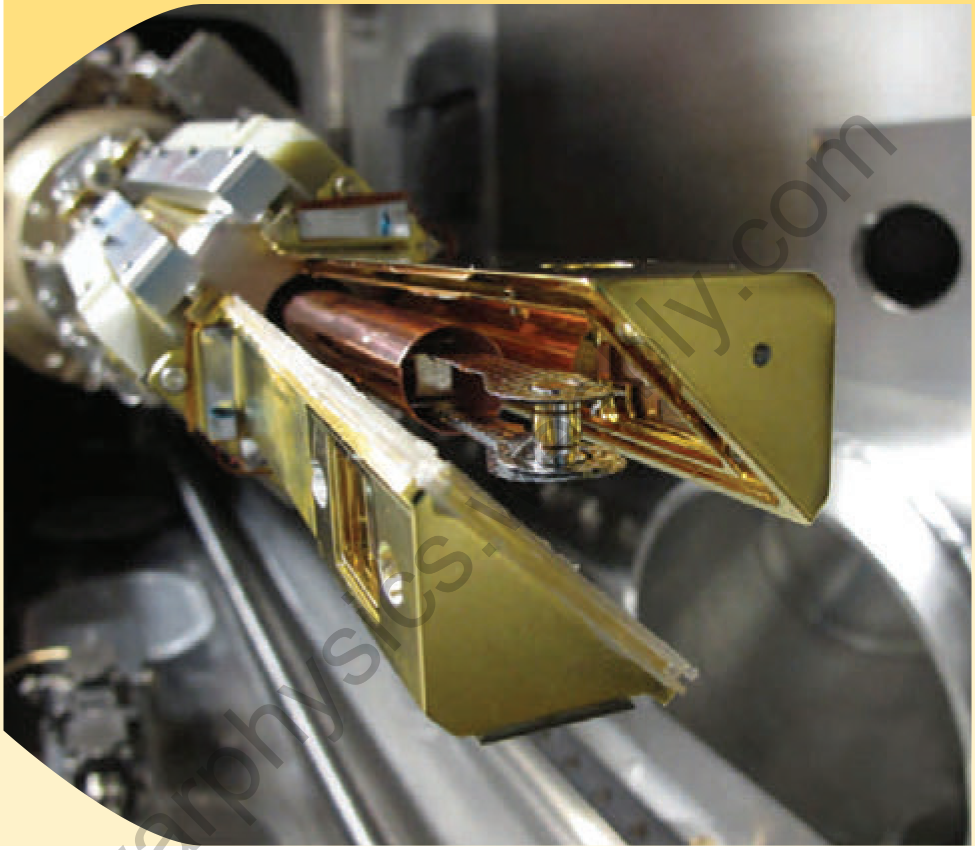
78. After determining that the Sun has existed for hundreds of millions of years, but before the discovery of nuclear physics, scientists could not explain why the Sun has continued to burn for such a long time interval. For example, if it were a coal fire, it would have burned up in about 3 000 yr. Assume the Sun, whose mass is equal to 1.99×10^{30} kg, originally consisted entirely of hydrogen and its total power output is 3.85×10^{26} W. (a) Assuming the energy-generating mechanism of the Sun is the fusion of hydrogen into helium via the net reaction

$$4(^{1}_{1}\text{H}) + 2(e^-) \rightarrow ^{4}_{2}\text{He} + 2\nu + \gamma$$

calculate the energy (in joules) given off by this reaction. (b) Take the mass of one hydrogen atom to be equal to 1.67×10^{-27} kg. Determine how many hydrogen atoms constitute the Sun. (c) If the total power output remains constant, after what time interval will all the hydrogen be converted into helium, making the Sun die? (d) How does your answer to part (c) compare with current estimates of the expected life of the Sun, which are 4 billion to 7 billion years?

CHAPTER

45 Applications of Nuclear Physics



In this chapter, we study both nuclear fission and nuclear fusion. The structure above is the target assembly for the inertial confinement procedure for initiating fusion by laser at the National Ignition Facility in Livermore, California. The triangle-shaped shrouds protect the fuel pellets and then open a few seconds before very powerful lasers bombard the target. *(Courtesy of Lawrence Livermore National Library)*

In this chapter, we study two means for deriving energy from nuclear reactions: fission, in which a large nucleus splits into two smaller nuclei, and fusion, in which two small nuclei fuse to form a larger one. In both cases, the released energy can be used either constructively (as in electric power plants) or destructively (as in nuclear weapons). We also examine the ways in which radiation interacts with matter and discuss the structure of fission and fusion reactors. The chapter concludes with a discussion of some industrial and biological applications of radiation.

45.1 Interactions Involving Neutrons

Nuclear fission is the process that occurs in present-day nuclear reactors and ultimately results in energy supplied to a community by electrical transmission. Nuclear fusion is an area of active research, but it has not yet been commercially developed for the supply of energy. We will discuss fission first and then explore fusion in Section 45.4.

To understand nuclear fission and the physics of nuclear reactors, we must first understand how neutrons interact with nuclei. Because of their charge neutrality, neutrons are not subject to Coulomb forces and as a result do not interact electrically with electrons or the nucleus. Therefore, neutrons can easily penetrate deep into an atom and collide with the nucleus.

A fast neutron (energy greater than approximately 1 MeV) traveling through matter undergoes many collisions with nuclei, giving up some of its kinetic energy in each collision. For fast neutrons in some materials, elastic collisions dominate. Materials for which that occurs are called **moderators** because they slow down (or moderate) the originally energetic neutrons very effectively. Moderator nuclei should be of low mass so that a large amount of kinetic energy is transferred to them when struck by neutrons. For this reason, materials that are abundant in hydrogen, such as paraffin and water, are good moderators for neutrons.

Eventually, most neutrons bombarding a moderator become **thermal neutrons,** which means they have given up so much of their energy that they are in thermal equilibrium with the moderator material. Their average kinetic energy at room temperature is, from Equation 21.19,

$$K_{\text{avg}} = \tfrac{3}{2}k_{\text{B}}T \approx \tfrac{3}{2}(1.38 \times 10^{-23}\text{ J/K})(300\text{ K}) = 6.21 \times 10^{-21}\text{ J} \approx 0.04\text{ eV}$$

which corresponds to a neutron root-mean-square speed of approximately 2 800 m/s. Thermal neutrons have a distribution of speeds, just as the molecules in a container of gas do (see Chapter 21). High-energy neutrons, those with energy of several MeV, *thermalize* (that is, their average energy reaches K_{avg}) in less than 1 ms when they are incident on a moderator.

Once the neutrons have thermalized and the energy of a particular neutron is sufficiently low, there is a high probability the neutron will be captured by a nucleus, an event that is accompanied by the emission of a gamma ray. This **neutron capture** reaction can be written

$$^{1}_{0}\text{n} + {}^{A}_{Z}\text{X} \rightarrow {}^{A+1}_{Z}\text{X}^* \rightarrow {}^{A+1}_{Z}\text{X} + \gamma \qquad (45.1)$$

◀ **Neutron capture reaction**

Once the neutron is captured, the nucleus ${}^{A+1}_{Z}\text{X}^*$ is in an excited state for a very short time before it undergoes gamma decay. The product nucleus ${}^{A+1}_{Z}\text{X}$ is usually radioactive and decays by beta emission.

The neutron-capture rate for neutrons passing through any sample depends on the type of atoms in the sample and on the energy of the incident neutrons. The interaction of neutrons with matter increases with decreasing neutron energy because a slow neutron spends a larger time interval in the vicinity of target nuclei.

45.2 Nuclear Fission

As mentioned in Section 44.2, nuclear **fission** occurs when a heavy nucleus, such as ^{235}U, splits into two smaller nuclei. Fission is initiated when a heavy nucleus captures a thermal neutron as described by the first step in Equation 45.1. The absorption of the neutron creates a nucleus that is unstable and can change to a lower-energy configuration by splitting into two smaller nuclei. In such a reaction, the combined mass of the daughter nuclei is less than the mass of the parent nucleus, and the difference in mass is called the **mass defect.** Multiplying the mass defect by c^2 gives the numerical value of the released energy. This energy is in the form of kinetic energy associated with the motion of the neutrons and the daughter nuclei after the fission event. Energy is released because the binding energy per nucleon of the daughter nuclei is approximately 1 MeV greater than that of the parent nucleus (see Fig. 44.5).

Nuclear fission was first observed in 1938 by Otto Hahn (1879–1968) and Fritz Strassmann (1902–1980) following some basic studies by Fermi. After bombarding uranium with neutrons, Hahn and Strassmann discovered among the reaction products two medium-mass elements, barium and lanthanum. Shortly thereafter, Lise Meitner (1878–1968) and her nephew Otto Frisch (1904–1979) explained what had happened. After absorbing a neutron, the uranium nucleus had split into two

Pitfall Prevention 45.1

Binding Energy Reminder Remember from Chapter 44 that binding energy is the absolute value of the system energy and is related to the system mass. Therefore, when considering Figure 44.5, imagine flipping it upside down for a graph representing system mass. In a fission reaction, the system mass decreases. This decrease in mass appears in the system as kinetic energy of the fission products.

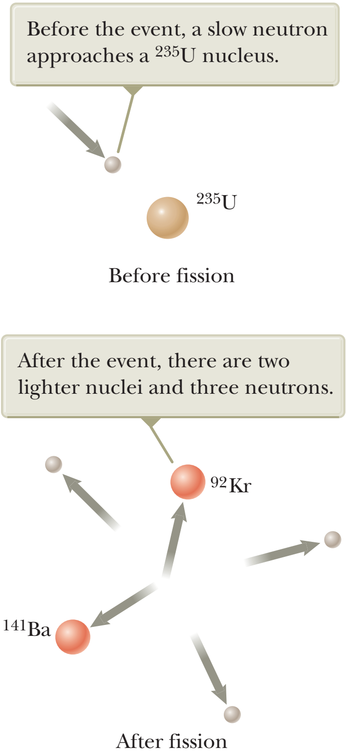


Figure 45.1 A nuclear fission event.

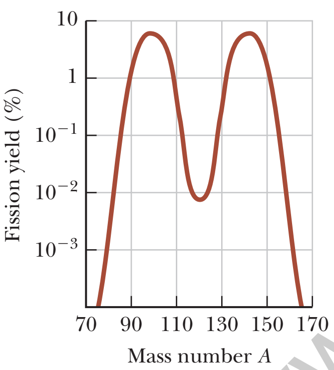


Figure 45.2 Distribution of fission products versus mass number for the fission of ^{235}U bombarded with thermal neutrons. Notice that the vertical axis is logarithmic.

nearly equal fragments plus several neutrons. Such an occurrence was of considerable interest to physicists attempting to understand the nucleus, but it was to have even more far-reaching consequences. Measurements showed that approximately 200 MeV of energy was released in each fission event, and this fact was to affect the course of history in World War II.

The fission of ^{235}U by thermal neutrons can be represented by the reaction

$$ {}^{1}_{0}\text{n} + {}^{235}_{92}\text{U} \rightarrow {}^{236}_{92}\text{U}^* \rightarrow \text{X} + \text{Y} + \text{neutrons} \qquad (45.2)$$

where ^{236}U* is an intermediate excited state that lasts for approximately 10^{-12} s before splitting into medium-mass nuclei X and Y, which are called **fission fragments.** In any fission reaction, there are many combinations of X and Y that satisfy the requirements of conservation of energy and charge. In the case of uranium, for example, approximately 90 daughter nuclei can be formed.

Fission also results in the production of several neutrons, typically two or three. On average, approximately 2.5 neutrons are released per event. A typical fission reaction for uranium is

$$ {}^{1}_{0}\text{n} + {}^{235}_{92}\text{U} \rightarrow {}^{141}_{56}\text{Ba} + {}^{92}_{36}\text{Kr} + 3({}^{1}_{0}\text{n}) \qquad (45.3)$$

Figure 45.1 shows a pictorial representation of the fission event in Equation 45.3.

Figure 45.2 is a graph of the distribution of fission products versus mass number A. The most probable products have mass numbers $A \approx 95$ and $A \approx 140$. Suppose these products are ${}^{95}_{39}$Y (with 56 neutrons) and ${}^{140}_{53}$I (with 87 neutrons). If these nuclei are located on the graph of Figure 44.4, it is seen that both are well above the line of stability. Because these fragments are very unstable owing to their unusually high number of neutrons, they almost instantaneously release two or three neutrons.

Let's estimate the disintegration energy Q released in a typical fission process. From Figure 44.5, we see that the binding energy per nucleon is approximately 7.2 MeV for heavy nuclei ($A \approx 240$) and approximately 8.2 MeV for nuclei of intermediate mass. The amount of energy released is 8.2 MeV − 7.2 MeV = 1 MeV per nucleon. Because there are a total of 235 nucleons in ${}^{235}_{92}$U, the energy released per fission event is approximately 235 MeV, a large amount of energy relative to the amount released in chemical processes. For example, the energy released in the combustion of one molecule of octane used in gasoline engines is about one-millionth of the energy released in a single fission event!

Quick Quiz 45.1 When a nucleus undergoes fission, the two daughter nuclei are generally radioactive. By which process are they most likely to decay? **(a)** alpha decay **(b)** beta decay (e^-) **(c)** beta decay (e^+)

Quick Quiz 45.2 Which of the following are possible fission reactions?

(a) ${}^{1}_{0}\text{n} + {}^{235}_{92}\text{U} \rightarrow {}^{140}_{54}\text{Xe} + {}^{94}_{38}\text{Sr} + 2({}^{1}_{0}\text{n})$

(b) ${}^{1}_{0}\text{n} + {}^{235}_{92}\text{U} \rightarrow {}^{132}_{50}\text{Sn} + {}^{101}_{42}\text{Mo} + 3({}^{1}_{0}\text{n})$

(c) ${}^{1}_{0}\text{n} + {}^{239}_{94}\text{Pu} \rightarrow {}^{137}_{53}\text{I} + {}^{97}_{41}\text{Nb} + 3({}^{1}_{0}\text{n})$

Example 45.1 The Energy Released in the Fission of ^{235}U

Calculate the energy released when 1.00 kg of ^{235}U fissions, taking the disintegration energy per event to be Q = 208 MeV.

SOLUTION

Conceptualize Imagine a nucleus of ^{235}U absorbing a neutron and then splitting into two smaller nuclei and several neutrons as in Figure 45.1.

45.1 continued

Categorize The problem statement tells us to categorize this example as one involving an energy analysis of nuclear fission.

Analyze Because $A = 235$ for uranium, one mole of this isotope has a mass of $M = 235$ g.

Find the number of nuclei in our sample in terms of the number of moles n and Avogadro's number, and then in terms of the sample mass m and the molar mass M of ^{235}U:

$$N = nN_A = \frac{m}{M}N_A$$

Find the total energy released when all nuclei undergo fission:

$$E = NQ = \frac{m}{M}N_AQ = \frac{1.00 \times 10^3 \text{ g}}{235 \text{ g/mol}}(6.02 \times 10^{23} \text{ mol}^{-1})(208 \text{ MeV})$$

$$= 5.33 \times 10^{26} \text{ MeV}$$

Finalize Convert this energy to kWh:

$$E = (5.33 \times 10^{26} \text{ MeV})\left(\frac{1.60 \times 10^{-13} \text{ J}}{1 \text{ MeV}}\right)\left(\frac{1 \text{ kWh}}{3.60 \times 10^6 \text{ J}}\right) = 2.37 \times 10^7 \text{ kWh}$$

which, if released slowly, is enough energy to keep a 100-W lightbulb operating for 30 000 years! If the available fission energy in 1 kg of ^{235}U were suddenly released, it would be equivalent to detonating about 20 000 tons of TNT.

45.3 Nuclear Reactors

In Section 45.2, we learned that when ^{235}U fissions, one incoming neutron results in an average of 2.5 neutrons emitted per event. These neutrons can trigger other nuclei to fission. Because more neutrons are produced by the event than are absorbed, there is the possibility of an ever-building chain reaction (Fig. 45.3). Experience shows that if the chain reaction is not controlled (that is, if it does not

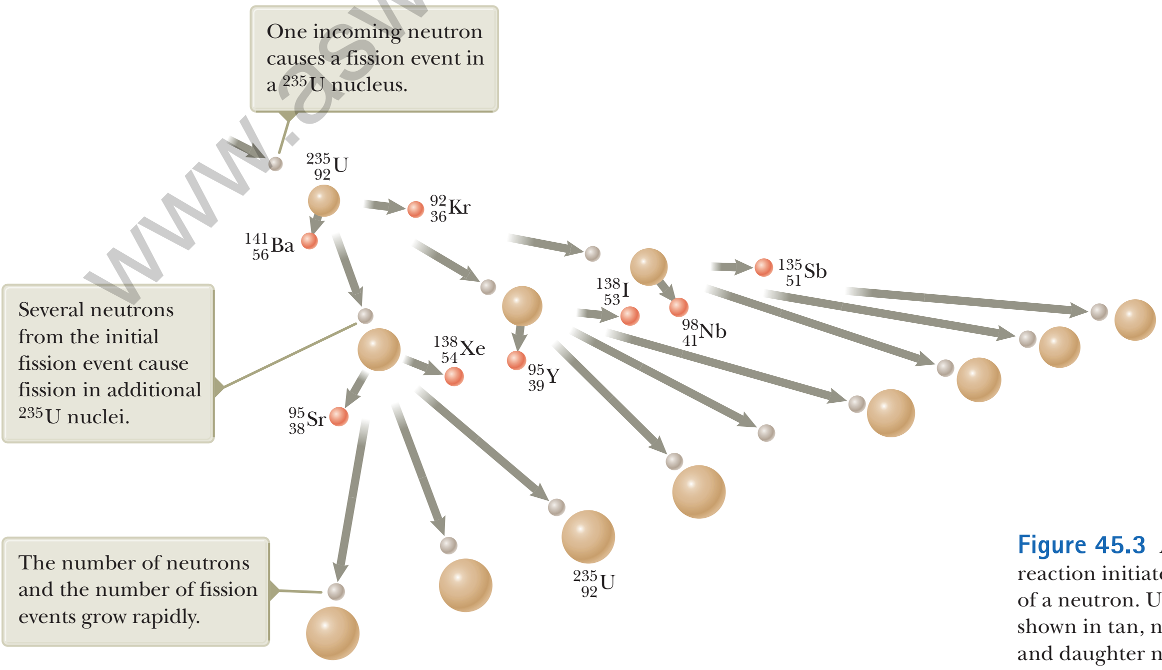


Figure 45.3 A nuclear chain reaction initiated by the capture of a neutron. Uranium nuclei are shown in tan, neutrons in gray, and daughter nuclei in orange.

Figure 45.4 Artist's rendition of the world's first nuclear reactor. Because of wartime secrecy, there are few photographs of the completed reactor, which was composed of layers of moderating graphite interspersed with uranium. A self-sustained chain reaction was first achieved on December 2, 1942. Word of the success was telephoned immediately to Washington, D.C., with this message: "The Italian navigator has landed in the New World and found the natives very friendly." The historic event took place in an improvised laboratory in the racquet court under the stands of the University of Chicago's Stagg Field, and the Italian navigator was Enrico Fermi.

proceed slowly), it can result in a violent explosion, with the sudden release of an enormous amount of energy. When the reaction is controlled, however, the energy released can be put to constructive use. In the United States, for example, nearly 20% of the electricity generated each year comes from nuclear power plants, and nuclear power is used extensively in many other countries, including France, Russia, and India.

A nuclear reactor is a system designed to maintain what is called a **self-sustained chain reaction.** This important process was first achieved in 1942 by Enrico Fermi and his team at the University of Chicago, using naturally occurring uranium as the fuel.[1] In the first nuclear reactor (Fig. 45.4), Fermi placed bricks of graphite (carbon) between the fuel elements. Carbon nuclei are about 12 times more massive than neutrons, but after several collisions with carbon nuclei, a neutron is slowed sufficiently to increase its likelihood of fission with ^{235}U. In this design, carbon is the moderator; most modern reactors use water as the moderator.

Most reactors in operation today also use uranium as fuel. Naturally occurring uranium contains only 0.7% of the ^{235}U isotope, however, with the remaining 99.3% being ^{238}U. This fact is important to the operation of a reactor because ^{238}U almost never fissions. Instead, it tends to absorb neutrons without a subsequent fission event, producing neptunium and plutonium. For this reason, reactor fuels must be artificially *enriched* to contain at least a few percent ^{235}U.

To achieve a self-sustained chain reaction, an average of one neutron emitted in each ^{235}U fission must be captured by another ^{235}U nucleus and cause that nucleus to undergo fission. A useful parameter for describing the level of reactor operation is the **reproduction constant** K, defined as **the average number of neutrons from each fission event that cause another fission event.** As we have seen, K has an average value of 2.5 in the uncontrolled fission of uranium.

A self-sustained and controlled chain reaction is achieved when $K = 1$. When in this condition, the reactor is said to be **critical.** When $K < 1$, the reactor is subcritical and the reaction dies out. When $K > 1$, the reactor is supercritical and a runaway reaction occurs. In a nuclear reactor used to furnish power to a utility company, it is necessary to maintain a value of K close to 1. If K rises above this value, the rest energy transformed to internal energy in the reaction could melt the reactor.

Several types of reactor systems allow the kinetic energy of fission fragments to be transformed to other types of energy and eventually transferred out of the

Enrico Fermi
Italian Physicist (1901–1954)
Fermi was awarded the Nobel Prize in Physics in 1938 for producing transuranic elements by neutron irradiation and for his discovery of nuclear reactions brought about by thermal neutrons. He made many other outstanding contributions to physics, including his theory of beta decay, the free-electron theory of metals, and the development of the world's first fission reactor in 1942. Fermi was truly a gifted theoretical and experimental physicist. He was also well known for his ability to present physics in a clear and exciting manner.

[1]Although Fermi's reactor was the first manufactured nuclear reactor, there is evidence that a natural fission reaction may have sustained itself for perhaps hundreds of thousands of years in a deposit of uranium in Gabon, West Africa. See G. Cowan, "A Natural Fission Reactor," *Scientific American* **235**(5): 36, 1976.

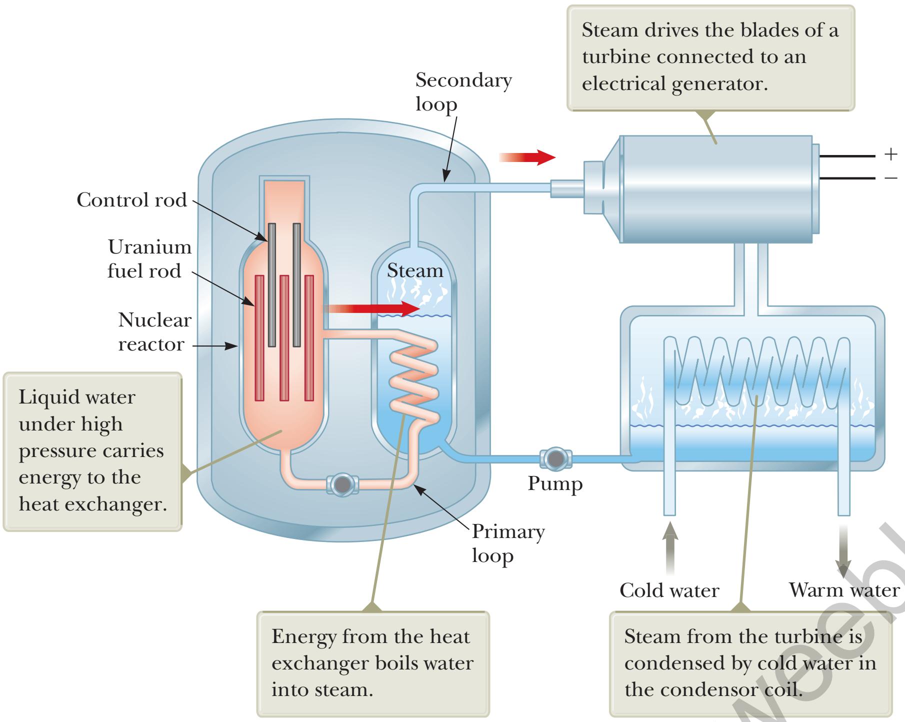


Figure 45.5 Main components of a pressurized-water nuclear reactor.

reactor plant by electrical transmission. The most common reactor in use in the United States is the pressurized-water reactor (Fig. 45.5). We shall examine this type because its main parts are common to all reactor designs. Fission events in the uranium **fuel elements** in the reactor core raise the temperature of the water contained in the primary loop, which is maintained at high pressure to keep the water from boiling. (This water also serves as the moderator to slow down the neutrons released in the fission events with energy of approximately 2 MeV.) The hot water is pumped through a heat exchanger, where the internal energy of the water is transferred by conduction to the water contained in the secondary loop. The hot water in the secondary loop is converted to steam, which does work to drive a turbine–generator system to create electric power. The water in the secondary loop is isolated from the water in the primary loop to avoid contamination of the secondary water and the steam by radioactive nuclei from the reactor core.

In any reactor, a fraction of the neutrons produced in fission leak out of the uranium fuel elements before inducing other fission events. If the fraction leaking out is too large, the reactor will not operate. The percentage lost is large if the fuel elements are very small because leakage is a function of the ratio of surface area to volume. Therefore, a critical feature of the reactor design is an optimal surface area–to–volume ratio of the fuel elements.

Control of Power Level

Safety is of critical importance in the operation of a nuclear reactor. The reproduction constant K must not be allowed to rise above 1, lest a runaway reaction occur. Consequently, reactor design must include a means of controlling the value of K.

The basic design of a nuclear reactor core is shown in Figure 45.6. The fuel elements consist of uranium that has been enriched in the ^{235}U isotope. To control the power level, **control rods** are inserted into the reactor core. These rods are made of materials such as cadmium that are very efficient in absorbing neutrons. By adjusting the number and position of the control rods in the reactor core, the K value

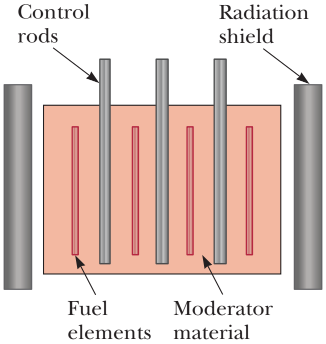


Figure 45.6 Cross section of a reactor core showing the control rods, fuel elements containing enriched fuel, and moderating material, all surrounded by a radiation shield.

can be varied and any power level within the design range of the reactor can be achieved.

Quick Quiz 45.3 To reduce the value of the reproduction constant K, do you **(a)** push the control rods deeper into the core or **(b)** pull the control rods farther out of the core?

Safety and Waste Disposal

The 1986 accident at the Chernobyl reactor in Ukraine and the 2011 nuclear disaster caused by the earthquake and tsunami in Japan rightfully focused attention on reactor safety. Unfortunately, at Chernobyl the activity of the materials released immediately after the accident totaled approximately 1.2×10^{19} Bq and resulted in the evacuation of 135 000 people. Thirty individuals died during the accident or shortly thereafter, and data from the Ukraine Radiological Institute suggest that more than 2 500 deaths could be attributed to the Chernobyl accident. In the period 1986–1997, there was a tenfold increase in the number of children contracting thyroid cancer from the ingestion of radioactive iodine in milk from cows that ate contaminated grass. One conclusion of an international conference studying the Ukraine accident was that the main causes of the Chernobyl accident were the coincidence of severe deficiencies in the reactor physical design and a violation of safety procedures. Most of these deficiencies have since been addressed at plants of similar design in Russia and neighboring countries of the former Soviet Union.

The March 2011 accident in Japan was caused by an unfortunate combination of a massive earthquake and subsequent tsunami. The most hard-hit power plant, Fukushima I, shut down automatically after the earthquake. Shutting down a nuclear power plant, however, is not an instantaneous process. Cooling water must continue to be circulated to carry the energy generated by beta decay of the fission by-products out of the reactor core. Unfortunately, the water from the tsunami broke the connection to the power grid, leaving the plant without outside electrical support for circulating the water. While the plant had emergency generators to take over in such a situation, the tsunami inundated the generator rooms, making the generators inoperable. Three of the six reactors at Fukushima experienced meltdown, and there were several explosions. Significant radiation was released into the environment. At the time of this printing, all 54 of Japan's nuclear power plants have been taken offline, and the Japanese public has expressed strong reluctance to continue with nuclear power.

Commercial reactors achieve safety through careful design and rigid operating protocol, and only when these variables are compromised do reactors pose a danger. Radiation exposure and the potential health risks associated with such exposure are controlled by three layers of containment. The fuel and radioactive fission products are contained inside the reactor vessel. Should this vessel rupture, the reactor building acts as a second containment structure to prevent radioactive material from contaminating the environment. Finally, the reactor facilities must be in a remote location to protect the general public from exposure should radiation escape the reactor building.

A continuing concern about nuclear fission reactors is the safe disposal of radioactive material when the reactor core is replaced. This waste material contains long-lived, highly radioactive isotopes and must be stored over long time intervals in such a way that there is no chance of environmental contamination. At present, sealing radioactive wastes in waterproof containers and burying them in deep geologic repositories seems to be the most promising solution.

Transport of reactor fuel and reactor wastes poses additional safety risks. Accidents during transport of nuclear fuel could expose the public to harmful levels of radiation. The U.S. Department of Energy requires stringent crash tests of all containers used to transport nuclear materials. Container manufacturers must demonstrate that their containers will not rupture even in high-speed collisions.

Despite these risks, there are advantages to the use of nuclear power to be weighed against the risks. For example, nuclear power plants do not produce air pollution and greenhouse gases as do fossil fuel plants, and the supply of uranium on the Earth is predicted to last longer than the supply of fossil fuels. For each source of energy—whether nuclear, hydroelectric, fossil fuel, wind, solar, or other—the risks must be weighed against the benefits and the availability of the energy source.

45.4 Nuclear Fusion

In Chapter 44, we found that the binding energy for light nuclei ($A < 20$) is much smaller than the binding energy for heavier nuclei, which suggests a process that is the reverse of fission. As mentioned in Section 39.8, when two light nuclei combine to form a heavier nucleus, the process is called nuclear **fusion.** Because the mass of the final nucleus is less than the combined masses of the original nuclei, there is a loss of mass accompanied by a release of energy.

Two examples of such energy-liberating fusion reactions are as follows:

$$^{1}_{1}\text{H} + {}^{1}_{1}\text{H} \rightarrow {}^{2}_{1}\text{H} + e^{+} + \nu$$

$$^{1}_{1}\text{H} + {}^{2}_{1}\text{H} \rightarrow {}^{3}_{2}\text{He} + \gamma$$

These reactions occur in the core of a star and are responsible for the outpouring of energy from the star. The second reaction is followed by either hydrogen–helium fusion or helium–helium fusion:

$$^{1}_{1}\text{H} + {}^{3}_{2}\text{He} \rightarrow {}^{4}_{2}\text{He} + e^{+} + \nu$$

$$^{3}_{2}\text{He} + {}^{3}_{2}\text{He} \rightarrow {}^{4}_{2}\text{He} + {}^{1}_{1}\text{H} + {}^{1}_{1}\text{H}$$

These fusion reactions are the basic reactions in the **proton–proton cycle,** believed to be one of the basic cycles by which energy is generated in the Sun and other stars that contain an abundance of hydrogen. Most of the energy production takes place in the Sun's interior, where the temperature is approximately 1.5×10^{7} K. Because such high temperatures are required to drive these reactions, they are called **thermonuclear fusion reactions.** All the reactions in the proton–proton cycle are exothermic. An overview of the cycle is that four protons combine to generate an alpha particle, positrons, gamma rays, and neutrinos.

Pitfall Prevention 45.2

Fission and Fusion The words *fission* and *fusion* sound similar, but they correspond to different processes. Consider the binding-energy graph in Figure 44.5. There are two directions from which you can approach the peak of the graph so that energy is released: combining two light nuclei, or fusion, and separating a heavy nucleus into two lighter nuclei, or fission.

Quick Quiz 45.4 In the core of a star, hydrogen nuclei combine in fusion reactions. Once the hydrogen has been exhausted, fusion of helium nuclei can occur. If the star is sufficiently massive, fusion of heavier and heavier nuclei can occur once the helium is used up. Consider a fusion reaction involving two nuclei with the same value of A. For this reaction to be exothermic, which of the following values of A are impossible? **(a)** 12 **(b)** 20 **(c)** 28 **(d)** 64

Example 45.2 Energy Released in Fusion

Find the total energy released in the fusion reactions in the proton–proton cycle.

SOLUTION

Conceptualize The net nuclear result of the proton–proton cycle is to fuse four protons to form an alpha particle. Study the reactions above for the proton–proton cycle to be sure you understand how four protons become an alpha particle.

Categorize We use concepts discussed in this section, so we categorize this example as a substitution problem.

continued

45.2 continued

Find the initial mass of the system using the atomic mass of hydrogen from Table 44.2: $4(1.007\ 825\ \text{u}) = 4.031\ 300\ \text{u}$

Find the change in mass of the system as this value minus the mass of a ^{4}He atom: $4.031\ 300\ \text{u} - 4.002\ 603\ \text{u} = 0.028\ 697\ \text{u}$

Convert this mass change into energy units: $E = 0.028\ 697\ \text{u} \times 931.494\ \text{MeV/u} = 26.7\ \text{MeV}$

This energy is shared among the alpha particle and other particles such as positrons, gamma rays, and neutrinos.

Terrestrial Fusion Reactions

The enormous amount of energy released in fusion reactions suggests the possibility of harnessing this energy for useful purposes. A great deal of effort is currently under way to develop a sustained and controllable thermonuclear reactor, a fusion power reactor. Controlled fusion is often called the ultimate energy source because of the availability of its fuel source: water. For example, if deuterium were used as the fuel, 0.12 g of it could be extracted from 1 gal of water at a cost of about four cents. This amount of deuterium would release approximately 10^{10} J if all nuclei underwent fusion. By comparison, 1 gal of gasoline releases approximately 10^8 J upon burning and costs far more than four cents.

An additional advantage of fusion reactors is that comparatively few radioactive by-products are formed. For the proton–proton cycle, for instance, the end product is safe, nonradioactive helium. Unfortunately, a thermonuclear reactor that can deliver a net power output spread over a reasonable time interval is not yet a reality, and many difficulties must be resolved before a successful device is constructed.

The Sun's energy is based in part on a set of reactions in which hydrogen is converted to helium. The proton–proton interaction is not suitable for use in a fusion reactor, however, because the event requires very high temperatures and densities. The process works in the Sun only because of the extremely high density of protons in the Sun's interior.

The reactions that appear most promising for a fusion power reactor involve deuterium ($^2_1$H) and tritium ($^3_1$H):

$$\begin{aligned} {}^2_1\text{H} + {}^2_1\text{H} &\rightarrow {}^3_2\text{He} + {}^1_0\text{n} \quad Q = 3.27\ \text{MeV} \\ {}^2_1\text{H} + {}^2_1\text{H} &\rightarrow {}^3_1\text{H} + {}^1_1\text{H} \quad Q = 4.03\ \text{MeV} \\ {}^2_1\text{H} + {}^3_1\text{H} &\rightarrow {}^4_2\text{He} + {}^1_0\text{n} \quad Q = 17.59\ \text{MeV} \end{aligned} \tag{45.4}$$

As noted earlier, deuterium is available in almost unlimited quantities from our lakes and oceans and is very inexpensive to extract. Tritium, however, is radioactive ($T_{1/2} = 12.3$ yr) and undergoes beta decay to ^{3}He. For this reason, tritium does not occur naturally to any great extent and must be artificially produced.

One major problem in obtaining energy from nuclear fusion is that the Coulomb repulsive force between two nuclei, which carry positive charges, must be overcome before they can fuse. Figure 45.7 is a graph of potential energy as a function of the separation distance between two deuterons (deuterium nuclei, each having charge $+e$). The potential energy is positive in the region $r > R$, where the Coulomb repulsive force dominates ($R \approx 1$ fm), and negative in the region $r < R$, where the nuclear force dominates. The fundamental problem then is to give the two nuclei enough kinetic energy to overcome this repulsive force. This requirement can be accomplished by raising the fuel to extremely high temperatures (to approximately 10^8 K). At these high temperatures, the atoms are ionized and the system consists of a collection of electrons and nuclei, commonly referred to as a *plasma*.

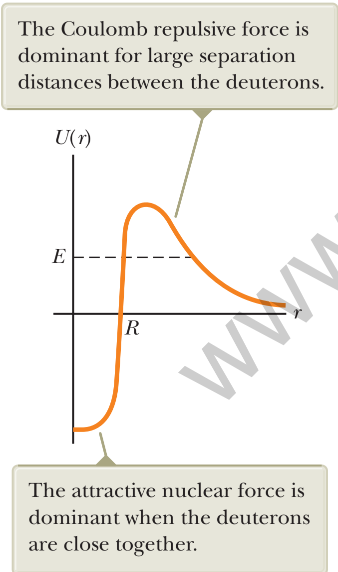


Figure 45.7 Potential energy as a function of separation distance between two deuterons. R is on the order of 1 fm. If we neglect tunneling, the two deuterons require an energy E greater than the height of the barrier to undergo fusion.

Example 45.3 The Fusion of Two Deuterons

For the nuclear force to overcome the repulsive Coulomb force, the separation distance between two deuterons must be approximately 1.0×10^{-14} m.

(A) Calculate the height of the potential barrier due to the repulsive force.

SOLUTION

Conceptualize Imagine moving two deuterons toward each other. As they move closer together, the Coulomb repulsion force becomes stronger. Work must be done on the system to push against this force, and this work appears in the system of two deuterons as electric potential energy.

Categorize We categorize this problem as one involving the electric potential energy of a system of two charged particles.

Analyze Evaluate the potential energy associated with two charges separated by a distance r (Eq. 25.13) for two deuterons:

$$U = k_e \frac{q_1 q_2}{r} = k_e \frac{(+e)^2}{r} = (8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(1.60 \times 10^{-19}\text{ C})^2}{1.0 \times 10^{-14}\text{ m}}$$

$$= 2.3 \times 10^{-14}\text{ J} = \boxed{0.14 \text{ MeV}}$$

(B) Estimate the temperature required for a deuteron to overcome the potential barrier, assuming an energy of $\frac{3}{2}k_B T$ per deuteron (where k_B is Boltzmann's constant).

SOLUTION

Because the total Coulomb energy of the pair is 0.14 MeV, the Coulomb energy per deuteron is equal to 0.07 MeV = 1.1×10^{-14} J.

Set this energy equal to the average energy per deuteron:

$$\tfrac{3}{2}k_B T = 1.1 \times 10^{-14}\text{ J}$$

Solve for T:

$$T = \frac{2(1.1 \times 10^{-14}\text{ J})}{3(1.38 \times 10^{-23}\text{ J/K})} = \boxed{5.6 \times 10^8 \text{ K}}$$

(C) Find the energy released in the deuterium–deuterium reaction

$$^2_1\text{H} + {}^2_1\text{H} \rightarrow {}^3_1\text{H} + {}^1_1\text{H}$$

SOLUTION

The mass of a single deuterium atom is equal to 2.014 102 u. Therefore, the total mass of the system before the reaction is 4.028 204 u.

Find the sum of the masses after the reaction:

$$3.016\,049\text{ u} + 1.007\,825\text{ u} = 4.023\,874\text{ u}$$

Find the change in mass and convert to energy units:

$$4.028\,204\text{ u} - 4.023\,874\text{ u} = 0.004\,33\text{ u}$$

$$= 0.004\,33\text{ u} \times 931.494\text{ MeV/u} = \boxed{4.03 \text{ MeV}}$$

Finalize The calculated temperature in part (B) is too high because the particles in the plasma have a Maxwellian speed distribution (Section 21.5) and therefore some fusion reactions are caused by particles in the high-energy tail of this distribution. Furthermore, even those particles that do not have enough energy to overcome the barrier have some probability of tunneling through (Section 41.5). When these effects are taken into account, a temperature of "only" 4×10^8 K appears adequate to fuse two deuterons in a plasma. In part (C), notice that the energy value is consistent with that already given in Equation 45.4.

WHAT IF? Suppose the tritium resulting from the reaction in part (C) reacts with another deuterium in the reaction

$$^2_1\text{H} + {}^3_1\text{H} \rightarrow {}^4_2\text{He} + {}^1_0\text{n}$$

How much energy is released in the sequence of two reactions?

continued

45.3 continued

Answer The overall effect of the sequence of two reactions is that three deuterium nuclei have combined to form a helium nucleus, a hydrogen nucleus, and a neutron. The initial mass is 3(2.014 102 u) = 6.042 306 u. After the reaction, the sum of the masses is 4.002 603 u + 1.007 825 u + 1.008 665 = 6.019 093 u. The excess mass is equal to 0.023 213 u, equivalent to an energy of 21.6 MeV. Notice that this value is the sum of the Q values for the second and third reactions in Equation 45.4.

The temperature at which the power generation rate in any fusion reaction exceeds the loss rate is called the **critical ignition temperature** T_{ignit}. This temperature for the deuterium–deuterium (D–D) reaction is 4×10^8 K. From the relationship $E \approx \frac{3}{2}k_BT$, the ignition temperature is equivalent to approximately 52 keV. The critical ignition temperature for the deuterium–tritium (D–T) reaction is approximately 4.5×10^7 K, or only 6 keV. A plot of the power P_{gen} generated by fusion versus temperature for the two reactions is shown in Figure 45.8. The straight green line represents the power P_{lost} lost via the radiation mechanism known as bremsstrahlung (Section 42.8). In this principal mechanism of energy loss, radiation (primarily x-rays) is emitted as the result of electron–ion collisions within the plasma. The intersections of the P_{lost} line with the P_{gen} curves give the critical ignition temperatures.

In addition to the high-temperature requirements, two other critical parameters determine whether or not a thermonuclear reactor is successful: the **ion density** n and **confinement time** τ, which is the time interval during which energy injected into the plasma remains within the plasma. British physicist J. D. Lawson (1923–2008) showed that both the ion density and confinement time must be large enough to ensure that more fusion energy is released than the amount required to raise the temperature of the plasma. For a given value of n, the probability of fusion between two particles increases as τ increases. For a given value of τ, the collision rate between nuclei increases as n increases. The product $n\tau$ is referred to as the **Lawson number** of a reaction. A graph of the value of $n\tau$ necessary to achieve a net energy output for the D–T and D–D reactions at different temperatures is shown in Figure 45.9. In particular, **Lawson's criterion** states that a net energy output is possible for values of $n\tau$ that meet the following conditions:

$$n\tau \geq 10^{14} \text{ s/cm}^3 \quad \text{(D–T)} \tag{45.5}$$

$$n\tau \geq 10^{16} \text{ s/cm}^3 \quad \text{(D–D)}$$

These values represent the minima of the curves in Figure 45.9.

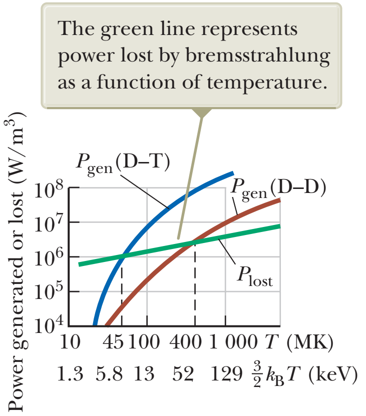


Figure 45.8 Power generated versus temperature for deuterium–deuterium (D–D) and deuterium–tritium (D–T) fusion. When the generation rate exceeds the loss rate, ignition takes place.

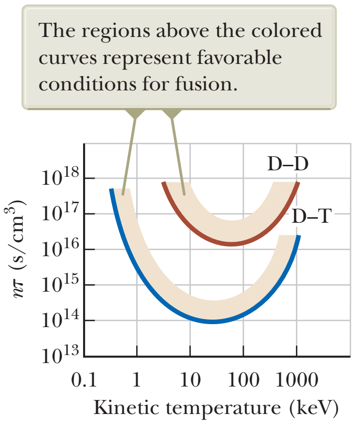


Figure 45.9 The Lawson number $n\tau$ at which net energy output is possible versus temperature for the D–T and D–D fusion reactions.

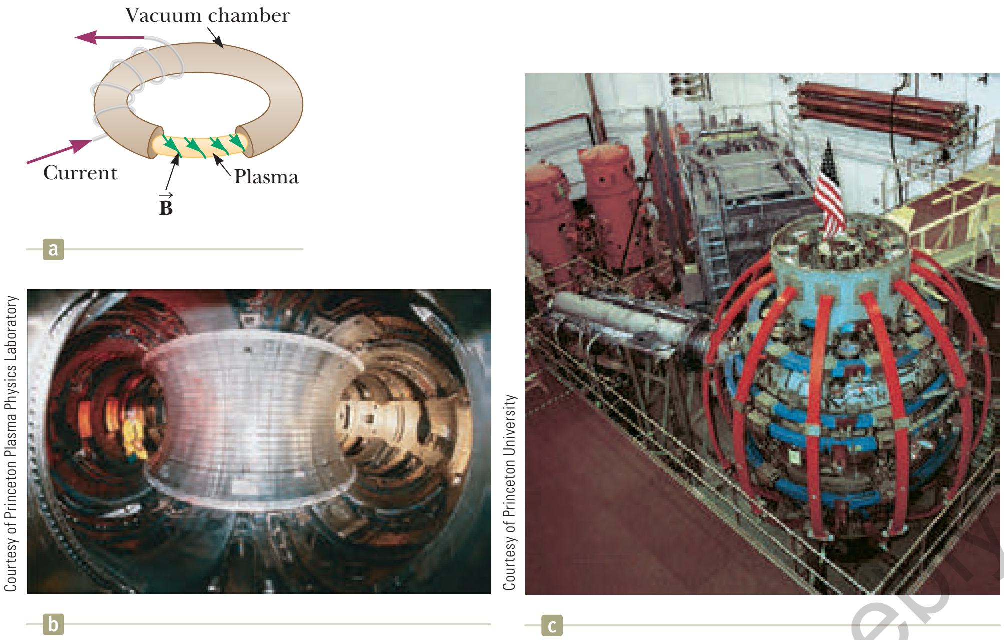


Figure 45.10 (a) Diagram of a tokamak used in the magnetic confinement scheme. (b) Interior view of the closed Tokamak Fusion Test Reactor (TFTR) vacuum vessel at the Princeton Plasma Physics Laboratory. (c) The National Spherical Torus Experiment (NSTX) that began operation in March 1999.

Lawson's criterion was arrived at by comparing the energy required to raise the temperature of a given plasma with the energy generated by the fusion process.[2] The energy E_{in} required to raise the temperature of the plasma is proportional to the ion density n, which we can express as $E_{in} = C_1 n$, where C_1 is some constant. The energy generated by the fusion process is proportional to $n^2\tau$, or $E_{gen} = C_2 n^2 \tau$. This dependence may be understood by realizing that the fusion energy released is proportional to both the rate at which interacting ions collide ($\propto n^2$) and the confinement time τ. Net energy is produced when $E_{gen} > E_{in}$. When the constants C_1 and C_2 are calculated for different reactions, the condition that $E_{gen} \geq E_{in}$ leads to Lawson's criterion.

Current efforts are aimed at meeting Lawson's criterion at temperatures exceeding T_{ignit}. Although the minimum required plasma densities have been achieved, the problem of confinement time is more difficult. The two basic techniques under investigation for solving this problem are *magnetic confinement* and *inertial confinement*.

Magnetic Confinement

Many fusion-related plasma experiments use **magnetic confinement** to contain the plasma. A toroidal device called a **tokamak,** first developed in Russia, is shown in Figure 45.10a. A combination of two magnetic fields is used to confine and stabilize the plasma: (1) a strong toroidal field produced by the current in the toroidal windings surrounding a doughnut-shaped vacuum chamber and (2) a weaker "poloidal" field produced by the toroidal current. In addition to confining the plasma, the toroidal current is used to raise its temperature. The resultant helical magnetic field lines spiral around the plasma and keep it from touching the walls of the vacuum chamber. (If the plasma touches the walls, its temperature is reduced and heavy impurities sputtered from the walls "poison" it, leading to large power losses.)

One major breakthrough in magnetic confinement in the 1980s was in the area of auxiliary energy input to reach ignition temperatures. Experiments have shown

[2]Lawson's criterion neglects the energy needed to set up the strong magnetic field used to confine the hot plasma in a magnetic confinement approach. This energy is expected to be about 20 times greater than the energy required to raise the temperature of the plasma. It is therefore necessary either to have a magnetic energy recovery system or to use superconducting magnets.

that injecting a beam of energetic neutral particles into the plasma is a very efficient method of raising it to ignition temperatures. Radio-frequency energy input will probably be needed for reactor-size plasmas.

When it was in operation from 1982 to 1997, the Tokamak Fusion Test Reactor (TFTR, Fig. 45.10b) at Princeton University reported central ion temperatures of 510 million degrees Celsius, more than 30 times greater than the temperature at the center of the Sun. The $n\tau$ values in the TFTR for the D–T reaction were well above 10^{13} s/cm^3 and close to the value required by Lawson's criterion. In 1991, reaction rates of 6×10^{17} D–T fusions per second were reached in the Joint European Torus (JET) tokamak at Abington, England.

One of the new generation of fusion experiments is the National Spherical Torus Experiment (NSTX) at the Princeton Plasma Physics Laboratory and shown in Figure 45.10c. This reactor was brought on line in February 1999 and has been running fusion experiments since then. Rather than the doughnut-shaped plasma of a tokamak, the NSTX produces a spherical plasma that has a hole through its center. The major advantage of the spherical configuration is its ability to confine the plasma at a higher pressure in a given magnetic field. This approach could lead to development of smaller, more economical fusion reactors.

An international collaborative effort involving the United States, the European Union, Japan, China, South Korea, India, and Russia is currently under way to build a fusion reactor called ITER. This acronym stands for International Thermonuclear Experimental Reactor, although recently the emphasis has shifted to interpreting "iter" in terms of its Latin meaning, "the way." One reason proposed for this change is to avoid public misunderstanding and negative connotations toward the word *thermonuclear.* This facility will address the remaining technological and scientific issues concerning the feasibility of fusion power. The design is completed, and Cadarache, France, was chosen in June 2005 as the reactor site. Construction began in 2007 and will require about 10 years, with fusion operation projected to begin in 2019. If the planned device works as expected, the Lawson number for ITER will be about six times greater than the current record holder, the JT-60U tokamak in Japan. ITER is expected to produce ten times as much output power as input power, and the energy content of the alpha particles inside the reactor will be so intense that they will sustain the fusion reaction, allowing the auxiliary energy sources to be turned off once the reaction is initiated.

Example 45.4 Inside a Fusion Reactor

In 1998, the JT-60U tokamak in Japan operated with a D–T plasma density of 4.8×10^{13} cm^{-3} at a temperature (in energy units) of 24.1 keV. It confined this plasma inside a magnetic field for 1.1 s.

(A) Do these data meet Lawson's criterion?

SOLUTION

Conceptualize With the help of the third of Equations 45.4, imagine many such reactions occurring in a plasma of high temperature and high density.

Categorize We use the concept of the Lawson number discussed in this section, so we categorize this example as a substitution problem.

Evaluate the Lawson number for the JT-60U: $n\tau = (4.8 \times 10^{13}\ \text{cm}^{-3})(1.1\ \text{s}) = 5.3 \times 10^{13}\ \text{s/cm}^3$

This value is close to meeting Lawson's criterion of 10^{14} s/cm^3 for a D–T plasma given in Equation 45.5. In fact, scientists recorded a power gain of 1.25, indicating that the reactor operated slightly past the break-even point and produced more energy than it required to maintain the plasma.

(B) How does the plasma density compare with the density of atoms in an ideal gas when the gas is under standard conditions ($T = 0°$C and $P = 1$ atm)?

45.4 continued

SOLUTION

Find the density of atoms in a sample of ideal gas by evaluating N_A/V_{mol}, where N_A is Avogadro's number and V_{mol} is the molar volume of an ideal gas under standard conditions, 2.24×10^{-2} m³/mol:

$$\frac{N_A}{V_{mol}} = \frac{6.02 \times 10^{23} \text{ atoms/mol}}{2.24 \times 10^{-2} \text{ m}^3\text{/mol}} = 2.7 \times 10^{25} \text{ atoms/m}^3$$

$$= 2.7 \times 10^{19} \text{ atoms/cm}^3$$

This value is more than 500 000 times greater than the plasma density in the reactor.

Inertial Confinement

The second technique for confining a plasma, called **inertial confinement,** makes use of a D–T target that has a very high particle density. In this scheme, the confinement time is very short (typically 10^{-11} to 10^{-9} s), and, because of their own inertia, the particles do not have a chance to move appreciably from their initial positions. Therefore, Lawson's criterion can be satisfied by combining a high particle density with a short confinement time.

Laser fusion is the most common form of inertial confinement. A small D–T pellet, approximately 1 mm in diameter, is struck simultaneously by several focused, high-intensity laser beams, resulting in a large pulse of input energy that causes the surface of the fuel pellet to evaporate (Fig. 45.11). The escaping particles exert a third-law reaction force on the core of the pellet, resulting in a strong, inwardly moving compressive shock wave. This shock wave increases the pressure and density of the core and produces a corresponding increase in temperature. When the temperature of the core reaches ignition temperature, fusion reactions occur.

One of the leading laser fusion laboratories in the United States is the Omega facility at the University of Rochester in New York. This facility focuses 24 laser beams on the target. Currently under operation at the Lawrence Livermore National Laboratory in Livermore, California, is the National Ignition Facility. The research apparatus there includes 192 laser beams that can be focused on a deuterium–tritium pellet. Construction was completed in early 2009, and a test firing of the lasers in March 2012 broke the record for lasers, delivering 1.87 MJ to a target. This energy is delivered in such a short time interval that the power is immense: 500 trillion watts, more than 1 000 times the power used in the United States at any moment.

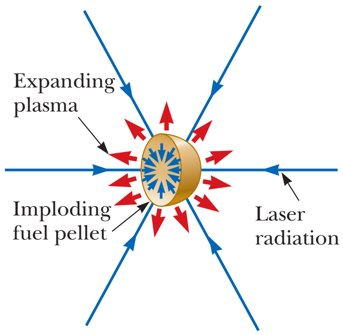


Figure 45.11 In inertial confinement, a D–T fuel pellet fuses when struck by several high-intensity laser beams simultaneously.

Fusion Reactor Design

In the D–T fusion reaction

$$^2_1\text{H} + {}^3_1\text{H} \rightarrow {}^4_2\text{He} + {}^1_0\text{n} \qquad Q = 17.59 \text{ MeV}$$

the alpha particle carries 20% of the energy and the neutron carries 80%, or approximately 14 MeV. A diagram of the deuterium–tritium fusion reaction is shown in Figure 45.12. Because the alpha particles are charged, they are primarily absorbed by the plasma, causing the plasma's temperature to increase. In contrast, the 14-MeV neutrons, being electrically neutral, pass through the plasma and are absorbed by a surrounding blanket material, where their large kinetic energy is extracted and used to generate electric power.

One scheme is to use molten lithium metal as the neutron-absorbing material and to circulate the lithium in a closed heat-exchange loop, thereby producing steam and driving turbines as in a conventional power plant. Figure 45.13 (page 1432) shows a diagram of such a reactor. It is estimated that a blanket of lithium approximately 1 m thick will capture nearly 100% of the neutrons from the fusion of a small D–T pellet.

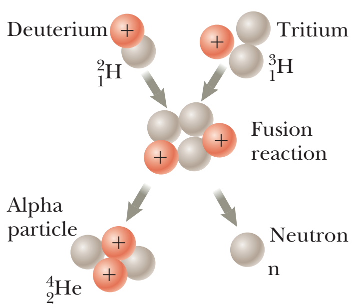


Figure 45.12 Deuterium–tritium fusion. Eighty percent of the energy released is in the 14-MeV neutron.

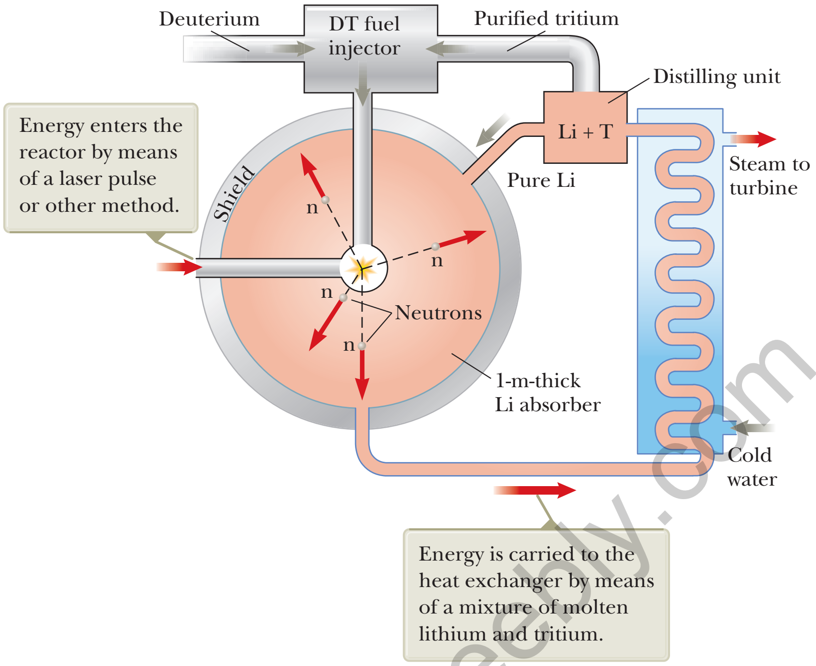


Figure 45.13 Diagram of a fusion reactor.

The capture of neutrons by lithium is described by the reaction

$$ {}^{1}_{0}\text{n} + {}^{6}_{3}\text{Li} \rightarrow {}^{3}_{1}\text{H} + {}^{4}_{2}\text{He} $$

where the kinetic energies of the charged tritium ${}^{3}_{1}\text{H}$ and alpha particle are transformed to internal energy in the molten lithium. An extra advantage of using lithium as the energy-transfer medium is that the tritium produced can be separated from the lithium and returned as fuel to the reactor.

Advantages and Problems of Fusion

If fusion power can ever be harnessed, it will offer several advantages over fission-generated power: (1) low cost and abundance of fuel (deuterium), (2) impossibility of runaway accidents, and (3) decreased radiation hazard. Some of the anticipated problems and disadvantages include (1) scarcity of lithium, (2) limited supply of helium, which is needed for cooling the superconducting magnets used to produce strong confining fields, and (3) structural damage and induced radioactivity caused by neutron bombardment. If such problems and the engineering design factors can be resolved, nuclear fusion may become a feasible source of energy in the twenty-first century.

45.5 Radiation Damage

In Chapter 34, we learned that electromagnetic radiation is all around us in the form of radio waves, microwaves, light waves, and so on. In this section, we describe forms of radiation that can cause severe damage as they pass through matter, such as radiation resulting from radioactive processes and radiation in the form of energetic particles such as neutrons and protons.

The degree and type of damage depend on several factors, including the type and energy of the radiation and the properties of the matter. The metals used in nuclear reactor structures can be severely weakened by high fluxes of energetic neutrons because these high fluxes often lead to metal fatigue. The damage in such situations is in the form of atomic displacements, often resulting in major alterations in the properties of the material.

Radiation damage in biological organisms is primarily due to ionization effects in cells. A cell's normal operation may be disrupted when highly reactive ions are formed as the result of ionizing radiation. For example, hydrogen and the hydroxyl radical OH^- produced from water molecules can induce chemical reactions that may break bonds in proteins and other vital molecules. Furthermore, the ionizing radiation may affect vital molecules directly by removing electrons from their structure. Large doses of radiation are especially dangerous because damage to a great number of molecules in a cell may cause the cell to die. Although the death of a single cell is usually not a problem, the death of many cells may result in irreversible damage to the organism. Cells that divide rapidly, such as those of the digestive tract, reproductive organs, and hair follicles, are especially susceptible. In addition, cells that survive the radiation may become defective. These defective cells can produce more defective cells and can lead to cancer.

In biological systems, it is common to separate radiation damage into two categories: somatic damage and genetic damage. *Somatic damage* is that associated with any body cell except the reproductive cells. Somatic damage can lead to cancer or can seriously alter the characteristics of specific organisms. *Genetic damage* affects only reproductive cells. Damage to the genes in reproductive cells can lead to defective offspring. It is important to be aware of the effect of diagnostic treatments, such as x-rays and other forms of radiation exposure, and to balance the significant benefits of treatment with the damaging effects.

Damage caused by radiation also depends on the radiation's penetrating power. Alpha particles cause extensive damage, but penetrate only to a shallow depth in a material due to the strong interaction with other charged particles. Neutrons do not interact via the electric force and hence penetrate deeper, causing significant damage. Gamma rays are high-energy photons that can cause severe damage, but often pass through matter without interaction.

Several units have been used historically to quantify the amount, or dose, of any radiation that interacts with a substance.

The **roentgen** (R) is that amount of ionizing radiation that produces an electric charge of 3.33×10^{-10} C in 1 cm^3 of air under standard conditions.

Equivalently, the roentgen is that amount of radiation that increases the energy of 1 kg of air by 8.76×10^{-3} J.

For most applications, the roentgen has been replaced by the rad (an acronym for *radiation absorbed dose*):

One **rad** is that amount of radiation that increases the energy of 1 kg of absorbing material by 1×10^{-2} J.

Although the rad is a perfectly good physical unit, it is not the best unit for measuring the degree of biological damage produced by radiation because damage depends not only on the dose but also on the type of the radiation. For example, a given dose of alpha particles causes about ten times more biological damage than an equal dose of x-rays. The **RBE** (relative biological effectiveness) factor for a given type of radiation is **the number of rads of x-radiation or gamma radiation that produces the same biological damage as 1 rad of the radiation being used.** The RBE factors for different types of radiation are given in Table 45.1 (page 1434). The values are only approximate because they vary with particle energy and with the form of the damage. The RBE factor should be considered only a first-approximation guide to the actual effects of radiation.

Finally, the **rem** (radiation equivalent in man) is the product of the dose in rad and the RBE factor:

$$\text{Dose in rem} \equiv \text{dose in rad} \times \text{RBE} \quad (45.6)$$

◀ **Radiation dose in rem**

Table 45.1 RBE Factors for Several Types of Radiation

Radiation	RBE Factor
X-rays and gamma rays	1.0
Beta particles	1.0–1.7
Alpha particles	10–20
Thermal neutrons	4–5
Fast neutrons and protons	10
Heavy ions	20

Note: RBE = relative biological effectiveness.

Table 45.2 Units for Radiation Dosage

Quantity	SI Unit	Symbol	Relations to Other SI Units	Older Unit	Conversion
Absorbed dose	gray	Gy	= 1 J/kg	rad	1 Gy = 100 rad
Dose equivalent	sievert	Sv	= 1 J/kg	rem	1 Sv = 100 rem

According to this definition, 1 rem of any two types of radiation produces the same amount of biological damage. Table 45.1 shows that a dose of 1 rad of fast neutrons represents an effective dose of 10 rem, but 1 rad of gamma radiation is equivalent to a dose of only 1 rem.

This discussion has focused on measurements of radiation dosage in units such as rads and rems because these units are still widely used. They have, however, been formally replaced with new SI units. The rad has been replaced with the *gray* (Gy), equal to 100 rad, and the rem has been replaced with the *sievert* (Sv), equal to 100 rem. Table 45.2 summarizes the older and the current SI units of radiation dosage.

Low-level radiation from natural sources such as cosmic rays and radioactive rocks and soil delivers to each of us a dose of approximately 2.4 mSv/yr. This radiation, called *background radiation,* varies with geography, with the main factors being altitude (exposure to cosmic rays) and geology (radon gas released by some rock formations, deposits of naturally radioactive minerals).

The upper limit of radiation dose rate recommended by the U.S. government (apart from background radiation) is approximately 5 mSv/yr. Many occupations involve much higher radiation exposures, so an upper limit of 50 mSv/yr has been set for combined whole-body exposure. Higher upper limits are permissible for certain parts of the body, such as the hands and the forearms. A dose of 4 to 5 Sv results in a mortality rate of approximately 50% (which means that half the people exposed to this radiation level die). The most dangerous form of exposure for most people is either ingestion or inhalation of radioactive isotopes, especially isotopes of those elements the body retains and concentrates, such as ^{90}Sr.

45.6 Uses of Radiation

Nuclear physics applications are extremely widespread in manufacturing, medicine, and biology. In this section, we present a few of these applications and the underlying theories supporting them.

Tracing

Radioactive tracers are used to track chemicals participating in various reactions. One of the most valuable uses of radioactive tracers is in medicine. For example, iodine, a nutrient needed by the human body, is obtained largely through the

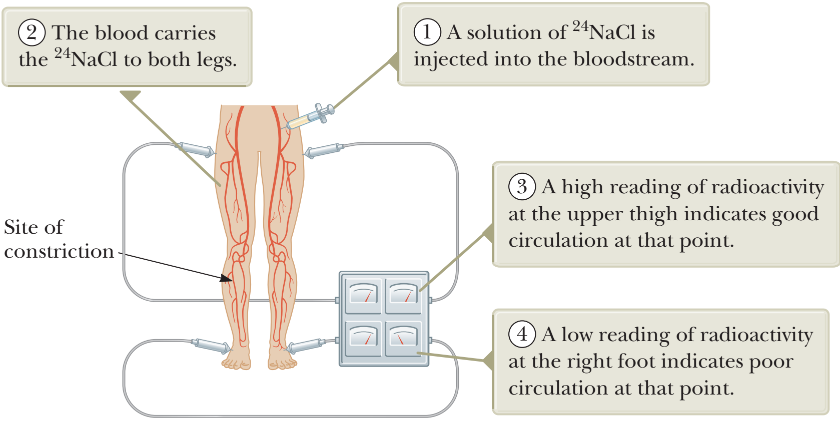


Figure 45.14 A tracer technique for determining the condition of the human circulatory system.

intake of iodized salt and seafood. To evaluate the performance of the thyroid, the patient drinks a very small amount of radioactive sodium iodide containing ^{131}I, an artificially produced isotope of iodine (the natural, nonradioactive isotope is ^{127}I). The amount of iodine in the thyroid gland is determined as a function of time by measuring the radiation intensity at the neck area. How much of the isotope ^{131}I remains in the thyroid is a measure of how well that gland is functioning.

A second medical application is indicated in Figure 45.14. A solution containing radioactive sodium is injected into a vein in the leg, and the time at which the radioisotope arrives at another part of the body is detected with a radiation counter. The elapsed time is a good indication of the presence or absence of constrictions in the circulatory system.

Tracers are also useful in agricultural research. Suppose the best method of fertilizing a plant is to be determined. A certain element in a fertilizer, such as nitrogen, can be *tagged* (identified) with one of its radioactive isotopes. The fertilizer is then sprayed on one group of plants, sprinkled on the ground for a second group, and raked into the soil for a third. A Geiger counter is then used to track the nitrogen through each of the three groups.

Tracing techniques are as wide ranging as human ingenuity can devise. Today, applications range from checking how teeth absorb fluoride to monitoring how cleansers contaminate food-processing equipment to studying deterioration inside an automobile engine. In this last case, a radioactive material is used in the manufacture of the car's piston rings and the oil is checked for radioactivity to determine the amount of wear on the rings.

Materials Analysis

For centuries, a standard method of identifying the elements in a sample of material has been chemical analysis, which involves determining how the material reacts with various chemicals. A second method is spectral analysis, which works because each element, when excited, emits its own characteristic set of electromagnetic wavelengths. These methods are now supplemented by a third technique, **neutron activation analysis.** A disadvantage of both chemical and spectral methods is that a fairly large sample of the material must be destroyed for the analysis. In addition, extremely small quantities of an element may go undetected by either method. Neutron activation analysis has an advantage over chemical analysis and spectral analysis in both respects.

When a material is irradiated with neutrons, nuclei in the material absorb the neutrons and are changed to different isotopes, most of which are radioactive. For example, ^{65}Cu absorbs a neutron to become ^{66}Cu, which undergoes beta decay:

$$^{1}_{0}\text{n} + ^{65}_{29}\text{Cu} \rightarrow ^{66}_{29}\text{Cu} \rightarrow ^{66}_{30}\text{Zn} + \text{e}^- + \bar{\nu}$$

The presence of the copper can be deduced because it is known that ^{66}Cu has a half-life of 5.1 min and decays with the emission of beta particles having a maximum energy of 2.63 MeV. Also emitted in the decay of ^{66}Cu is a 1.04-MeV gamma ray. By examining the radiation emitted by a substance after it has been exposed to neutron irradiation, one can detect extremely small amounts of an element in that substance.

Neutron activation analysis is used routinely in a number of industries. In commercial aviation, for example, it is used to check airline luggage for hidden explosives. One nonroutine use is of historical interest. Napoleon died on the island of St. Helena in 1821, supposedly of natural causes. Over the years, suspicion has existed that his death was not all that natural. After his death, his head was shaved and locks of his hair were sold as souvenirs. In 1961, the amount of arsenic in a sample of this hair was measured by neutron activation analysis, and an unusually large quantity of arsenic was found. (Activation analysis is so sensitive that very small pieces of a single hair could be analyzed.) Results showed that the arsenic was fed to him irregularly. In fact, the arsenic concentration pattern corresponded to the fluctuations in the severity of Napoleon's illness as determined from historical records.

Art historians use neutron activation analysis to detect forgeries. The pigments used in paints have changed throughout history, and old and new pigments react differently to neutron activation. The method can even reveal hidden works of art behind existing paintings because an older, hidden layer of paint reacts differently than the surface layer to neutron activation.

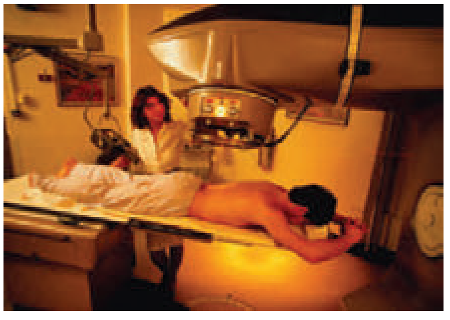
Radiation Therapy

Radiation causes much damage to rapidly dividing cells. Therefore, it is useful in cancer treatment because tumor cells divide extremely rapidly. Several mechanisms can be used to deliver radiation to a tumor. In Section 42.8, we discussed the use of high-energy x-rays in the treatment of cancerous tissue. Other treatment protocols include the use of narrow beams of radiation from a radioactive source. As an example, Figure 45.15 shows a machine that uses ^{60}Co as a source. The ^{60}Co isotope emits gamma rays with photon energies higher than 1 MeV.

In other situations, a technique called *brachytherapy* is used. In this treatment plan, thin radioactive needles called *seeds* are implanted in the cancerous tissue. The energy emitted from the seeds is delivered directly to the tumor, reducing the exposure of surrounding tissue to radiation damage. In the case of prostate cancer, the active isotopes used in brachytherapy include ^{125}I and ^{103}Pd.

Food Preservation

Radiation is finding increasing use as a means of preserving food because exposure to high levels of radiation can destroy or incapacitate bacteria and mold spores (Fig. 45.16). Techniques include exposing foods to gamma rays, high-energy electron beams, and x-rays. Food preserved by such exposure can be placed in a sealed container (to keep out new spoiling agents) and stored for long periods of time. There is little or no evidence of adverse effect on the taste or nutritional value of food



Martin Dohrn/Photo Researchers, Inc.

Figure 45.15 This large machine is being set to deliver a dose of radiation from ^{60}Co in an effort to destroy a cancerous tumor. Cancer cells are especially susceptible to this type of therapy because they tend to divide more often than cells of healthy tissue nearby.

Council for Agricultural Science & Technology

Figure 45.16 The strawberries on the left are untreated and have become moldy. The unspoiled strawberries on the right have been irradiated. The radiation has killed or incapacitated the mold spores that have spoiled the strawberries on the left.

from irradiation. The safety of irradiated foods has been endorsed by the World Health Organization, the Centers for Disease Control and Prevention, the U.S. Department of Agriculture, and the Food and Drug Administration. Irradiation of food is presently permitted in more than 50 countries. Some estimates place the amount of irradiated food in the world as high as 500 000 metric tons each year.

Summary

Concepts and Principles

The probability that neutrons are captured as they move through matter generally increases with decreasing neutron energy. A **thermal neutron** is a slow-moving neutron that has a high probability of being captured by a nucleus in a **neutron capture event:**

$$^{1}_{0}\text{n} + {}^{A}_{Z}\text{X} \rightarrow {}^{A+1}_{Z}\text{X}^* \rightarrow {}^{A+1}_{Z}\text{X} + \gamma \quad \textbf{(45.1)}$$

where ${}^{A+1}_{Z}\text{X}^*$ is an excited intermediate nucleus that rapidly emits a photon.

Nuclear fission occurs when a very heavy nucleus, such as ^{235}U, splits into two smaller **fission fragments.** Thermal neutrons can create fission in ^{235}U:

$$^{1}_{0}\text{n} + {}^{235}_{92}\text{U} \rightarrow {}^{236}_{92}\text{U}^* \rightarrow \text{X} + \text{Y} + \text{neutrons} \quad \textbf{(45.2)}$$

where ^{236}U* is an intermediate excited state and X and Y are the fission fragments. On average, 2.5 neutrons are released per fission event. The fragments then undergo a series of beta and gamma decays to various stable isotopes. The energy released per fission event is approximately 200 MeV.

The **reproduction constant** K is the average number of neutrons released from each fission event that cause another event. In a fission reactor, it is necessary to maintain $K \approx 1$. The value of K is affected by such factors as reactor geometry, mean neutron energy, and probability of neutron capture.

In **nuclear fusion,** two light nuclei fuse to form a heavier nucleus and release energy. The major obstacle in obtaining useful energy from fusion is the large Coulomb repulsive force between the charged nuclei at small separation distances. The temperature required to produce fusion is on the order of 10^8 K, and at this temperature, all matter occurs as a plasma.

In a fusion reactor, the plasma temperature must reach the **critical ignition temperature,** the temperature at which the power generated by the fusion reactions exceeds the power lost in the system. The most promising fusion reaction is the D–T reaction, which has a critical ignition temperature of approximately 4.5×10^7 K. Two critical parameters in fusion reactor design are **ion density** n and **confinement time** τ, the time interval during which the interacting particles must be maintained at $T > T_{\text{ignit}}$. **Lawson's criterion** states that for the D–T reaction, $n\tau \geq 10^{14}$ s/cm^3.

Objective Questions

1. denotes answer available in *Student Solutions Manual/Study Guide*

1. In a certain fission reaction, a ^{235}U nucleus captures a neutron. This process results in the creation of the products ^{137}I and ^{96}Y along with how many neutrons? (a) 1 (b) 2 (c) 3 (d) 4 (e) 5

2. Which particle is most likely to be captured by a ^{235}U nucleus and cause it to undergo fission? (a) an energetic proton (b) an energetic neutron (c) a slow-moving alpha particle (d) a slow-moving neutron (e) a fast-moving electron

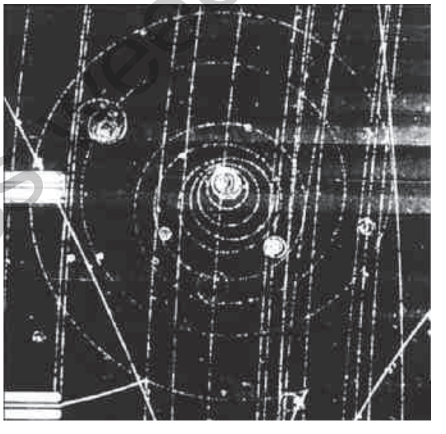
3. In the first nuclear weapon test carried out in New Mexico, the energy released was equivalent to approximately 17 kilotons of TNT. Estimate the mass decrease in the nuclear fuel representing the energy converted from rest energy into other forms in this event. *Note:* One ton of TNT has the energy equivalent of 4.2×10^9 J. (a) 1 μg (b) 1 mg (c) 1 g (d) 1 kg (e) 20 kg

4. Working with radioactive materials at a laboratory over one year, (a) Tom received 1 rem of alpha radiation, (b) Karen received 1 rad of fast neutrons, (c) Paul received 1 rad of thermal neutrons as a whole-body dose, and (d) Ingrid received 1 rad of thermal neutrons to her hands only. Rank these four doses according to the likely amount of biological damage from the greatest to the least, noting any cases of equality.

5. If the moderator were suddenly removed from a nuclear reactor in an electric generating station, what is the most likely consequence? (a) The reactor would go supercritical, and a runaway reaction would occur. (b) The nuclear reaction would proceed in the same way, but the reactor would overheat. (c) The reactor would become subcritical, and the reaction would die out. (d) No change would occur in the reactor's operation.

6. You may use Figure 44.5 to answer this question. Three nuclear reactions take place, each involving 108 nucleons: (1) eighteen ^{6}Li nuclei fuse in pairs to form nine ^{12}C nuclei, (2) four nuclei each with 27 nucleons fuse in pairs to form two nuclei with 54 nucleons, and (3) one nucleus with 108 nucleons fissions to form two nuclei with 54 nucleons. Rank these three reactions from the largest positive Q value (representing energy output) to the largest negative value (representing energy input). Also include $Q = 0$ in your ranking to make clear which of the reactions put out energy and which absorb energy. Note any cases of equality in your ranking.

7. A device called a *bubble chamber* uses a liquid (usually liquid hydrogen) maintained near its boiling point. Ions produced by incoming charged particles from nuclear decays leave bubble tracks, which can be photographed. Figure OQ45.7 shows particle tracks in a bubble chamber immersed in a magnetic field. The tracks are generally spirals rather than sections of circles. What is the primary reason for this shape? (a) The magnetic field is not perpendicular to the velocity of the particles. (b) The magnetic field is not uniform in space. (c) The forces on the particles increase with time. (d) The speeds of the particles decrease with time.



Courtesy Lawrence Berkeley Laboratory, University of California, Photographic Services

Figure OQ45.7

8. If an alpha particle and an electron have the same kinetic energy, which undergoes the greater deflection when passed through a magnetic field? (a) The alpha particle does. (b) The electron does. (c) They undergo the same deflection. (d) Neither is deflected.

9. Which of the following fuel conditions is *not* necessary to operate a self-sustained controlled fusion reactor? (a) The fuel must be at a sufficiently high temperature. (b) The fuel must be radioactive. (c) The fuel must be at a sufficiently high density. (d) The fuel must be confined for a sufficiently long period of time. (e) Conditions (a) through (d) are all necessary.

Conceptual Questions

1. denotes answer available in *Student Solutions Manual/Study Guide*

1. What factors make a terrestrial fusion reaction difficult to achieve?

2. Lawson's criterion states that the product of ion density and confinement time must exceed a certain number before a break-even fusion reaction can occur. Why should these two parameters determine the outcome?

3. Why would a fusion reactor produce less radioactive waste than a fission reactor?

4. Discuss the advantages and disadvantages of fission reactors from the point of view of safety, pollution, and resources. Make a comparison with power generated from the burning of fossil fuels.

5. Discuss the similarities and differences between fusion and fission.

6. If a nucleus captures a slow-moving neutron, the product is left in a highly excited state, with an energy approximately 8 MeV above the ground state. Explain the source of the excitation energy.

7. Discuss the advantages and disadvantages of fusion power from the viewpoint of safety, pollution, and resources.

8. A scintillation crystal can be a detector of radiation when combined with a photomultiplier tube (Section 40.2). The scintillator is usually a solid or liquid material whose atoms are easily excited by radiation. The excited atoms then emit photons when they return to their ground state. The design of the radiation detector in Figure CQ45.8 might suggest that any number of dynodes may be used to amplify a weak signal. What factors do you suppose would limit the amplification in this device?

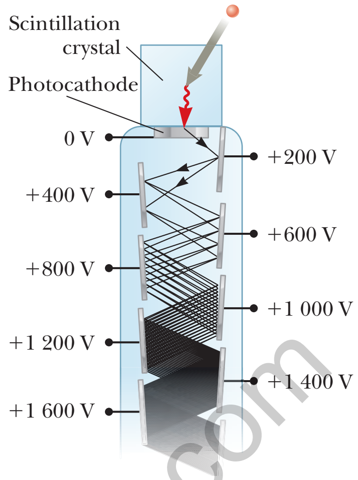


Figure CQ45.8

9. Why is water a better shield against neutrons than lead or steel?

Problems

The problems found in this chapter may be assigned online in Enhanced WebAssign

1. straightforward; **2.** intermediate; **3.** challenging

1. full solution available in the *Student Solutions Manual/Study Guide*

AMT Analysis Model tutorial available in Enhanced WebAssign

GP Guided Problem

M Master It tutorial available in Enhanced WebAssign

W Watch It video solution available in Enhanced WebAssign

Section 45.1 Interactions Involving Neutrons

Section 45.2 Nuclear Fission

Problem 57 in Chapter 25 and Problems 22 and 78 in Chapter 44 can be assigned with this chapter.

1. M If the average energy released in a fission event is 208 MeV, find the total number of fission events required to operate a 100-W lightbulb for 1.0 h.

2. Burning one metric ton (1 000 kg) of coal can yield an energy of 3.30×10^{10} J. Fission of one nucleus of uranium-235 yields an average of approximately 200 MeV. What mass of uranium produces the same energy in fission as burning one metric ton of coal?

3. Strontium-90 is a particularly dangerous fission product of ^{235}U because it is radioactive and it substitutes for calcium in bones. What other direct fission products would accompany it in the neutron-induced fission of ^{235}U? *Note:* This reaction may release two, three, or four free neutrons.

4. A typical nuclear fission power plant produces approximately 1.00 GW of electrical power. Assume the plant has an overall efficiency of 40.0% and each fission reaction produces 200 MeV of energy. Calculate the mass of ^{235}U consumed each day.

5. List the nuclear reactions required to produce ^{233}U from ^{232}Th under fast neutron bombardment.

6. The following fission reaction is typical of those occurring in a nuclear electric generating station:

$$^{1}_{0}\text{n} + ^{235}_{92}\text{U} \rightarrow ^{141}_{56}\text{Ba} + ^{92}_{36}\text{Kr} + 3(^{1}_{0}\text{n})$$

(a) Find the energy released in the reaction. The masses of the products are 140.914 411 u for $^{141}_{56}$Ba and 91.926 156 u for $^{92}_{36}$Kr. (b) What fraction of the initial rest energy of the system is transformed to other forms?

7. Find the energy released in the fission reaction

$$^{1}_{0}\text{n} + ^{235}_{92}\text{U} \rightarrow ^{88}_{38}\text{Sr} + ^{136}_{54}\text{Xe} + 12(^{1}_{0}\text{n})$$

8. A 2.00-MeV neutron is emitted in a fission reactor. If it loses half its kinetic energy in each collision with a moderator atom, how many collisions does it undergo as it becomes a thermal neutron, with energy 0.039 eV?

9. Find the energy released in the fission reaction

$$^{1}_{0}\text{n} + ^{235}_{92}\text{U} \rightarrow ^{98}_{40}\text{Zr} + ^{135}_{52}\text{Te} + 3(^{1}_{0}\text{n})$$

The atomic masses of the fission products are 97.912 735 u for $^{98}_{40}$Zr and 134.916 450 u for $^{135}_{52}$Te.

10. Seawater contains 3.00 mg of uranium per cubic meter. (a) Given that the average ocean depth is about 4.00 km and water covers two-thirds of the Earth's surface, estimate the amount of uranium dissolved in the ocean. (b) About 0.700% of naturally occurring uranium is the fissionable isotope ^{235}U. Estimate how long the uranium in the oceans could supply the world's energy needs at the current usage of 1.50×10^{13} J/s. (c) Where does the dissolved uranium come from? (d) Is it a renewable energy source?.

11. AMT M **Review.** Suppose seawater exerts an average frictional drag force of 1.00×10^5 N on a nuclear-powered ship. The fuel consists of enriched uranium containing 3.40% of the fissionable isotope $^{235}_{92}$U, and the ship's reactor has an efficiency of 20.0%. Assuming 200 MeV is released per fission event, how far can the ship travel per kilogram of fuel?

Section 45.3 Nuclear Reactors

12. M Assume ordinary soil contains natural uranium in an amount of 1 part per million by mass. (a) How much uranium is in the top 1.00 m of soil on a 1-acre (43 560-ft^2) plot of ground, assuming the specific gravity of soil is 4.00? (b) How much of the isotope ^{235}U, appropriate for nuclear reactor fuel, is in this soil? *Hint:* See Table 44.2 for the percent abundance of $^{235}_{92}$U.

13. If the reproduction constant is 1.000 25 for a chain reaction in a fission reactor and the average time interval between successive fissions is 1.20 ms, by what factor does the reaction rate increase in one minute?

14. To minimize neutron leakage from a reactor, the ratio of the surface area to the volume should be a minimum. For a given volume V, calculate this ratio for (a) a sphere, (b) a cube, and (c) a parallelepiped of dimensions $a \times a \times 2a$. (d) Which of these shapes would have minimum leakage? Which would have maximum leakage? Explain your answers.

15. The probability of a nuclear reaction increases dramatically when the incident particle is given energy above the "Coulomb barrier," which is the electric potential energy of the two nuclei when their surfaces barely touch. Compute the Coulomb barrier for the absorption of an alpha particle by a gold nucleus.

16. A large nuclear power reactor produces approximately 3 000 MW of power in its core. Three months after a reactor is shut down, the core power from radioactive by-products is 10.0 MW. Assuming each emission delivers 1.00 MeV of energy to the power, find the activity in becquerels three months after the reactor is shut down.

17. GP According to one estimate, there are 4.40×10^6 metric tons of world uranium reserves extractable at \$130/kg or less. We wish to determine if these reserves are sufficient to supply all the world's energy needs. About 0.700% of naturally occurring uranium is the fissionable isotope ^{235}U. (a) Calculate the mass of ^{235}U in the reserve in grams. (b) Find the number of moles of ^{235}U in the reserve. (c) Find the number of ^{235}U nuclei in the reserve. (d) Assuming 200 MeV is obtained from each fission reaction and all this energy is captured, calculate the total energy in joules that can be extracted from the reserve. (e) Assuming the rate of world power consumption remains constant at 1.5×10^{13} J/s, how many years could the uranium reserve provide for all the world's energy needs? (f) What conclusion can be drawn?

18. *Why is the following situation impossible?* An engineer working on nuclear power makes a breakthrough so that he is able to control what daughter nuclei are created in a fission reaction. By carefully controlling the process, he is able to restrict the fission reactions to just this single possibility: the uranium-235 nucleus absorbs a slow neutron and splits into lanthanum-141 and bromine-94. Using this breakthrough, he is able to design and build a successful nuclear reactor in which only this single process occurs.

19. M An all-electric home uses approximately 2 000 kWh of electric energy per month. How much uranium-235 would be required to provide this house with its energy needs for one year? Assume 100% conversion efficiency and 208 MeV released per fission.

20. A particle cannot generally be localized to distances much smaller than its de Broglie wavelength. This fact can be taken to mean that a slow neutron appears to be larger to a target particle than does a fast neutron in the sense that the slow neutron has probabilities of being found over a larger volume of space. For a thermal neutron at room temperature of 300 K, find (a) the linear momentum and (b) the de Broglie wavelength. (c) State how this effective size compares with both nuclear and atomic dimensions.

Section 45.4 Nuclear Fusion

21. When a star has exhausted its hydrogen fuel, it may fuse other nuclear fuels. At temperatures above 1.00×10^8 K, helium fusion can occur. Consider the following processes. (a) Two alpha particles fuse to produce a nucleus A and a gamma ray. What is nucleus A? (b) Nucleus A from part (a) absorbs an alpha particle to produce nucleus B and a gamma ray. What is nucleus B? (c) Find the total energy released in the sequence of reactions given in parts (a) and (b).

22. An all-electric home uses 2 000 kWh of electric energy per month. Assuming all energy released from fusion could be captured, how many fusion events described by the reaction $^2_1\text{H} + ^3_1\text{H} \rightarrow ^4_2\text{He} + ^1_0\text{n}$ would be required to keep this home running for one year?

23. Find the energy released in the fusion reaction

$$^1_1\text{H} + ^2_1\text{H} \rightarrow ^3_2\text{He} + \gamma$$

24. Two nuclei having atomic numbers Z_1 and Z_2 approach each other with a total energy E. (a) When they are far apart, they interact only by electric repulsion. If they approach to a distance of 1.00×10^{-14} m, the nuclear force suddenly takes over to make them fuse. Find the minimum value of E, in terms of Z_1 and Z_2, required to produce fusion. (b) State how E depends on the atomic numbers. (c) If $Z_1 + Z_2$ is to have a certain target value such as 60, would it be energetically favorable to take $Z_1 = 1$ and $Z_2 = 59$, or $Z_1 = Z_2 = 30$, or some other choice? Explain your answer. (d) Evaluate from your expression the minimum energy for fusion for the D–D and D–T reactions (the first and third reactions in Eq. 45.4).

25. (a) Consider a fusion generator built to create 3.00 GW of power. Determine the rate of fuel burning in grams per hour if the D–T reaction is used. (b) Do the same for the D–D reaction, assuming the reaction products are split evenly between (n, ^{3}He) and (p, ^{3}H).

26. **Review.** Consider the deuterium–tritium fusion reaction with the tritium nucleus at rest:

$$^2_1\text{H} + ^3_1\text{H} \rightarrow ^4_2\text{He} + ^1_0\text{n}$$

(a) Suppose the reactant nuclei will spontaneously fuse if their surfaces touch. From Equation 44.1, determine the required distance of closest approach between their centers. (b) What is the electric potential energy (in electron volts) at this distance? (c) Suppose the deuteron is fired straight at an originally stationary tritium nucleus with just enough energy to reach the required distance of closest approach. What is the common speed of the deuterium and tritium nuclei, in terms of the initial deuteron speed v_i, as they touch? (d) Use energy methods to find the minimum initial deuteron energy required to achieve fusion. (e) Why does the fusion reaction actually occur at much lower deuteron energies than the energy calculated in part (d)?

27. Of all the hydrogen in the oceans, 0.030 0% of the mass is deuterium. The oceans have a volume of 317 million mi^3. (a) If nuclear fusion were controlled and all the deuterium in the oceans were fused to $^4_2$He, how many joules of energy would be released? (b) **What If?** World power consumption is approximately 1.50×10^{13} W. If consumption were 100 times greater, how many years would the energy calculated in part (a) last?

28. It has been suggested that fusion reactors are safe from explosion because the plasma never contains enough energy to do much damage. (a) In 1992, the TFTR reactor, with a plasma volume of approximately 50.0 m^3, achieved an ion temperature of 4.00×10^8 K, an ion density of 2.00×10^{13} cm^{-3}, and a confinement time of 1.40 s. Calculate the amount of energy stored in the plasma of the TFTR reactor. (b) How many kilograms of water at 27.0°C could be boiled away by this much energy?

29. M To understand why plasma containment is necessary, consider the rate at which an unconfined plasma would be lost. (a) Estimate the rms speed of deuterons in a plasma at a temperature of 4.00×10^8 K. (b) **What If?** Estimate the order of magnitude of the time interval during which such a plasma would remain in a 10.0-cm cube if no steps were taken to contain it.

30. Another series of nuclear reactions that can produce energy in the interior of stars is the carbon cycle first proposed by Hans Bethe in 1939, leading to his Nobel Prize in Physics in 1967. This cycle is most efficient when the central temperature in a star is above 1.6×10^7 K. Because the temperature at the center of the Sun is only 1.5×10^7 K, the following cycle produces less than 10% of the Sun's energy. (a) A high-energy proton is absorbed by ^{12}C. Another nucleus, A, is produced in the reaction, along with a gamma ray. Identify nucleus A. (b) Nucleus A decays through positron emission to form nucleus B. Identify nucleus B. (c) Nucleus B absorbs a proton to produce nucleus C and a gamma ray. Identify nucleus C. (d) Nucleus C absorbs a proton to produce nucleus D and a gamma ray. Identify nucleus D. (e) Nucleus D decays through positron emission to produce nucleus E. Identify nucleus E. (f) Nucleus E absorbs a proton to produce nucleus F plus an alpha particle. Identify nucleus F. (g) What is the significance of the final nucleus in the last step of the cycle outlined in part (f)?

31. **Review.** To confine a stable plasma, the magnetic energy density in the magnetic field (Eq. 32.14) must exceed the pressure $2nk_BT$ of the plasma by a factor of at least 10. In this problem, assume a confinement time $\tau = 1.00$ s. (a) Using Lawson's criterion, determine the ion density required for the D–T reaction. (b) From the ignition-temperature criterion, determine the required plasma pressure. (c) Determine the magnitude of the magnetic field required to contain the plasma.

Section 45.5 Radiation Damage

32. Assume an x-ray technician takes an average of eight x-rays per workday and receives a dose of 5.0 rem/yr as a result. (a) Estimate the dose in rem per x-ray taken. (b) Explain how the technician's exposure compares with low-level background radiation.

33. When gamma rays are incident on matter, the intensity of the gamma rays passing through the material varies with depth x as $I(x) = I_0 e^{-\mu x}$, where I_0 is the intensity of the radiation at the surface of the material (at $x = 0$) and μ is the linear absorption coefficient. For 0.400-MeV gamma rays in lead, the linear absorption coefficient is 1.59 cm^{-1}. (a) Determine the "half-thickness" for lead, that is, the thickness of lead that would absorb half the incident gamma rays. (b) What thickness reduces the radiation by a factor of 10^4?

34. When gamma rays are incident on matter, the intensity of the gamma rays passing through the material varies with depth x as $I(x) = I_0 e^{-\mu x}$, where I_0 is the intensity of the radiation at the surface of the material (at $x = 0$) and μ is the linear absorption coefficient. (a) Determine the "half-thickness" for a material with linear absorption coefficient μ, that is, the thickness of the material that would absorb half the incident gamma rays. (b) What thickness changes the radiation by a factor of f?

35. Review. A particular radioactive source produces 100 mrad of 2.00-MeV gamma rays per hour at a distance of 1.00 m from the source. (a) How long could a person stand at this distance before accumulating an intolerable dose of 1.00 rem? (b) **What If?** Assuming the radioactive source is a point source, at what distance would a person receive a dose of 10.0 mrad/h?

36. M A person whose mass is 75.0 kg is exposed to a whole-body dose of 0.250 Gy. How many joules of energy are deposited in the person's body?

37. Review. The danger to the body from a high dose of gamma rays is not due to the amount of energy absorbed; rather, it is due to the ionizing nature of the radiation. As an illustration, calculate the rise in body temperature that results if a "lethal" dose of 1 000 rad is absorbed strictly as internal energy. Take the specific heat of living tissue as 4 186 J/kg · °C.

38. Review. *Why is the following situation impossible?* A "clever" technician takes his 20-min coffee break and boils some water for his coffee with an x-ray machine. The machine produces 10.0 rad/s, and the temperature of the water in an insulated cup is initially 50.0°C.

39. W A small building has become accidentally contaminated with radioactivity. The longest-lived material in the building is strontium-90. ($^{90}_{38}$Sr has an atomic mass 89.907 7 u, and its half-life is 29.1 yr. It is particularly dangerous because it substitutes for calcium in bones.) Assume the building initially contained 5.00 kg of this substance uniformly distributed throughout the building and the safe level is defined as less than 10.0 decays/min (which is small compared with background radiation). How long will the building be unsafe?

40. Technetium-99 is used in certain medical diagnostic procedures. Assume 1.00×10^{-8} g of ^{99}Tc is injected into a 60.0-kg patient and half of the 0.140-MeV gamma rays are absorbed in the body. Determine the total radiation dose received by the patient.

41. W To destroy a cancerous tumor, a dose of gamma radiation with a total energy of 2.12 J is to be delivered in 30.0 days from implanted sealed capsules containing palladium-103. Assume this isotope has a half-life of 17.0 d and emits gamma rays of energy 21.0 keV, which are entirely absorbed within the tumor. (a) Find the initial activity of the set of capsules. (b) Find the total mass of radioactive palladium these "seeds" should contain.

42. Strontium-90 from the testing of nuclear bombs can still be found in the atmosphere. Each decay of ^{90}Sr releases 1.10 MeV of energy into the bones of a person who has had strontium replace his or her body's calcium. Assume a 70.0-kg person receives 1.00 ng of ^{90}Sr from contaminated milk. Take the half-life of ^{90}Sr to be 29.1 yr. Calculate the absorbed dose rate (in joules per kilogram) in one year.

Section 45.6 Uses of Radiation

43. M When gamma rays are incident on matter, the intensity of the gamma rays passing through the material varies with depth x as $I(x) = I_0 e^{-\mu x}$, where I_0 is the intensity of the radiation at the surface of the material (at $x = 0$) and μ is the linear absorption coefficient. For low-energy gamma rays in steel, take the absorption coefficient to be 0.720 mm^{-1}. (a) Determine the "half-thickness" for steel, that is, the thickness of steel that would absorb half the incident gamma rays. (b) In a steel mill, the thickness of sheet steel passing into a roller is measured by monitoring the intensity of gamma radiation reaching a detector below the rapidly moving metal from a small source immediately above the metal. If the thickness of the sheet changes from 0.800 mm to 0.700 mm, by what percentage does the gamma-ray intensity change?

44. A method called *neutron activation analysis* can be used for chemical analysis at the level of isotopes. When a sample is irradiated by neutrons, radioactive atoms are produced continuously and then decay according to their characteristic half-lives. (a) Assume one species of radioactive nuclei is produced at a constant rate R and its decay is described by the conventional radioactive decay law. Assuming irradiation begins at time $t = 0$, show that the number of radioactive atoms accumulated at time t is

$$N = \frac{R}{\lambda}(1 - e^{-\lambda t})$$

(b) What is the maximum number of radioactive atoms that can be produced?

45. You want to find out how many atoms of the isotope ^{65}Cu are in a small sample of material. You bombard the sample with neutrons to ensure that on the order of 1% of these copper nuclei absorb a neutron. After activation, you turn off the neutron flux and then use a highly efficient detector to monitor the gamma radiation that comes out of the sample. Assume half of the ^{66}Cu nuclei emit a 1.04-MeV gamma ray in their decay. (The other half of the activated nuclei decay directly to the ground state of ^{66}Ni.) If after 10 min (two half-lives) you have detected 1.00×10^4 MeV of photon energy at 1.04 MeV, (a) approximately how many ^{65}Cu atoms are in the sample? (b) Assume the sample contains natural copper. Refer to the isotopic abundances listed in Table 44.2 and estimate the total mass of copper in the sample.

Additional Problems

46. A fusion reaction that has been considered as a source of energy is the absorption of a proton by a boron-11 nucleus to produce three alpha particles:

$$^1_1\text{H} + {}^{11}_5\text{B} \rightarrow 3(^4_2\text{He})$$

This reaction is an attractive possibility because boron is easily obtained from the Earth's crust. A disadvantage is that the protons and boron nuclei must have large kinetic energies for the reaction to take place. This requirement contrasts with the initiation of uranium fission by slow neutrons. (a) How much energy is released in each reaction? (b) Why must the reactant particles have high kinetic energies?

47. Review. A very slow neutron (with speed approximately equal to zero) can initiate the reaction

$$^{1}_{0}\text{n} + {}^{10}_{5}\text{B} \rightarrow {}^{7}_{3}\text{Li} + {}^{4}_{2}\text{He}$$

The alpha particle moves away with speed 9.25×10^6 m/s. Calculate the kinetic energy of the lithium nucleus. Use nonrelativistic equations.

48. Review. The first nuclear bomb was a fissioning mass of plutonium-239 that exploded in the Trinity test before dawn on July 16, 1945, at Alamogordo, New Mexico. Enrico Fermi was 14 km away, lying on the ground facing away from the bomb. After the whole sky had flashed with unbelievable brightness, Fermi stood up and began dropping bits of paper to the ground. They first fell at his feet in the calm and silent air. As the shock wave passed, about 40 s after the explosion, the paper then in flight jumped approximately 2.5 m away from ground zero. (a) Equation 17.10 describes the relationship between the pressure amplitude ΔP_{max} of a sinusoidal air compression wave and its displacement amplitude s_{max}. The compression pulse produced by the bomb explosion was not a sinusoidal wave, but let's use the same equation to compute an estimate for the pressure amplitude, taking $\omega \sim 1\ \text{s}^{-1}$ as an estimate for the angular frequency at which the pulse ramps up and down. (b) Find the change in volume ΔV of a sphere of radius 14 km when its radius increases by 2.5 m. (c) The energy carried by the blast wave is the work done by one layer of air on the next as the wave crest passes. An extension of the logic used to derive Equation 20.8 shows that this work is given by $(\Delta P_{\text{max}})(\Delta V)$. Compute an estimate for this energy. (d) Assume the blast wave carried on the order of one-tenth of the explosion's energy. Make an order-of-magnitude estimate of the bomb yield. (e) One ton of exploding TNT releases 4.2 GJ of energy. What was the order of magnitude of the energy of the Trinity test in equivalent tons of TNT? Fermi's immediate knowledge of the bomb yield agreed with that determined days later by analysis of elaborate measurements.

49. On August 6, 1945, the United States dropped on Hiroshima a nuclear bomb that released 5×10^{13} J of energy, equivalent to that from 12 000 tons of TNT. The fission of one $^{235}_{92}$U nucleus releases an average of 208 MeV. Estimate (a) the number of nuclei fissioned and (b) the mass of this $^{235}_{92}$U.

50. (a) A student wishes to measure the half-life of a radioactive substance using a small sample. Consecutive clicks of her radiation counter are randomly spaced in time. The counter registers 372 counts during one 5.00-min interval and 337 counts during the next 5.00 min. The average background rate is 15 counts per minute. Find the most probable value for the half-life. (b) Estimate the uncertainty in the half-life determination in part (a). Explain your reasoning.

51. M In a Geiger–Mueller tube for detecting radiation (see Problem 68 in Chapter 25), the voltage between the electrodes is typically 1.00 kV and the current pulse discharges a 5.00-pF capacitor. (a) What is the energy amplification of this device for a 0.500-MeV electron? (b) How many electrons participate in the avalanche caused by the single initial electron?

52. Review. Consider a nucleus at rest, which then spontaneously splits into two fragments of masses m_1 and m_2. (a) Show that the fraction of the total kinetic energy carried by fragment m_1 is

$$\frac{K_1}{K_{\text{tot}}} = \frac{m_2}{m_1 + m_2}$$

and the fraction carried by m_2 is

$$\frac{K_2}{K_{\text{tot}}} = \frac{m_1}{m_1 + m_2}$$

assuming relativistic corrections can be ignored. A stationary $^{236}_{92}$U nucleus fissions spontaneously into two primary fragments, $^{87}_{35}$Br and $^{149}_{57}$La. (b) Calculate the disintegration energy. The required atomic masses are 86.920 711 u for $^{87}_{35}$Br, 148.934 370 u for $^{149}_{57}$La, and 236.045 562 u for $^{236}_{92}$U. (c) How is the disintegration energy split between the two primary fragments? (d) Calculate the speed of each fragment immediately after the fission.

53. Consider the carbon cycle in Problem 30. (a) Calculate the Q value for each of the six steps in the carbon cycle listed in Problem 30. (b) In the second and fifth steps of the cycle, the positron that is ejected combines with an electron to form two photons. The energies of these photons must be included in the energy released in the cycle. How much energy is released by these annihilations in each of the two steps? (c) What is the overall energy released in the carbon cycle? (d) Do you think that the energy carried off by the neutrinos is deposited in the star? Explain.

54. A fission reactor is hit by a missile, and 5.00×10^6 Ci of ^{90}Sr, with half-life 29.1 yr, evaporates into the air. The strontium falls out over an area of 10^4 km^2. After what time interval will the activity of the ^{90}Sr reach the agriculturally "safe" level of 2.00 μCi/m^2?

55. W The alpha-emitter plutonium-238 ($^{238}_{94}$Pu, atomic mass 238.049 560 u, half-life 87.7 yr) was used in a nuclear energy source on the Apollo Lunar Surface Experiments Package (Fig. P45.55, page 1444). The energy source, called the Radioisotope Thermoelectric Generator, is the small gray object to the left of the gold-shrouded Central Station in the photograph. Assume the source contains 3.80 kg of ^{238}Pu and the efficiency for conversion of radioactive decay energy to energy transferred by electrical transmission is 3.20%. Determine the initial power output of the source.

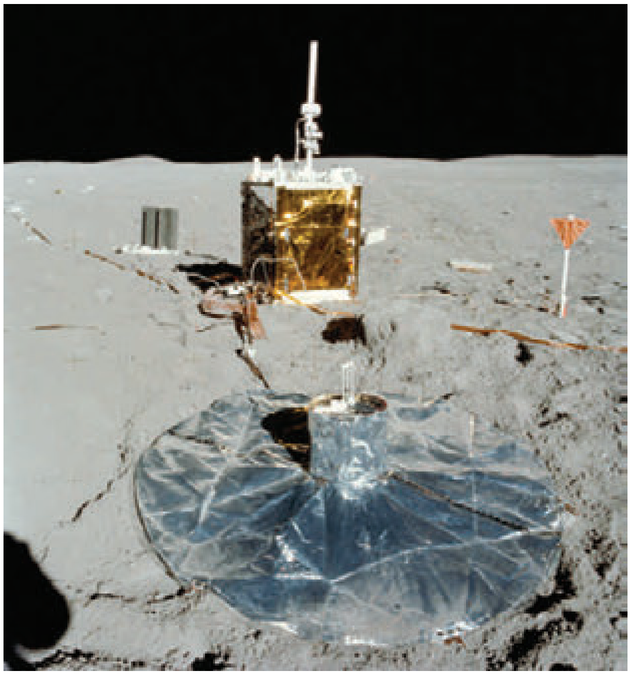


Figure P45.55

56. The half-life of tritium is 12.3 yr. (a) If the TFTR fusion reactor contained 50.0 m^3 of tritium at a density equal to 2.00×10^{14} ions/cm^3, how many curies of tritium were in the plasma? (b) State how this value compares with a fission inventory (the estimated supply of fissionable material) of 4.00×10^{10} Ci.

57. Review. A nuclear power plant operates by using the energy released in nuclear fission to convert 20°C water into 400°C steam. How much water could theoretically be converted to steam by the complete fissioning of 1.00 g of ^{235}U at 200 MeV/fission?

58. Review. A nuclear power plant operates by using the energy released in nuclear fission to convert liquid water at T_c into steam at T_h. How much water could theoretically be converted to steam by the complete fissioning of a mass m of ^{235}U if the energy released per fission event is E?

59. Consider the two nuclear reactions

$$\begin{aligned} A + B &\rightarrow C + E \\ C + D &\rightarrow F + G \end{aligned}$$

(a) Show that the net disintegration energy for these two reactions ($Q_{\text{net}} = Q_{\text{I}} + Q_{\text{II}}$) is identical to the disintegration energy for the net reaction

$$A + B + D \rightarrow E + F + G$$

(b) One chain of reactions in the proton–proton cycle in the Sun's core is

$$\begin{aligned} {}^1_1\text{H} + {}^1_1\text{H} &\rightarrow {}^2_1\text{H} + {}^0_1\text{e} + \nu \\ {}^0_1\text{e} + {}^{\ 0}_{-1}\text{e} &\rightarrow 2\gamma \\ {}^1_1\text{H} + {}^2_1\text{H} &\rightarrow {}^3_2\text{He} + \gamma \\ {}^1_1\text{H} + {}^3_2\text{He} &\rightarrow {}^4_2\text{He} + {}^0_1\text{e} + \nu \\ {}^0_1\text{e} + {}^{\ 0}_{-1}\text{e} &\rightarrow 2\gamma \end{aligned}$$

Based on part (a), what is Q_{net} for this sequence?

60. Natural uranium must be processed to produce uranium enriched in ^{235}U for weapons and power plants. The processing yields a large quantity of nearly pure ^{238}U as a by-product, called "depleted uranium." Because of its high mass density, ^{238}U is used in armor-piercing artillery shells. (a) Find the edge dimension of a 70.0-kg cube of ^{238}U ($\rho = 19.1 \times 10^3$ kg/m^3). (b) The isotope ^{238}U has a long half-life of 4.47×10^9 yr. As soon as one nucleus decays, a relatively rapid series of 14 steps begins that together constitute the net reaction

$$^{238}_{92}\text{U} \rightarrow 8({}^4_2\text{He}) + 6({}^{\ 0}_{-1}\text{e}) + {}^{206}_{82}\text{Pb} + 6\bar{\nu} + Q_{\text{net}}$$

Find the net decay energy. (Refer to Table 44.2.) (c) Argue that a radioactive sample with decay rate R and decay energy Q has power output $P = QR$. (d) Consider an artillery shell with a jacket of 70.0 kg of ^{238}U. Find its power output due to the radioactivity of the uranium and its daughters. Assume the shell is old enough that the daughters have reached steady-state amounts. Express the power in joules per year. (e) **What If?** A 17-year-old soldier of mass 70.0 kg works in an arsenal where many such artillery shells are stored. Assume his radiation exposure is limited to 5.00 rem per year. Find the rate in joules per year at which he can absorb energy of radiation. Assume an average RBE factor of 1.10.

61. Suppose the target in a laser fusion reactor is a sphere of solid hydrogen that has a diameter of 1.50×10^{-4} m and a density of 0.200 g/cm^3. Assume half of the nuclei are ^{2}H and half are ^{3}H. (a) If 1.00% of a 200-kJ laser pulse is delivered to this sphere, what temperature does the sphere reach? (b) If all the hydrogen fuses according to the D–T reaction, how many joules of energy are released?

62. When photons pass through matter, the intensity I of the beam (measured in watts per square meter) decreases exponentially according to

$$I = I_0 e^{-\mu x}$$

where I is the intensity of the beam that just passed through a thickness x of material and I_0 is the intensity of the incident beam. The constant μ is known as the linear absorption coefficient, and its value depends on the absorbing material and the wavelength of the photon beam. This wavelength (or energy) dependence allows us to filter out unwanted wavelengths from a broad-spectrum x-ray beam. (a) Two x-ray beams of wavelengths λ_1 and λ_2 and equal incident intensities pass through the same metal plate. Show that the ratio of the emergent beam intensities is

$$\frac{I_2}{I_1} = e^{-(\mu_2 - \mu_1)x}$$

(b) Compute the ratio of intensities emerging from an aluminum plate 1.00 mm thick if the incident beam contains equal intensities of 50 pm and 100 pm x-rays. The values of μ for aluminum at these two wavelengths are $\mu_1 = 5.40\ \text{cm}^{-1}$ at 50 pm and $\mu_2 = 41.0\ \text{cm}^{-1}$ at 100 pm. (c) Repeat part (b) for an aluminum plate 10.0 mm thick.

63. Assume a deuteron and a triton are at rest when they fuse according to the reaction

$${}^2_1\text{H} + {}^3_1\text{H} \rightarrow {}^4_2\text{He} + {}^1_0\text{n}$$

Determine the kinetic energy acquired by the neutron.

64. (a) Calculate the energy (in kilowatt-hours) released if 1.00 kg of ^{239}Pu undergoes complete fission and the energy released per fission event is 200 MeV. (b) Calculate the energy (in electron volts) released in the deuterium–tritium fusion reaction

$${}^2_1\text{H} + {}^3_1\text{H} \rightarrow {}^4_2\text{He} + {}^1_0\text{n}$$

(c) Calculate the energy (in kilowatt-hours) released if 1.00 kg of deuterium undergoes fusion according to this reaction. (d) **What If?** Calculate the energy (in kilowatt-hours) released by the combustion of 1.00 kg of carbon in coal if each $C + O_2 \rightarrow CO_2$ reaction yields 4.20 eV. (e) List advantages and disadvantages of each of these methods of energy generation.

65. Consider a 1.00-kg sample of natural uranium composed primarily of ^{238}U, a smaller amount (0.720% by mass) of ^{235}U, and a trace (0.005 00%) of ^{234}U, which has a half-life of 2.44×10^5 yr. (a) Find the activity in curies due to each of the isotopes. (b) What fraction of the total activity is due to each isotope? (c) Explain whether the activity of this sample is dangerous.

66. Approximately 1 of every 3 300 water molecules contains one deuterium atom. (a) If all the deuterium nuclei in 1 L of water are fused in pairs according to the D–D fusion reaction $^2\text{H} + {}^2\text{H} \rightarrow {}^3\text{He} + \text{n} + 3.27$ MeV, how much energy in joules is liberated? (b) **What If?** Burning gasoline produces approximately 3.40×10^7 J/L. State how the energy obtainable from the fusion of the deuterium in 1 L of water compares with the energy liberated from the burning of 1 L of gasoline.

67. Carbon detonations are powerful nuclear reactions that temporarily tear apart the cores inside massive stars late in their lives. These blasts are produced by carbon fusion, which requires a temperature of approximately 6×10^8 K to overcome the strong Coulomb repulsion between carbon nuclei. (a) Estimate the repulsive energy barrier to fusion, using the temperature required for carbon fusion. (In other words, what is the average kinetic energy of a carbon nucleus at 6×10^8 K?) (b) Calculate the energy (in MeV) released in each of these "carbon-burning" reactions:

$$^{12}\text{C} + {}^{12}\text{C} \rightarrow {}^{20}\text{Ne} + {}^4\text{He}$$
$$^{12}\text{C} + {}^{12}\text{C} \rightarrow {}^{24}\text{Mg} + \gamma$$

(c) Calculate the energy in kilowatt-hours given off when 2.00 kg of carbon completely fuse according to the first reaction.

68. A sealed capsule containing the radiopharmaceutical phosphorus-32, an e^- emitter, is implanted into a patient's tumor. The average kinetic energy of the beta particles is 700 keV. The initial activity is 5.22 MBq. Assume the beta particles are completely absorbed in 100 g of tissue. Determine the absorbed dose during a 10.0-day period.

69. A certain nuclear plant generates internal energy at a rate of 3.065 GW and transfers energy out of the plant by electrical transmission at a rate of 1.000 GW. Of the waste energy, 3.0% is ejected to the atmosphere and the remainder is passed into a river. A state law requires that the river water be warmed by no more than 3.50°C when it is returned to the river. (a) Determine the amount of cooling water necessary (in kilograms per hour and cubic meters per hour) to cool the plant. (b) Assume fission generates 7.80×10^{10} J/g of ^{235}U. Determine the rate of fuel burning (in kilograms per hour) of ^{235}U.

70. The Sun radiates energy at the rate of 3.85×10^{26} W. Suppose the net reaction $4({}^1_1\text{H}) + 2({}^{\ 0}_{-1}\text{e}) \rightarrow {}^4_2\text{He} + 2\nu + \gamma$ accounts for all the energy released. Calculate the number of protons fused per second.

Challenge Problems

71. During the manufacture of a steel engine component, radioactive iron (^{59}Fe) with a half-life of 45.1 d is included in the total mass of 0.200 kg. The component is placed in a test engine when the activity due to this isotope is 20.0 μCi. After a 1 000-h test period, some of the lubricating oil is removed from the engine and found to contain enough ^{59}Fe to produce 800 disintegrations/min/L of oil. The total volume of oil in the engine is 6.50 L. Calculate the total mass worn from the engine component per hour of operation.

72. (a) At time $t = 0$, a sample of uranium is exposed to a neutron source that causes N_0 nuclei to undergo fission. The sample is in a supercritical state, with a reproduction constant $K > 1$. A chain reaction occurs that proliferates fission throughout the mass of uranium. The chain reaction can be thought of as a succession of *generations*. The N_0 fissions produced initially are the zeroth generation of fissions. From this generation, N_0K neutrons go off to produce fission of new uranium nuclei. The N_0K fissions that occur subsequently are the first generation of fissions, and from this generation N_0K^2 neutrons go in search of uranium nuclei in which to cause fission. The subsequent N_0K^2 fissions are the second generation of fissions. This process can continue until all the uranium nuclei have fissioned. Show that the cumulative total of fissions N that have occurred up to and including the nth generation after the zeroth generation is given by

$$N = N_0\left(\frac{K^{n+1} - 1}{K - 1}\right)$$

(b) Consider a hypothetical uranium weapon made from 5.50 kg of isotopically pure ^{235}U. The chain reaction has a reproduction constant of 1.10 and starts with a zeroth generation of 1.00×10^{20} fissions. The average time interval between one fission generation and the next is 10.0 ns. How long after the zeroth generation does it take the uranium in this weapon to fission completely? (c) Assume the bulk modulus of uranium is 150 GPa. Find the speed of sound in uranium. You may ignore the density difference between ^{235}U and natural uranium. (d) Find the time interval required for a compressional wave to cross the radius of a 5.50-kg sphere of uranium. This time interval indicates how quickly the motion of explosion begins. (e) Fission must occur in a time interval that is short compared with that in part (d); otherwise, most of the uranium will disperse in small chunks without having fissioned. Can the weapon considered in part (b) release the explosive energy of all its uranium? If so, how much energy does it release in equivalent tons of TNT? Assume one ton of TNT releases 4.20 GJ and each uranium fission releases 200 MeV of energy.

73. Assume a photomultiplier tube for detecting radiation has seven dynodes with potentials of 100, 200, 300, . . . , 700 V as shown in Figure P45.73. The average energy required to free an electron from the dynode surface is 10.0 eV. Assume only one electron is incident and the tube functions with 100% efficiency. (a) How many electrons are freed at the first dynode at 100 V? (b) How many electrons are collected at the last dynode? (c) What is the energy available to the counter for all the electrons arriving at the last dynode?

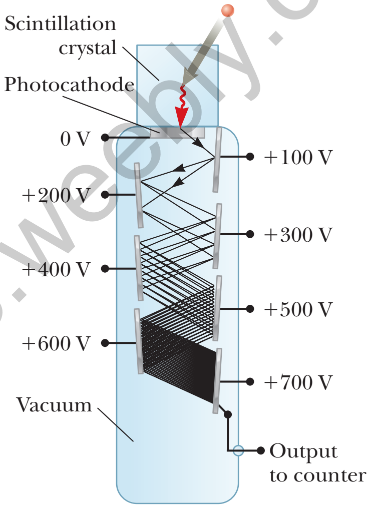
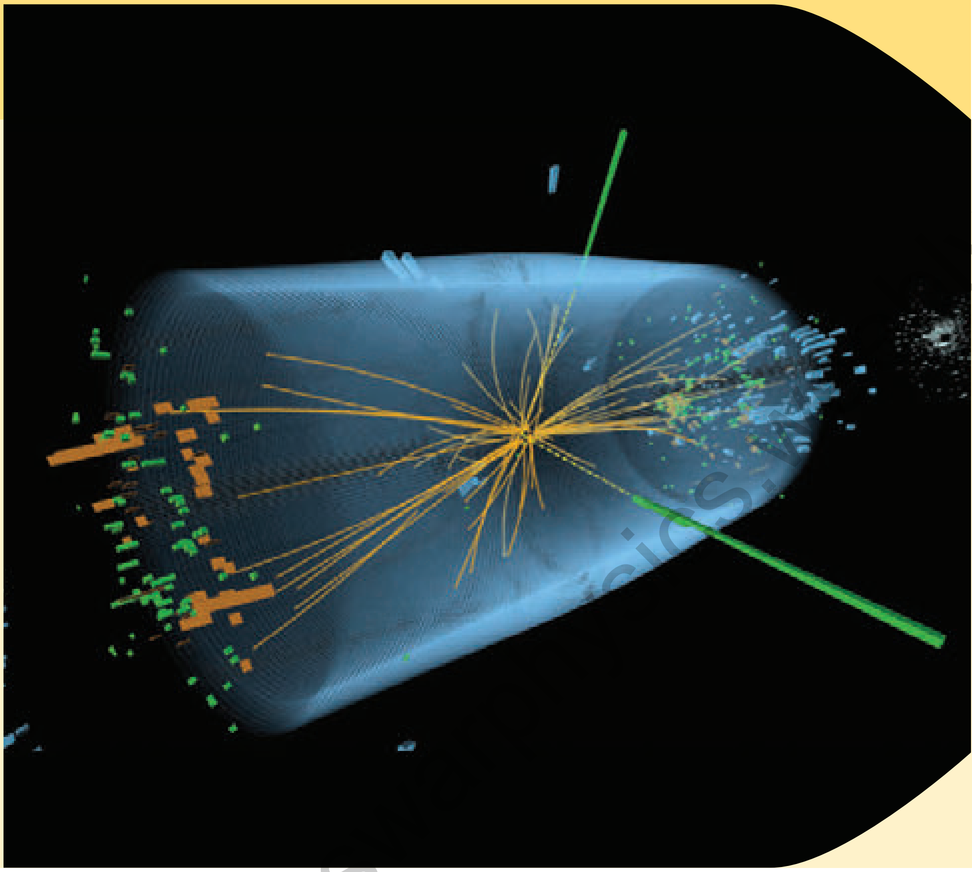


Figure P45.73

CHAPTER

46

Particle Physics and Cosmology



One of the most intense areas of current research is the hunt for the Higgs boson, discussed in Section 46.10. The photo shows an event recorded at the Large Hadron Collider in July 2012 that shows particles consistent with the creation of a Higgs boson. The data is not entirely conclusive, however, and the hunt continues. *(CERN)*

The word *atom* comes from the Greek *atomos,* which means "indivisible." The early Greeks believed that atoms were the indivisible constituents of matter; that is, they regarded them as elementary particles. After 1932, physicists viewed all matter as consisting of three constituent particles: electrons, protons, and neutrons. Beginning in the 1940s, many "new" particles were discovered in experiments involving high-energy collisions between known particles. The new particles are characteristically very unstable and have very short half-lives, ranging between 10^{-6} s and 10^{-23} s. So far, more than 300 of these particles have been catalogued.

Until the 1960s, physicists were bewildered by the great number and variety of subatomic particles that were being discovered. They wondered whether the particles had no systematic relationship connecting them or whether a pattern was emerging that would provide a better understanding of the elaborate structure in the subatomic world. For example, that the neutron has a magnetic moment despite having zero electric charge (Section 44.8) suggests an underlying structure to the neutron. The periodic table explains how more than 100 elements can be formed from three types of particles (electrons, protons, and neutrons), which suggests there is, perhaps, a means of forming more than 300 subatomic particles from a small number of basic building blocks.

Recall Figure 1.2, which illustrated the various levels of structure in matter. We studied the atomic structure of matter in Chapter 42. In Chapter 44, we investigated the substructure of the atom by describing the structure of the nucleus. As mentioned in Section 1.2, the protons and neutrons in the nucleus, and a host of other exotic particles, are now known to be composed of six different varieties of particles called *quarks.* In this concluding chapter, we examine the current theory of elementary particles, in which all matter is constructed from only two families of particles, quarks and leptons. We also discuss how clarifications of such models might help scientists understand the birth and evolution of the Universe.

46.1 The Fundamental Forces in Nature

As noted in Section 5.1, all natural phenomena can be described by four fundamental forces acting between particles. In order of decreasing strength, they are the nuclear force, the electromagnetic force, the weak force, and the gravitational force.

The nuclear force discussed in Chapter 44 is an attractive force between nucleons. It has a very short range and is negligible for separation distances between nucleons greater than approximately 10^{-15} m (about the size of the nucleus). The electromagnetic force, which binds atoms and molecules together to form ordinary matter, has a strength of approximately 10^{-2} times that of the nuclear force. This long-range force decreases in magnitude as the inverse square of the separation between interacting particles. The weak force is a short-range force that tends to produce instability in certain nuclei. It is responsible for decay processes, and its strength is only about 10^{-5} times that of the nuclear force. Finally, the gravitational force is a long-range force that has a strength of only about 10^{-39} times that of the nuclear force. Although this familiar interaction is the force that holds the planets, stars, and galaxies together, its effect on elementary particles is negligible.

In Section 13.3, we discussed the difficulty early scientists had with the notion of the gravitational force acting at a distance, with no physical contact between the interacting objects. To resolve this difficulty, the concept of the gravitational field was introduced. Similarly, in Chapter 23, we introduced the electric field to describe the electric force acting between charged objects, and we followed that with a discussion of the magnetic field in Chapter 29. For each of these types of fields, we developed a particle in a field analysis model. In modern physics, the nature of the interaction between particles is carried a step further. These interactions are described in terms of the exchange of entities called **field particles** or **exchange particles.** Field particles are also called **gauge bosons.**[1] The interacting particles continuously emit and absorb field particles. The emission of a field particle by one particle and its absorption by another manifests as a force between the two interacting particles. In the case of the electromagnetic interaction, for instance, the field particles are photons. In the language of modern physics, the electromagnetic force is said to be *mediated* by photons, and photons are the field particles of the electromagnetic field. Likewise, the nuclear force is mediated by field particles called *gluons.* The weak force is mediated by field particles called *W* and *Z bosons,* and the gravitational force is proposed to be mediated by field particles called *gravitons.* These interactions, their ranges, and their relative strengths are summarized in Table 46.1.

[1]The word *bosons* suggests that the field particles have integral spin as discussed in Section 43.8. The word *gauge* comes from *gauge theory,* which is a sophisticated mathematical analysis that is beyond the scope of this book.

Table 46.1 Particle Interactions

Interactions	Relative Strength	Range of Force	Mediating Field Particle	Mass of Field Particle (GeV/c^2)
Nuclear	1	Short ($\approx$ 1 fm)	Gluon	0
Electromagnetic	10^{-2}	∞	Photon	0
Weak	10^{-5}	Short ($\approx 10^{-3}$ fm)	$W^{\pm}$, Z^0 bosons	80.4, 80.4, 91.2
Gravitational	10^{-39}	∞	Graviton	0

46.2 Positrons and Other Antiparticles

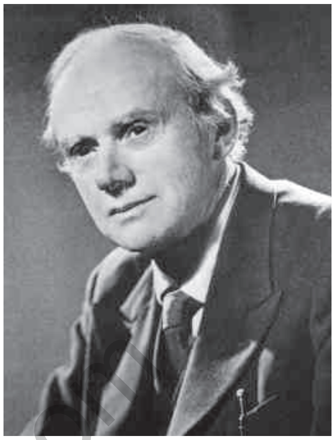
In the 1920s, Paul Dirac developed a relativistic quantum-mechanical description of the electron that successfully explained the origin of the electron's spin and its magnetic moment. His theory had one major problem, however: its relativistic wave equation required solutions corresponding to negative energy states, and if negative energy states existed, an electron in a state of positive energy would be expected to make a rapid transition to one of these states, emitting a photon in the process.

Dirac circumvented this difficulty by postulating that all negative energy states are filled. The electrons occupying these negative energy states are collectively called the *Dirac sea*. Electrons in the Dirac sea (the blue area in Fig. 46.1) are not directly observable because the Pauli exclusion principle does not allow them to react to external forces; there are no available states to which an electron can make a transition in response to an external force. Therefore, an electron in such a state acts as an isolated system unless an interaction with the environment is strong enough to excite the electron to a positive energy state. Such an excitation causes one of the negative energy states to be vacant as in Figure 46.1, leaving a hole in the sea of filled states. This process is described by the nonisolated system model: as energy enters the system by some transfer mechanism, the system energy increases and the electron is excited to a higher energy level. *The hole can react to external forces and is observable.* The hole reacts in a way similar to that of the electron except that it has a positive charge: it is the *antiparticle* to the electron.

This theory strongly suggested that *an antiparticle exists for every particle,* not only for fermions such as electrons but also for bosons. It has subsequently been verified that practically every known elementary particle has a distinct antiparticle. Among the exceptions are the photon and the neutral pion (π^0; see Section 46.3). Following the construction of high-energy accelerators in the 1950s, many other antiparticles were revealed. They included the antiproton, discovered by Emilio Segré (1905–1989) and Owen Chamberlain (1920–2006) in 1955, and the antineutron, discovered shortly thereafter. The antiparticle for a charged particle has the same mass as the particle but opposite charge.[2] For example, the electron's antiparticle (the *positron* mentioned in Section 44.4) has a rest energy of 0.511 MeV and a positive charge of $+1.60 \times 10^{-19}$ C.

Carl Anderson (1905–1991) observed the positron experimentally in 1932 and was awarded a Nobel Prize in Physics in 1936 for this achievement. Anderson discovered the positron while examining tracks created in a cloud chamber by electron-like particles of positive charge. (These early experiments used cosmic rays—mostly energetic protons passing through interstellar space—to initiate high-energy reactions on the order of several GeV.) To discriminate between positive and negative charges, Anderson placed the cloud chamber in a magnetic field,

[2]Antiparticles for uncharged particles, such as the neutron, are a little more difficult to describe. One basic process that can detect the existence of an antiparticle is pair annihilation. For example, a neutron and an antineutron can annihilate to form two gamma rays. Because the photon and the neutral pion do not have distinct antiparticles, pair annihilation is not observed with either of these particles.



Paul Adrien Maurice Dirac
British Physicist (1902–1984)
Dirac was instrumental in the understanding of antimatter and the unification of quantum mechanics and relativity. He made many contributions to the development of quantum physics and cosmology. In 1933, Dirac won a Nobel Prize in Physics.

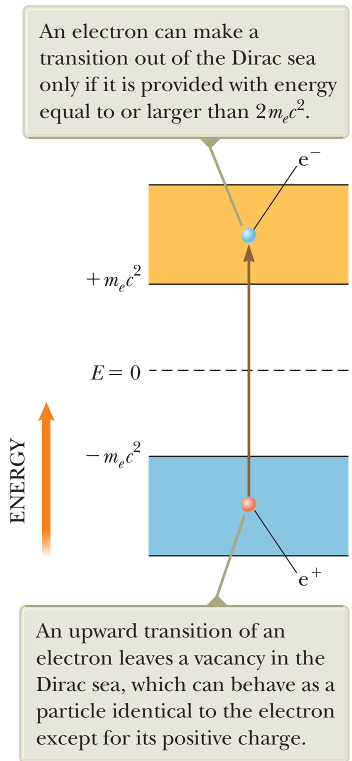


Figure 46.1 Dirac's model for the existence of antielectrons (positrons). The minimum energy for an electron to exist in the gold band is its rest energy m_ec^2. The blue band of negative energies is filled with electrons.

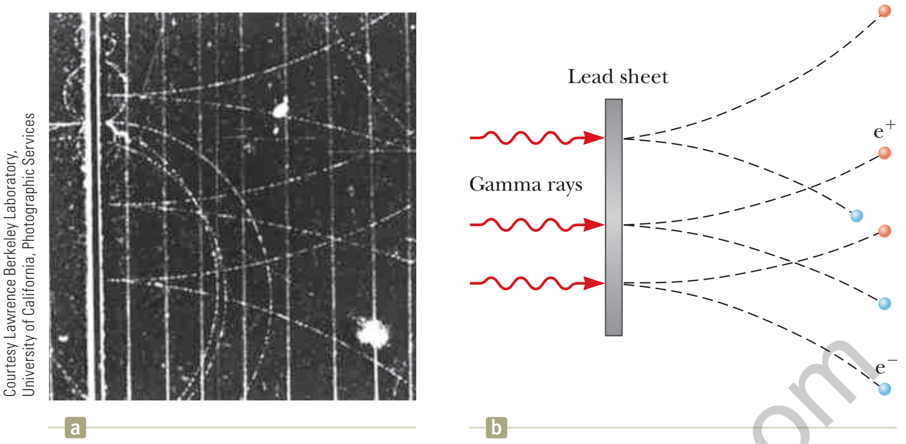


Figure 46.2 (a) Bubble-chamber tracks of electron–positron pairs produced by 300-MeV gamma rays striking a lead sheet from the left. (b) The pertinent pair-production events. The positrons deflect upward and the electrons downward in an applied magnetic field.

Pitfall Prevention 46.1

Antiparticles An antiparticle is not identified solely on the basis of opposite charge; even neutral particles have antiparticles, which are defined in terms of other properties, such as spin.

causing moving charges to follow curved paths. He noted that some of the electron-like tracks deflected in a direction corresponding to a positively charged particle.

Since Anderson's discovery, positrons have been observed in a number of experiments. A common source of positrons is **pair production.** In this process, a gamma-ray photon with sufficiently high energy interacts with a nucleus and an electron–positron pair is created from the photon. (The presence of the nucleus allows the principle of conservation of momentum to be satisfied.) Because the total rest energy of the electron–positron pair is $2m_ec^2 = 1.02$ MeV (where m_e is the mass of the electron), the photon must have at least this much energy to create an electron–positron pair. The energy of a photon is converted to rest energy of the electron and positron in accordance with Einstein's relationship $E_R = mc^2$. If the gamma-ray photon has energy in excess of the rest energy of the electron–positron pair, the excess appears as kinetic energy of the two particles. Figure 46.2 shows early observations of tracks of electron–positron pairs in a bubble chamber created by 300-MeV gamma rays striking a lead sheet.

Quick Quiz 46.1 Given the identification of the particles in Figure 46.2b, is the direction of the external magnetic field in Figure 46.2a **(a)** into the page, **(b)** out of the page, or **(c)** impossible to determine?

The reverse process can also occur. Under the proper conditions, an electron and a positron can annihilate each other to produce two gamma-ray photons that have a combined energy of at least 1.02 MeV:

$$e^- + e^+ \rightarrow 2\gamma$$

Because the initial momentum of the electron–positron system is approximately zero, the two gamma rays travel in opposite directions after the annihilation, satisfying the principle of conservation of momentum for the isolated system.

Electron–positron annihilation is used in the medical diagnostic technique called *positron-emission tomography* (PET). The patient is injected with a glucose solution containing a radioactive substance that decays by positron emission, and the material is carried throughout the body by the blood. A positron emitted during a decay event in one of the radioactive nuclei in the glucose solution annihilates with an electron in the surrounding tissue, resulting in two gamma-ray photons emitted in opposite directions. A gamma detector surrounding the patient pinpoints the source of the photons and, with the assistance of a computer, displays an image of the sites at which the glucose accumulates. (Glucose metabolizes rapidly in cancerous tumors and accumulates at those sites, providing a strong signal for a PET detector system.) The images from a PET scan can indicate a wide variety of disorders in the brain, including Alzheimer's disease (Fig. 46.3). In addition, because glucose metabolizes more rapidly in active areas

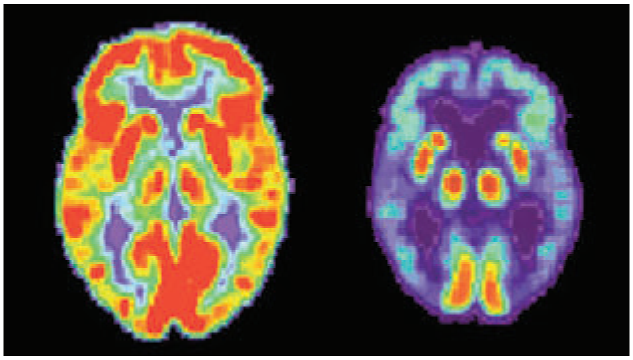


Figure 46.3 PET scans of the brain of a healthy older person *(left)* and that of a patient suffering from Alzheimer's disease *(right)*. Lighter regions contain higher concentrations of radioactive glucose, indicating higher metabolism rates and therefore increased brain activity.

National Institutes of Health

of the brain, a PET scan can indicate areas of the brain involved in the activities in which the patient is engaging at the time of the scan, such as language use, music, and vision.

46.3 Mesons and the Beginning of Particle Physics

Physicists in the mid-1930s had a fairly simple view of the structure of matter. The building blocks were the proton, the electron, and the neutron. Three other particles were either known or postulated at the time: the photon, the neutrino, and the positron. Together these six particles were considered the fundamental constituents of matter. With this simple picture, however, no one was able to answer the following important question: the protons in any nucleus should strongly repel one another due to their charges of the same sign, so what is the nature of the force that holds the nucleus together? Scientists recognized that this mysterious force must be much stronger than anything encountered in nature up to that time. This force is the nuclear force discussed in Section 44.1 and examined in historical perspective in the following paragraphs.

The first theory to explain the nature of the nuclear force was proposed in 1935 by Japanese physicist Hideki Yukawa, an effort that earned him a Nobel Prize in Physics in 1949. To understand Yukawa's theory, recall the introduction of field particles in Section 46.1, which stated that each fundamental force is mediated by a field particle exchanged between the interacting particles. Yukawa used this idea to explain the nuclear force, proposing the existence of a new particle whose exchange between nucleons in the nucleus causes the nuclear force. He established that the range of the force is inversely proportional to the mass of this particle and predicted the mass to be approximately 200 times the mass of the electron. (Yukawa's predicted particle is *not* the gluon mentioned in Section 46.1, which is massless and is today considered to be the field particle for the nuclear force.) Because the new particle would have a mass between that of the electron and that of the proton, it was called a **meson** (from the Greek *meso,* "middle").

In efforts to substantiate Yukawa's predictions, physicists began experimental searches for the meson by studying cosmic rays entering the Earth's atmosphere. In 1937, Carl Anderson and his collaborators discovered a particle of mass 106 MeV/c^2, approximately 207 times the mass of the electron. This particle was thought to be Yukawa's meson. Subsequent experiments, however, showed that the particle interacted very weakly with matter and hence could not be the field particle for the nuclear force. That puzzling situation inspired several theoreticians to propose two mesons having slightly different masses equal to approximately 200 times that of the electron, one having been discovered by Anderson and the other, still undiscovered, predicted by Yukawa. This idea was confirmed in 1947 with the discovery of the **pi meson** (π), or simply **pion.** The particle discovered by Anderson in 1937, the one initially thought to be Yukawa's meson, is not really a

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Hideki Yukawa
Japanese Physicist (1907–1981)
Yukawa was awarded the Nobel Prize in Physics in 1949 for predicting the existence of mesons. This photograph of him at work was taken in 1950 in his office at Columbia University. Yukawa came to Columbia in 1949 after spending the early part of his career in Japan.

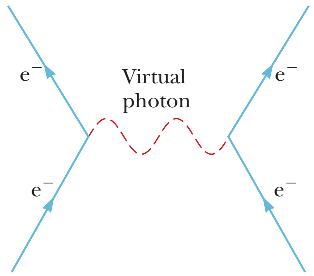


Figure 46.4 Feynman diagram representing a photon mediating the electromagnetic force between two electrons.

meson. (We shall discuss the characteristics of mesons in Section 46.4.) Instead, it takes part in the weak and electromagnetic interactions only and is now called the **muon** (μ).

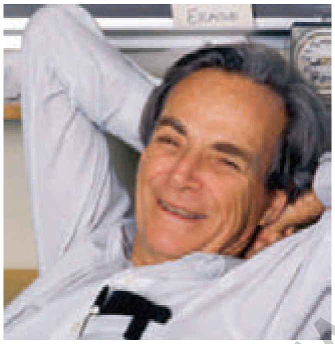
The pion comes in three varieties, corresponding to three charge states: π^+, π^-, and π^0. The π^+ and π^- particles (π^- is the antiparticle of π^+) each have a mass of 139.6 MeV/c^2, and the π^0 mass is 135.0 MeV/c^2. Two muons exist: μ^- and its antiparticle μ^+.

Pions and muons are very unstable particles. For example, the π^-, which has a mean lifetime of 2.6×10^{-8} s, decays to a muon and an antineutrino.[3] The muon, which has a mean lifetime of 2.2 μs, then decays to an electron, a neutrino, and an antineutrino:

$$\begin{aligned} \pi^- &\rightarrow \mu^- + \bar{\nu} \\ \mu^- &\rightarrow e^- + \nu + \bar{\nu} \end{aligned} \qquad \textbf{(46.1)}$$

For chargeless particles (as well as some charged particles, such as the proton), a bar over the symbol indicates an antiparticle, as for the neutrino in beta decay (see Section 44.5). Other antiparticles, such as e^+ and μ^+, use a different notation.

The interaction between two particles can be represented in a simple diagram called a **Feynman diagram,** developed by American physicist Richard P. Feynman. Figure 46.4 is such a diagram for the electromagnetic interaction between two electrons. A Feynman diagram is a qualitative graph of time on the vertical axis versus space on the horizontal axis. It is qualitative in the sense that the actual values of time and space are not important, but the overall appearance of the graph provides a pictorial representation of the process.



Richard Feynman
American Physicist (1918–1988)
Inspired by Dirac, Feynman developed quantum electrodynamics, the theory of the interaction of light and matter on a relativistic and quantum basis. In 1965, Feynman won the Nobel Prize in Physics. The prize was shared by Feynman, Julian Schwinger, and Sin Itiro Tomonaga. Early in Feynman's career, he was a leading member of the team developing the first nuclear weapon in the Manhattan Project. Toward the end of his career, he worked on the commission investigating the 1986 *Challenger* tragedy and demonstrated the effects of cold temperatures on the rubber O-rings used in the space shuttle.

In the simple case of the electron–electron interaction in Figure 46.4, a photon (the field particle) mediates the electromagnetic force between the electrons. Notice that the entire interaction is represented in the diagram as occurring at a single point in time. Therefore, the paths of the electrons appear to undergo a discontinuous change in direction at the moment of interaction. The electron paths shown in Figure 46.4 are different from the *actual* paths, which would be curved due to the continuous exchange of large numbers of field particles.

In the electron–electron interaction, the photon, which transfers energy and momentum from one electron to the other, is called a *virtual photon* because it vanishes during the interaction without having been detected. In Chapter 40, we discussed that a photon has energy $E = hf$, where f is its frequency. Consequently, for a system of two electrons initially at rest, the system has energy $2m_ec^2$ before a virtual photon is released and energy $2m_ec^2 + hf$ after the virtual photon is released (plus any kinetic energy of the electron resulting from the emission of the photon). Is that a violation of the law of conservation of energy for an isolated system? No; this process does *not* violate the law of conservation of energy because the virtual

[3]The antineutrino is another zero-charge particle for which the identification of the antiparticle is more difficult than that for a charged particle. Although the details are beyond the scope of this book, the neutrino and antineutrino can be differentiated by means of the relationship between the linear momentum and the spin angular momentum of the particles.

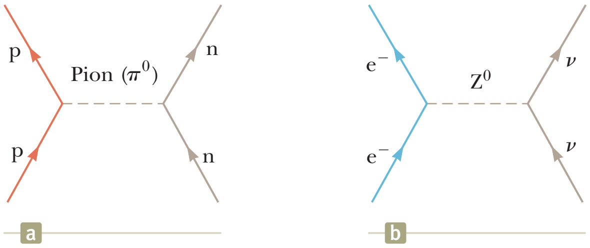


Figure 46.5 (a) Feynman diagram representing a proton and a neutron interacting via the nuclear force with a neutral pion mediating the force. (This model is *not* the current model for nucleon interaction.) (b) Feynman diagram for an electron and a neutrino interacting via the weak force, with a Z^0 boson mediating the force.

photon has a very short lifetime Δt that makes the uncertainty in the energy $\Delta E \approx \hbar/2\ \Delta t$ of the system greater than the photon energy. Therefore, within the constraints of the uncertainty principle, the energy of the system is conserved.

Now consider a pion exchange between a proton and a neutron according to Yukawa's model (Fig. 46.5a). The energy ΔE_R needed to create a pion of mass m_π is given by Einstein's equation $\Delta E_R = m_\pi c^2$. As with the photon in Figure 46.4, the very existence of the pion would appear to violate the law of conservation of energy if the particle existed for a time interval greater than $\Delta t \approx \hbar/2\,\Delta E_R$ (from the uncertainty principle), where Δt is the time interval required for the pion to transfer from one nucleon to the other. Therefore,

$$\Delta t \approx \frac{\hbar}{2\,\Delta E_R} = \frac{\hbar}{2m_\pi c^2}$$

and the rest energy of the pion is

$$m_\pi c^2 = \frac{\hbar}{2\,\Delta t} \tag{46.2}$$

Because the pion cannot travel faster than the speed of light, the maximum distance d it can travel in a time interval Δt is $c\,\Delta t$. Therefore, using Equation 46.2 and $d = c\,\Delta t$, we find

$$m_\pi c^2 = \frac{\hbar c}{2d} \tag{46.3}$$

From Table 46.1, we know that the range of the nuclear force is on the order of 10^{-15} fm. Using this value for d in Equation 46.3, we estimate the rest energy of the pion to be

$$m_p c^2 \approx \frac{(1.055 \times 10^{-34}\ \text{J}\cdot\text{s})(3.00 \times 10^{8}\ \text{m/s})}{2(1 \times 10^{-15}\ \text{m})}$$

$$= 1.6 \times 10^{-11}\ \text{J} \approx 100\ \text{MeV}$$

which corresponds to a mass of 100 MeV/c^2 (approximately 200 times the mass of the electron). This value is in reasonable agreement with the observed pion mass.

The concept just described is quite revolutionary. In effect, it says that a system of two nucleons can change into two nucleons plus a pion as long as it returns to its original state in a very short time interval. (Remember that this description is the older historical model, which assumes the pion is the field particle for the nuclear force; the gluon is the actual field particle in current models.) Physicists often say that a nucleon undergoes *fluctuations* as it emits and absorbs field particles. These fluctuations are a consequence of a combination of quantum mechanics (through the uncertainty principle) and special relativity (through Einstein's energy–mass relationship $E_R = mc^2$).

Pitfall Prevention 46.2

The Nuclear Force and the Strong Force The nuclear force discussed in Chapter 44 was historically called the strong force. Once the quark theory (Section 46.8) was established, however, the phrase *strong force* was reserved for the force between quarks. We shall follow this convention: the strong force is between quarks or particles built from quarks, and the nuclear force is between nucleons in a nucleus. The nuclear force is a secondary result of the strong force as discussed in Section 46.9. It is sometimes called the *residual strong force*. Because of this historical development of the names for these forces, other books sometimes refer to the nuclear force as the strong force.

In this section, we discussed the field particles that were originally proposed to mediate the nuclear force (pions) and those that mediate the electromagnetic force (photons). The graviton, the field particle for the gravitational force, has yet to be observed. In 1983, $W^{\pm}$ and Z^0 particles, which mediate the weak force, were discovered by Italian physicist Carlo Rubbia (b. 1934) and his associates, using a proton–antiproton collider. Rubbia and Simon van der Meer (1925–2011), both at CERN,[4] shared the 1984 Nobel Prize in Physics for the discovery of the $W^{\pm}$ and Z^0 particles and the development of the proton–antiproton collider. Figure 46.5b shows a Feynman diagram for a weak interaction mediated by a Z^0 boson.

46.4 Classification of Particles

All particles other than field particles can be classified into two broad categories, *hadrons* and *leptons*. The criterion for separating these particles into categories is whether or not they interact via the strong force. The nuclear force between nucleons in a nucleus is a particular manifestation of the strong force, but we will use the term *strong force* to refer to any interaction between particles made up of quarks. (For more detail on quarks and the strong force, see Section 46.8.) Table 46.2 provides a summary of the properties of hadrons and leptons.

Table 46.2 Some Particles and Their Properties

Category	Particle Name	Symbol	Antiparticle	Mass (MeV/c^2)	B	L_e	L_μ	L_τ	S	Lifetime(s)	Spin
Leptons	Electron	e^-	e^+	0.511	0	+1	0	0	0	Stable	$\frac{1}{2}$
	Electron–neutrino	ν_e	$\bar{\nu}_e$	$< 2\ \text{eV}/c^2$	0	+1	0	0	0	Stable	$\frac{1}{2}$
	Muon	μ^-	μ^+	105.7	0	0	+1	0	0	2.20×10^{-6}	$\frac{1}{2}$
	Muon–neutrino	ν_μ	$\bar{\nu}_\mu$	< 0.17	0	0	+1	0	0	Stable	$\frac{1}{2}$
	Tau	τ^-	τ^+	1 784	0	0	0	+1	0	$< 4 \times 10^{-13}$	$\frac{1}{2}$
	Tau–neutrino	ν_τ	$\bar{\nu}_\tau$	< 18	0	0	0	+1	0	Stable	$\frac{1}{2}$
Hadrons											
Mesons	Pion	π^+	π^-	139.6	0	0	0	0	0	2.60×10^{-8}	0
		π^0	Self	135.0	0	0	0	0	0	0.83×10^{-16}	0
	Kaon	K^+	K^-	493.7	0	0	0	0	+1	1.24×10^{-8}	0
		K^0_S	$\bar{K}^0_S$	497.7	0	0	0	0	+1	0.89×10^{-10}	0
		K^0_L	$\bar{K}^0_L$	497.7	0	0	0	0	+1	5.2×10^{-8}	0
	Eta	η	Self	548.8	0	0	0	0	0	$< 10^{-18}$	0
		η'	Self	958	0	0	0	0	0	2.2×10^{-21}	0
Baryons	Proton	p	$\bar{p}$	938.3	+1	0	0	0	0	Stable	$\frac{1}{2}$
	Neutron	n	$\bar{n}$	939.6	+1	0	0	0	0	614	$\frac{1}{2}$
	Lambda	Λ^0	$\bar{\Lambda}^0$	1 115.6	+1	0	0	0	−1	2.6×10^{-10}	$\frac{1}{2}$
	Sigma	Σ^+	$\bar{\Sigma}^-$	1 189.4	+1	0	0	0	−1	0.80×10^{-10}	$\frac{1}{2}$
		Σ^0	$\bar{\Sigma}^0$	1 192.5	+1	0	0	0	−1	6×10^{-20}	$\frac{1}{2}$
		Σ^-	$\bar{\Sigma}^+$	1 197.3	+1	0	0	0	−1	1.5×10^{-10}	$\frac{1}{2}$
	Delta	Δ^{++}	$\bar{\Delta}^{--}$	1 230	+1	0	0	0	0	6×10^{-24}	$\frac{3}{2}$
		Δ^+	$\bar{\Delta}^-$	1 231	+1	0	0	0	0	6×10^{-24}	$\frac{3}{2}$
		Δ^0	$\bar{\Delta}^0$	1 232	+1	0	0	0	0	6×10^{-24}	$\frac{3}{2}$
		Δ^-	$\bar{\Delta}^+$	1 234	+1	0	0	0	0	6×10^{-24}	$\frac{3}{2}$
	Xi	Ξ^0	$\bar{\Xi}^0$	1 315	+1	0	0	0	−2	2.9×10^{-10}	$\frac{1}{2}$
		Ξ^-	$\bar{\Xi}^+$	1 321	+1	0	0	0	−2	1.64×10^{-10}	$\frac{1}{2}$
	Omega	Ω^-	Ω^+	1 672	+1	0	0	0	−3	0.82×10^{-10}	$\frac{3}{2}$

[4]CERN was originally the Conseil Européen pour la Recherche Nucléaire; the name has been altered to the European Organization for Nuclear Research, and the laboratory operated by CERN is called the European Laboratory for Particle Physics. The CERN acronym has been retained and is commonly used to refer to both the organization and the laboratory.

Hadrons

Particles that interact through the strong force (as well as through the other fundamental forces) are called **hadrons.** The two classes of hadrons, *mesons* and *baryons,* are distinguished by their masses and spins.

Mesons all have zero or integer spin (0 or 1). As indicated in Section 46.3, the name comes from the expectation that Yukawa's proposed meson mass would lie between the masses of the electron and the proton. Several meson masses do lie in this range, although mesons having masses greater than that of the proton have been found to exist.

All mesons decay finally into electrons, positrons, neutrinos, and photons. The pions are the lightest known mesons and have masses of approximately 1.4×10^2 MeV/c^2, and all three pions—π^+, π^-, and π^0—have a spin of 0. (This spin-0 characteristic indicates that the particle discovered by Anderson in 1937, the muon, is not a meson. The muon has spin $\frac{1}{2}$ and belongs in the *lepton* classification, described below.)

Baryons, the second class of hadrons, have masses equal to or greater than the proton mass (the name *baryon* means "heavy" in Greek), and their spin is always a half-integer value ($\frac{1}{2}$, $\frac{3}{2}$, . . .). Protons and neutrons are baryons, as are many other particles. With the exception of the proton, all baryons decay in such a way that the end products include a proton. For example, the baryon called the Ξ^0 hyperon (Greek letter xi) decays to the Λ^0 baryon (Greek letter lambda) in approximately 10^{-10} s. The Λ^0 then decays to a proton and a π^- in approximately 3×10^{-10} s.

Today it is believed that hadrons are not elementary particles but instead are composed of more elementary units called quarks, per Section 46.8.

Leptons

Leptons (from the Greek *leptos,* meaning "small" or "light") are particles that do not interact by means of the strong force. All leptons have spin $\frac{1}{2}$. Unlike hadrons, which have size and structure, leptons appear to be truly elementary, meaning that they have no structure and are point-like.

Quite unlike the case with hadrons, the number of known leptons is small. Currently, scientists believe that only six leptons exist: the electron, the muon, the tau, and a neutrino associated with each: e^-, μ^-, τ^-, ν_e, ν_μ, and ν_τ. The tau lepton, discovered in 1975, has a mass about twice that of the proton. Direct experimental evidence for the neutrino associated with the tau was announced by the Fermi National Accelerator Laboratory (Fermilab) in July 2000. Each of the six leptons has an antiparticle.

Current studies indicate that neutrinos have a small but nonzero mass. If they do have mass, they cannot travel at the speed of light. In addition, because so many neutrinos exist, their combined mass may be sufficient to cause all the matter in the Universe to eventually collapse into a single point, which might then explode and create a completely new Universe! We shall discuss this possibility in more detail in Section 46.11.

46.5 Conservation Laws

The laws of conservation of energy, linear momentum, angular momentum, and electric charge for an isolated system provide us with a set of rules that all processes must follow. In Chapter 44, we learned that conservation laws are important for understanding why certain radioactive decays and nuclear reactions occur and others do not. In the study of elementary particles, a number of additional conservation laws are important. Although the two described here have no theoretical foundation, they are supported by abundant empirical evidence.

Baryon Number

Experimental results show that whenever a baryon is created in a decay or nuclear reaction, an antibaryon is also created. This scheme can be quantified by assigning every particle a quantum number, the **baryon number,** as follows: $B = +1$ for all baryons, $B = -1$ for all antibaryons, and $B = 0$ for all other particles. (See Table 46.2.) The **law of conservation of baryon number** states that

Conservation of baryon number

> whenever a nuclear reaction or decay occurs, the sum of the baryon numbers before the process must equal the sum of the baryon numbers after the process.

If baryon number is conserved, the proton must be absolutely stable. For example, a decay of the proton to a positron and a neutral pion would satisfy conservation of energy, momentum, and electric charge. Such a decay has never been observed, however. The law of conservation of baryon number would be consistent with the absence of this decay because the proposed decay would involve the loss of a baryon. Based on experimental observations as pointed out in Example 46.2, all we can say at present is that protons have a half-life of at least 10^{33} years (the estimated age of the Universe is only 10^{10} years). Some recent theories, however, predict that the proton is unstable. According to this theory, baryon number is not absolutely conserved.

Quick Quiz 46.2 Consider the decays **(i)** $n \rightarrow \pi^+ + \pi^- + \mu^+ + \mu^-$ and **(ii)** $n \rightarrow p + \pi^-$. From the following choices, which conservation laws are violated by each decay? (a) energy (b) electric charge (c) baryon number (d) angular momentum (e) no conservation laws

Example 46.1 Checking Baryon Numbers

Use the law of conservation of baryon number to determine whether each of the following reactions can occur:

(A) $p + n \rightarrow p + p + n + \overline{p}$

SOLUTION

Conceptualize The mass on the right is larger than the mass on the left. Therefore, one might be tempted to claim that the reaction violates energy conservation. The reaction can indeed occur, however, if the initial particles have sufficient kinetic energy to allow for the increase in rest energy of the system.

Categorize We use a conservation law developed in this section, so we categorize this example as a substitution problem.

Evaluate the total baryon number for the left side of the reaction: $1 + 1 = 2$

Evaluate the total baryon number for the right side of the reaction: $1 + 1 + 1 + (-1) = 2$

Therefore, baryon number is conserved and the reaction can occur.

(B) $p + n \rightarrow p + p + \overline{p}$

SOLUTION

Evaluate the total baryon number for the left side of the reaction: $1 + 1 = 2$

Evaluate the total baryon number for the right side of the reaction: $1 + 1 + (-1) = 1$

Because baryon number is not conserved, the reaction cannot occur.

Example 46.2 Detecting Proton Decay

Measurements taken at two neutrino detection facilities, the Irvine–Michigan–Brookhaven detector (Fig. 46.6) and the Super Kamiokande in Japan, indicate that the half-life of protons is at least 10^{33} yr.

(A) Estimate how long we would have to watch, on average, to see a proton in a glass of water decay.

Figure 46.6 (Example 46.2) A diver swims through ultrapure water in the Irvine–Michigan–Brookhaven neutrino detector. This detector holds almost 7 000 metric tons of water and is lined with over 2 000 photomultiplier tubes, many of which are visible in the photograph.

SOLUTION

Conceptualize Imagine the number of protons in a glass of water. Although this number is huge, the probability of a single proton undergoing decay is small, so we would expect to wait for a long time interval before observing a decay.

Categorize Because a half-life is provided in the problem, we categorize this problem as one in which we can apply our statistical analysis techniques from Section 44.4.

Analyze Let's estimate that a drinking glass contains a number of moles n of water, with a mass of m = 250 g and a molar mass M = 18 g/mol.

Find the number of molecules of water in the glass:

$$N_{\text{molecules}} = nN_A = \frac{m}{M}N_A$$

Each water molecule contains one proton in each of its two hydrogen atoms plus eight protons in its oxygen atom, for a total of ten protons. Therefore, there are $N = 10N_{\text{molecules}}$ protons in the glass of water.

Find the activity of the protons from Equation 44.7:

$$(1)\quad R = \lambda N = \frac{\ln 2}{T_{1/2}}\left(10\,\frac{m}{M}N_A\right) = \frac{\ln 2}{10^{33}\text{ yr}}(10)\left(\frac{250\text{ g}}{18\text{ g/mol}}\right)(6.02\times10^{23}\text{ mol}^{-1})$$

$$= 5.8\times10^{-8}\text{ yr}^{-1}$$

Finalize The decay constant represents the probability that *one* proton decays in one year. The probability that *any* proton in our glass of water decays in the one-year interval is given by Equation (1). Therefore, we must watch our glass of water for $1/R \approx$ 17 million years! That indeed is a long time interval, as expected.

(B) The Super Kamiokande neutrino facility contains 50 000 metric tons of water. Estimate the average time interval between detected proton decays in this much water if the half-life of a proton is 10^{33} yr.

SOLUTION

Analyze The proton decay rate R in a sample of water is proportional to the number N of protons. Set up a ratio of the decay rate in the Super Kamiokande facility to that in a glass of water:

$$\frac{R_{\text{Kamiokande}}}{R_{\text{glass}}} = \frac{N_{\text{Kamiokande}}}{N_{\text{glass}}} \rightarrow R_{\text{Kamiokande}} = \frac{N_{\text{Kamiokande}}}{N_{\text{glass}}}R_{\text{glass}}$$

The number of protons is proportional to the mass of the sample, so express the decay rate in terms of mass:

$$R_{\text{Kamiokande}} = \frac{m_{\text{Kamiokande}}}{m_{\text{glass}}}R_{\text{glass}}$$

Substitute numerical values:

$$R_{\text{Kamiokande}} = \left(\frac{50\,000\text{ metric tons}}{0.250\text{ kg}}\right)\left(\frac{1\,000\text{ kg}}{1\text{ metric ton}}\right)(5.8\times10^{-8}\text{ yr}^{-1}) \approx 12\text{ yr}^{-1}$$

Finalize The average time interval between decays is about one-twelfth of a year, or approximately one month. That is much shorter than the time interval in part (A) due to the tremendous amount of water in the detector facility. Despite this rosy prediction of one proton decay per month, a proton decay has never been observed. This suggests that the half-life of the proton may be larger than 10^{33} years or that proton decay simply does not occur.

Lepton Number

There are three conservation laws involving lepton numbers, one for each variety of lepton. The **law of conservation of electron lepton number** states that

Conservation of electron lepton number

> whenever a nuclear reaction or decay occurs, the sum of the electron lepton numbers before the process must equal the sum of the electron lepton numbers after the process.

The electron and the electron neutrino are assigned an electron lepton number $L_e = +1$, and the antileptons e^+ and $\bar{\nu}_e$ are assigned an electron lepton number $L_e = -1$. All other particles have $L_e = 0$. For example, consider the decay of the neutron:

$$n \rightarrow p + e^- + \bar{\nu}_e$$

Before the decay, the electron lepton number is $L_e = 0$; after the decay, it is $0 + 1 + (-1) = 0$. Therefore, electron lepton number is conserved. (Baryon number must also be conserved, of course, and it is: before the decay, $B = +1$, and after the decay, $B = +1 + 0 + 0 = +1$.)

Similarly, when a decay involves muons, the muon lepton number L_μ is conserved. The μ^- and the ν_μ are assigned a muon lepton number $L_\mu = +1$, and the antimuons μ^+ and $\bar{\nu}_\mu$ are assigned a muon lepton number $L_\mu = -1$. All other particles have $L_\mu = 0$.

Finally, tau lepton number L_τ is conserved with similar assignments made for the tau lepton, its neutrino, and their two antiparticles.

Quick Quiz 46.3 Consider the following decay: $\pi^0 \rightarrow \mu^- + e^+ + \nu_\mu$. What conservation laws are violated by this decay? **(a)** energy **(b)** angular momentum **(c)** electric charge **(d)** baryon number **(e)** electron lepton number **(f)** muon lepton number **(g)** tau lepton number **(h)** no conservation laws

Quick Quiz 46.4 Suppose a claim is made that the decay of the neutron is given by $n \rightarrow p + e^-$. What conservation laws are violated by this decay? **(a)** energy **(b)** angular momentum **(c)** electric charge **(d)** baryon number **(e)** electron lepton number **(f)** muon lepton number **(g)** tau lepton number **(h)** no conservation laws

Example 46.3 Checking Lepton Numbers

Use the law of conservation of lepton numbers to determine whether each of the following decay schemes (A) and (B) can occur:

(A) $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$

SOLUTION

Conceptualize Because this decay involves a muon and an electron, L_μ and L_e must each be conserved separately if the decay is to occur.

Categorize We use a conservation law developed in this section, so we categorize this example as a substitution problem.

Evaluate the lepton numbers before the decay: $L_\mu = +1 \qquad L_e = 0$

Evaluate the total lepton numbers after the decay: $L_\mu = 0 + 0 + 1 = +1 \qquad L_e = +1 + (-1) + 0 = 0$

Therefore, both numbers are conserved and on this basis the decay is possible.

(B) $\pi^+ \rightarrow \mu^+ + \nu_\mu + \nu_e$

▶ **46.3** continued

SOLUTION

Evaluate the lepton numbers before the decay: $L_\mu = 0 \qquad L_e = 0$

Evaluate the total lepton numbers after the decay: $L_\mu = -1 + 1 + 0 = 0 \qquad L_e = 0 + 0 + 1 = 1$

Therefore, the decay is not possible because electron lepton number is not conserved.

46.6 Strange Particles and Strangeness

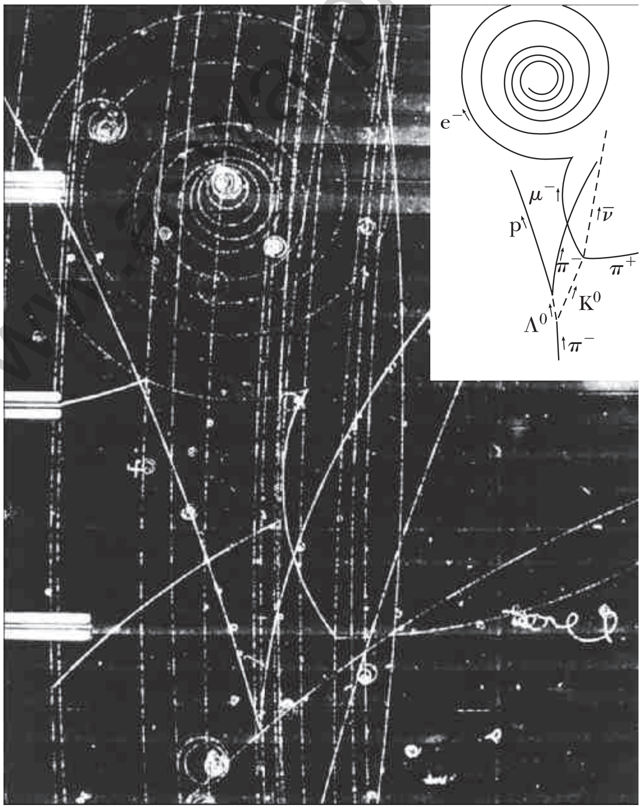
Many particles discovered in the 1950s were produced by the interaction of pions with protons and neutrons in the atmosphere. A group of these—the kaon (K), lambda (Λ), and sigma (Σ) particles—exhibited unusual properties both as they were created and as they decayed; hence, they were called *strange particles.*

One unusual property of strange particles is that they are always produced in pairs. For example, when a pion collides with a proton, a highly probable result is the production of two neutral strange particles (Fig. 46.7):

$$\pi^- + p \rightarrow K^0 + \Lambda^0$$

The reaction $\pi^- + p \rightarrow K^0 + n$, where only one final particle is strange, never occurs, however, even though no previously known conservation laws would be violated and even though the energy of the pion is sufficient to initiate the reaction.

The second peculiar feature of strange particles is that although they are produced in reactions involving the strong interaction at a high rate, they do not decay into particles that interact via the strong force at a high rate. Instead, they decay very slowly, which is characteristic of the weak interaction. Their half-lives are in



Courtesy Lawrence Berkeley Laboratory, University of California, Photographic Services

Figure 46.7 This bubble-chamber photograph shows many events, and the inset is a drawing of identified tracks. The strange particles Λ^0 and K^0 are formed at the bottom as a π^- particle interacts with a proton in the reaction $\pi^- + p \rightarrow K^0 + \Lambda^0$. (Notice that the neutral particles leave no tracks, as indicated by the dashed lines in the inset.) The Λ^0 then decays in the reaction $\Lambda^0 \rightarrow \pi^- + p$ and the K^0 in the reaction $K^0 \rightarrow \pi^+ + \mu^- + \bar{\nu}_\mu$.

the range 10^{-10} s to 10^{-8} s, whereas most other particles that interact via the strong force have much shorter lifetimes on the order of 10^{-23} s.

To explain these unusual properties of strange particles, a new quantum number S, called **strangeness,** was introduced, together with a conservation law. The strangeness numbers for some particles are given in Table 46.2. The production of strange particles in pairs is handled mathematically by assigning $S = +1$ to one of the particles, $S = -1$ to the other, and $S = 0$ to all nonstrange particles. The **law of conservation of strangeness** states that

Conservation of strangeness ▶

> in a nuclear reaction or decay that occurs via the strong force, strangeness is conserved; that is, the sum of the strangeness numbers before the process must equal the sum of the strangeness numbers after the process. In processes that occur via the weak interaction, strangeness may not be conserved.

The low decay rate of strange particles can be explained by assuming the strong and electromagnetic interactions obey the law of conservation of strangeness but the weak interaction does not. Because the decay of a strange particle involves the loss of one strange particle, it violates strangeness conservation and hence proceeds slowly via the weak interaction.

Example 46.4 Is Strangeness Conserved?

(A) Use the law of strangeness conservation to determine whether the reaction $\pi^0 + \text{n} \rightarrow \text{K}^+ + \Sigma^-$ occurs.

SOLUTION

Conceptualize We recognize that there are strange particles appearing in this reaction, so we see that we will need to investigate conservation of strangeness.

Categorize We use a conservation law developed in this section, so we categorize this example as a substitution problem.

Evaluate the strangeness for the left side of the reaction using Table 46.2: $S = 0 + 0 = 0$

Evaluate the strangeness for the right side of the reaction: $S = +1 - 1 = 0$

Therefore, strangeness is conserved and the reaction is allowed.

(B) Show that the reaction $\pi^- + \text{p} \rightarrow \pi^- + \Sigma^+$ does not conserve strangeness.

SOLUTION

Evaluate the strangeness for the left side of the reaction: $S = 0 + 0 = 0$

Evaluate the strangeness for the right side of the reaction: $S = 0 + (-1) = -1$

Therefore, strangeness is not conserved.

46.7 Finding Patterns in the Particles

One tool scientists use is the detection of patterns in data, patterns that contribute to our understanding of nature. For example, Table 21.2 shows a pattern of molar specific heats of gases that allows us to understand the differences among monatomic, diatomic, and polyatomic gases. Figure 42.20 shows a pattern of peaks in the ionization energy of atoms that relate to the quantized energy levels in the

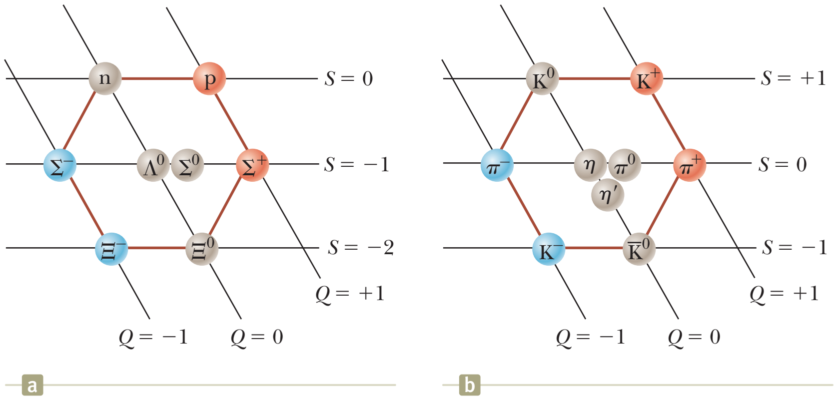


Figure 46.8 (a) The hexagonal eightfold-way pattern for the eight spin-$\frac{1}{2}$ baryons. This strangeness-versus-charge plot uses a sloping axis for charge number Q and a horizontal axis for strangeness S. (b) The eightfold-way pattern for the nine spin-zero mesons.

atoms. Figure 44.7 shows a pattern of peaks in the binding energy that suggest a shell structure within the nucleus. One of the best examples of this tool's use is the development of the periodic table, which provides a fundamental understanding of the chemical behavior of the elements. As mentioned in the introduction, the periodic table explains how more than 100 elements can be formed from three particles, the electron, the proton, and the neutron. The table of nuclides, part of which is shown in Table 44.2, contains hundreds of nuclides, but all can be built from protons and neutrons.

The number of particles observed by particle physicists is in the hundreds. Is it possible that a small number of entities exist from which all these particles can be built? Taking a hint from the success of the periodic table and the table of nuclides, let explore the historical search for patterns among the particles.

Many classification schemes have been proposed for grouping particles into families. Consider, for instance, the baryons listed in Table 46.2 that have spins of $\frac{1}{2}$: p, n, Λ^0, Σ^+, Σ^0, Σ^-, Ξ^0, and Ξ^-. If we plot strangeness versus charge for these baryons using a sloping coordinate system as in Figure 46.8a, a fascinating pattern is observed: six of the baryons form a hexagon, and the remaining two are at the hexagon's center.

As a second example, consider the following nine spin-zero mesons listed in Table 46.2: π^+, π^0, π^-, K^+, K^0, K^-, η, η', and the antiparticle $\overline{K}^0$. Figure 46.8b is a plot of strangeness versus charge for this family. Again, a hexagonal pattern emerges. In this case, each particle on the perimeter of the hexagon lies opposite its antiparticle and the remaining three (which form their own antiparticles) are at the center of the hexagon. These and related symmetric patterns were developed independently in 1961 by Murray Gell-Mann and Yuval Ne'eman (1925–2006). Gell-Mann called the patterns the **eightfold way,** after the eightfold path to nirvana in Buddhism.

Groups of baryons and mesons can be displayed in many other symmetric patterns within the framework of the eightfold way. For example, the family of spin-$\frac{3}{2}$ baryons known in 1961 contains nine particles arranged in a pattern like that of the pins in a bowling alley as in Figure 46.9. (The particles Σ^{*+}, Σ^{*0}, Σ^{*-}, Ξ^{*0},

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Murray Gell-Mann
American Physicist (b. 1929)
In 1969, Murray Gell-Mann was awarded the Nobel Prize in Physics for his theoretical studies dealing with subatomic particles.

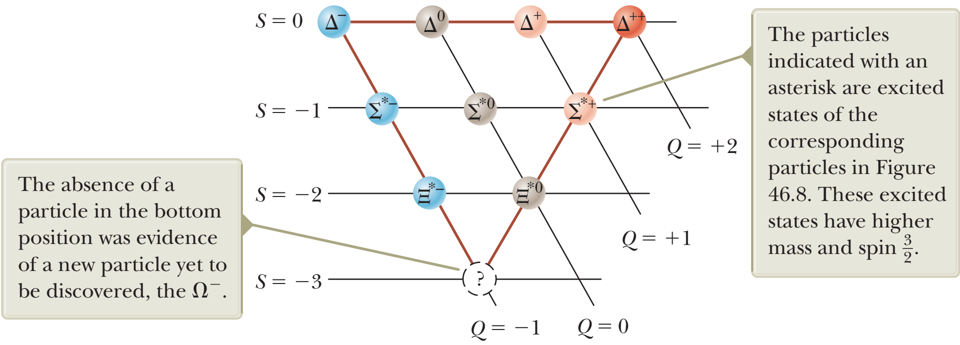


Figure 46.9 The pattern for the higher-mass, spin-$\frac{3}{2}$ baryons known at the time the pattern was proposed.

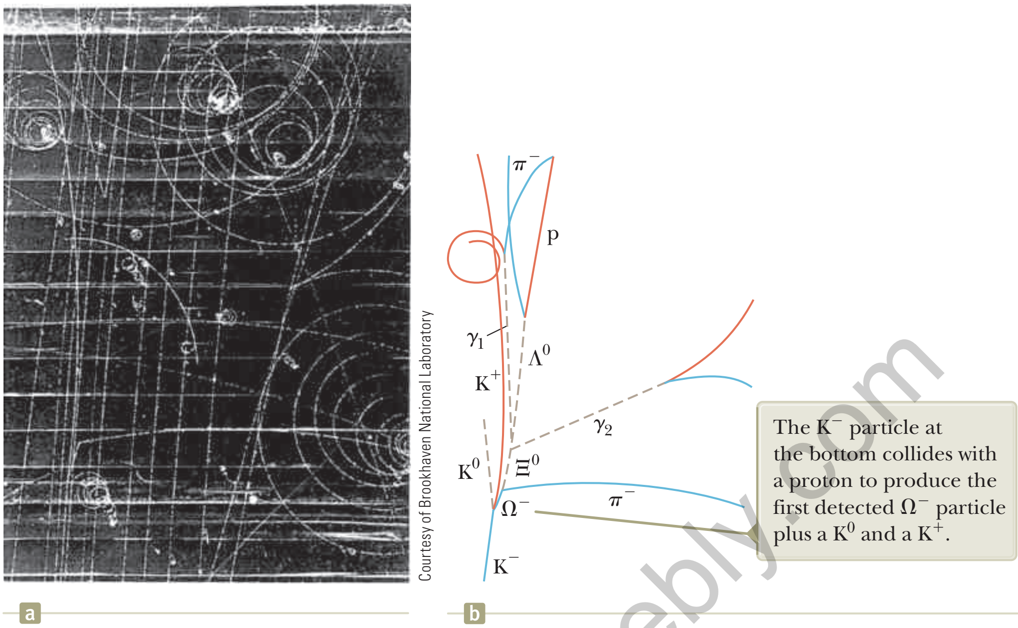


Figure 46.10 Discovery of the Ω^- particle. The photograph on the left shows the original bubble-chamber tracks. The drawing on the right isolates the tracks of the important events.

and Ξ^{*-} are excited states of the particles Σ^+, Σ^0, Σ^-, Ξ^0, and Ξ^-. In these higher-energy states, the spins of the three quarks—see Section 46.8—making up the particle are aligned so that the total spin of the particle is $\frac{3}{2}$.) When this pattern was proposed, an empty spot occurred in it (at the bottom position), corresponding to a particle that had never been observed. Gell-Mann predicted that the missing particle, which he called the omega minus (Ω^-), should have spin $\frac{3}{2}$, charge -1, strangeness -3, and rest energy of approximately 1 680 MeV. Shortly thereafter, in 1964, scientists at the Brookhaven National Laboratory found the missing particle through careful analyses of bubble-chamber photographs (Fig. 46.10) and confirmed all its predicted properties.

The prediction of the missing particle in the eightfold way has much in common with the prediction of missing elements in the periodic table. Whenever a vacancy occurs in an organized pattern of information, experimentalists have a guide for their investigations.

46.8 Quarks

As mentioned earlier, leptons appear to be truly elementary particles because there are only a few types of them, and experiments indicate that they have no measurable size or internal structure. Hadrons, on the other hand, are complex particles having size and structure. The existence of the strangeness–charge patterns of the eightfold way suggests that hadrons have substructure. Furthermore, hundreds of types of hadrons exist and many decay into other hadrons.

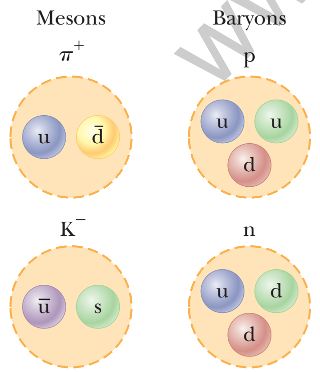


Figure 46.11 Quark composition of two mesons and two baryons.

The Original Quark Model

In 1963, Gell-Mann and George Zweig (b. 1937) independently proposed a model for the substructure of hadrons. According to their model, all hadrons are composed of two or three elementary constituents called **quarks.** (Gell-Mann borrowed the word *quark* from the passage "Three quarks for Muster Mark" in James Joyce's *Finnegans Wake.* In Zweig's model, he called the constituents "aces.") The model has three types of quarks, designated by the symbols u, d, and s, that are given the arbitrary names **up, down,** and **strange.** The various types of quarks are called **flavors.** Figure 46.11 is a pictorial representation of the quark compositions of several hadrons.

Table 46.3 **Properties of Quarks and Antiquarks**

Quarks

Name	Symbol	Spin	Charge	Baryon Number	Strangeness	Charm	Bottomness	Topness
Up	u	$\frac{1}{2}$	$+\frac{2}{3}e$	$\frac{1}{3}$	0	0	0	0
Down	d	$\frac{1}{2}$	$-\frac{1}{3}e$	$\frac{1}{3}$	0	0	0	0
Strange	s	$\frac{1}{2}$	$-\frac{1}{3}e$	$\frac{1}{3}$	−1	0	0	0
Charmed	c	$\frac{1}{2}$	$+\frac{2}{3}e$	$\frac{1}{3}$	0	+1	0	0
Bottom	b	$\frac{1}{2}$	$-\frac{1}{3}e$	$\frac{1}{3}$	0	0	+1	0
Top	t	$\frac{1}{2}$	$+\frac{2}{3}e$	$\frac{1}{3}$	0	0	0	+1

Antiquarks

Name	Symbol	Spin	Charge	Baryon Number	Strangeness	Charm	Bottomness	Topness
Anti-up	$\bar{u}$	$\frac{1}{2}$	$-\frac{2}{3}e$	$-\frac{1}{3}$	0	0	0	0
Anti-down	$\bar{d}$	$\frac{1}{2}$	$+\frac{1}{3}e$	$-\frac{1}{3}$	0	0	0	0
Anti-strange	$\bar{s}$	$\frac{1}{2}$	$+\frac{1}{3}e$	$-\frac{1}{3}$	+1	0	0	0
Anti-charmed	$\bar{c}$	$\frac{1}{2}$	$-\frac{2}{3}e$	$-\frac{1}{3}$	0	−1	0	0
Anti-bottom	$\bar{b}$	$\frac{1}{2}$	$+\frac{1}{3}e$	$-\frac{1}{3}$	0	0	−1	0
Anti-top	$\bar{t}$	$\frac{1}{2}$	$-\frac{2}{3}e$	$-\frac{1}{3}$	0	0	0	−1

An unusual property of quarks is that they carry a fractional electric charge. The u, d, and s quarks have charges of $+2e/3$, $-e/3$, and $-e/3$, respectively, where e is the elementary charge 1.60×10^{-19} C. These and other properties of quarks and antiquarks are given in Table 46.3. Quarks have spin $\frac{1}{2}$, which means that all quarks are fermions, defined as any particle having half-integral spin, as pointed out in Section 43.8. As Table 46.3 shows, associated with each quark is an antiquark of opposite charge, baryon number, and strangeness.

The compositions of all hadrons known when Gell-Mann and Zweig presented their model can be completely specified by three simple rules:

- A meson consists of one quark and one antiquark, giving it a baryon number of 0, as required.
- A baryon consists of three quarks.
- An antibaryon consists of three antiquarks.

The theory put forth by Gell-Mann and Zweig is referred to as the *original quark model.*

Quick Quiz 46.5 Using a coordinate system like that in Figure 46.8, draw an eightfold-way diagram for the three quarks in the original quark model.

Charm and Other Developments

Although the original quark model was highly successful in classifying particles into families, some discrepancies occurred between its predictions and certain experimental decay rates. Consequently, several physicists proposed a fourth quark flavor in 1967. They argued that if four types of leptons exist (as was thought at the time), there should also be four flavors of quarks because of an underlying symmetry in nature. The fourth quark, designated c, was assigned a property called **charm.** A *charmed* quark has charge $+2e/3$, just as the up quark does, but its charm distinguishes it from the other three quarks. This introduces a new quantum number C, representing charm. The new quark has charm $C = +1$, its antiquark has charm of $C = -1$, and all other quarks have $C = 0$. Charm, like strangeness, is conserved in strong and electromagnetic interactions but not in weak interactions.

Table 46.4 Quark Composition of Mesons

		Antiquarks									
		$\bar{b}$		$\bar{c}$		$\bar{s}$		$\bar{d}$		$\bar{u}$	
Quarks	**b**	Υ	$(\bar{b}b)$	B_c^-	$(\bar{c}b)$	$\bar{B}_s^0$	$(\bar{s}b)$	$\bar{B}_d^0$	$(\bar{d}b)$	B^-	$(\bar{u}b)$
	c	B_c^+	$(\bar{b}c)$	J/Ψ	$(\bar{c}c)$	D_s^+	$(\bar{s}c)$	D^+	$(\bar{d}c)$	D^0	$(\bar{u}c)$
	s	B_s^0	$(\bar{b}s)$	D_s^-	$(\bar{c}s)$	η, η'	$(\bar{s}s)$	$\bar{K}^0$	$(\bar{d}s)$	K^-	$(\bar{u}s)$
	d	B_d^0	$(\bar{b}d)$	D^-	$(\bar{c}d)$	K^0	$(\bar{s}d)$	π^0, η, η'	$(\bar{d}d)$	π^-	$(\bar{u}d)$
	u	B^+	$(\bar{b}u)$	$\bar{D}^0$	$(\bar{c}u)$	K^+	$(\bar{s}u)$	π^+	$(\bar{d}u)$	π^0, η, η'	$(\bar{u}u)$

Note: The top quark does not form mesons because it decays too quickly.

Evidence that the charmed quark exists began to accumulate in 1974, when a heavy meson called the J/Ψ particle (or simply Ψ, Greek letter psi) was discovered independently by two groups, one led by Burton Richter (b. 1931) at the Stanford Linear Accelerator (SLAC), and the other led by Samuel Ting (b. 1936) at the Brookhaven National Laboratory. In 1976, Richter and Ting were awarded the Nobel Prize in Physics for this work. The J/Ψ particle does not fit into the three-quark model; instead, it has properties of a combination of the proposed charmed quark and its antiquark ($c\bar{c}$). It is much more massive than the other known mesons (~3 100 MeV/c^2), and its lifetime is much longer than the lifetimes of particles that interact via the strong force. Soon, related mesons were discovered, corresponding to such quark combinations as $\bar{c}d$ and $c\bar{d}$, all of which have great masses and long lifetimes. The existence of these new mesons provided firm evidence for the fourth quark flavor.

In 1975, researchers at Stanford University reported strong evidence for the tau (τ) lepton, mass 1 784 MeV/c^2. This fifth type of lepton led physicists to propose that more flavors of quarks might exist, on the basis of symmetry arguments similar to those leading to the proposal of the charmed quark. These proposals led to more elaborate quark models and the prediction of two new quarks, **top** (t) and **bottom** (b). (Some physicists prefer *truth* and *beauty*.) To distinguish these quarks from the others, quantum numbers called *topness* and *bottomness* (with allowed values +1, 0, −1) were assigned to all quarks and antiquarks (see Table 46.3). In 1977, researchers at the Fermi National Laboratory, under the direction of Leon Lederman (b. 1922), reported the discovery of a very massive new meson Υ (Greek letter upsilon), whose composition is considered to be $b\bar{b}$, providing evidence for the bottom quark. In March 1995, researchers at Fermilab announced the discovery of the top quark (supposedly the last of the quarks to be found), which has a mass of 173 GeV/c^2.

Table 46.4 lists the quark compositions of mesons formed from the up, down, strange, charmed, and bottom quarks. Table 46.5 shows the quark combinations for the baryons listed in Table 46.2. Notice that only two flavors of quarks, u and d, are contained in all hadrons encountered in ordinary matter (protons and neutrons).

Will the discoveries of elementary particles ever end? How many "building blocks" of matter actually exist? At present, physicists believe that the elementary particles in nature are six quarks and six leptons, together with their antiparticles, and the four field particles listed in Table 46.1. Table 46.6 lists the rest energies and charges of the quarks and leptons.

Despite extensive experimental effort, no isolated quark has ever been observed. Physicists now believe that at ordinary temperatures, quarks are permanently confined inside ordinary particles because of an exceptionally strong force that prevents them from escaping, called (appropriately) the **strong force**[5] (which we

Table 46.5 Quark Composition of Several Baryons

Particle	Quark Composition
p	uud
n	udd
Λ^0	uds
Σ^+	uus
Σ^0	uds
Σ^-	dds
Δ^{++}	uuu
Δ^+	uud
Δ^0	udd
Δ^-	ddd
Ξ^0	uss
Ξ^-	dss
Ω^-	sss

Note: Some baryons have the same quark composition, such as the p and the Δ^+ and the n and the Δ^0. In these cases, the Δ particles are considered to be excited states of the proton and neutron.

[5]As a reminder, the original meaning of the term *strong force* was the short-range attractive force between nucleons, which we have called the *nuclear force*. The nuclear force between nucleons is a secondary effect of the strong force between quarks.

Table 46.6 The Elementary Particles and Their Rest Energies and Charges

Particle	Approximate Rest Energy	Charge
Quarks		
u	2.4 MeV	$+\frac{2}{3}e$
d	4.8 MeV	$-\frac{1}{3}e$
s	104 MeV	$-\frac{1}{3}e$
c	1.27 GeV	$+\frac{2}{3}e$
b	4.2 GeV	$-\frac{1}{3}e$
t	173 GeV	$+\frac{2}{3}e$
Leptons		
e^-	511 keV	$-e$
μ^-	105.7 MeV	$-e$
τ^-	1.78 GeV	$-e$
ν_e	< 2 eV	0
ν_μ	< 0.17 MeV	0
ν_τ	< 18 MeV	0

introduced at the beginning of Section 46.4 and will discuss further in Section 46.10). This force increases with separation distance, similar to the force exerted by a stretched spring. Current efforts are under way to form a **quark–gluon plasma,** a state of matter in which the quarks are freed from neutrons and protons. In 2000, scientists at CERN announced evidence for a quark–gluon plasma formed by colliding lead nuclei. In 2005, experiments at the Relativistic Heavy Ion Collider (RHIC) at Brookhaven suggested the creation of a quark–gluon plasma. Neither laboratory has provided definitive data to verify the existence of a quark–gluon plasma. Experiments continue, and the ALICE project (A Large Ion Collider Experiment) at the Large Hadron Collider at CERN has joined the search.

Quick Quiz 46.6 Doubly charged baryons, such as the Δ^{++}, are known to exist. True or False: Doubly charged mesons also exist.

46.9 Multicolored Quarks

Shortly after the concept of quarks was proposed, scientists recognized that certain particles had quark compositions that violated the exclusion principle. In Section 42.7, we applied the exclusion principle to electrons in atoms. The principle is more general, however, and applies to all particles with half-integral spin ($\frac{1}{2}$, $\frac{3}{2}$, etc.), which are collectively called fermions. Because all quarks are fermions having spin $\frac{1}{2}$, they are expected to follow the exclusion principle. One example of a particle that appears to violate the exclusion principle is the Ω^- (sss) baryon, which contains three strange quarks having parallel spins, giving it a total spin of $\frac{3}{2}$. All three quarks have the same spin quantum number, in violation of the exclusion principle. Other examples of baryons made up of identical quarks having parallel spins are the Δ^{++} (uuu) and the Δ^- (ddd).

To resolve this problem, it was suggested that quarks possess an additional property called **color charge.** This property is similar in many respects to electric charge except that it occurs in six varieties rather than two. The colors assigned to quarks are red, green, and blue, and antiquarks have the colors antired, antigreen, and antiblue. Therefore, the colors red, green, and blue serve as the "quantum numbers" for the color of the quark. To satisfy the exclusion principle, the three quarks in any baryon must all have different colors. Look again at the quarks in the baryons in Figure 46.11 and notice the colors. The three colors "neutralize" to white.

Pitfall Prevention 46.3

Color Charge Is Not Really Color The description of color for a quark has nothing to do with visual sensation from light. It is simply a convenient name for a property that is analogous to electric charge.

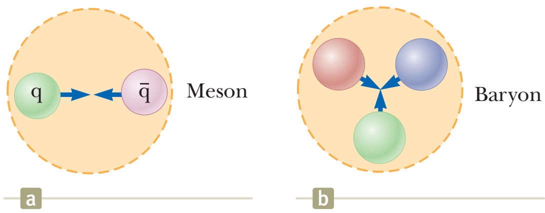


Figure 46.12 (a) A green quark is attracted to an antigreen quark. This forms a meson whose quark structure is $(q\bar{q})$. (b) Three quarks of different colors attract one another to form a baryon.

A quark and an antiquark in a meson must be of a color and the corresponding anticolor and will consequently neutralize to white, similar to the way electric charges + and − neutralize to zero net charge. (See the mesons in Fig. 46.11.) The apparent violation of the exclusion principle in the Ω^- baryon is removed because the three quarks in the particle have different colors.

The new property of color increases the number of quarks by a factor of 3 because each of the six quarks comes in three colors. Although the concept of color in the quark model was originally conceived to satisfy the exclusion principle, it also provided a better theory for explaining certain experimental results. For example, the modified theory correctly predicts the lifetime of the π^0 meson.

The theory of how quarks interact with each other is called **quantum chromodynamics,** or QCD, to parallel the name *quantum electrodynamics* (the theory of the electrical interaction between light and matter). In QCD, each quark is said to carry a color charge, in analogy to electric charge. The strong force between quarks is often called the **color force.** Therefore, the terms *strong force* and *color force* are used interchangeably.

In Section 46.1, we stated that the nuclear interaction between hadrons is mediated by massless field particles called **gluons.** As mentioned earlier, the nuclear force is actually a secondary effect of the strong force between quarks. The gluons are the mediators of the strong force. When a quark emits or absorbs a gluon, the quark's color may change. For example, a blue quark that emits a gluon may become a red quark and a red quark that absorbs this gluon becomes a blue quark.

The color force between quarks is analogous to the electric force between charges: particles with the same color repel, and those with opposite colors attract. Therefore, two green quarks repel each other, but a green quark is attracted to an antigreen quark. The attraction between quarks of opposite color to form a meson $(q\bar{q})$ is indicated in Figure 46.12a. Differently colored quarks also attract one another, although with less intensity than the oppositely colored quark and antiquark. For example, a cluster of red, blue, and green quarks all attract one another to form a baryon as in Figure 46.12b. Therefore, every baryon contains three quarks of three different colors.

Although the nuclear force between two colorless hadrons is negligible at large separations, the net strong force between their constituent quarks is not exactly zero at small separations. This residual strong force is the nuclear force that binds protons and neutrons to form nuclei. It is similar to the force between two electric dipoles. Each dipole is electrically neutral. An electric field surrounds the dipoles, however, because of the separation of the positive and negative charges (see Section 23.6). As a result, an electric interaction occurs between the dipoles that is weaker than the force between single charges. In Section 43.1, we explored how this interaction results in the Van der Waals force between neutral molecules.

According to QCD, a more basic explanation of the nuclear force can be given in terms of quarks and gluons. Figure 46.13a shows the nuclear interaction between a neutron and a proton by means of Yukawa's pion, in this case a π^-. This drawing differs from Figure 46.5a, in which the field particle is a π^0; there is no transfer of charge from one nucleon to the other in Figure 46.5a. In Figure 46.13a, the charged pion carries charge from one nucleon to the other, so the nucleons change identities, with the proton becoming a neutron and the neutron becoming a proton.

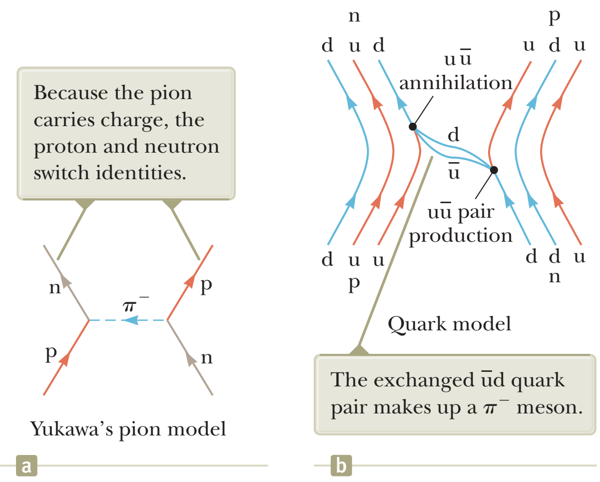


Figure 46.13 (a) A nuclear interaction between a proton and a neutron explained in terms of Yukawa's pion-exchange model. (b) The same interaction, explained in terms of quarks and gluons.

Let's look at the same interaction from the viewpoint of the quark model, shown in Figure 46.13b. In this Feynman diagram, the proton and neutron are represented by their quark constituents. Each quark in the neutron and proton is continuously emitting and absorbing gluons. The energy of a gluon can result in the creation of quark–antiquark pairs. This process is similar to the creation of electron–positron pairs in pair production, which we investigated in Section 46.2. When the neutron and proton approach to within 1 fm of each other, these gluons and quarks can be exchanged between the two nucleons, and such exchanges produce the nuclear force. Figure 46.13b depicts one possibility for the process shown in Figure 46.13a. A down quark in the neutron on the right emits a gluon. The energy of the gluon is then transformed to create a $u\bar{u}$ pair. The u quark stays within the nucleon (which has now changed to a proton), and the recoiling d quark and the $\bar{u}$ antiquark are transmitted to the proton on the left side of the diagram. Here the $\bar{u}$ annihilates a u quark within the proton and the d is captured. The net effect is to change a u quark to a d quark, and the proton on the left has changed to a neutron.

As the d quark and $\bar{u}$ antiquark in Figure 46.13b transfer between the nucleons, the d and $\bar{u}$ exchange gluons with each other and can be considered to be bound to each other by means of the strong force. Looking back at Table 46.4, we see that this combination is a π^-, or Yukawa's field particle! Therefore, the quark model of interactions between nucleons is consistent with the pion-exchange model.

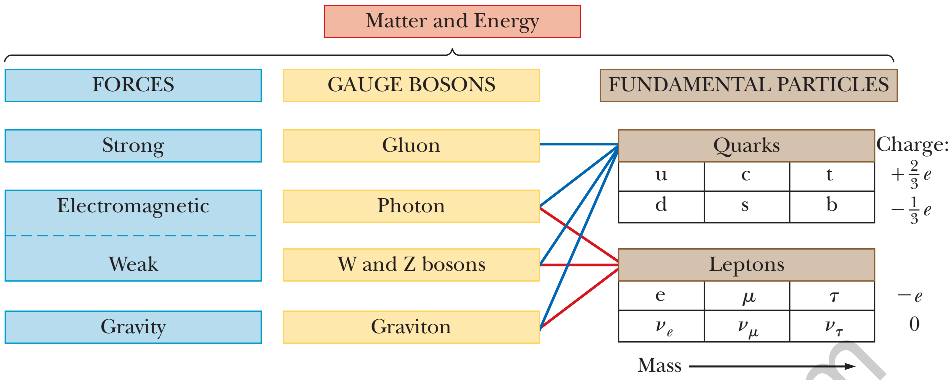
46.10 The Standard Model

Scientists now believe there are three classifications of truly elementary particles: leptons, quarks, and field particles. These three types of particles are further classified as either fermions or bosons. Quarks and leptons have spin $\frac{1}{2}$ and hence are fermions, whereas the field particles have integral spin of 1 or higher and are bosons.

Recall from Section 46.1 that the weak force is believed to be mediated by the W^+, W^-, and Z^0 bosons. These particles are said to have *weak charge,* just as quarks have color charge. Therefore, each elementary particle can have mass, electric charge, color charge, and weak charge. Of course, one or more of these could be zero.

In 1979, Sheldon Glashow (b. 1932), Abdus Salam (1926–1996), and Steven Weinberg (b. 1933) won the Nobel Prize in Physics for developing a theory that unifies the electromagnetic and weak interactions. This **electroweak theory** postulates that the weak and electromagnetic interactions have the same strength when the particles involved have very high energies. The two interactions are viewed as different manifestations of a single unifying electroweak interaction. The theory makes many concrete predictions, but perhaps the most spectacular is the prediction of

Figure 46.14 The Standard Model of particle physics.



the masses of the W and Z particles at approximately 82 GeV/c^2 and 93 GeV/c^2, respectively. These predictions are close to the masses in Table 46.1 determined by experiment.

The combination of the electroweak theory and QCD for the strong interaction is referred to in high-energy physics as the **Standard Model.** Although the details of the Standard Model are complex, its essential ingredients can be summarized with the help of Fig. 46.14. (Although the Standard Model does not include the gravitational force at present, we include gravity in Fig. 46.14 because physicists hope to eventually incorporate this force into a unified theory.) This diagram shows that quarks participate in all the fundamental forces and that leptons participate in all except the strong force.

The Standard Model does not answer all questions. A major question still unanswered is why, of the two mediators of the electroweak interaction, the photon has no mass but the W and Z bosons do. Because of this mass difference, the electromagnetic and weak forces are quite distinct at low energies but become similar at very high energies, when the rest energy is negligible relative to the total energy. The behavior as one goes from high to low energies is called *symmetry breaking* because the forces are similar, or symmetric, at high energies but are very different at low energies. The nonzero rest energies of the W and Z bosons raise the question of the origin of particle masses. To resolve this problem, a hypothetical particle called the **Higgs boson,** which provides a mechanism for breaking the electroweak symmetry, has been proposed. The Standard Model modified to include the Higgs boson provides a logically consistent explanation of the massive nature of the W and Z bosons. In July 2012, announcements from the ATLAS (A Toroidal LHC Apparatus) and CMS (Compact Muon Solenoid) experiments at the Large Hadron Collider (LHC) at CERN claimed the discovery of a new particle having properties consistent with that of a Higgs boson. The mass of the particle is 125–127 GeV, within the range of predictions made from theoretical considerations using the Standard Model.

Because of the limited energy available in conventional accelerators using fixed targets, it is necessary to employ colliding-beam accelerators called **colliders.** The concept of colliders is straightforward. Particles that have equal masses and equal kinetic energies, traveling in opposite directions in an accelerator ring, collide head-on to produce the required reaction and form new particles. Because the total momentum of the interacting particles is zero, all their kinetic energy is available for the reaction.

Several colliders provided important data for understanding the Standard Model in the latter part of the 20th century and the first decade of the 21st century: the Large Electron–Positron (LEP) Collider and the Super Proton Synchrotron at CERN, the Stanford Linear Collider, and the Tevatron at the Fermi National Laboratory in Illinois. The Relativistic Heavy Ion Collider at Brookhaven National Laboratory is the sole remaining collider in operation in the United States. The Large Hadron Collider at CERN, which began collision operations in March 2010, has

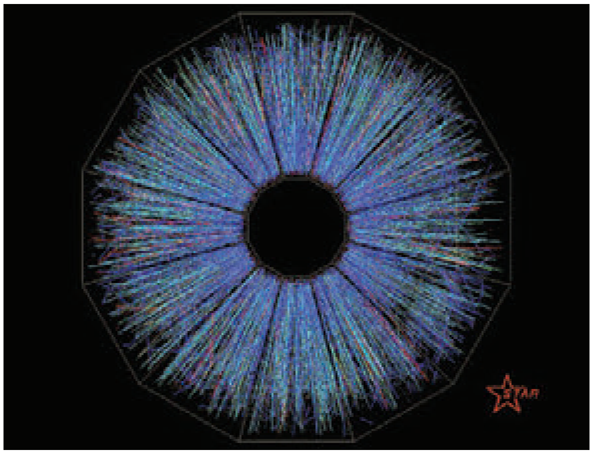


Figure 46.15 A shower of particle tracks from a head-on collision of gold nuclei, each moving with energy 100 GeV. This collision occurred at the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory and was recorded with the STAR (Solenoidal Tracker at RHIC) detector. The tracks represent many fundamental particles arising from the energy of the collision.

taken the lead in particle studies due to its extremely high energy capabilities. The expected upper limit for the LHC is a center-of-mass energy of 14 TeV. (See page 868 for a photo of a magnet used by the LHC.)

In addition to increasing energies in modern accelerators, detection techniques have become increasingly sophisticated. We saw simple bubble-chamber photographs earlier in this chapter that required hours of analysis by hand. Figure 46.15 shows a complex set of tracks from a collision of gold nuclei.

46.11 The Cosmic Connection

In this section, we describe one of the most fascinating theories in all science—the Big Bang theory of the creation of the Universe—and the experimental evidence that supports it. This theory of cosmology states that the Universe had a beginning and furthermore that the beginning was so cataclysmic that it is impossible to look back beyond it. According to this theory, the Universe erupted from an infinitely dense singularity about 14 billion years ago. The first few moments after the Big Bang saw such extremely high energy that it is believed that all four interactions of physics were unified and all matter was contained in a quark–gluon plasma.

The evolution of the four fundamental forces from the Big Bang to the present is shown in Figure 46.16 (page 1470). During the first 10^{-43} s (the ultrahot epoch, $T \sim 10^{32}$ K), it is presumed the strong, electroweak, and gravitational forces were joined to form a completely unified force. In the first 10^{-35} s following the Big Bang (the hot epoch, $T \sim 10^{29}$ K), symmetry breaking occurred for gravity while the strong and electroweak forces remained unified. It was a period when particle energies were so great ($> 10^{16}$ GeV) that very massive particles as well as quarks, leptons, and their antiparticles existed. Then, after 10^{-35} s, the Universe rapidly expanded and cooled (the warm epoch, $T \sim 10^{29}$ to 10^{15} K) and the strong and electroweak forces parted company. As the Universe continued to cool, the electroweak force split into the weak force and the electromagnetic force approximately 10^{-10} s after the Big Bang.

After a few minutes, protons and neutrons condensed out of the plasma. For half an hour, the Universe underwent thermonuclear fusion, exploding as a hydrogen bomb and producing most of the helium nuclei that now exist. The Universe continued to expand, and its temperature dropped. Until about 700 000 years after the Big Bang, the Universe was dominated by radiation. Energetic radiation prevented matter from forming single hydrogen atoms because photons would instantly ionize any atoms that happened to form. Photons experienced continuous Compton scattering from the vast numbers of free electrons, resulting in a Universe that was opaque to radiation. By the time the Universe was about 700 000 years old, it had

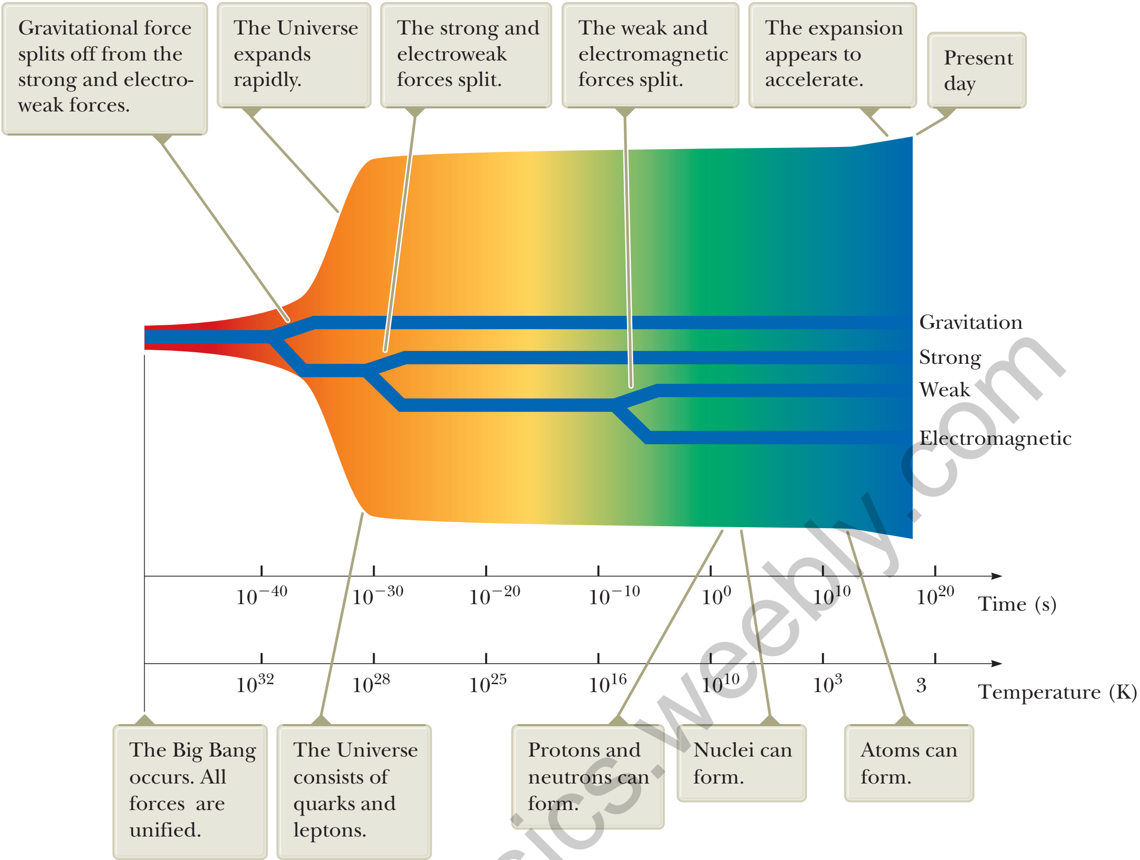


Figure 46.16 A brief history of the Universe from the Big Bang to the present. The four forces became distinguishable during the first nanosecond. Following that, all the quarks combined to form particles that interact via the nuclear force. The leptons, however, remained separate and to this day exist as individual, observable particles.

expanded and cooled to approximately 3 000 K and protons could bind to electrons to form neutral hydrogen atoms. Because of the quantized energies of the atoms, far more wavelengths of radiation were not absorbed by atoms than were absorbed, and the Universe suddenly became transparent to photons. Radiation no longer dominated the Universe, and clumps of neutral matter steadily grew: first atoms, then molecules, gas clouds, stars, and finally galaxies.

Observation of Radiation from the Primordial Fireball

In 1965, Arno A. Penzias (b. 1933) and Robert W. Wilson (b. 1936) of Bell Laboratories were testing a sensitive microwave receiver and made an amazing discovery. A pesky signal producing a faint background hiss was interfering with their satellite communications experiments. The microwave horn that served as their receiving antenna is shown in Figure 46.17. Evicting a flock of pigeons from the 20-ft horn and cooling the microwave detector both failed to remove the signal.

AT&T Bell Laboratories

Figure 46.17 Robert W. Wilson *(left)* and Arno A. Penzias with the Bell Telephone Laboratories horn-reflector antenna.

The intensity of the detected signal remained unchanged as the antenna was pointed in different directions. That the radiation had equal strengths in all directions suggested that the entire Universe was the source of this radiation. Ultimately, it became clear that they were detecting microwave background radiation (at a wavelength of 7.35 cm), which represented the leftover "glow" from the Big Bang. Through a casual conversation, Penzias and Wilson discovered that a group at Princeton University had predicted the residual radiation from the Big Bang and were planning an experiment to attempt to confirm the theory. The excitement in the scientific community was high when Penzias and Wilson announced that they had already observed an excess microwave background compatible with a 3-K

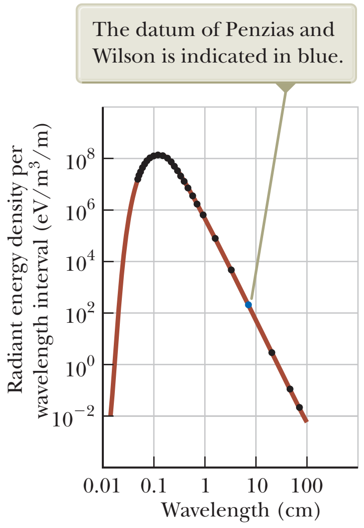


Figure 46.18 Theoretical blackbody (brown curve) and measured radiation spectra (black points) of the Big Bang. Most of the data were collected from the COsmic Background Explorer, or COBE, satellite.

blackbody source, which was consistent with the predicted temperature of the Universe at this time after the Big Bang.

Because Penzias and Wilson made their measurements at a single wavelength, they did not completely confirm the radiation as 3-K blackbody radiation. Subsequent experiments by other groups added intensity data at different wavelengths as shown in Figure 46.18. The results confirm that the radiation is that of a black body at 2.7 K. This figure is perhaps the most clear-cut evidence for the Big Bang theory. The 1978 Nobel Prize in Physics was awarded to Penzias and Wilson for this most important discovery.

In the years following Penzias and Wilson's discovery, other researchers made measurements at different wavelengths. In 1989, the COBE (COsmic Background Explorer) satellite was launched by NASA and added critical measurements at wavelengths below 0.1 cm. The results of these measurements led to a Nobel Prize in Physics for the principal investigators in 2006. Several data points from COBE are shown in Figure 46.18. The Wilkinson Microwave Anisotropy Probe, launched in June 2001, exhibits data that allow observation of temperature differences in the cosmos in the microkelvin range. Ongoing observations are also being made from Earth-based facilities, associated with projects such as QUaD, Qubic, and the South Pole Telescope. In addition, the Planck satellite was launched in May 2009 by the European Space Agency. This space-based observatory has been measuring the cosmic background radiation with higher sensitivity than the Wilkinson probe. The series of measurements taken since 1965 are consistent with thermal radiation associated with a temperature of 2.7 K. The whole story of the cosmic temperature is a remarkable example of science at work: building a model, making a prediction, taking measurements, and testing the measurements against the predictions.

Other Evidence for an Expanding Universe

The Big Bang theory of cosmology predicts that the Universe is expanding. Most of the key discoveries supporting the theory of an expanding Universe were made in the 20th century. Vesto Melvin Slipher (1875–1969), an American astronomer, reported in 1912 that most galaxies are receding from the Earth at speeds up to several million miles per hour. Slipher was one of the first scientists to use Doppler shifts (see Section 17.4) in spectral lines to measure galaxy velocities.

In the late 1920s, Edwin P. Hubble (1889–1953) made the bold assertion that the whole Universe is expanding. From 1928 to 1936, until they reached the limits of the 100-inch telescope, Hubble and Milton Humason (1891–1972) worked at Mount Wilson in California to prove this assertion. The results of that work and of its continuation with the use of a 200-inch telescope in the 1940s showed that the speeds at which galaxies are receding from the Earth increase in direct proportion to their distance R from us. This linear relationship, known as **Hubble's law,** may be written

Hubble's law ▶

$$v = HR \quad (46.4)$$

where H, called the **Hubble constant,** has the approximate value

$$H \approx 22 \times 10^{-3} \text{ m/(s} \cdot \text{ly)}$$

Example 46.5 Recession of a Quasar AM

A quasar is an object that appears similar to a star and is very distant from the Earth. Its speed can be determined from Doppler-shift measurements in the light it emits. A certain quasar recedes from the Earth at a speed of $0.55c$. How far away is it?

SOLUTION

Conceptualize A common mental representation for the Hubble law is that of raisin bread cooking in an oven. Imagine yourself at the center of the loaf of bread. As the entire loaf of bread expands upon heating, raisins near you move slowly with respect to you. Raisins far away from you on the edge of the loaf move at a higher speed.

Categorize We use a concept developed in this section, so we categorize this example as a substitution problem.

Find the distance through Hubble's law:

$$R = \frac{v}{H} = \frac{(0.55)(3.00 \times 10^8 \text{ m/s})}{22 \times 10^{-3} \text{ m/(s} \cdot \text{ly)}} = 7.5 \times 10^9 \text{ ly}$$

WHAT IF? Suppose the quasar has moved at this speed ever since the Big Bang. With this assumption, estimate the age of the Universe.

Answer Let's approximate the distance from the Earth to the quasar as the distance the quasar has moved from the singularity since the Big Bang. We can then find the time interval from the *particle under constant speed* model: $\Delta t = d/v = R/v = 1/H \approx$ 14 billion years, which is in approximate agreement with other calculations.

Will the Universe Expand Forever?

In the 1950s and 1960s, Allan R. Sandage (1926–2010) used the 200-inch telescope at Mount Palomar to measure the speeds of galaxies at distances of up to 6 billion light-years away from the Earth. These measurements showed that these very distant galaxies were moving approximately 10 000 km/s faster than Hubble's law predicted. According to this result, the Universe must have been expanding more rapidly 1 billion years ago, and consequently we conclude from these data that the expansion rate is slowing.[6] Today, astronomers and physicists are trying to determine the rate of expansion. If the average mass density of the Universe is less than some critical value ρ_c, the galaxies will slow in their outward rush but still escape to infinity. If the average density exceeds the critical value, the expansion will eventually stop and contraction will begin, possibly leading to a superdense state followed by another expansion. In this scenario, we have an oscillating Universe.

Example 46.6 The Critical Density of the Universe AM

(A) Starting from energy conservation, derive an expression for the critical mass density of the Universe ρ_c in terms of the Hubble constant H and the universal gravitational constant G.

[6]The data at large distances have large observational uncertainties and may be systematically in error from effects such as abnormal brightness in the most distant visible clusters.

SOLU IO

Conceptualize Figure 46.19 shows a large section of the Universe, contained within a sphere of radius *R*. The total mass in this volume is A galaxy of mass that has a speed at a distance from the center of the sphere escapes to infinity (at which its speed approaches zero) if the sum of its kinetic energy and the gravitational potential energy of the system is zero.

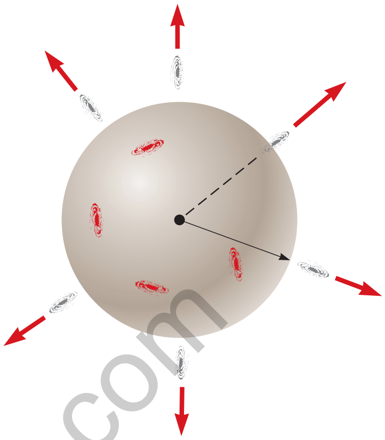


Figure 46.19 (Example 46.6) The galaxy marked with mass is escaping from a large cluster of galaxies contained within a spherical volume of radius . Only the mass within slows the escaping galaxy.

Categorize The Universe may be infinite in spatial extent, but Gauss's law for gravitation (an analog to Gauss's law for electric fields in Chapter 24) implies that only the mass inside the sphere contributes to the gravitational poten tial energy of the galaxy–sphere system. Therefore, we categorize this problem as one in which we apply Gauss's law for gravitation. We model the sphere in Figure 46.19 and the escaping galaxy as an *isolated system* for *energy*.

Analyze Write the appropriate reduction of Equation 8.2, assuming that the galaxy leaves the spherical volume while moving at the escape speed:

$$-mv \qquad \frac{GmM}{}\Big)\Big]$$

Substitute for the mass contained within the sphere the product of the critical density and the volume of the sphere:

$$-mv \qquad \frac{Gm\ -}{}$$

Solve for the critical density:

$$\frac{}{}$$

From Hubble's law, substitute for the ratio

$$(1) \qquad \frac{}{} \quad - \quad \frac{}{}$$

(B) Estimate a numerical value for the critical density in grams per cubic centimeter.

SOLU IO

In Equation (1), substitute numerical values for and

$$\frac{}{} \quad \frac{22 \quad 10 \quad \text{m} \qquad \text{ly})]}{6.67 \quad 10^{\ 11}\ \text{N} \qquad \text{kg}} \quad 8.7 \quad 10\ \ \text{kg} \qquad \text{ly}$$

Reconcile the units by converting light-years to meters:

$$8.7 \quad 10\ \ \text{kg} \qquad \text{ly} \quad \frac{1\ \text{ly}}{9.46 \quad 10^{15}\ \text{m}}$$

$$9.7 \qquad \text{kg/m} \qquad 9.7 \quad 10^{\ 30}\ \text{g} \quad \text{cm}$$

Finalize Because the mass of a hydrogen atom is 1.67 ^{24} g, this value of corresponds to 6 hydrogen atoms per cubic centimeter or 6 atoms per cubic meter.

Missing Mass in the Universe?

The luminous matter in galaxies averages out to a Universe density of about g/cm . The radiation in the Universe has a mass equivalent of approxi mately 2% of the luminous matter. The total mass of all nonluminous matter (such as interstellar gas and black holes) may be estimated from the speeds of galaxies orbiting each other in a cluster. The higher the galaxy speeds, the more mass in the cluster. Measurements on the Coma cluster of galaxies indicate, surprisingly, that the amount of nonluminous matter is 20 to 30 times the amount of luminous matter present in stars and luminous gas clouds. Yet even this large, invisible component of *dark matter* (see Section 13.6), if extrapolated to the Universe as a whole, leaves the observed mass density a factor of 10 less than ρ_c calculated in Example 46.6. The deficit, called *missing mass,* has been the subject of intense theoretical and experimental work, with exotic particles such as axions, photinos, and superstring particles suggested as candidates for the missing mass. Some researchers have made the more mundane proposal that the missing mass is present in neutrinos. In fact, neutrinos are so abundant that a tiny neutrino rest energy on the order of only 20 eV would furnish the missing mass and "close" the Universe. Current experiments designed to measure the rest energy of the neutrino will have an effect on predictions for the future of the Universe.

Mysterious Energy in the Universe?

A surprising twist in the story of the Universe arose in 1998 with the observation of a class of supernovae that have a fixed absolute brightness. By combining the apparent brightness and the redshift of light from these explosions, their distance and speed of recession from the Earth can be determined. These observations led to the conclusion that the expansion of the Universe is not slowing down, but is accelerating! Observations by other groups also led to the same interpretation.

To explain this acceleration, physicists have proposed *dark energy,* which is energy possessed by the vacuum of space. In the early life of the Universe, gravity dominated over the dark energy. As the Universe expanded and the gravitational force between galaxies became smaller because of the great distances between them, the dark energy became more important. The dark energy results in an effective repulsive force that causes the expansion rate to increase.[7]

Although there is some degree of certainty about the beginning of the Universe, we are uncertain about how the story will end. Will the Universe keep on expanding forever, or will it someday collapse and then expand again, perhaps in an endless series of oscillations? Results and answers to these questions remain inconclusive, and the exciting controversy continues.

46.12 Problems and Perspectives

While particle physicists have been exploring the realm of the very small, cosmologists have been exploring cosmic history back to the first microsecond of the Big Bang. Observation of the events that occur when two particles collide in an accelerator is essential for reconstructing the early moments in cosmic history. For this reason, perhaps the key to understanding the early Universe is to first understand the world of elementary particles. Cosmologists and physicists now find that they have many common goals and are joining hands in an attempt to understand the physical world at its most fundamental level.

Our understanding of physics at short distances is far from complete. Particle physics is faced with many questions. Why does so little antimatter exist in the Universe? Is it possible to unify the strong and electroweak theories in a logical and consistent manner? Why do quarks and leptons form three similar but distinct families? Are muons the same as electrons apart from their difference in mass, or do they have other subtle differences that have not been detected? Why are some particles charged and others neutral? Why do quarks carry a fractional charge? What determines the masses of the elementary constituents of matter? Can isolated quarks exist? Why do electrons and protons have *exactly* the same magnitude of

[7]For an overview of dark energy, see S. Perlmutter, "Supernovae, Dark Energy, and the Accelerating Universe," *Physics Today* **56**(4): 53–60, April 2003.

charge when one is a truly fundamental particle and the other is built from smaller particles?

An important and obvious question that remains is whether leptons and quarks have an underlying structure. If they do, we can envision an infinite number of deeper structure levels. If leptons and quarks are indeed the ultimate constituents of matter, however, scientists hope to construct a final theory of the structure of matter, just as Einstein dreamed of doing. This theory, whimsically called the The ory of Everything, is a combination of the Standard Model and a quantum theory of gravity.

String Theory: A New Perspective

Let's briefly discuss one current effort at answering some of these questions by proposing a new perspective on particles. While reading this book, you may recall starting off with the *particle* model in Chapter 2 and doing quite a bit of physics with it. In Chapter 16, we introduced the *wave* model, and there was more physics to be investigated via the properties of waves. We used a *wave* model for light in Chapter 35; in Chapter 40, however, we saw the need to return to the *particle* model for light. Furthermore, we found that material particles had wave-like characteristics. The quantum particle model discussed in Chapter 40 allowed us to build particles out of waves, suggesting that a *wave* is the fundamental entity. In the current Chap ter 46, however, we introduced elementary *particles* as the fundamental entities. It seems as if we cannot make up our mind! In this final section, we discuss a current research effort to build particles out of waves and vibrations on strings!

String theory is an effort to unify the four fundamental forces by modeling all particles as various quantized vibrational modes of a single entity, an incredibly small string. The typical length of such a string is on the order of 10 m, called the **Planck length.** We have seen quantized modes before in the frequencies of vibrating guitar strings in Chapter 18 and the quantized energy levels of atoms in Chapter 42. In string theory, each quantized mode of vibration of the string cor responds to a different elementary particle in the Standard Model.

One complicating factor in string theory is that it requires space–time to have ten dimensions. Despite the theoretical and conceptual difficulties in dealing with ten dimensions, string theory holds promise in incorporating gravity with the other forces. Four of the ten dimensions—three space dimensions and one time dimen sion—are visible to us. The other six are said to be *compactified;* that is, the six dimen sions are curled up so tightly that they are not visible in the macroscopic world.

As an analogy, consider a soda straw. You can build a soda straw by cutting a rectangular piece of paper (Fig. 46.20a), which clearly has two dimensions, and rolling it into a small tube (Fig. 46.20b). From far away, the soda straw looks like a one-dimensional straight line. The second dimension has been curled up and is not visible. String theory claims that six space–time dimensions are curled up in an analogous way, with the curling being on the size of the Planck length and impos sible to see from our viewpoint.

Another complicating factor with string theory is that it is difficult for string theorists to guide experimentalists as to what to look for in an experiment. The

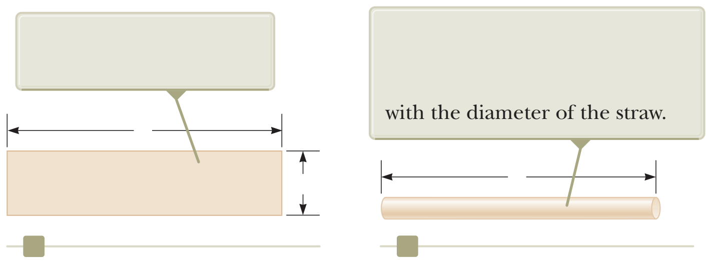


Figure 46.20 (a) A piece of paper is cut into a rectangular shape. (b) The paper is rolled up into a soda straw.

Planck length is so small that direct experimentation on strings is impossible. Until the theory has been further developed, string theorists are restricted to applying the theory to known results and testing for consistency.

One of the predictions of string theory, called **supersymmetry,** or SUSY, suggests that every elementary particle has a superpartner that has not yet been observed. It is believed that supersymmetry is a broken symmetry (like the broken electroweak symmetry at low energies) and the masses of the superpartners are above our current capabilities of detection by accelerators. Some theorists claim that the mass of superpartners is the missing mass discussed in Section 46.11. Keeping with the whimsical trend in naming particles and their properties, superpartners are given names such as the *squark* (the superpartner to a quark), the *selectron* (electron), and the *gluino* (gluon).

Other theorists are working on **M-theory,** which is an eleven-dimensional theory based on membranes rather than strings. In a way reminiscent of the correspondence principle, M-theory is claimed to reduce to string theory if one compactifies from eleven dimensions to ten dimensions.

The questions listed at the beginning of this section go on and on. Because of the rapid advances and new discoveries in the field of particle physics, many of these questions may be resolved in the next decade and other new questions may emerge.

Summary

Concepts and Principles

Before quark theory was developed, the four fundamental forces in nature were identified as nuclear, electromagnetic, weak, and gravitational. All the interactions in which these forces take part are mediated by **field particles.** The electromagnetic interaction is mediated by photons; the weak interaction is mediated by the $W^{\pm}$ and Z^0 bosons; the gravitational interaction is mediated by gravitons; and the nuclear interaction is mediated by gluons.

A charged particle and its **antiparticle** have the same mass but opposite charge, and other properties will have opposite values, such as lepton number and baryon number. It is possible to produce particle–antiparticle pairs in nuclear reactions if the available energy is greater than $2mc^2$, where m is the mass of the particle (or antiparticle).

Particles other than field particles are classified as hadrons or leptons. **Hadrons** interact via all four fundamental forces. They have size and structure and are not elementary particles. There are two types, **baryons** and **mesons.** Baryons, which generally are the most massive particles, have nonzero **baryon number** and a spin of $\frac{1}{2}$ or $\frac{3}{2}$. Mesons have baryon number zero and either zero or integral spin.

Leptons have no structure or size and are considered truly elementary. They interact only via the weak, gravitational, and electromagnetic forces. Six types of leptons exist: the electron e^-, the muon μ^-, and the tau τ^-, and their neutrinos ν_e, ν_μ, and ν_τ.

In all reactions and decays, quantities such as energy, linear momentum, angular momentum, electric charge, baryon number, and lepton number are strictly conserved. Certain particles have properties called **strangeness** and **charm.** These unusual properties are conserved in all decays and nuclear reactions except those that occur via the weak force.

Theorists in elementary particle physics have postulated that all hadrons are composed of smaller units known as **quarks,** and experimental evidence agrees with this model. Quarks have fractional electric charge and come in six **flavors:** up (u), down (d), strange (s), charmed (c), top (t), and bottom (b). Each baryon contains three quarks, and each meson contains one quark and one antiquark.

According to the theory of **quantum chromodynamics,** quarks have a property called **color;** the force between quarks is referred to as the **strong force** or the **color force.** The strong force is now considered to be a fundamental force. The nuclear force, which was originally considered to be fundamental, is now understood to be a secondary effect of the strong force due to gluon exchanges between hadrons.

The electromagnetic and weak forces are now considered to be manifestations of a single force called the **electroweak force.** The combination of quantum chromodynamics and the electroweak theory is called the **Standard Model.**

The background microwave radiation discovered by Penzias and Wilson strongly suggests that the Universe started with a Big Bang about 14 billion years ago. The background radiation is equivalent to that of a black body at 3 K. Various astronomical measurements strongly suggest that the Universe is expanding. According to **Hubble's law,** distant galaxies are receding from the Earth at a speed $v = HR$, where H is the **Hubble constant,** $H \approx 22 \times 10^{-3}$ m/(s · ly), and R is the distance from the Earth to the galaxy.

Objective Questions

1. denotes answer available in *Student Solutions Manual/Study Guide*

1. What interactions affect protons in an atomic nucleus? More than one answer may be correct. (a) the nuclear interaction (b) the weak interaction (c) the electromagnetic interaction (d) the gravitational interaction

2. In one experiment, two balls of clay of the same mass travel with the same speed v toward each other. They collide head-on and come to rest. In a second experiment, two clay balls of the same mass are again used. One ball hangs at rest, suspended from the ceiling by a thread. The second ball is fired toward the first at speed v, to collide, stick to the first ball, and continue to move forward. Is the kinetic energy that is transformed into internal energy in the first experiment (a) one-fourth as much as in the second experiment, (b) one-half as much as in the second experiment, (c) the same as in the second experiment, (d) twice as much as in the second experiment, or (e) four times as much as in the second experiment?

3. The Ω^- particle is a baryon with spin $\frac{3}{2}$. Does the Ω^- particle have (a) three possible spin states in a magnetic field, (b) four possible spin states, (c) three times the charge of a spin $-\frac{1}{2}$ particle, or (d) three times the mass of a spin $-\frac{1}{2}$ particle, or (e) are none of those choices correct?

4. Which of the following field particles mediates the strong force? (a) photon (b) gluon (c) graviton (d) $W^\pm$ and Z bosons (e) none of those field particles

5. An isolated stationary muon decays into an electron, an electron antineutrino, and a muon neutrino. Is the total kinetic energy of these three particles (a) zero, (b) small, or (c) large compared to their rest energies, or (d) none of those choices are possible?

6. Define the average density of the solar system ρ_{SS} as the total mass of the Sun, planets, satellites, rings, asteroids, icy outliers, and comets, divided by the volume of a sphere around the Sun large enough to contain all these objects. The sphere extends about halfway to the nearest star, with a radius of approximately 2×10^{16} m, about two light-years. How does this average density of the solar system compare with the critical density ρ_c required for the Universe to stop its Hubble's-law expansion? (a) ρ_{SS} is much greater than ρ_c. (b) ρ_{SS} is approximately or precisely equal to ρ_c. (c) ρ_{SS} is much less than ρ_c. (d) It is impossible to determine.

7. When an electron and a positron meet at low speed in empty space, they annihilate each other to produce two 0.511-MeV gamma rays. What law would be violated if they produced one gamma ray with an energy of 1.02 MeV? (a) conservation of energy (b) conservation of momentum (c) conservation of charge (d) conservation of baryon number (e) conservation of electron lepton number

8. Place the following events into the correct sequence from the earliest in the history of the Universe to the latest. (a) Neutral atoms form. (b) Protons and neutrons are no longer annihilated as fast as they form. (c) The Universe is a quark–gluon soup. (d) The Universe is like the core of a normal star today, forming helium by nuclear fusion. (e) The Universe is like the surface of a hot star today, consisting of a plasma of ionized atoms. (f) Polyatomic molecules form. (g) Solid materials form.

Conceptual Questions

1. denotes answer available in *Student Solutions Manual/Study Guide*

1. The W and Z bosons were first produced at CERN in 1983 by causing a beam of protons and a beam of antiprotons to meet at high energy. Why was this discovery important?

2. What are the differences between hadrons and leptons?

3. Neutral atoms did not exist until hundreds of thousands of years after the Big Bang. Why?

4. Describe the properties of baryons and mesons and the important differences between them.

5. The Ξ^0 particle decays by the weak interaction according to the decay mode $\Xi^0 \rightarrow \Lambda^0 + \pi^0$. Would you expect this decay to be fast or slow? Explain.

6. In the theory of quantum chromodynamics, quarks come in three colors. How would you justify the statement that "all baryons and mesons are colorless"?

7. An antibaryon interacts with a meson. Can a baryon be produced in such an interaction? Explain.

8. Describe the essential features of the Standard Model of particle physics.

9. How many quarks are in each of the following: (a) a baryon, (b) an antibaryon, (c) a meson, (d) an antimeson? (e) How do you explain that baryons have half-integral spins, whereas mesons have spins of 0 or 1?

10. Are the laws of conservation of baryon number, lepton number, and strangeness based on fundamental properties of nature (as are the laws of conservation of momentum and energy, for example)? Explain.

11. Name the four fundamental interactions and the field particle that mediates each.

12. How did Edwin Hubble determine in 1928 that the Universe is expanding?

13. Kaons all decay into final states that contain no protons or neutrons. What is the baryon number for kaons?

Problems

ENHANCED WebAssign The problems found in this chapter may be assigned online in Enhanced WebAssign

1. straightforward; **2.** intermediate; **3.** challenging

1. full solution available in the *Student Solutions Manual/Study Guide*

AMT Analysis Model tutorial available in Enhanced WebAssign

GP Guided Problem

M Master It tutorial available in Enhanced WebAssign

W Watch It video solution available in Enhanced WebAssign

Section 46.1 The Fundamental Forces in Nature

Section 46.2 Positrons and Other Antiparticles

1. Model a penny as 3.10 g of pure copper. Consider an anti-penny minted from 3.10 g of copper anti-atoms, each with 29 positrons in orbit around a nucleus comprising 29 antiprotons and 34 or 36 antineutrons. (a) Find the energy released if the two coins collide. (b) Find the value of this energy at the unit price of \$0.11/kWh, a representative retail rate for energy from the electric company.

2. Two photons are produced when a proton and an antiproton annihilate each other. In the reference frame in which the center of mass of the proton–antiproton system is stationary, what are (a) the minimum frequency and (b) the corresponding wavelength of each photon?

3. A photon produces a proton–antiproton pair according to the reaction $\gamma \rightarrow p + \bar{p}$. (a) What is the minimum possible frequency of the photon? (b) What is its wavelength?

4. At some time in your life, you may find yourself in a hospital to have a PET, or positron-emission tomography, scan. In the procedure, a radioactive element that undergoes e^+ decay is introduced into your body. The equipment detects the gamma rays that result from pair annihilation when the emitted positron encounters an electron in your body's tissue. During such a scan, suppose you receive an injection of glucose containing on the order of 10^{10} atoms of ^{14}O, with half-life 70.6 s. Assume the oxygen remaining after 5 min is uniformly distributed through 2 L of blood. What is then the order of magnitude of the oxygen atoms' activity in 1 cm^3 of the blood?

5. M A photon with an energy E_γ = 2.09 GeV creates a proton–antiproton pair in which the proton has a kinetic energy of 95.0 MeV. What is the kinetic energy of the antiproton? *Note:* $m_p c^2 = 938.3$ MeV.

Section 46.3 Mesons and the Beginning of Particle Physics

6. One mediator of the weak interaction is the Z^0 boson, with mass 91 GeV/c^2. Use this information to find the order of magnitude of the range of the weak interaction.

7. (a) Prove that the exchange of a virtual particle of mass m can be associated with a force with a range given by

$$d \approx \frac{1\,240}{4\pi mc^2} = \frac{98.7}{mc^2}$$

where d is in nanometers and mc^2 is in electron volts. (b) State the pattern of dependence of the range on the mass. (c) What is the range of the force that might be produced by the virtual exchange of a proton?

Section 46.4 Classification of Particles

Section 46.5 Conservation Laws

8. The first of the following two reactions can occur, but the second cannot. Explain.

$$K_S^0 \rightarrow \pi^+ + \pi^- \quad \text{(can occur)}$$

$$\Lambda^0 \rightarrow \pi^+ + \pi^- \quad \text{(cannot occur)}$$

9. A neutral pion at rest decays into two photons according to $\pi^0 \rightarrow \gamma + \gamma$. Find the (a) energy, (b) momentum, and (c) frequency of each photon.

10. When a high-energy proton or pion traveling near the speed of light collides with a nucleus, it travels an average distance of 3×10^{-15} m before interacting. From this information, find the order of magnitude of the time interval required for the strong interaction to occur.

11. Each of the following reactions is forbidden. Determine what conservation laws are violated for each reaction.

(a) $p + \overline{p} \rightarrow \mu^+ + e^-$

(b) $\pi^- + p \rightarrow p + \pi^+$

(c) $p + p \rightarrow p + p + n$

(d) $\gamma + p \rightarrow n + \pi^0$

(e) $\nu_e + p \rightarrow n + e^+$

12. (a) Show that baryon number and charge are conserved in the following reactions of a pion with a proton:

(1) $\pi^+ + p \rightarrow K^+ + \Sigma^+$

(2) $\pi^+ + p \rightarrow \pi^+ + \Sigma^+$

(b) The first reaction is observed, but the second never occurs. Explain.

13. The following reactions or decays involve one or more neutrinos. In each case, supply the missing neutrino (ν_e, ν_μ, or ν_τ) or antineutrino.

(a) $\pi^- \rightarrow \mu^- + ?$ (b) $K^+ \rightarrow \mu^+ + ?$

(c) $? + p \rightarrow n + e^+$ (d) $? + n \rightarrow p + e^-$

(e) $? + n \rightarrow p + \mu^-$ (f) $\mu^- \rightarrow e^- + ? + ?$

14. Determine the type of neutrino or antineutrino involved in each of the following processes.

(a) $\pi^+ \rightarrow \pi^0 + e^+ + ?$ (b) $? + p \rightarrow \mu^- + p + \pi^+$

(c) $\Lambda^0 \rightarrow p + \mu^- + ?$ (d) $\tau^+ \rightarrow \mu^+ + ? + ?$

15. Determine which of the following reactions can occur. For those that cannot occur, determine the conservation law (or laws) violated.

(a) $p \rightarrow \pi^+ + \pi^0$ (b) $p + p \rightarrow p + p + \pi^0$

(c) $p + p \rightarrow p + \pi^+$ (d) $\pi^+ \rightarrow \mu^+ + \nu_\mu$

(e) $n \rightarrow p + e^- + \overline{\nu}_e$ (f) $\pi^+ \rightarrow \mu^+ + n$

16. Occasionally, high-energy muons collide with electrons and produce two neutrinos according to the reaction $\mu^+ + e^- \rightarrow 2\nu$. What kind of neutrinos are they?

17. A K_S^0 particle at rest decays into a π^+ and a π^-. The mass of the K_S^0 is 497.7 MeV/c^2, and the mass of each π meson is 139.6 MeV/c^2. What is the speed of each pion?

18. (a) Show that the proton-decay $p \rightarrow e^+ + \gamma$ cannot occur because it violates the conservation of baryon number. (b) **What If?** Imagine that this reaction does occur and the proton is initially at rest. Determine the energies and magnitudes of the momentum of the positron and photon after the reaction. (c) Determine the speed of the positron after the reaction.

19. A Λ^0 particle at rest decays into a proton and a π^- meson. (a) Use the data in Table 46.2 to find the Q value for this decay in MeV. (b) What is the total kinetic energy shared by the proton and the π^- meson after the decay? (c) What is the total momentum shared by the proton and the π^- meson? (d) The proton and the π^- meson have momenta with the same magnitude after the decay. Do they have equal kinetic energies? Explain.

Section 46.6 Strange Particles and Strangeness

20. The neutral meson ρ^0 decays by the strong interaction into two pions:

$$\rho^0 \rightarrow \pi^+ + \pi^- \quad (T_{1/2} \sim 10^{-23} \text{ s})$$

The neutral kaon also decays into two pions:

$$K_S^0 \rightarrow \pi^+ + \pi^- \quad (T_{1/2} \sim 10^{-10} \text{ s})$$

How do you explain the difference in half-lives?

21. Which of the following processes are allowed by the strong interaction, the electromagnetic interaction, the weak interaction, or no interaction at all?

(a) $\pi^- + p \rightarrow 2\eta$ (b) $K^- + n \rightarrow \Lambda^0 + \pi^-$

(c) $K^- \rightarrow \pi^- + \pi^0$ (d) $\Omega^- \rightarrow \Xi^- + \pi^0$

(e) $\eta \rightarrow 2\gamma$

22. For each of the following forbidden decays, determine what conservation laws are violated.

(a) $\mu^- \rightarrow e^- + \gamma$ (b) $n \rightarrow p + e^- + \nu_e$

(c) $\Lambda^0 \rightarrow p + \pi^0$ (d) $p \rightarrow e^+ + \pi^0$

(e) $\Xi^0 \rightarrow n + \pi^0$

23. Fill in the missing particle. Assume reaction (a) occurs via the strong interaction and reactions (b) and (c) involve the weak interaction. Assume also the total strangeness changes by one unit if strangeness is not conserved.

(a) $K^+ + p \rightarrow ? + p$ (c) $K^+ \rightarrow ? + \mu^+ + \nu_\mu$

(b) $\Omega^- \rightarrow ? + \pi^-$

24. Identify the conserved quantities in the following processes.

(a) $\Xi^- \rightarrow \Lambda^0 + \mu^- + \nu_\mu$ (b) $K_S^0 \rightarrow 2\pi^0$
(c) $K^- + p \rightarrow \Sigma^0 + n$ (d) $\Sigma^0 \rightarrow \Lambda^0 + \gamma$
(e) $e^+ + e^- \rightarrow \mu^+ + \mu^-$ (f) $\overline{p} + n \rightarrow \overline{\Lambda^0} + \Sigma^-$
(g) Which reactions cannot occur? Why not?

25. Determine whether or not strangeness is conserved in the following decays and reactions.

(a) $\Lambda^0 \rightarrow p + \pi^-$ (b) $\pi^- + p \rightarrow \Lambda^0 + K^0$
(c) $\overline{p} + p \rightarrow \overline{\Lambda^0} + \Lambda^0$ (d) $\pi^- + p \rightarrow \pi^- + \Sigma^+$
(e) $\Xi^- \rightarrow \Lambda^0 + \pi^-$ (f) $\Xi^0 \rightarrow p + \pi^-$

26. GP The particle decay $\Sigma^+ \rightarrow \pi^+ + n$ is observed in a bubble chamber. Figure P46.26 represents the curved tracks of the particles Σ^+ and π^+ and the invisible track of the neutron in the presence of a uniform magnetic field of 1.15 T directed out of the page. The measured radii of curvature are 1.99 m for the Σ^+ particle and 0.580 m for the π^+ particle. From this information, we wish to determine the mass of the Σ^+ particle. (a) Find the magnitudes of the momenta of the Σ^+ and the π^+ particles in units of MeV/c. (b) The angle between the momenta of the Σ^+ and the π^+ particles at the moment of decay is $\theta = 64.5°$. Find the magnitude of the momentum of the neutron. (c) Calculate the total energy of the π^+ particle and of the neutron from their known masses ($m_\pi = 139.6$ MeV/c^2, $m_n = 939.6$ MeV/c^2) and the relativistic energy–momentum relation. (d) What is the total energy of the Σ^+ particle? (e) Calculate the mass of the Σ^+ particle. (f) Compare the mass with the value in Table 46.2.

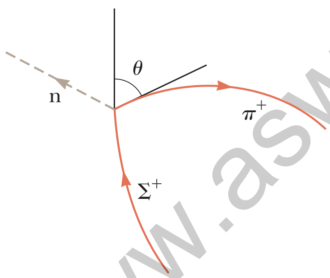


Figure P46.26

27. M If a K_S^0 meson at rest decays in 0.900×10^{-10} s, how far does a K_S^0 meson travel if it is moving at $0.960c$?

Section 46.7 Finding Patterns in the Particles

Section 46.8 Quarks

Section 46.9 Multicolored Quarks

Section 46.10 The Standard Model

Problem 89 in Chapter 39 can be assigned with Section 46.10.

28. The quark compositions of the K^0 and Λ^0 particles are $d\overline{s}$ and uds, respectively. Show that the charge, baryon number, and strangeness of these particles equal the sums of these numbers for the quark constituents.

29. The reaction $\pi^- + p \rightarrow K^0 + \Lambda^0$ occurs with high probability, whereas the reaction $\pi^- + p \rightarrow K^0 + n$ never occurs. Analyze these reactions at the quark level. Show that the first reaction conserves the total number of each type of quark and the second reaction does not.

30. Identify the particles corresponding to the quark states (a) suu, (b) $\overline{u}d$, (c) $\overline{s}d$, and (d) ssd.

31. The quark composition of the proton is uud, whereas that of the neutron is udd. Show that the charge, baryon number, and strangeness of these particles equal the sums of these numbers for their quark constituents.

32. Analyze each of the following reactions in terms of constituent quarks and show that each type of quark is conserved. (a) $\pi^+ + p \rightarrow K^+ + \Sigma^+$ (b) $K^- + p \rightarrow K^+ + K^0 + \Omega^-$ (c) Determine the quarks in the final particle for this reaction: $p + p \rightarrow K^0 + p + \pi^+ + ?$ (d) In the reaction in part (c), identify the mystery particle.

33. What is the electrical charge of the baryons with the quark compositions (a) $\overline{u}\,\overline{u}\,\overline{d}$ and (b) $\overline{u}\,\overline{d}\,\overline{d}$ (c) What are these baryons called?

34. M Find the number of electrons, and of each species of quark, in 1 L of water.

35. A Σ^0 particle traveling through matter strikes a proton; then a Σ^+ and a gamma ray as well as a third particle emerge. Use the quark model of each to determine the identity of the third particle.

36. **What If?** Imagine that binding energies could be ignored. Find the masses of the u and d quarks from the masses of the proton and neutron.

Section 46.11 The Cosmic Connection

Problem 21 in Chapter 39 can be assigned with this section.

37. **Review.** Refer to Section 39.4. Prove that the Doppler shift in wavelength of electromagnetic waves is described by

$$\lambda' = \lambda\sqrt{\frac{1 + v/c}{1 - v/c}}$$

where λ' is the wavelength measured by an observer moving at speed v away from a source radiating waves of wavelength λ.

38. Gravitation and other forces prevent Hubble's-law expansion from taking place except in systems larger than clusters of galaxies. **What If?** Imagine that these forces could be ignored and all distances expanded at a rate described by the Hubble constant of 22×10^{-3} m/(s · ly). (a) At what rate would the 1.85-m height of a basketball player be increasing? (b) At what rate would the distance between the Earth and the Moon be increasing?

39. **Review.** The cosmic background radiation is blackbody radiation from a source at a temperature of 2.73 K.

(a) Use Wien's law to determine the wavelength at which this radiation has its maximum intensity. (b) In what part of the electromagnetic spectrum is the peak of the distribution?

40. Assume dark matter exists throughout space with a uniform density of 6.00×10^{-28} kg/m^3. (a) Find the amount of such dark matter inside a sphere centered on the Sun, having the Earth's orbit as its equator. (b) Explain whether the gravitational field of this dark matter would have a measurable effect on the Earth's revolution.

41. The early Universe was dense with gamma-ray photons of energy $\sim k_B T$ and at such a high temperature that protons and antiprotons were created by the process $\gamma \rightarrow \text{p} + \bar{\text{p}}$ as rapidly as they annihilated each other. As the Universe cooled in adiabatic expansion, its temperature fell below a certain value and proton pair production became rare. At that time, slightly more protons than antiprotons existed, and essentially all the protons in the Universe today date from that time. (a) Estimate the order of magnitude of the temperature of the Universe when protons condensed out. (b) Estimate the order of magnitude of the temperature of the Universe when electrons condensed out.

42. If the average density of the Universe is small compared with the critical density, the expansion of the Universe described by Hubble's law proceeds with speeds that are nearly constant over time. (a) Prove that in this case the age of the Universe is given by the inverse of the Hubble constant. (b) Calculate $1/H$ and express it in years.

43. Review. A star moving away from the Earth at $0.280c$ emits radiation that we measure to be most intense at the wavelength 500 nm. Determine the surface temperature of this star.

44. Review. Use Stefan's law to find the intensity of the cosmic background radiation emitted by the fireball of the Big Bang at a temperature of 2.73 K.

45. M The first quasar to be identified and the brightest found to date, 3C 273 in the constellation Virgo, was observed to be moving away from the Earth at such high speed that the observed blue 434-nm H_γ line of hydrogen is Doppler-shifted to 510 nm, in the green portion of the spectrum. (a) How fast is the quasar receding? (b) Edwin Hubble discovered that all objects outside the local group of galaxies are moving away from us, with speeds v proportional to their distances R. Hubble's law is expressed as $v = HR$, where the Hubble constant has the approximate value $H \approx 22 \times 10^{-3}$ m/(s · ly). Determine the distance from the Earth to this quasar.

46. The various spectral lines observed in the light from a distant quasar have longer wavelengths λ'_n than the wavelengths λ_n measured in light from a stationary source. Here n is an index taking different values for different spectral lines. The fractional change in wavelength toward the red is the same for all spectral lines. That is, the Doppler redshift parameter Z defined by

$$Z = \frac{\lambda'_n - \lambda_n}{\lambda_n}$$

is common to all spectral lines for one object. In terms of Z, use Hubble's law to determine (a) the speed of recession of the quasar and (b) the distance from the Earth to this quasar.

47. W Using Hubble's law, find the wavelength of the 590-nm sodium line emitted from galaxies (a) 2.00×10^6 ly, (b) 2.00×10^8 ly, and (c) 2.00×10^9 ly away from the Earth.

48. The visible section of the Universe is a sphere centered on the bridge of your nose, with radius 13.7 billion light-years. (a) Explain why the visible Universe is getting larger, with its radius increasing by one light-year in every year. (b) Find the rate at which the volume of the visible section of the Universe is increasing.

49. In Section 13.6, we discussed dark matter along with one proposal for the origin of dark matter: WIMPs, or *weakly interacting massive particles.* Another proposal is that dark matter consists of large planet-sized objects, called MACHOs, or *massive astrophysical compact halo objects,* that drift through interstellar space and are not bound to a solar system. Whether WIMPs or MACHOs, suppose astronomers perform theoretical calculations and determine the average density of the observable Universe to be $1.20\rho_c$. If this value were correct, how many times larger will the Universe become before it begins to collapse? That is, by what factor will the distance between remote galaxies increase in the future?

Section 46.12 Problems and Perspectives

50. Classical general relativity views the structure of space–time as deterministic and well defined down to arbitrarily small distances. On the other hand, quantum general relativity forbids distances smaller than the Planck length given by $L = (\hbar G/c^3)^{1/2}$. (a) Calculate the value of the Planck length. The quantum limitation suggests that after the Big Bang, when all the presently observable section of the Universe was contained within a point-like singularity, nothing could be observed until that singularity grew larger than the Planck length. Because the size of the singularity grew at the speed of light, we can infer that no observations were possible during the time interval required for light to travel the Planck length. (b) Calculate this time interval, known as the Planck time T, and state how it compares with the ultrahot epoch mentioned in the text.

Additional Problems

51. For each of the following decays or reactions, name at least one conservation law that prevents it from occurring.

(a) $\pi^- + \text{p} \rightarrow \Sigma^+ + \pi^0$

(b) $\mu^- \rightarrow \pi^- + \nu_e$

(c) $\text{p} \rightarrow \pi^+ + \pi^+ + \pi^-$

52. Identify the unknown particle on the left side of the following reaction:

53. Assume that the half-life of free neutrons is 614 s. What fraction of a group of free thermal neutrons with kinetic energy 0.040 0 eV will decay before traveling a distance of 10.0 km?

54. *Why is the following situation impossible?* A gamma-ray photon with energy 1.05 MeV strikes a stationary electron, causing the following reaction to occur:

Assume all three final particles move with the same speed in the same direction after the reaction.

55. **Review.** Supernova Shelton 1987A, located approximately 170 000 ly from the Earth, is estimated to have emitted a burst of neutrinos carrying energy (Fig. P46.55). Suppose the average neutrino energy was 6 MeV and your mother's body presented cross-sectional area 5 000 cm . To an order of magnitude, how many of these neutrinos passed through her?

Australian Astronomical Observatory, photography by David Malin from AAT Plates

Figure P46.55 Problems 55 and 72.

The energy flux carried by neutrinos from the Sun is estimated to be on the order of 0.400 W/m at the Earth's surface. Estimate the fractional mass loss of the Sun over 10 yr due to the emission of neutrinos. The mass of the Sun is 1.989 kg. The Earth–Sun distance is equal to 1.496

57. Hubble's law can be stated in vector form as . Outside the local group of galaxies, all objects are moving away from us with velocities proportional to their positions relative to us. In this form, it sounds as if our location in the Universe is specially privileged. Prove that Hubble's law is equally true for an observer elsewhere in the Universe. Proceed as follows. Assume we are at the origin of coordinates, one galaxy cluster is at location and has velocity relative to us, and another galaxy cluster has position vector and velocity Suppose the speeds are nonrelativistic. Consider the frame of reference of an observer in the first of these galaxy clusters. (a) Show that our velocity relative to her, together with the position vector of our galaxy cluster from hers, satisfies Hubble's law. (b) Show that the position and velocity of cluster 2 relative to cluster 1 satisfy Hubble's law.

58. meson at rest decays according to → − . Assume the antineutrino has no mass and moves off with the speed of light. Take 139.6 MeV and 105.7 MeV. What is the energy carried off by the neutrino?

59. AMT An unstable particle, initially at rest, decays into a proton (rest energy 938.3 MeV) and a negative pion (rest energy 139.6 MeV). A uniform magnetic field of 0.250 T exists perpendicular to the velocities of the created particles. The radius of curvature of each track is found to be 1.33 m. What is the mass of the original unstable particle?

60. An unstable particle, initially at rest, decays into a positively charged particle of charge and rest energy and a negatively charged particle of charge and rest energy . A uniform magnetic field of magnitude exists perpendicular to the velocities of the created particles. The radius of curvature of each track is What is the mass of the original unstable particle?

61. (a) What processes are described by the Feynman diagrams in Figure P46.61? (b) What is the exchanged particle in each process?

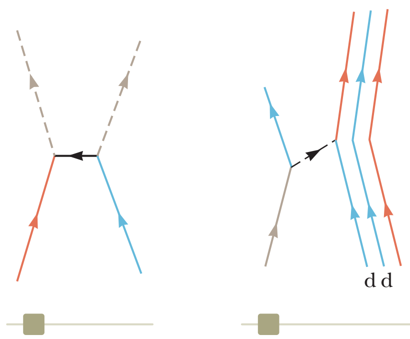


Figure P46.61

62. Identify the mediators for the two interactions described in the Feynman diagrams shown in Figure P46.62.

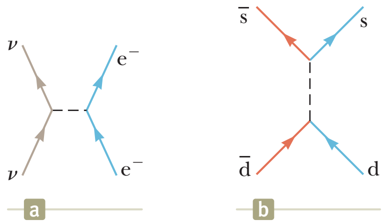


Figure P46.62

63. **Review.** The energy required to excite an atom is on the order of 1 eV. As the temperature of the Universe dropped below a threshold, neutral atoms could form from plasma and the Universe became transparent. Use the Boltzmann distribution function e^{-E/k_BT} to find the order of magnitude of the threshold temperature at which 1.00% of a population of photons has energy greater than 1.00 eV.

64. A Σ^0 particle at rest decays according to $\Sigma^0 \rightarrow \Lambda^0 + \gamma$. Find the gamma-ray energy.

65. Two protons approach each other head-on, each with 70.4 MeV of kinetic energy, and engage in a reaction in which a proton and positive pion emerge at rest. What third particle, obviously uncharged and therefore difficult to detect, must have been created?

66. Two protons approach each other with velocities of equal magnitude in opposite directions. What is the minimum kinetic energy of each proton if the two are to produce a π^+ meson at rest in the reaction $p + p \rightarrow p + n + \pi^+$?

Challenge Problems

67. Determine the kinetic energies of the proton and pion resulting from the decay of a Λ^0 at rest:

$$\Lambda^0 \rightarrow p + \pi^-$$

68. A particle of mass m_1 is fired at a stationary particle of mass m_2, and a reaction takes place in which new particles are created out of the incident kinetic energy. Taken together, the product particles have total mass m_3. The minimum kinetic energy the bombarding particle must have so as to induce the reaction is called the threshold energy. At this energy, the kinetic energy of the products is a minimum, so the fraction of the incident kinetic energy that is available to create new particles is a maximum. This condition is met when all the product particles have the same velocity and the particles have no kinetic energy of motion relative to one another. (a) By using conservation of relativistic energy and momentum and the relativistic energy–momentum relation, show that the threshold kinetic energy is

$$K_{\min} = \frac{[m_3^{\,2} - (m_1 + m_2)^2]c^2}{2m_2}$$

Calculate the threshold kinetic energy for each of the following reactions: (b) $p + p \rightarrow p + p + p + \bar{p}$ (one of the initial protons is at rest, and antiprotons are produced); (c) $\pi^- + p \rightarrow K^0 + \Lambda^0$ (the proton is at rest, and strange particles are produced); (d) $p + p \rightarrow p + p + \pi^0$ (one of the initial protons is at rest, and pions are produced); and (e) $p + \bar{p} \rightarrow Z^0$ (one of the initial particles is at rest, and Z^0 particles of mass 91.2 GeV/c^2 are produced).

69. W A free neutron beta decays by creating a proton, an electron, and an antineutrino according to the reaction $n \rightarrow p + e^- + \bar{\nu}$. **What If?** Imagine that a free neutron were to decay by creating a proton and electron according to the reaction $n \rightarrow p + e^-$ and assume the neutron is initially at rest in the laboratory. (a) Determine the energy released in this reaction. (b) Energy and momentum are conserved in the reaction. Determine the speeds of the proton and the electron after the reaction. (c) Is either of these particles moving at a relativistic speed? Explain.

70. The cosmic rays of highest energy are mostly protons, accelerated by unknown sources. Their spectrum shows a cutoff at an energy on the order of 10^{20} eV. Above that energy, a proton interacts with a photon of cosmic microwave background radiation to produce mesons, for example, according to $p + \gamma \rightarrow p + \pi^0$. Demonstrate this fact by taking the following steps. (a) Find the minimum photon energy required to produce this reaction in the reference frame where the total momentum of the photon–proton system is zero. The reaction was observed experimentally in the 1950s with photons of a few hundred MeV. (b) Use Wien's displacement law to find the wavelength of a photon at the peak of the blackbody spectrum of the primordial microwave background radiation, with a temperature of 2.73 K. (c) Find the energy of this photon. (d) Consider the reaction in part (a) in a moving reference frame so that the photon is the same as that in part (c). Calculate the energy of the proton in this frame, which represents the Earth reference frame.

71. Assume the average density of the Universe is equal to the critical density. (a) Prove that the age of the Universe is given by $2/(3H)$. (b) Calculate $2/(3H)$ and express it in years.

72. The most recent naked-eye supernova was Supernova Shelton 1987A (Fig. P46.55). It was 170 000 ly away in the Large Magellanic Cloud, a satellite galaxy of the Milky Way. Approximately 3 h before its optical brightening was noticed, two neutrino detection experiments simultaneously registered the first neutrinos from an identified source other than the Sun. The Irvine–Michigan–Brookhaven experiment in a salt mine in Ohio registered eight neutrinos over a 6-s period, and the Kamiokande II experiment in a zinc mine in Japan counted eleven neutrinos in 13 s. (Because the supernova is far south in the sky, these neutrinos entered the detectors from below. They passed through the Earth before they were by chance absorbed by nuclei in the detectors.) The neutrino energies were between approximately 8 MeV and

40 MeV. If neutrinos have no mass, neutrinos of all energies should travel together at the speed of light, and the data are consistent with this possibility. The arrival times could vary simply because neutrinos were created at different moments as the core of the star collapsed into a neutron star. If neutrinos have nonzero mass, lower-energy neutrinos should move comparatively slowly. The data are consistent with a 10-MeV neutrino requiring at most approximately 10 s more than a photon would require to travel from the supernova to us. Find the upper limit that this observation sets on the mass of a neutrino. (Other evidence sets an even tighter limit.)

73. A rocket engine for space travel using photon drive and matter–antimatter annihilation has been suggested. Suppose the fuel for a short-duration burn consists of N protons and N antiprotons, each with mass m. (a) Assume all the fuel is annihilated to produce photons. When the photons are ejected from the rocket, what momentum can be imparted to it? (b) **What If?** If half the protons and antiprotons annihilate each other and the energy released is used to eject the remaining particles, what momentum could be given to the rocket? (c) Which scheme results in the greater change in speed for the rocket?

APPENDIX A

Tables

Table A.1 Conversion Factors

Length

	m	cm	km	in.	ft	mi
1 meter	1	10^2	10^{-3}	39.37	3.281	6.214×10^{-4}
1 centimeter	10^{-2}	1	10^{-5}	0.393 7	3.281×10^{-2}	6.214×10^{-6}
1 kilometer	10^3	10^5	1	3.937×10^4	3.281×10^3	0.621 4
1 inch	2.540×10^{-2}	2.540	2.540×10^{-5}	1	8.333×10^{-2}	1.578×10^{-5}
1 foot	0.304 8	30.48	3.048×10^{-4}	12	1	1.894×10^{-4}
1 mile	1 609	1.609×10^5	1.609	6.336×10^4	5 280	1

Mass

	kg	g	slug	u
1 kilogram	1	10^3	6.852×10^{-2}	6.024×10^{26}
1 gram	10^{-3}	1	6.852×10^{-5}	6.024×10^{23}
1 slug	14.59	1.459×10^4	1	8.789×10^{27}
1 atomic mass unit	1.660×10^{-27}	1.660×10^{-24}	1.137×10^{-28}	1

Note: 1 metric ton = 1 000 kg.

Time

	s	min	h	day	yr
1 second	1	1.667×10^{-2}	2.778×10^{-4}	1.157×10^{-5}	3.169×10^{-8}
1 minute	60	1	1.667×10^{-2}	6.994×10^{-4}	1.901×10^{-6}
1 hour	3 600	60	1	4.167×10^{-2}	1.141×10^{-4}
1 day	8.640×10^4	1 440	24	1	2.738×10^{-5}
1 year	3.156×10^7	5.259×10^5	8.766×10^3	365.2	1

Speed

	m/s	cm/s	ft/s	mi/h
1 meter per second	1	10^2	3.281	2.237
1 centimeter per second	10^{-2}	1	3.281×10^{-2}	2.237×10^{-2}
1 foot per second	0.304 8	30.48	1	0.681 8
1 mile per hour	0.447 0	44.70	1.467	1

Note: 1 mi/min = 60 mi/h = 88 ft/s.

Force

	N	lb
1 newton	1	0.224 8
1 pound	4.448	1

(*Continued*)

Table A.1 Conversion Factors *(continued)*

Energy, Energy Transfer

	J	ft · lb	eV
1 joule	1	0.737 6	6.242×10^{18}
1 foot-pound	1.356	1	8.464×10^{18}
1 electron volt	1.602×10^{-19}	1.182×10^{-19}	1
1 calorie	4.186	3.087	2.613×10^{19}
1 British thermal unit	1.055×10^{3}	7.779×10^{2}	6.585×10^{21}
1 kilowatt-hour	3.600×10^{6}	2.655×10^{6}	2.247×10^{25}

	cal	Btu	kWh
1 joule	0.238 9	9.481×10^{-4}	2.778×10^{-7}
1 foot-pound	0.323 9	1.285×10^{-3}	3.766×10^{-7}
1 electron volt	3.827×10^{-20}	1.519×10^{-22}	4.450×10^{-26}
1 calorie	1	3.968×10^{-3}	1.163×10^{-6}
1 British thermal unit	2.520×10^{2}	1	2.930×10^{-4}
1 kilowatt-hour	8.601×10^{5}	3.413×10^{2}	1

Pressure

	Pa	atm
1 pascal	1	9.869×10^{-6}
1 atmosphere	1.013×10^{5}	1
1 centimeter mercury[a]	1.333×10^{3}	1.316×10^{-2}
1 pound per square inch	6.895×10^{3}	6.805×10^{-2}
1 pound per square foot	47.88	4.725×10^{-4}

	cm Hg	lb/in.²	lb/ft²
1 pascal	7.501×10^{-4}	1.450×10^{-4}	2.089×10^{-2}
1 atmosphere	76	14.70	2.116×10^{3}
1 centimeter mercury[a]	1	0.194 3	27.85
1 pound per square inch	5.171	1	144
1 pound per square foot	3.591×10^{-2}	6.944×10^{-3}	1

[a]At 0°C and at a location where the free-fall acceleration has its "standard" value, 9.806 65 m/s².

Table A.2 Symbols, Dimensions, and Units of Physical Quantities

Quantity	Common Symbol	Unit[a]	Dimensions[b]	Unit in Terms of Base SI Units
Acceleration	$\vec{\mathbf{a}}$	m/s²	L/T²	m/s²
Amount of substance	n	MOLE		mol
Angle	θ, ϕ	radian (rad)	1	
Angular acceleration	$\vec{\boldsymbol{\alpha}}$	rad/s²	T^{-2}	s^{-2}
Angular frequency	ω	rad/s	T^{-1}	s^{-1}
Angular momentum	$\vec{\mathbf{L}}$	kg · m²/s	ML²/T	kg · m²/s
Angular velocity	$\vec{\boldsymbol{\omega}}$	rad/s	T^{-1}	s^{-1}
Area	A	m²	L²	m²
Atomic number	Z			
Capacitance	C	farad (F)	Q²T²/ML²	A² · s⁴/kg · m²
Charge	q, Q, e	coulomb (C)	Q	A · s

(Continued)

Table A.2 Symbols, Dimensions, and Units of Physical Quantities *(continued)*

Quantity	Common Symbol	Unit[a]	Dimensions[b]	Unit in Terms of Base SI Units
Charge density				
Line	λ	C/m	Q/L	A · s/m
Surface	σ	C/m^2	Q/L^2	A · s/m^2
Volume	ρ	C/m^3	Q/L^3	A · s/m^3
Conductivity	σ	1/Ω · m	Q^{2}T/ML^3	A^2 · s^3/kg · m^3
Current	I	AMPERE	Q/T	A
Current density	J	A/m^2	Q/TL^2	A/m^2
Density	ρ	kg/m^3	M/L^3	kg/m^3
Dielectric constant	κ			
Electric dipole moment	$\vec{\mathbf{p}}$	C · m	QL	A · s · m
Electric field	$\vec{\mathbf{E}}$	V/m	ML/QT^2	kg · m/A · s^3
Electric flux	Φ_E	V · m	ML^3/QT^2	kg · m^3/A · s^3
Electromotive force	$\mathcal{E}$	volt (V)	ML^2/QT^2	kg · m^2/A · s^3
Energy	E, U, K	joule (J)	ML^2/T^2	kg · m^2/s^2
Entropy	S	J/K	ML^2/T^{2}K	kg · m^2/s^2 · K
Force	$\vec{\mathbf{F}}$	newton (N)	ML/T^2	kg · m/s^2
Frequency	f	hertz (Hz)	T^{-1}	s^{-1}
Heat	Q	joule (J)	ML^2/T^2	kg · m^2/s^2
Inductance	L	henry (H)	ML^2/Q^2	kg · m^2/A^2 · s^2
Length	ℓ, L	METER	L	m
Displacement	$\Delta x, \Delta\vec{\mathbf{r}}$			
Distance	d, h			
Position	$x, y, z, \vec{\mathbf{r}}$			
Magnetic dipole moment	$\vec{\boldsymbol{\mu}}$	N · m/T	QL^2/T	A · m^2
Magnetic field	$\vec{\mathbf{B}}$	tesla (T) (= Wb/m^2)	M/QT	kg/A · s^2
Magnetic flux	Φ_B	weber (Wb)	ML^2/QT	kg · m^2/A · s^2
Mass	m, M	KILOGRAM	M	kg
Molar specific heat	C	J/mol · K		kg · m^2/s^2 · mol · K
Moment of inertia	I	kg · m^2	ML^2	kg · m^2
Momentum	$\vec{\mathbf{p}}$	kg · m/s	ML/T	kg · m/s
Period	T	s	T	s
Permeability of free space	μ_0	N/A^2 (= H/m)	ML/Q^2	kg · m/A^2 · s^2
Permittivity of free space	ϵ_0	C^2/N · m^2 (= F/m)	Q^{2}T^2/ML^3	A^2 · s^4/kg · m^3
Potential	V	volt (V)(= J/C)	ML^2/QT^2	kg · m^2/A · s^3
Power	P	watt (W)(= J/s)	ML^2/T^3	kg · m^2/s^3
Pressure	P	pascal (Pa)(= N/m^2)	M/LT^2	kg/m · s^2
Resistance	R	ohm (Ω)(= V/A)	ML^2/Q^{2}T	kg · m^2/A^2 · s^3
Specific heat	c	J/kg · K	L^2/T^{2}K	m^2/s^2 · K
Speed	v	m/s	L/T	m/s
Temperature	T	KELVIN	K	K
Time	t	SECOND	T	s
Torque	$\vec{\boldsymbol{\tau}}$	N · m	ML^2/T^2	kg · m^2/s^2
Velocity	$\vec{\mathbf{v}}$	m/s	L/T	m/s
Volume	V	m^3	L^3	m^3
Wavelength	λ	m	L	m
Work	W	joule (J)(= N · m)	ML^2/T^2	kg · m^2/s^2

[a]The base SI units are given in uppercase letters.

[b]The symbols M, L, T, K, and Q denote mass, length, time, temperature, and charge, respectively.

APPENDIX

B Mathematics Review

This appendix in mathematics is intended as a brief review of operations and methods. Early in this course, you should be totally familiar with basic algebraic techniques, analytic geometry, and trigonometry. The sections on differential and integral calculus are more detailed and are intended for students who have difficulty applying calculus concepts to physical situations.

B.1 Scientific Notation

Many quantities used by scientists often have very large or very small values. The speed of light, for example, is about 300 000 000 m/s, and the ink required to make the dot over an *i* in this textbook has a mass of about 0.000 000 001 kg. Obviously, it is very cumbersome to read, write, and keep track of such numbers. We avoid this problem by using a method incorporating powers of the number 10:

$$10^0 = 1$$
$$10^1 = 10$$
$$10^2 = 10 \times 10 = 100$$
$$10^3 = 10 \times 10 \times 10 = 1\,000$$
$$10^4 = 10 \times 10 \times 10 \times 10 = 10\,000$$
$$10^5 = 10 \times 10 \times 10 \times 10 \times 10 = 100\,000$$

and so on. The number of zeros corresponds to the power to which ten is raised, called the **exponent** of ten. For example, the speed of light, 300 000 000 m/s, can be expressed as 3.00×10^8 m/s.

In this method, some representative numbers smaller than unity are the following:

$$10^{-1} = \frac{1}{10} = 0.1$$
$$10^{-2} = \frac{1}{10 \times 10} = 0.01$$
$$10^{-3} = \frac{1}{10 \times 10 \times 10} = 0.001$$
$$10^{-4} = \frac{1}{10 \times 10 \times 10 \times 10} = 0.000\,1$$
$$10^{-5} = \frac{1}{10 \times 10 \times 10 \times 10 \times 10} = 0.000\,01$$

In these cases, the number of places the decimal point is to the left of the digit 1 equals the value of the (negative) exponent. Numbers expressed as some power of ten multiplied by another number between one and ten are said to be in **scientific notation.** For example, the scientific notation for 5 943 000 000 is 5.943×10^9 and that for 0.000 083 2 is 8.32×10^{-5}.

When numbers expressed in scientific notation are being multiplied, the following general rule is very useful:

$$10^n \times 10^m = 10^{n+m} \quad \textbf{(B.1)}$$

where n and m can be *any* numbers (not necessarily integers). For example, $10^2 \times 10^5 = 10^7$. The rule also applies if one of the exponents is negative: $10^3 \times 10^{-8} = 10^{-5}$.

When dividing numbers expressed in scientific notation, note that

$$\frac{10^n}{10^m} = 10^n \times 10^{-m} = 10^{n-m} \quad \textbf{(B.2)}$$

Exercises

With help from the preceding rules, verify the answers to the following equations:

1. $86\,400 = 8.64 \times 10^4$
2. $9\,816\,762.5 = 9.816\,762\,5 \times 10^6$
3. $0.000\,000\,039\,8 = 3.98 \times 10^{-8}$
4. $(4.0 \times 10^8)(9.0 \times 10^9) = 3.6 \times 10^{18}$
5. $(3.0 \times 10^7)(6.0 \times 10^{-12}) = 1.8 \times 10^{-4}$
6. $\dfrac{75 \times 10^{-11}}{5.0 \times 10^{-3}} = 1.5 \times 10^{-7}$
7. $\dfrac{(3 \times 10^6)(8 \times 10^{-2})}{(2 \times 10^{17})(6 \times 10^5)} = 2 \times 10^{-18}$

B.2 Algebra

Some Basic Rules

When algebraic operations are performed, the laws of arithmetic apply. Symbols such as x, y, and z are usually used to represent unspecified quantities, called the **unknowns.**

First, consider the equation

$$8x = 32$$

If we wish to solve for x, we can divide (or multiply) each side of the equation by the same factor without destroying the equality. In this case, if we divide both sides by 8, we have

$$\frac{8x}{8} = \frac{32}{8}$$

$$x = 4$$

Next consider the equation

$$x + 2 = 8$$

In this type of expression, we can add or subtract the same quantity from each side. If we subtract 2 from each side, we have

$$x + 2 - 2 = 8 - 2$$

$$x = 6$$

In general, if $x + a = b$, then $x = b - a$.

Now consider the equation

$$\frac{x}{5} = 9$$

If we multiply each side by 5, we are left with x on the left by itself and 45 on the right:

$$\left(\frac{x}{5}\right)(5) = 9 \times 5$$
$$x = 45$$

In all cases, *whatever operation is performed on the left side of the equality must also be performed on the right side.*

The following rules for multiplying, dividing, adding, and subtracting fractions should be recalled, where a, b, c, and d are four numbers:

	Rule	Example
Multiplying	$\left(\frac{a}{b}\right)\left(\frac{c}{d}\right) = \frac{ac}{bd}$	$\left(\frac{2}{3}\right)\left(\frac{4}{5}\right) = \frac{8}{15}$
Dividing	$\frac{(a/b)}{(c/d)} = \frac{ad}{bc}$	$\frac{2/3}{4/5} = \frac{(2)(5)}{(4)(3)} = \frac{10}{12}$
Adding	$\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}$	$\frac{2}{3} - \frac{4}{5} = \frac{(2)(5) - (4)(3)}{(3)(5)} = -\frac{2}{15}$

Exercises

In the following exercises, solve for x.

	Answers
1. $a = \frac{1}{1 + x}$	$x = \frac{1 - a}{a}$
2. $3x - 5 = 13$	$x = 6$
3. $ax - 5 = bx + 2$	$x = \frac{7}{a - b}$
4. $\frac{5}{2x + 6} = \frac{3}{4x + 8}$	$x = -\frac{11}{7}$

Powers

When powers of a given quantity x are multiplied, the following rule applies:

$$x^n x^m = x^{n+m} \tag{B.3}$$

For example, $x^2x^4 = x^{2+4} = x^6$.

When dividing the powers of a given quantity, the rule is

$$\frac{x^n}{x^m} = x^{n-m} \tag{B.4}$$

For example, $x^8/x^2 = x^{8-2} = x^6$.

A power that is a fraction, such as $\frac{1}{3}$, corresponds to a root as follows:

$$x^{1/n} = \sqrt[n]{x} \tag{B.5}$$

For example, $4^{1/3} = \sqrt[3]{4} = 1.587\,4$. (A scientific calculator is useful for such calculations.)

Finally, any quantity x^n raised to the mth power is

$$(x^n)^m = x^{nm} \tag{B.6}$$

Table B.1 summarizes the rules of exponents.

Table B.1 Rules of Exponents

$x^0 = 1$
$x^1 = x$
$x^n x^m = x^{n+m}$
$x^n/x^m = x^{n-m}$
$x^{1/n} = \sqrt[n]{x}$
$(x^n)^m = x^{nm}$

Exercises

Verify the following equations:

1. $3^2 \times 3^3 = 243$

2. $x^5x^{-8} = x^{-3}$

3. $x^{10}/x^{-5} = x^{15}$
4. $5^{1/3} = 1.709\ 976$ (Use your calculator.)
5. $60^{1/4} = 2.783\ 158$ (Use your calculator.)
6. $(x^4)^3 = x^{12}$

Factoring

Some useful formulas for factoring an equation are the following:

$ax + ay + az = a(x + y + z)$ common factor

$a^2 + 2ab + b^2 = (a + b)^2$ perfect square

$a^2 - b^2 = (a + b)(a - b)$ differences of squares

Quadratic Equations

The general form of a quadratic equation is

$$ax^2 + bx + c = 0 \tag{B.7}$$

where x is the unknown quantity and a, b, and c are numerical factors referred to as **coefficients** of the equation. This equation has two roots, given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{B.8}$$

If $b^2 \geq 4ac$, the roots are real.

Example B.1

The equation $x^2 + 5x + 4 = 0$ has the following roots corresponding to the two signs of the square-root term:

$$x = \frac{-5 \pm \sqrt{5^2 - (4)(1)(4)}}{2(1)} = \frac{-5 \pm \sqrt{9}}{2} = \frac{-5 \pm 3}{2}$$

$$x_+ = \frac{-5 + 3}{2} = -1 \qquad x_- = \frac{-5 - 3}{2} = -4$$

where x_+ refers to the root corresponding to the positive sign and x_- refers to the root corresponding to the negative sign.

Exercises

Solve the following quadratic equations:

		Answers
1. $x^2 + 2x - 3 = 0$	$x_+ = 1$	$x_- = -3$
2. $2x^2 - 5x + 2 = 0$	$x_+ = 2$	$x_- = \frac{1}{2}$
3. $2x^2 - 4x - 9 = 0$	$x_+ = 1 + \sqrt{22}/2$	$x_- = 1 - \sqrt{22}/2$

Linear Equations

A linear equation has the general form

$$y = mx + b \tag{B.9}$$

where and are constants. This equation is referred to as linear because the graph of versus is a straight line as shown in Figure B.1. The constant , called the **-intercept,** represents the value of at which the straight line intersects the axis. The constant is equal to the **slope** of the straight line. If any two points on the straight line are specified by the coordinates ( ) and ( ) as in Figure B.1, the slope of the straight line can be expressed as

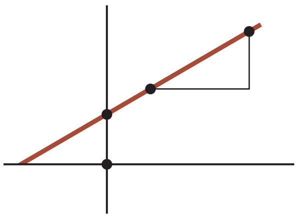


Figure B.1 A straight line graphed on an coordinate sys tem. The slope of the line is the ratio of to

Slope ——— —— **(B.10)**

Note that and can have either positive or negative values. If 0, the straight line has a *positive* slope as in Figure B.1. If 0, the straight line has a *negative* slope. In Figure B.1, both and are positive. Three other possible situations are shown in Figure B.2.

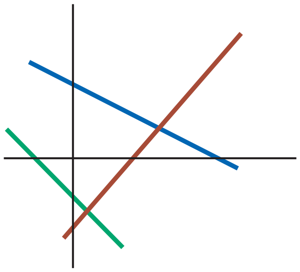


Figure B.2 The brown line has a positive slope and a negative intercept. The blue line has a negative slope and a positive -intercept. The green line has a negative slope and a negative -intercept.

Exercises

Draw graphs of the following straight lines: (a) 3 (b) (c)

2. Find the slopes of the straight lines described in Exercise 1.

Answers (a) 5 (b) 2 (c)

3. Find the slopes of the straight lines that pass through the following sets of points: (a) (0, 4) and (4, 2) (b) (0, 0) and (2, 5) (c) (5, 2) and (4,

Answers (a) – (b) – (c) –

Solving Simultaneous Linear Equations

Consider the equation 3 = 15, which has two unknowns, and Such an equation does not have a unique solution. For example, (= 0, = 3), (= 5, 0), and 2, –) are all solutions to this equation.

If a problem has two unknowns, a unique solution is possible only if we have *two* pieces of information. In most common cases, those two pieces of information are equations. In general, if a problem has unknowns, its solution requires equations. To solve two simultaneous equations involving two unknowns, and , we solve one of the equations for in terms of and substitute this expression into the other equation.

In some cases, the two pieces of information may be (1) one equation and (2) a condition on the solutions. For example, suppose we have the equation and the condition that and must be the smallest positive nonzero integers possible. Then, the single equation does not allow a unique solution, but the addition of the condition gives us that 1 and

Example B.2

Solve the two simultaneous equations

(1) = −8

(2) 4

SOLUTION

From Equation (2), 2. Substitution of this equation into Equation (1) gives

= −8

= −18

B.2

3

1

Alternative Solution Multiply each term in Equation (1) by the factor 2 and add the result to Equation (2):

$$10 \qquad = -16$$
$$4$$
$$12 \qquad = -12$$

1

3

Two linear equations containing two unknowns can also be solved by a graphical method. If the straight lines corresponding to the two equations are plotted in a conventional coordinate system, the intersection of the two lines represents the solution. For example, consider the two equations

$$2$$
$$= -1$$

These equations are plotted in Figure B.3. The intersection of the two lines has the coordinates 5 and = 3, which represents the solution to the equations. You should check this solution by the analytical technique discussed earlier.

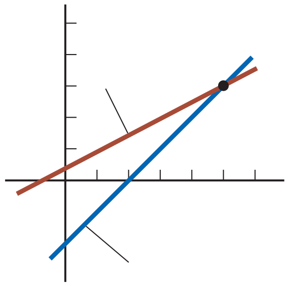


Figure B.3 A graphical solution for two linear equations.

Exercises

Solve the following pairs of simultaneous equations involving two unknowns:

	Answers
	5,
2.	65, 3.27
3.	2,

Logarithms

Suppose a quantity is expressed as a power of some quantity

(B.11)

The number is called the **base** number. The **logarithm** of with respect to the base is equal to the exponent to which the base must be raised to satisfy the expression

$$\log \qquad \textbf{(B.12)}$$

Conversely, the **antilogarithm** of is the number

$$\text{antilog} \qquad \textbf{(B.13)}$$

In practice, the two bases most often used are base 10, called the *common* logarithm base, and base = 2.718 282, called Euler's constant or the *natural* logarithm base. When common logarithms are used,

$$\log_{10} \quad \text{or} \quad 10 \qquad \textbf{(B.14)}$$

When natural logarithms are used,

$$\ln \quad \text{or} \qquad \text{(B.15)}$$

For example, log = 1.716, so antilog 1.716 = $^{1.716}$ = 52. Likewise, ln 52 3.951, so antiln 3.951 $^{3.951}$52.

In general, note you can convert between base 10 and base with the equality

$$\ln \quad 2.302\,585 \;\log_{10} \qquad \text{(B.16)}$$

Finally, some useful properties of logarithms are the following:

$$\log ab \quad \log \quad \log$$
$$\log \quad \log \quad \log \qquad \text{any base}$$
$$\log \quad \log$$
$$\ln$$
$$\ln$$
$$\ln - = -\ln$$

B.3 Geometry

The **distance** between two points having coordinates ( ) and ( ) is

$$\text{(B.17)}$$

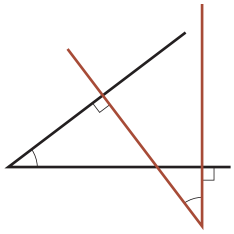


Figure B.4 The angles are equal because their sides are perpendicular.

Two angles are equal if their sides are perpendicular, right side to right side and left side to left side. For example, the two angles marked in Figure B.4 are the same because of the perpendicularity of the sides of the angles. To distinguish the left and right sides of an angle, imagine standing at the angle's apex and facing into the angle.

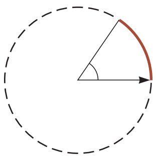


Figure B.5 The angle in radians is the ratio of the arc length to the radius of the circle.

Radian measure: The arc length of a circular arc (Fig. B.5) is proportional to the radius for a fixed value of (in radians):

$$\theta = - \qquad \text{(B.18)}$$

Table B.2 gives the **areas** and **volumes** for several geometric shapes used throughout this text.

The equation of a **straight line** (Fig. B.6) is

$$mx \qquad \text{(B.19)}$$

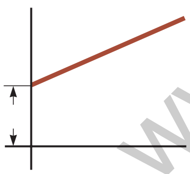


Figure B.6 A straight line with a slope of and a -intercept of

where is the -intercept and is the slope of the line.

The equation of a **circle** of radius centered at the origin is

$$\qquad \text{(B.20)}$$

The equation of an **ellipse** having the origin at its center (Fig. B.7) is

$$\text{—} \quad \text{—} \qquad \text{(B.21)}$$

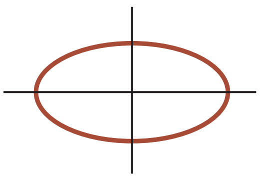


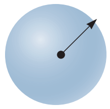
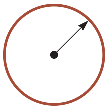
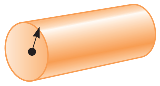
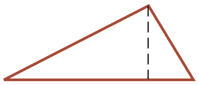
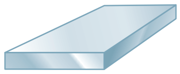
Figure B.7 An ellipse with semimajor axis and semiminor axis

where is the length of the semimajor axis (the longer one) and is the length of the semiminor axis (the shorter one).

The equation of a **parabola** the vertex of which is at (Fig. B.8) is

$$ax \qquad \text{(B.22)}$$

Table B.2 **Useful Information for Geometry**

Shape	Area or Volume	Shape	Area or Volume
			Surface area Volume
			Lateral surface Volume
 Triangle			Surface area Volume

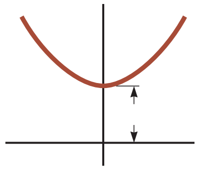


Figure B.8 A parabola with its vertex at

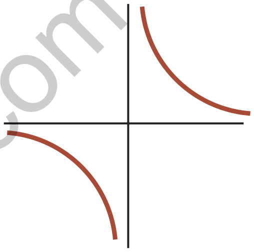


Figure B.9 A hyperbola.

The equation of a **rectangular hyperbola** (Fig. B.9) is

$$xy \quad \text{constant} \tag{B.23}$$

B.4 Trigonometry

That portion of mathematics based on the special properties of the right triangle is called trigonometry. By definition, a right triangle is a triangle containing a 90 angle. Consider the right triangle shown in Figure B.10, where side is opposite the angle , side is adjacent to the angle , and side is the hypotenuse of the triangle. The three basic trigonometric functions defined by such a triangle are the sine (sin), cosine (cos), and tangent (tan). In terms of the angle , these functions are defined as follows:

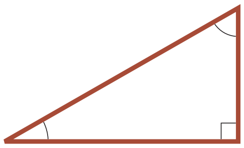


Figure B.10 A right triangle, used to define the basic functions of trigonometry.

$$\sin\theta = \frac{\text{side opposite}}{\text{hypotenuse}} \tag{B.24}$$

$$\cos\theta = \frac{\text{side adjacent to}}{\text{hypotenuse}} \tag{B.25}$$

$$\tan\theta = \frac{\text{side opposite}}{\text{side adjacent to}} \tag{B.26}$$

The Pythagorean theorem provides the following relationship among the sides of a right triangle:

(B.27)

From the preceding definitions and the Pythagorean theorem, it follows that

$$\sin \ \theta + \cos \ \theta = 1$$

$$\tan\theta = \frac{\sin}{\cos}$$

The cosecant, secant, and cotangent functions are defined by

$$\csc\theta = \frac{}{\sin} \qquad \sec\theta = \frac{}{\cos} \qquad \cot\theta = \frac{}{\tan}$$

Table B.3 Some Trigonometric Identities

sin θ + cos θ =	csc θ = cot
sec θ = tan	sin − − cos
sin 2θ = 2 sin cos	cos − − cos
cos 2θ = cos θ − sin	cos θ = 2 sin −
tan 2θ = $\frac{2 \tan}{\tan}$	tan − $\frac{\cos}{\cos}$
sin sin cos cos sin	
cos cos cos sin sin	
sin sin 2 sin −)] cos −)]	
cos cos 2 cos −)] cos −)]	
cos cos 2 sin −)] sin −)]	

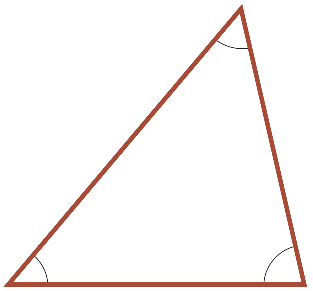


Figure B.11 An arbitrary, non-right triangle.

The following relationships are derived directly from the right triangle shown in Figure B.10:

$$\sin\theta = \cos\ 90° - \theta$$
$$\cos\theta = \sin\ 90° - \theta$$
$$\cot\theta = \tan\ 90° - \theta$$

Some properties of trigonometric functions are the following:

$$\sin\ -\theta\ = -\sin$$
$$\cos\ -\theta\ \ \cos$$
$$\tan\ -\theta\ = -\tan$$

The following relationships apply to *any* triangle as shown in Figure B.11:

$$\alpha + \beta + \gamma = 180$$

Law of cosines

bc cos

ac cos

ab cos

Law of sines $\frac{}{\sin}\ \frac{}{\sin}\ \frac{}{\sin}$

Table B.3 lists a number of useful trigonometric identities.

Example B.3

Consider the right triangle in Figure B.12 in which = 2.00, 5.00, and is unknown. From the Pythagorean theorem, we have

$$2.00\quad 5.00\quad 4.00\quad 25.0\quad 29.0$$

$$\overline{29.0}\quad 5.39$$

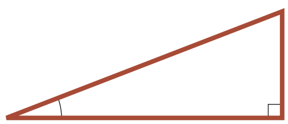


Figure B.12 (Example B.3)

To find the angle , note that

$$\tan\theta = - \frac{2.00}{5.00}\ 0.400$$

B.3

Using a calculator, we find that

$$\theta = \quad 0.400 \quad 21.8$$

where tan (0.400) is the notation for "angle whose tangent is 0.400," sometimes written as arctan (0.400).

Exercises

In Figure B.13, identify (a) the side opposite (b) the side adjacent to and then find (c) cos , (d) sin , and (e) tan

Answers (a) 3 (b) 3 (c) - (d) - (e) -

2. In a certain right triangle, the two sides that are perpendicular to each other are 5.00 m and 7.00 m long. What is the length of the third side?

Answer 8.60 m

3. A right triangle has a hypotenuse of length 3.0 m, and one of its angles is 30 . (a) What is the length of the side opposite the 30 angle? (b) What is the side adjacent to the 30 angle?

Answers (a) 1.5 m (b) 2.6 m

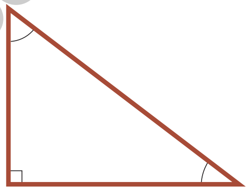


Figure B.13 (Exercise 1)

B.5 Series Expansions

nx

ln $= \pm$ - -

sin

cos in radians

tan 15

For << 1, the following approximations can be used:

nx sin

cos

ln tan

B.6 Differential Calculus

In various branches of science, it is sometimes necessary to use the basic tools of calculus, invented by Newton, to describe physical phenomena. The use of calculus is fundamental in the treatment of various problems in Newtonian mechanics, electricity, and magnetism. In this section, we simply state some basic properties and "rules of thumb" that should be a useful review to the student.

The approximations for the functions sin , cos , and tan are for 0.1 rad.

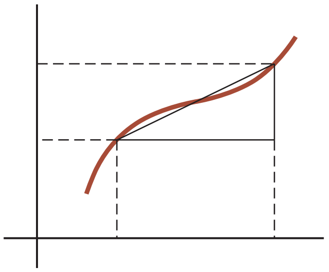


Figure B.14 The lengths and are used to define the derivative of this function at a point.

First, a **function** must be specified that relates one variable to another (e.g., a coordinate as a function of time). Suppose one of the variables is called (the dependent variable), and the other (the independent variable). We might have a function relationship such as

$$ax \quad bx \quad cx$$

If , and are specified constants, can be calculated for any value of We usually deal with continuous functions, that is, those for which varies "smoothly" with

The **derivative** of with respect to is defined as the limit as approaches zero of the slopes of chords drawn between two points on the versus curve. Mathematically, we write this definition as

$$\frac{dy}{dx} \quad \lim \quad \frac{}{} \quad \lim \quad \frac{+\Delta}{} \qquad \text{(B.28)}$$

where and are defined as $=$ $-$ and $=$ $-$ (Fig. B.14). Note that *dx does not* mean divided by , but rather is simply a notation of the limiting process of the derivative as defined by Equation B.28.

A useful expression to remember when $=$, where is a *constant* and is *any* positive or negative number (integer or fraction), is

$$\frac{dy}{dx} \quad nax \qquad \text{(B.29)}$$

If) is a polynomial or algebraic function of , we apply Equation B.29 to *each* term in the polynomial and take [constant]/ = 0. In Examples B.4 through B.7, we evaluate the derivatives of several functions.

Table B.4 **Derivative for Several Functions**

$\frac{}{dx}$	
$\frac{}{dx}$ ax	nax
$\frac{}{dx}$ ^{ax}	ae^{ax}
$\frac{}{dx}$ $\sin ax$	$\cos ax$
$\frac{}{dx}$ $\cos ax = -$	$\sin ax$
$\frac{}{dx}$ $\tan ax$	$\sec \; ax$
$\frac{}{dx}$ $\cot ax = -$	$\csc \; ax$
$\frac{}{dx}$ $\sec$	$\tan \; \sec$
$\frac{}{dx}$ $\csc$	$= -\cot \; \csc$
$\frac{}{dx}$ $\ln ax$	$-$
$\frac{}{dx}$ $\sin \; ax$	
$\frac{}{dx}$ $\cos \; ax$	
$\frac{}{dx}$ $\tan \; ax$	

Note: The symbols and represent constants.

Special Properties of the Derivative

A. Derivative of the product of two functions If a function) is given by the product of two functions—say,) and)—the derivative of) is defined as

$$\frac{}{dx} \quad \frac{}{dx} \quad)] \quad \frac{dh}{dx} \quad \frac{dg}{dx} \qquad \text{(B.30)}$$

B. Derivative of the sum of two functions If a function) is equal to the sum of two functions, the derivative of the sum is equal to the sum of the derivatives:

$$\frac{}{dx} \quad \frac{}{dx} \quad)] \quad \frac{dg}{dx} \quad \frac{dh}{dx} \qquad \text{(B.31)}$$

C. Chain rule of differential calculus If) and), then can be written as the product of two derivatives:

$$\frac{dy}{dz} \quad \frac{dy}{dx} \frac{dx}{dz} \qquad \text{(B.32)}$$

D. The second derivative The second derivative of with respect to is defined as the derivative of the function (the derivative of the derivative). It is usually written as

$$\frac{}{dx} \quad \frac{}{dx} \frac{dy}{dx} \qquad \text{(B.33)}$$

Some of the more commonly used derivatives of functions are listed in Table B.4.

Example B.4

Suppose $y(x)$ (that is, y as a function of x) is given by

$$y(x) = ax^3 + bx + c$$

where a and b are constants. It follows that

$$y(x + \Delta x) = a(x + \Delta x)^3 + b(x + \Delta x) + c$$
$$= a(x^3 + 3x^2\,\Delta x + 3x\,\Delta x^2 + \Delta x^3) + b(x + \Delta x) + c$$

so

$$\Delta y = y(x + \Delta x) - y(x) = a(3x^2\,\Delta x + 3x\,\Delta x^2 + \Delta x^3) + b\Delta x$$

Substituting this into Equation B.28 gives

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} [3ax^2 + 3ax\,\Delta x + a\,\Delta x^2] + b$$

$$\frac{dy}{dx} = \boxed{3ax^2 + b}$$

Example B.5

Find the derivative of

$$y(x) = 8x^5 + 4x^3 + 2x + 7$$

SOLUTION

Applying Equation B.29 to each term independently and remembering that d/dx (constant) $= 0$, we have

$$\frac{dy}{dx} = 8(5)x^4 + 4(3)x^2 + 2(1)x^0 + 0$$

$$\frac{dy}{dx} = \boxed{40x^4 + 12x^2 + 2}$$

Example B.6

Find the derivative of $y(x) = x^3/(x + 1)^2$ with respect to x.

SOLUTION

We can rewrite this function as $y(x) = x^3(x + 1)^{-2}$ and apply Equation B.30:

$$\frac{dy}{dx} = (x + 1)^{-2}\frac{d}{dx}(x^3) + x^3\frac{d}{dx}(x + 1)^{-2}$$
$$= (x + 1)^{-2}\,3x^2 + x^3\,(-2)(x + 1)^{-3}$$

$$\frac{dy}{dx} = \boxed{\frac{3x^2}{(x + 1)^2} - \frac{2x^3}{(x + 1)^3}} = \boxed{\frac{x^2(x + 3)}{(x + 1)^3}}$$

Example B.7

A useful formula that follows from Equation B.30 is the derivative of the quotient of two functions. Show that

$$\frac{d}{dx}\left[\frac{g(x)}{h(x)}\right] = \frac{h\dfrac{dg}{dx} - g\dfrac{dh}{dx}}{h^2}$$

SOLUTION

We can write the quotient as gh^{-1} and then apply Equations B.29 and B.30:

$$\frac{d}{dx}\left(\frac{g}{h}\right) = \frac{d}{dx}(gh^{-1}) = g\frac{d}{dx}(h^{-1}) + h^{-1}\frac{d}{dx}(g)$$

$$= -gh^{-2}\frac{dh}{dx} + h^{-1}\frac{dg}{dx}$$

$$= \frac{h\dfrac{dg}{dx} - g\dfrac{dh}{dx}}{h^2}$$

B.7 Integral Calculus

We think of integration as the inverse of differentiation. As an example, consider the expression

$$f(x) = \frac{dy}{dx} = 3ax^2 + b \tag{B.34}$$

which was the result of differentiating the function

$$y(x) = ax^3 + bx + c$$

in Example B.4. We can write Equation B.34 as $dy = f(x)\,dx = (3ax^2 + b)\,dx$ and obtain $y(x)$ by "summing" over all values of x. Mathematically, we write this inverse operation as

$$y(x) = \int f(x)\,dx$$

For the function $f(x)$ given by Equation B.34, we have

$$y(x) = \int (3ax^2 + b)\,dx = ax^3 + bx + c$$

where c is a constant of the integration. This type of integral is called an *indefinite integral* because its value depends on the choice of c.

A general **indefinite integral** $I(x)$ is defined as

$$I(x) = \int f(x)\,dx \tag{B.35}$$

where $f(x)$ is called the *integrand* and $f(x) = dI(x)/dx$.

For a *general continuous* function $f(x)$, the integral can be interpreted geometrically as the area under the curve bounded by $f(x)$ and the x axis, between two specified values of x, say, x_1 and x_2, as in Figure B.15.

The area of the blue element in Figure B.15 is approximately $f(x_i)\,\Delta x_i$. If we sum all these area elements between x_1 and x_2 and take the limit of this sum as $\Delta x_i \rightarrow 0$,

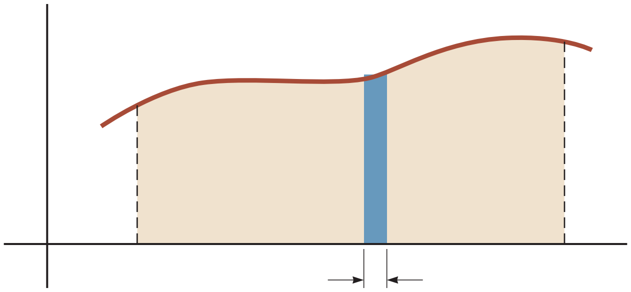


Figure B.15 The definite integral of a function is the area under the curve of the function between the limits and

we obtain the *true* area under the curve bounded by) and the axis, between the limits and

$$\text{Area} \quad \lim \qquad dx \tag{B.36}$$

Integrals of the type defined by Equation B.36 are called **definite integrals.**

One common integral that arises in practical situations has the form

$$dx \quad \frac{}{} \tag{B.37}$$

This result is obvious, being that differentiation of the right-hand side with respect to gives = directly. If the limits of the integration are known, this integral becomes a *definite integral* and is written

$$dx \quad \frac{}{} \qquad \frac{}{} \tag{B.38}$$

Examples

1. dx — —
2. dx ——
3. dx — ——

Partial Integration

Sometimes it is useful to apply the method of *partial integration* (also called "inte grating by parts") to evaluate certain integrals. This method uses the property

$$dv \quad uv \qquad du \tag{B.39}$$

where and are *carefully* chosen so as to reduce a complex integral to a simpler one. In many cases, several reductions have to be made. Consider the function

$$dx$$

which can be evaluated by integrating by parts twice. First, if we choose = = , we obtain

$$dx \qquad\qquad dx$$

Now, in the second term, choose $u = x$, $v = e^x$, which gives

$$\int x^2 e^x \, dx = x^2 e^x - 2x e^x + 2\int e^x \, dx + c_1$$

or

$$\int x^2 e^x \, dx = x^2 e^x - 2xe^x + 2e^x + c_2$$

The Perfect Differential

Another useful method to remember is that of the *perfect differential,* in which we look for a change of variable such that the differential of the function is the differential of the independent variable appearing in the integrand. For example, consider the integral

$$I(x) = \int \cos^2 x \sin x \, dx$$

This integral becomes easy to evaluate if we rewrite the differential as $d(\cos x) = -\sin x \, dx$. The integral then becomes

$$\int \cos^2 x \sin x \, dx = -\int \cos^2 x \, d(\cos x)$$

If we now change variables, letting $y = \cos x$, we obtain

$$\int \cos^2 x \sin x \, dx = -\int y^2 \, dy = -\frac{y^3}{3} + c = -\frac{\cos^3 x}{3} + c$$

Table B.5 lists some useful indefinite integrals. Table B.6 gives Gauss's probability integral and other definite integrals. A more complete list can be found in various handbooks, such as *The Handbook of Chemistry and Physics* (Boca Raton, FL: CRC Press, published annually).

Table B.5 Some Indefinite Integrals (An arbitrary constant should be added to each of these integrals.)

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} \text{ (provided } n \neq 1)$$

$$\int \frac{dx}{x} = \int x^{-1} \, dx = \ln x$$

$$\int \frac{dx}{a + bx} = \frac{1}{b} \ln(a + bx)$$

$$\int \frac{x \, dx}{a + bx} = \frac{x}{b} - \frac{a}{b^2} \ln(a + bx)$$

$$\int \frac{dx}{x(x + a)} = -\frac{1}{a} \ln \frac{x + a}{x}$$

$$\int \frac{dx}{(a + bx)^2} = -\frac{1}{b(a + bx)}$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a}$$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \frac{a + x}{a - x} \quad (a^2 - x^2 > 0)$$

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \frac{x - a}{x + a} \quad (x^2 - a^2 > 0)$$

$$\int \ln ax \, dx = (x \ln ax) - x$$

$$\int xe^{ax} \, dx = \frac{e^{ax}}{a^2}(ax - 1)$$

$$\int \frac{dx}{a + be^{cx}} = \frac{x}{a} - \frac{1}{ac} \ln(a + be^{cx})$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax$$

$$\int \tan ax \, dx = -\frac{1}{a} \ln(\cos ax) = \frac{1}{a} \ln(\sec ax)$$

$$\int \cot ax \, dx = \frac{1}{a} \ln(\sin ax)$$

$$\int \sec ax \, dx = \frac{1}{a} \ln(\sec ax + \tan ax) = \frac{1}{a} \ln\left[\tan\left(\frac{ax}{2} + \frac{\pi}{4}\right)\right]$$

$$\int \csc ax \, dx = \frac{1}{a} \ln(\csc ax - \cot ax) = \frac{1}{a} \ln\left(\tan \frac{ax}{2}\right)$$

(Continued)

Table B.5 Some Indefinite Integrals *(continued)*

$$\int \frac{x\,dx}{a^2 \pm x^2} = \pm\tfrac{1}{2}\ln(a^2 \pm x^2)$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} = -\cos^{-1}\frac{x}{a} \quad (a^2 - x^2 > 0)$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln(x + \sqrt{x^2 \pm a^2})$$

$$\int \frac{x\,dx}{\sqrt{a^2 - x^2}} = -\sqrt{a^2 - x^2}$$

$$\int \frac{x\,dx}{\sqrt{x^2 \pm a^2}} = \sqrt{x^2 \pm a^2}$$

$$\int \sqrt{a^2 - x^2}\,dx = \tfrac{1}{2}\left(x\sqrt{a^2 - x^2} + a^2\sin^{-1}\frac{x}{|a|}\right)$$

$$\int x\sqrt{a^2 - x^2}\,dx = -\tfrac{1}{3}(a^2 - x^2)^{3/2}$$

$$\int \sqrt{x^2 \pm a^2}\,dx = \tfrac{1}{2}\,x\sqrt{x^2 \pm a^2} \pm a^2\ln(x + \sqrt{x^2 \pm a^2})$$

$$\int x(\sqrt{x^2 \pm a^2})\,dx = \tfrac{1}{3}(x^2 \pm a^2)^{3/2}$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int \sin^2 ax\,dx = \frac{x}{2} - \frac{\sin 2ax}{4a}$$

$$\int \cos^2 ax\,dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$$

$$\int \frac{dx}{\sin^2 ax} = -\frac{1}{a}\cot ax$$

$$\int \frac{dx}{\cos^2 ax} = \frac{1}{a}\tan ax$$

$$\int \tan^2 ax\,dx = \frac{1}{a}(\tan ax) - x$$

$$\int \cot^2 ax\,dx = -\frac{1}{a}(\cot ax) - x$$

$$\int \sin^{-1} ax\,dx = x(\sin^{-1} ax) + \frac{\sqrt{1 - a^2x^2}}{a}$$

$$\int \cos^{-1} ax\,dx = x(\cos^{-1} ax) - \frac{\sqrt{1 - a^2x^2}}{a}$$

$$\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{x}{a^2\sqrt{x^2 + a^2}}$$

$$\int \frac{x\,dx}{(x^2 + a^2)^{3/2}} = -\frac{1}{\sqrt{x^2 + a^2}}$$

Table B.6 Gauss's Probability Integral and Other Definite Integrals

$$\int_0^\infty x^n e^{-ax}\,dx = \frac{n!}{a^{n+1}}$$

$$I_0 = \int_0^\infty e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}} \quad \text{(Gauss's probability integral)}$$

$$I_1 = \int_0^\infty xe^{-ax^2}\,dx = \frac{1}{2a}$$

$$I_2 = \int_0^\infty x^2 e^{-ax^2}\,dx = -\frac{dI_0}{da} = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}$$

$$I_3 = \int_0^\infty x^3 e^{-ax^2}\,dx = -\frac{dI_1}{da} = \frac{1}{2a^2}$$

$$I_4 = \int_0^\infty x^4 e^{-ax^2}\,dx = \frac{d^2 I_0}{da^2} = \frac{3}{8}\sqrt{\frac{\pi}{a^5}}$$

$$I_5 = \int_0^\infty x^5 e^{-ax^2}\,dx = \frac{d^2 I_1}{da^2} = \frac{1}{a^3}$$

$$\vdots$$

$$I_{2n} = (-1)^n \frac{d^n}{da^n} I_0$$

$$I_{2n+1} = (-1)^n \frac{d^n}{da^n} I_1$$

B.8 Propagation of Uncertainty

In laboratory experiments, a common activity is to take measurements that act as raw data. These measurements are of several types—length, time interval, temperature, voltage, and so on—and are taken by a variety of instruments. Regardless of the measurement and the quality of the instrumentation, **there is always uncertainty associated with a physical measurement.** This uncertainty is a combination of that associated with the instrument and that related to the system being measured. An example of the former is the inability to exactly determine the position of a length measurement between the lines on a meterstick. An example of uncertainty related to the system being measured is the variation of temperature within a sample of water so that a single temperature for the sample is difficult to determine.

Uncertainties can be expressed in two ways. **Absolute uncertainty** refers to an uncertainty expressed in the same units as the measurement. Therefore, the length of a computer disk label might be expressed as (5.5 ± 0.1) cm. The uncertainty of ± 0.1 cm by itself is not descriptive enough for some purposes, however. This uncertainty is large if the measurement is 1.0 cm, but it is small if the measurement is 100 m. To give a more descriptive account of the uncertainty, **fractional uncertainty** or **percent uncertainty** is used. In this type of description, the uncertainty is divided by the actual measurement. Therefore, the length of the computer disk label could be expressed as

$$\ell = 5.5\text{ cm} \pm \frac{0.1\text{ cm}}{5.5\text{ cm}} = 5.5\text{ cm} \pm 0.018 \quad \text{(fractional uncertainty)}$$

or as

$$\ell = 5.5\text{ cm} \pm 1.8\% \quad \text{(percent uncertainty)}$$

When combining measurements in a calculation, the percent uncertainty in the final result is generally larger than the uncertainty in the individual measurements. This is called **propagation of uncertainty** and is one of the challenges of experimental physics.

Some simple rules can provide a reasonable estimate of the uncertainty in a calculated result:

Multiplication and division: When measurements with uncertainties are multiplied or divided, add the *percent uncertainties* to obtain the percent uncertainty in the result.

Example: The Area of a Rectangular Plate

$$A = \ell w = (5.5\text{ cm} \pm 1.8\%) \times (6.4\text{ cm} \pm 1.6\%) = 35\text{ cm}^2 \pm 3.4\%$$
$$= (35 \pm 1)\text{ cm}^2$$

Addition and subtraction: When measurements with uncertainties are added or subtracted, add the *absolute uncertainties* to obtain the absolute uncertainty in the result.

Example: A Change in Temperature

$$\Delta T = T_2 - T_1 = (99.2 \pm 1.5)°\text{C} - (27.6 \pm 1.5)°\text{C} = (71.6 \pm 3.0)°\text{C}$$
$$= 71.6°\text{C} \pm 4.2\%$$

Powers: If a measurement is taken to a power, the percent uncertainty is multiplied by that power to obtain the percent uncertainty in the result.

Example: The Volume of a Sphere

$$V = \tfrac{4}{3}\pi r^3 = \tfrac{4}{3}\pi(6.20 \text{ cm} \pm 2.0\%)^3 = 998 \text{ cm}^3 \pm 6.0\%$$
$$= (998 \pm 60) \text{ cm}^3$$

For complicated calculations, many uncertainties are added together, which can cause the uncertainty in the final result to be undesirably large. Experiments should be designed such that calculations are as simple as possible.

Notice that uncertainties in a calculation always add. As a result, an experiment involving a subtraction should be avoided if possible, especially if the measurements being subtracted are close together. The result of such a calculation is a small difference in the measurements and uncertainties that add together. It is possible that the uncertainty in the result could be larger than the result itself!

APPENDIX

C Periodic Table of the Elements

Group I	Group II	Transition elements						
H 1 1.007 9 $1s$								
Li 3 6.941 $2s^1$	**Be** 4 9.0122 $2s^2$							
Na 11 22.990 $3s^1$	**Mg** 12 24.305 $3s^2$							
K 19 39.098 $4s^1$	**Ca** 20 40.078 $4s^2$	**Sc** 21 44.956 $3d^14s^2$	**Ti** 22 47.867 $3d^24s^2$	**V** 23 50.942 $3d^34s^2$	**Cr** 24 51.996 $3d^54s^1$	**Mn** 25 54.938 $3d^54s^2$	**Fe** 26 55.845 $3d^64s^2$	**Co** 27 58.933 $3d^74s^2$
Rb 37 85.468 $5s^1$	**Sr** 38 87.62 $5s^2$	**Y** 39 88.906 $4d^15s^2$	**Zr** 40 91.224 $4d^25s^2$	**Nb** 41 92.906 $4d^45s^1$	**Mo** 42 95.94 $4d^55s^1$	**Tc** 43 (98) $4d^55s^2$	**Ru** 44 101.07 $4d^75s^1$	**Rh** 45 102.91 $4d^85s^1$
Cs 55 132.91 $6s^1$	**Ba** 56 137.33 $6s^2$	57–71*	**Hf** 72 178.49 $5d^26s^2$	**Ta** 73 180.95 $5d^36s^2$	**W** 74 183.84 $5d^46s^2$	**Re** 75 186.21 $5d^56s^2$	**Os** 76 190.23 $5d^66s^2$	**Ir** 77 192.2 $5d^76s^2$
Fr 87 (223) $7s^1$	**Ra** 88 (226) $7s^2$	89–103**	**Rf** 104 (261) $6d^27s^2$	**Db** 105 (262) $6d^37s^2$	**Sg** 106 (266)	**Bh** 107 (264)	**Hs** 108 (277)	**Mt** 109 (268)

Symbol — **Ca** 20 — Atomic number
Atomic mass† — 40.078
$4s^2$ — Electron configuration

*Lanthanide series	**La** 57 138.91 $5d^16s^2$	**Ce** 58 140.12 $5d^14f^16s^2$	**Pr** 59 140.91 $4f^36s^2$	**Nd** 60 144.24 $4f^46s^2$	**Pm** 61 (145) $4f^56s^2$	**Sm** 62 150.36 $4f^66s^2$
**Actinide series	**Ac** 89 (227) $6d^17s^2$	**Th** 90 232.04 $6d^27s^2$	**Pa** 91 231.04 $5f^26d^17s^2$	**U** 92 238.03 $5f^36d^17s^2$	**Np** 93 (237) $5f^46d^17s^2$	**Pu** 94 (244) $5f^67s^2$

Note: Atomic mass values given are averaged over isotopes in the percentages in which they exist in nature.
† For an unstable element, mass number of the most stable known isotope is given in parentheses.

			Group III	Group IV	Group V	Group VI	Group VII	Group 0
							H 1 1.007 9 $1s^1$	**He** 2 4.002 6 $1s^2$
			B 5 10.811 $2p^1$	**C** 6 12.011 $2p^2$	**N** 7 14.007 $2p^3$	**O** 8 15.999 $2p^4$	**F** 9 18.998 $2p^5$	**Ne** 10 20.180 $2p^6$
			Al 13 26.982 $3p^1$	**Si** 14 28.086 $3p^2$	**P** 15 30.974 $3p^3$	**S** 16 32.066 $3p^4$	**Cl** 17 35.453 $3p^5$	**Ar** 18 39.948 $3p^6$
Ni 28 58.693 $3d^8 4s^2$	**Cu** 29 63.546 $3d^{10} 4s^1$	**Zn** 30 65.41 $3d^{10} 4s^2$	**Ga** 31 69.723 $4p^1$	**Ge** 32 72.64 $4p^2$	**As** 33 74.922 $4p^3$	**Se** 34 78.96 $4p^4$	**Br** 35 79.904 $4p^5$	**Kr** 36 83.80 $4p^6$
Pd 46 106.42 $4d^{10}$	**Ag** 47 107.87 $4d^{10} 5s^1$	**Cd** 48 112.41 $4d^{10} 5s^2$	**In** 49 114.82 $5p^1$	**Sn** 50 118.71 $5p^2$	**Sb** 51 121.76 $5p^3$	**Te** 52 127.60 $5p^4$	**I** 53 126.90 $5p^5$	**Xe** 54 131.29 $5p^6$
Pt 78 195.08 $5d^9 6s^1$	**Au** 79 196.97 $5d^{10} 6s^1$	**Hg** 80 200.59 $5d^{10} 6s^2$	**Tl** 81 204.38 $6p^1$	**Pb** 82 207.2 $6p^2$	**Bi** 83 208.98 $6p^3$	**Po** 84 (209) $6p^4$	**At** 85 (210) $6p^5$	**Rn** 86 (222) $6p^6$
Ds 110 (271)	**Rg** 111 (272)	**Cn** 112 (285)	113†† (284)	**Fl** 114 (289)	115†† (288)	**Lv** 116 (293)	117†† (294)	118†† (294)

Eu 63 151.96 $4f^7 6s^2$	**Gd** 64 157.25 $4f^7 5d^1 6s^2$	**Tb** 65 158.93 $4f^8 5d^1 6s^2$	**Dy** 66 162.50 $4f^{10} 6s^2$	**Ho** 67 164.93 $4f^{11} 6s^2$	**Er** 68 167.26 $4f^{12} 6s^2$	**Tm** 69 168.93 $4f^{13} 6s^2$	**Yb** 70 173.04 $4f^{14} 6s^2$	**Lu** 71 174.97 $4f^{14} 5d^1 6s^2$
Am 95 (243) $5f^7 7s^2$	**Cm** 96 (247) $5f^7 6d^1 7s^2$	**Bk** 97 (247) $5f^8 6d^1 7s^2$	**Cf** 98 (251) $5f^{10} 7s^2$	**Es** 99 (252) $5f^{11} 7s^2$	**Fm** 100 (257) $5f^{12} 7s^2$	**Md** 101 (258) $5f^{13} 7s^2$	**No** 102 (259) $5f^{14} 7s^2$	**Lr** 103 (262) $5f^{14} 6d^1 7s^2$

†† Elements 113, 115, 117, and 118 have not yet been officially named. Only small numbers of atoms of these elements have been observed.
Note: For a description of the atomic data, visit *physics.nist.gov/PhysRefData/Elements/per_text.html.*

APPENDIX

D SI Units

Table D.1 SI Units

	SI Base Unit	
Base Quantity	**Name**	**Symbol**
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Temperature	kelvin	K
Amount of substance	mole	mol
Luminous intensity	candela	cd

Table D.2 Some Derived SI Units

Other Quantity	Name	Symbol	Expression in Terms of Base Units	Expression in Terms of SI Units
Plane angle	radian	rad	m/m	
Frequency	hertz	Hz	s^{-1}	
Force	newton	N	$kg \cdot m/s^2$	J/m
Pressure	pascal	Pa	$kg/m \cdot s^2$	N/m^2
Energy	joule	J	$kg \cdot m^2/s^2$	$N \cdot m$
Power	watt	W	$kg \cdot m^2/s^3$	J/s
Electric charge	coulomb	C	$A \cdot s$	
Electric potential	volt	V	$kg \cdot m^2/A \cdot s^3$	W/A
Capacitance	farad	F	$A^2 \cdot s^4/kg \cdot m^2$	C/V
Electric resistance	ohm	Ω	$kg \cdot m^2/A^2 \cdot s^3$	V/A
Magnetic flux	weber	Wb	$kg \cdot m^2/A \cdot s^2$	$V \cdot s$
Magnetic field	tesla	T	$kg/A \cdot s^2$	
Inductance	henry	H	$kg \cdot m^2/A^2 \cdot s^2$	$T \cdot m^2/A$

Answers to Quick Quizzes and Odd-Numbered Problems

Chapter 1

Answers to Quick Quizzes

(a)
2. False
3. (b)

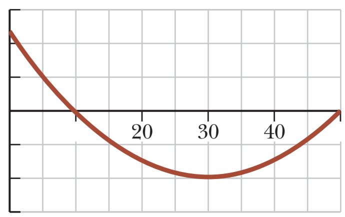
Answers to Odd-Numbered Problems

(a) 5.52 kg/m (b) It is between the density of aluminum and that of iron and is greater than the densities of typical surface rocks.
3. 23.0 kg
5. 7.69 cm
0.141 nm
9. (b) only
11. (a) kg m/s (b) N · s
13. No.
15. 11.4 kg/m
17. 871 m
19. By measuring the pages, we find that each page has area 0.277 m 0.217 m 0.060 m . The room has wall area 37 m , requiring 616 sheets that would be counted as 232 pages. Volume 1 of this textbook contains only 784 pages.
21. 1.00
23. 4.05
25. 2.86 cm
27. 151
29. (a) 507 years (b) 2.48 bills
31. balls in a room 4 m by 4 m by 3 m
33. piano tuners
35. (209 4) cm
37. 31 556 926.0 s
39.
41. 8.80%
43.
45. (a) 6.71 m (b) 0.894 (c) 0.745
47. 48.6 kg
49. 3.46
51. Answers may vary somewhat due to variation in reading precise numbers off the graph. (a) 0.015 g (b) 8% (c) 5.2 g/m (d) For shapes cut from this copy paper, the mass of the cutout is proportional to its area. The proportionality constant is 5.2 g/m 8%, where the uncertainty is estimated. (e) This result is to be expected if the paper has thickness and density that are uniform within the experimental uncertainty. (f) The slope is the areal density of the paper, its mass per unit area.
53. 5.2 m , 3%
55. 316 m
57. 5.0 m
59. 3.41 m
61. (a) aluminum, 2.75 g/cm ; copper, 9.36 g/cm ; brass, 8.91 g/cm ; tin, 7.68 g/cm ; iron, 7.88 g/cm
(b) The tabulated values are smaller by 2% for aluminum, by 5% for copper, by 6% for brass, by 5% for tin, and by 0.3% for iron.
63. gal/yr
65. Answers may vary. (a) prokaryotes (b)
67. (a) 2.70 g/cm 1.19 g/cm (b) 1.39 kg
69. 0.579 (1.19 , where is in cubic feet and is in seconds
71. (a) 0.529 cm/s (b) 11.5 cm/s
73. (a) 12.1 m (b) 135° (c) 25.2° (d) 135°

Chapter 2

Answers to Quick Quizzes

(c)
2. (b)
3. False. Your graph should look something like the one shown below. This graph shows that the maximum speed is about 5.0 m/s, which is 18 km/h (11 mi/h), so the driver was not speeding.



4. (b)
5. (c)
6. (a)–(e), (b)–(d), (c)–(f)
(i) (e) **(ii)** (d)

Answers to Odd-Numbered Problems

(a) 5 m/s (b) 1.2 m/s (c) 2.5 m/s (d) 3.3 m/s (e) 0
3. (a) 3.75 m/s (b) 0

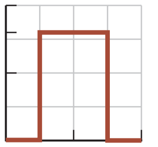
5. (a) 2.30 m/s (b) 16.1 m/s (c) 11.5 m/s
(a) 2.4 m/s (b) 3.8 m/s (c) 4.0 s

9. (a) 5.0 m/s (b) 2.5 m/s (c) 0 (d) 5.0 m/s

11. (a) 5.00 m (b) 4.88

13. (a) 2.80 h (b) 218 km

15. (a)



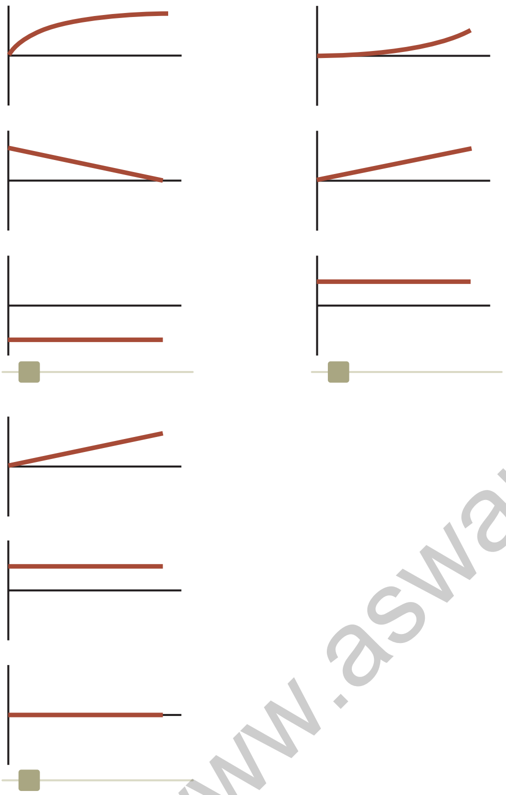
(b) 1.60 m/s (c) 0.800 m/s

17. (a) 1.3 m/s (b) 3 s, 2 m/s (c) 6 s, 10 s
(d) 1.5 m/s

19. (a) 20 m/s, 5 m/s (b) 263 m

21. (a) 2.00 m (b) 3.00 m/s (c) 2.00 m/s

23.



25. (a) 4.98 s (b) 1.20 m/s

27. (a) 9.00 m/s (b) 3.00 m/s (c) 17.0 m/s (d) The graph of velocity versus time is a straight line passing through 13 m/s at 10:05 a.m. and sloping downward, decreasing by 4 m/s for each second thereafter. (e) If and only if we know the object's velocity at one instant of time, knowing its acceleration tells us its velocity at every other moment as long as the acceleration is constant.

29. 16.0 cm/s

31. (a) 202 m/s (b) 198 m

33. (a) 35.0 s (b) 15.7 m/s

35. 3.10 m/s

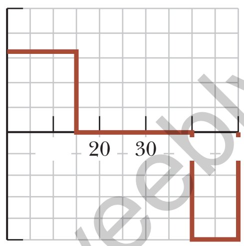
37. (a)

(b) Particle under constant acceleration
(c)) (Equation 2.17)
(d) (e) 1.25 m/s (f) 8.00 s

39. (a) The idea is false unless the acceleration is zero. We define constant acceleration to mean that the velocity is changing steadily in time. So, the velocity cannot be changing steadily in space.
(b) This idea is true. Because the velocity is changing steadily in time, the velocity halfway through an interval is equal to the average of its initial and final values.

41. (a) 13.5 m (b) 13.5 m (c) 13.5 m (d) 22.5 m

43. (a) 1.88 km (b) 1.46 km
(c)



(d) 0 1.67 *ab* 50 375; 250 2.5 375 (In all three expressions, is in meters and is in seconds.) (e) 37.5 m/s

45. (a) 0.231 m (b) 0.364 m (c) 0.399 m (d) 0.175 m

47. David will be unsuccessful. The average human reaction time is about 0.2 s (research on the Internet) and a dol lar bill is about 15.5 cm long, so David's fingers are about 8 cm from the end of the bill before it is dropped. The bill will fall about 20 cm before he can close his fingers.

49. (a) 510 m (b) 20.4 s

51. 1.79 s

53. (a) 10.0 m/s up (b) 4.68 m/s down

55. (a) 7.82 m (b) 0.782 s

57. (a)

59. (a) (10.0 3.00 (1.67 (1.50 (In these expressions, is in m/s is in meters, and is in seconds.) (b) 3.00 ms (c) 450 m/s (d) 0.900 m

61. (a) 4.00 m/s (b) 1.00 ms (c) 0.816 m

63. (a) 3.00 s (b) 15.3 m/s
(c) 31.4 m/s down and 34.8 m/s down

65. (a) 3.00 m/s (b) 6.00 s (c) −0.300 m/s (d) 2.05 m/s

67. (a) 2.83 s (b) It is exactly the same situation as in Example 2.8 except that this problem is in the vertical direction. The descending elevator plays the role of the speeding car, and the falling bolt plays the role of the accelerating trooper. Turn Figure 2.13 through 90° clockwise to visualize the elevator–bolt problem! (c) If each floor is 3 m high, the highest floor that can be reached is the 13th floor.

69. (a) From the graph, we see that the Acela is cruising at a constant positive velocity in the positive direction from about 50 s to 50 s. From 50 s to 200 s, the Acela accel erates in the positive direction reaching a top speed of about 170 mi/h. Around 200 s, the engineer applies the brakes, and the train, still traveling in the positive direction, slows down and then stops at 350 s. Just after

350 s, the train reverses direction (becomes negative) and steadily gains speed in the negative direction. (b) approximately 2.2 mi/h/s (c) approximately 6.7 mi

71. (a) Here, must be greater than and the distance between the leading athlete and the finish line must be great enough so that the trailing athlete has time to catch up.

(b) (c)

73. (a) 5.46 s (b) 73.0 m
(c) $_{Stan}$ 22.6 m/s, $_{Kathy}$ 26.7 m/s

75. (a) (1/tan (b) The velocity starts off larger than for small values of and then decreases, approaching zero as approaches 90°.

77. (a) 15.0 s (b) 30.0 m/s (c) 225 m

79. 1.60 m/s

81. (a) 35.9 m (b) 4.04 s (c) 45.8 m (d) 22.6 m/s

83. (a) 5.32 m/s for Laura and 3.75 m/s for Healan
(b) 10.6 m/s for Laura and 11.2 m/s for Healan
(c) Laura, by 2.63 m (d) 4.47 m at 2.84 s

85. (a) 26.4 m (b) 6.8%

Chapter 3

Answers to Quick Quizzes

vectors: (b), (c); scalars: (a), (d), (e)

2. (c)

3. (b) and (c)

4. (b)

5. (c)

Answers to Odd-Numbered Problems

2.75, 4.76) m

3. (a) 8.60 m (b) 4.47 m, 63.4°; 4.24 m, 135°

5. (a) (3.56 cm, 2.40 cm) (b) (4.30 cm, 326°)
(c) (8.60 cm, 34.0°) (d) (12.9 cm, 146°)

70.0 m

9. This situation can *never* be true because the distance is the length of an arc of a circle between two points, whereas the magnitude of the displacement vector is a straight-line chord of the circle between the same points.

11. (a) 5.2 m at 60° (b) 3.0 m at 330° (c) 3.0 m at 150°
(d) 5.2 m at 300°

13. approximately 420 ft at

15. 47.2 units at 122°

17. (a) yes (b) The speed of the camper should be 28.3 m/s or more to satisfy this requirement.

19. (a) (11.1 6.40) m (b) (1.65 2.86) cm
(c) (18.0 12.6) in.

21. 358 m at 2.00° S of E

23. (a) 2.00 6.00 (b) 4.00 2.00 (c) 6.32 (d) 4.47
(e) 288°; 26.6°

25. 9.48 m at 166°

27. 4.64 m at 78.6° N of E

29. (a) 185 N at 77.8° from the positive axis
(b) (39.3 181

31. (a) 2.83 m at 315° (b) 13.4 m at 117°

33. (a) 8.00 12.0 4.00 (b) 2.00 3.00 1.00
(c) 24.0 36.0 12.0

35. (a) 3.00 2.00 (b) 3.61 at 146° (c) 3.00 6.00

37. (a) 5.00 and 7.00 (b) For vectors to be equal, all their components must be equal. A vector equation contains more information than a scalar equation.

39. 196 cm at 345°

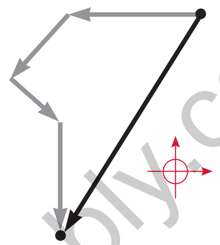
41. (a) 15.1 7.72 cm (b) 7.72 15.1 cm
(c) 7.72 15.1

43. (a) 20.5 35.5 m (b) 25.0 m
(c) 61.5 107 m (d) 37.5 m (e) 157 km

45. 1.43 m at 32.2° above the horizontal

47. (a) 10.4 cm (b) 35.5°

49. (a)



(b) 18.3 b (c) 12.4 b at 233° counterclockwise from east

51. 240 m at 237°

53. (a) 25.4 s (b) 15.0 km/h

55. (a) 0.079 8 N (b) 57.9° (c) 32.1°

57. (a) The , and components are, respectively, 2.00, 1.00, and 3.00. (b) 3.74 (c) 57.7°, 74.5°, 36.7°

59. 1.15°

61. (a) (10 000 9 600 sin $^{1/2}$ cm (b) 270°; 140 cm (c) 90°; 20.0 cm (d) They do make sense. The maximum value is attained when and are in the same direction, and it is 60 cm 80 cm. The minimum value is attained when and are in opposite directions, and it is 80 cm 60 cm.

63. (a) 2.00 m/s (b) its velocity vector

65. (a) (b) $^{1/2}$
(c)

67. (a) (10.0 m, 16.0 m) (b) This center of mass of the tree distribution is the same location whatever order we take the trees in. (We will study center of mass in Chapter 9.)

Chapter 4

Answers to Quick Quizzes

(a)

2. (i) (b) **(ii)** (a)

3. 15°, 30°, 45°, 60°, 75°

4. **(i)** (d) **(ii)** (b)

5. **(i)** (b) **(ii)** (d)

Answers to Odd-Numbered Problems

(a) 4.87 km at 209° from east (b) 23.3 m/s
(c) 13.5 m/s at 209°

3. (a) (1.00 0.750) m/s (b) (1.00 0.500) m/s, 1.12 m/s

5. (a) 18.0 4.00 4.90 , where is in meters and is in seconds
(b) 18.0 4.00 9.80 , where is in meters per second and is in seconds
(c) = −9.80
(d) 54.0 32.1 18.0 25.4 m s; = −9.80

7. (a) $\vec{\mathbf{v}} = -12.0t\,\hat{\mathbf{j}}$, where $\vec{\mathbf{v}}$ is in meters per second and t is in seconds (b) $\vec{\mathbf{a}} = -12.0\hat{\mathbf{j}}$ m/s^2 (c) $\vec{\mathbf{r}} = (3.00\hat{\mathbf{i}} - 6.00\hat{\mathbf{j}})$ m; $\vec{\mathbf{v}} = -12.0\hat{\mathbf{j}}$ m/s
9. (a) $(0.800\hat{\mathbf{i}} - 0.300\hat{\mathbf{j}})$ m/s^2 (b) 339° (c) $(360\hat{\mathbf{i}} - 72.7\hat{\mathbf{j}})$ m, −15.2°
11. 12.0 m/s
13. (a) 2.81 m/s horizontal (b) 60.2° below the horizontal
15. 53.1°
17. (a) 3.96 m/s horizontally forward (b) 9.6%
19. 67.8°
21. $d \tan \theta_i - \dfrac{gd^2}{2v_i^2 \cos^2 \theta_i}$
23. (a) The ball clears by 0.89 m. (b) while descending
25. (a) 18.1 m/s (b) 1.13 m (c) 2.79 m
27. 9.91 m/s
29. (a) (0, 50.0 m) (b) $v_{xi} = 18.0$ m/s; $v_{yi} = 0$ (c) Particle under constant acceleration (d) Particle under constant velocity (e) $v_{xf} = v_{xi}$; $v_{yf} = -gt$ (f) $x_f = v_{xi}t$; $y_f = y_i - \frac{1}{2}gt^2$ (g) 3.19 s (h) 36.1 m/s, −60.1°
31. 1.92 s
33. 377 m/s^2
35. 2.06×10^3 rev/min
37. 0.749 rev/s
39. 7.58×10^3 m/s, 5.80×10^3 s
41. 1.48 m/s^2 inward and 29.9° backward
43. (a) Yes. The particle can be either speeding up or slowing down, with a tangential component of acceleration of magnitude $\sqrt{6^2 - 4.5^2} = 3.97$ m/s^2. (b) No. The magnitude of the acceleration cannot be less than $v^2/r = 4.5$ m/s^2.
45. (a) 1.26 h (b) 1.13 h (c) 1.19 h
47. (a) 15.0 km/h east (b) 15.0 km/h west (c) 0.016 7 h = 60.0 s
49. (a) 9.80 m/s^2 down and 2.50 m/s^2 south (b) 9.80 m/s^2 down (c) The bolt moves on a parabola with its axis downward and tilting to the south. It lands south of the point directly below its starting point. (d) The bolt moves on a parabola with a vertical axis.
51. (a) $\dfrac{2d/c}{1 - v^2/c^2}$ (b) $\dfrac{2d}{c}$

(c) The trip in flowing water takes a longer time interval. The swimmer travels at the low upstream speed for a longer time interval, so his average speed is reduced below c. Mathematically, $1/(1 - v^2/c^2)$ is always greater than 1. In the extreme, as $v \to c$, the time interval becomes infinite. In that case, the student can never return to the starting point because he cannot swim fast enough to overcome the river current.
53. 15.3 m
55. 54.4 m/s^2
57. The relationship between the height h and the walking speed is $h = (4.16 \times 10^{-3})v_x^2$, where h is in meters and v_x is in meters per second. At a typical walking speed of 4 to 5 km/h, the ball would have to be dropped from a height of about 1 cm, clearly much too low for a person's hand. Even at Olympic-record speed for the 100-m run (confirm on the Internet), this situation would only occur if the ball is dropped from about 0.4 m, which is also below the hand of a normally proportioned person.
59. (a) 101 m/s (b) 3.27×10^4 ft (c) 20.6 s
61. (a) 26.9 m/s (b) 67.3 m (c) $(2.00\hat{\mathbf{i}} - 5.00\hat{\mathbf{j}})$ m/s^2
63. (a) $(7.62\hat{\mathbf{i}} - 6.48\hat{\mathbf{j}})$ cm (b) $(10.0\hat{\mathbf{i}} - 7.05\hat{\mathbf{j}})$ cm
65. (a) 1.52 km (b) 36.1 s (c) 4.05 km
67. The initial height of the ball when struck is 3.94 m, which is too high for the batter to hit the ball.
69. (a) 1.69 km/s (b) 1.80 h
71. (a) 46.5 m/s (b) −77.6° (c) 6.34 s
73. (a) $x = v_i(0.164\,3 + 0.002\,299v_i^2)^{1/2} + 0.047\,94v_i^2$, where x is in meters and v_i is in meters per second (b) 0.041 0 m (c) 961 m (d) $x \approx 0.405v_i$ (e) $x \approx 0.095\,9v_i^2$ (f) The graph of x versus v_i starts from the origin as a straight line with slope 0.405 s. Then it curves upward above this tangent line, becoming closer and closer to the parabola $x = 0.095\,9v_i^2$, where x is in meters and v_i is in meters per second.
75. (a) 6.80 km (b) 3.00 km vertically above the impact point (c) 66.2°
77. (a) 20.0 m/s (b) 5.00 s (c) $(16.0\hat{\mathbf{i}} - 27.1\hat{\mathbf{j}})$ m/s (d) 6.53 s (e) $24.5\hat{\mathbf{i}}$ m
79. (a) 4.00 km/h (b) 4.00 km/h
81. (a) 43.2 m (b) $(9.66\hat{\mathbf{i}} - 25.6\hat{\mathbf{j}})$ m/s (c) Air resistance would ordinarily make the jump distance smaller and the final horizontal and vertical velocity components both somewhat smaller. If a skilled jumper shapes her body into an airfoil, however, she can deflect downward the air through which she passes so that it deflects her upward, giving her more time in the air and a longer jump.
83. (a) swim perpendicular to the banks (b) 133 m (c) 53.1° (d) 107 m
85. 33.5° below the horizontal
87. $\tan^{-1}\left(\dfrac{\sqrt{2gh}}{v}\right)$
89. Safe distances are less than 270 m or greater than 3.48×10^3 m from the western shore.

Chapter 5

Answers to Quick Quizzes

1. (d)
2. (a)
3. (d)
4. (b)
5. **(i)** (c) **(ii)** (a)
6. (b)
7. (b) Pulling up on the rope decreases the normal force, which, in turn, decreases the force of kinetic friction.

Answers to Odd-Numbered Problems

1. (a) 534 N (b) 54.5 kg
3. (a) $(6.00\hat{\mathbf{i}} + 15.0\hat{\mathbf{j}})$ N (b) 16.2 N
5. (a) $(2.50\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}})$ N (b) 5.59 N
7. 2.58 N
9. (a) 1.53 m (b) 24.0 N forward and upward at 5.29° with the horizontal
11. (a) 3.64×10^{-18} N (b) 8.93×10^{-30} N is 408 billion times smaller
13. (a) force exerted by spring on hand, to the left; force exerted by spring on wall, to the right (b) force exerted

by wagon on handle, downward to the left; force exerted by wagon on planet, upward; force exerted by wagon on ground, downward (c) force exerted by football on player, downward to the right; force exerted by football on planet, upward (d) force exerted by small-mass object on large-mass object, to the left (e) force exerted by negative charge on positive charge, to the left (f) force exerted by iron on magnet, to the left

15. (a) 45.0 15.0 m/s (b) 162° from the + axis (c) 225 75.0 m (d) 227 79.0

17. (a) (b) (c) $\frac{Fh}{mg}$ (d)

19. (a) 5.00 m/s at 36.9° (b) 6.08 m/s at 25.3°

21. (a) 15.0 lb up (b) 5.00 lb up (c) 0

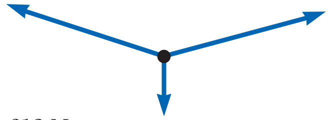
23. (a) 2.15 N forward (b) 645 N forward (c) 645 N toward the rear (d) 1.02 10 N at 74.1° below the horizontal and rearward

25. (a) 3.43 kN (b) 0.967 m/s horizontally forward

27. (a) cos 40° 0 and sin 40° 220 N 0; 342 N and 262 N (b) cos 40° (220 N) sin 40° 0 and sin 40 (220 N) cos 40° 0; 262 N and 342 N (c) The results agree. The methods are of the same level of difficulty. Each involves one equation in one unknown and one equation in two unknowns. If we are interested in finding without finding , method (b) is simpler.

29. (a) 7.0 m/s horizontal and to the right (b) 21 N (c) 14 N horizontal and to the right

31. (a)



(b) 613 N

33. 253 N, 165 N, 325 N

35. 100 N and 204 N

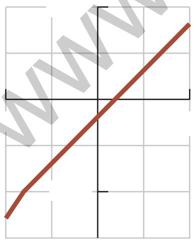
37. 8.66 N east

39. (a) tan (b) 4.16 m/s

41. (a) 646 N up (b) 646 N up (c) 627 N up (d) 589 N up

43. (a) 79.8 N, 39.9 N (b) 2.34 m/s

45. (a) 19.6 N (b) 78.4 N (c)



47. 3.73 m

49. (a) 2.20 m/s (b) 27.4 N

51. (a) 706 N (b) 814 N (c) 706 N (d) 648 N

53. 1.76 kN to the left

55. a) 0.306 (b) 0.245

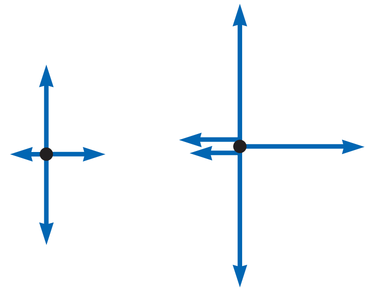
57. = 0.727, 0.577

59. (a) 1.11 s (b) 0.875 s

61. (a) 1.78 m/s (b) 0.368 (c) 9.37 N (d) 2.67 m/s

63. 37.8 N

65. (a)



(b) 1.29 m/s to the right (c) 27.2 N

67. 6.84 m

69. 0.060 0 m

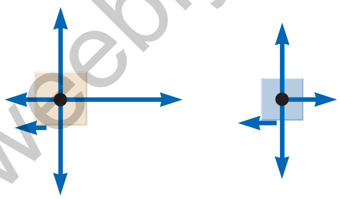
71. (a) 0.087 1 (b) 27.4 N

73. (a) Removing mass (b) 13.7 mi/h · s

75. (a) (b)

77. (a) 2.22 m (b) 8.74 m/s down the incline

79. (a)

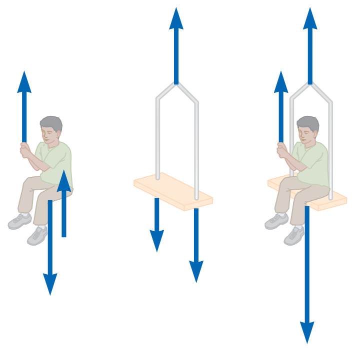


(b) (c) (d) (e)

(f) $- \mu$ $- \mu$

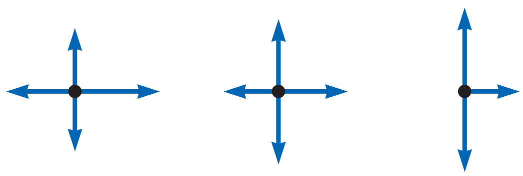
(g) $- \mu$

81. (a)



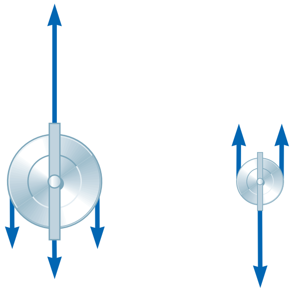
(b) 0.408 m/s (c) 83.3 N

83. (a)



(b) 2.00 m/s to the right (c) 4.00 N on , 6.00 N right on , 8.00 N right on (d) 14.0 N between and , 8.00 N between and (e) The block models the heavy block of wood. The contact force on your back is modeled by the force between the and the blocks, which is much less than the force . The difference between and this contact force is the net force causing the acceleration of the 5-kg pair of objects. The acceleration is real and nonzero, but it lasts for so short a time that it is never associated with a large velocity. The frame of the building and your legs exert forces, small in magnitude relative to the hammer blow, to bring the partition, block, and you to rest again over a time interval large relative to the hammer blow.

85. (a) *Upper pulley:* *Lower pulley:*



(b) /2, /2, /2, 3 /2, (c)

87. 0.287

89. (b) If is greater than tan (1/), motion is impossible.

91. (a) The net force on the cushion is in a fixed direction, downward and forward making angle tan) with the vertical. Starting from rest, it will move along this line with (b) increasing speed. Its velocity changes in magni tude. (c) 1.63 m (d) It will move along a parabola. The axis of the parabola is parallel to the line described in part (a). If the cushion is thrown in a direction above this line, its path will be concave downward, making its veloc ity become more and more nearly parallel to the line over time. If the cushion is thrown down more steeply, its path will be concave upward, again making its velocity turn toward the fixed direction of its acceleration.

95. (a) 30.7° (b) 0.843 N

97. 72.0 N

99. (a) 0.931 m/s (b) From a value of 0.625 m/s for large , the acceleration gradually increases, passes through a maximum, and then drops more rapidly, becoming nega tive and reaching 2.10 m/s at 0. (c) 0.976 m/s at 25.0 cm (d) 6.10 cm

101. (a) 4.90 m/s (b) 3.13 m/s at 30.0° below the horizontal (c) 1.35 m (d) 1.14 s (e) The mass of the block makes no difference.

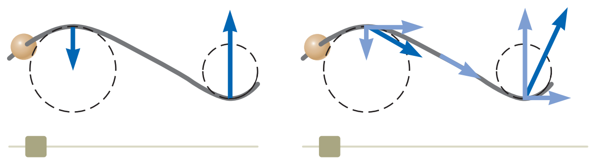
103. (a) 2.13 s (b) 1.66 m

Chapter 6

Answers to Quick Quizzes

(i) (a) **(ii)** (b)

(i) Because the speed is constant, the only direction the force can have is that of the centripetal acceleration. The force is larger at than at because the radius at is smaller. There is no force at because the wire is straight. **(ii)** In addition to the forces in the centripetal direction in part (a), there are now tangential forces to provide the tangential acceleration. The tangential force is the same at all three points because the tangential acceleration is constant.



3. (c)

4. (a)

Answers to Odd-Numbered Problems

any speed up to 8.08 m/s

(a) 8.33 N toward the nucleus

(b) 9.15 m/s inward

5. 6.22

2.14 rev/min

9. (a) static friction (b) 0.085 0

11. 14.3 m/s

13. (a) 1.33 m/s (b) 1.79 m/s at 48.0° inward from the direc tion of the velocity

15. (a) — (b) 2

17. (a) 8.62 m (b) , downward (c) 8.45 m/s (d) Calcu lation of the normal force shows it to be negative, which is impossible. We interpret it to mean that the normal force goes to zero at some point and the passengers will fall out of their seats near the top of the ride if they are not restrained in some way. We could arrive at this same result without calculating the normal force by noting that the acceleration in part (c) is smaller than that due to gravity. The teardrop shape has the advantage of a larger acceleration of the riders at the top of the arc for a path having the same height as the circular path, so the pas sengers stay in the cars.

19. No. The archeologist needs a vine of tensile strength equal to or greater than 1.38 kN to make it across.

21. (a) 17.0° (b) 5.12 N

23. (a) 491 N (b) 50.1 kg (c) 2.00 m/s

25. 0.527

27. 0.212 m/s , opposite the velocity vector

29. 3.01 N up

31. (a) 1.47 N s/m (b) 2.04 s (c) 2.94

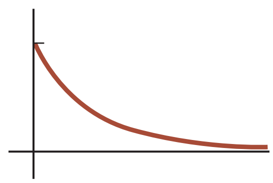
35. (a) 0.034 7 s (b) 2.50 m/s (c)

37. (a) At , the velocity is eastward and the acceleration is southward. (b) At , the velocity is southward and the acceleration is westward.

39. 781 N

41. (a) mg $\frac{mv}{}$ (b) $\sqrt{gR}$

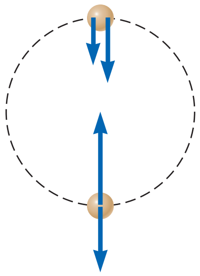
43. (a) $bt/$ (b)



(c) In this model, the object keeps moving forever. (d) It travels a finite distance in an infinite time interval.

45. (a) the downward gravitational force and the tension force in the string, always directed toward the center of the path

(b)



(c) 6.05 N (d) 7.82 m/s

47. (a) 106 N up the incline (b) 0.396

49. (a) 0.016 2 kg/m (b) - (c) 0.778 (d) 1.5% (e) For nested coffee filters falling in air at terminal speed, the graph of air resistance force as a function of the square of speed demonstrates that the force is proportional to the speed squared, within the experimental uncertainty estimated as 2%. This proportionality agrees with the theoretical model of air resistance at high speeds. The drag coefficient of a coffee filter is 0.78 2%.

51. (cos tan sin

53. (a) The only horizontal force on the car is the force of friction, with a maximum value determined by the sur face roughness (described by the coefficient of static friction) and the normal force (here equal to the gravita tional force on the car). (b) 34.3 m (c) 68.6 m (d) Brak ing is better. You should not turn the wheel. If you used any of the available friction force to change the direction of the car, it would be unavailable to slow the car and the stopping distance would be greater. (e) The conclusion is true in general. The radius of the curve you can barely make is twice your minimum stopping distance.

55. (a) 735 N (b) 732 N (c) The gravitational force is larger. The normal force is smaller, just like it is when going over the top of a Ferris wheel.

57. (a) 5.19 m/s (b) (c) 555 N

59. (b) The gravitational and friction forces remain constant, the normal force increases, and the person remains in motion with the wall. (c) The gravitational force remains constant, the normal and friction forces decrease, and the person slides relative to the wall and downward into the pit.

61. (a) min $\frac{\tan\theta - \mu}{+ \mu \tan}$ max $\frac{\tan\theta + \mu}{- \mu \tan}$

(b) tan

63. 12.8 N

65. (a) 78.3 m/s (b) 11.1 s (c) 121 m

67. (a) 8.04 s (b) 379 m/s (c) 1.19 m/s (d) 9.55 cm

69. (a) 0.013 2 m/s (b) 1.03 m/s (c) 6.87 m/s

Chapter 7

Answers to Quick Quizzes

(a)

2. (c), (a), (d), (b)

3. (d)

4. (a)

5. (b)

6. (c)

(i) (c) **(ii)** (a)

8. (d)

Answers to Odd-Numbered Problems

(a) 1.59 J (b) smaller (c) the same

3. (a) 472 J (b) 2.76 kN

5. (a) 31.9 J (b) 0 (c) 0 (d) 31.9 J

9. 16.0

11. (a) 16.0 J (b) 36.9°

13. 7.05 m at 28.4°

15. (a) 7.50 J (b) 15.0 J (c) 7.50 J (d) 30.0 J

17. (a) 0.938 cm (b) 1.25 J

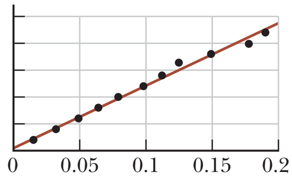
19. (a) 575 N/m (b) 46.0 J

21. (a) *mg* — — (b) — —

23. (a) Design the spring constant so that the weight of one tray removed from the pile causes an extension of the springs equal to the thickness of one tray. (b) 316 N/m (c) We do not need to know the length and width of the tray.

25. (b) *mgR*

27. (a)



(b) The slope of the line is 116 N/m. (c) We use all the points listed and also the origin. There is no visible evi dence for a bend in the graph or nonlinearity near either end. (d) 116 N/m (e) 12.7 N

29. 50.0 J

31. (a) 60.0 J (b) 60.0 J

33. (a) 1.20 J (b) 5.00 m/s (c) 6.30 J

35. 878 kN up

37. (a) 4.56 kJ (b) 4.56 kJ (c) 6.34 kN (d) 422 km/s (e) 6.34 kN (f) The two theories agree.

39. (a) 97.8 J (b) 4.31 31.6 N (c) 8.73 m/s

41. (a) 2.5 J (b) 9.8 J (c) 12 J

43. (a) 196 J (b) 196 J (c) 196 J (d) The gravitational force is conservative.

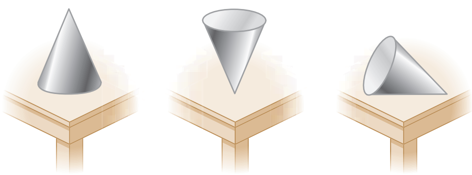
45. (a) 125 J (b) 50.0 J (c) 66.7 J (d) nonconservative (e) The work done on the particle depends on the path followed by the particle.

47. away from the other particle

49.

51. (a) 40.0 J (b) 40.0 J (c) 62.5 J

53.



55. 90.0 J
57. (a) 8 N/m (b) It lasts for a time interval. If the interaction occupied no time interval, the force exerted by each ball on the other would be infinite, and that can not happen. (c) 0.8 J (d) 0.15 mm (e) 10
59. 0.299 m/s
61. (a) 20.5 14.3 N 36.4 21.0 N
(b) 15.9 35.3 N
(c) 3.18 7.07 m
(d) 5.54 23.7 m
(e) 2.30 39.3 m (f) 1.48 kJ (g) 1.48 kJ
(h) The work–kinetic energy theorem is consistent with Newton's second law.
63. 0.131 m
65. (a) (b) The force must be conservative because the work the force does on the particle on which it acts depends only on the original and final positions of the particle, not on the path between them.
67. (a) 3.62 /(4.30 23.4), where is in meters and is in kilograms (b) 0.095 1 m (c) 0.492 m (d) 6.85 m (e) The situation is impossible. (f) The extension is directly proportional to when is only a few grams. Then it grows faster and faster, diverging to infinity for 0.184 kg.

Chapter 8

Answers to Quick Quizzes

(a) For the television set, energy enters by electrical transmission (through the power cord). Energy leaves by heat (from hot surfaces into the air), mechanical waves (sound from the speaker), and electromagnetic radia tion (from the screen). (b) For the gasoline-powered lawn mower, energy enters by matter transfer (gasoline). Energy leaves by work (on the blades of grass), mechani cal waves (sound), and heat (from hot surfaces into the air). (c) For the hand-cranked pencil sharpener, energy enters by work (from your hand turning the crank). Energy leaves by work (done on the pencil), mechanical waves (sound), and heat due to the temperature increase from friction.
2. **(i)** (b) **(ii)** (b) **(iii)** (a)
3. (a)
4.
5. (c)

Answers to Odd-Numbered Problems

(a) $_{int}$ $_{ER}$
(b) $_{int}$
(c) (d) 0 $_{ER}$
3. 10.2 m
5. (a) $^{1/2}$ (b) 0.098 0 N down
(a) 4.43 m/s (b) 5.00 m
9. 5.49 m/s
11. $\frac{\sqrt{gh}}{15}$
13. ——
15. (a) 0.791 m/s (b) 0.531 m/s
17. (a) 5.60 J (b) 2.29 rev
19. (a) 168 J
21. (a) 1.40 m/s (b) 4.60 cm after release (c) 1.79 m/s
23. (a) 160 J (b) 73.5 J (c) 28.8 N (d) 0.679
25. (a) 4.12 m (b) 3.35 m
27. (a) Isolated. The only external influence on the system is the normal force from the slide, but this force is always perpendicular to its displacement so it performs no work on the system. (b) No, the slide is frictionless. (c) $_{system}$ mgh (d) $_{system}$ $-mgh$ $-$
(e) $_{system}$ mgy $_{max}$ $-$
(f) $\sqrt{gh}$ (g) $_{max}$ $-\cos$) (h) If friction is present, mechanical energy of the system would *not* be conserved, so the child's kinetic energy at all points after leaving the top of the waterslide would be reduced when compared with the frictionless case. Consequently, her launch speed and maximum height would be reduced as well.
29. 1.23 kW
31. 4.5
33. $145
35.
37. (a) 423 mi/gal (b) 776 mi/gal
39. 236 s or 3.93 min
41. (a) 10.2 kW (b) 10.6 kW (c) 5.82 MJ
43. (a) 0.588 J (b) 0.588 J (c) 2.42 m/s
(d) 0.196 J, 0.392 J
45. ——
47. (a) , where is in seconds and is in joules (b) 12 and 48 , where is in seconds, is in m/s , and is in newtons (c) P 48 288 , where is in seconds and is in watts (d) 1.25
49. (a) 11.1 m/s (b) 1.00 J (c) 1.35 m
51. (a) 6.08 J (b) 4.59 J (c) 4.59
53. (a) 4.0 mm (b) 1.0 cm
55. (a) 2.17 kW (b) 58.6 kW
57. (a) 1.38 J (b) 5.51
(c) The value in part (b) represents only energy that leaves the engine and is transformed to kinetic energy of the car. Additional energy leaves the energy by sound and heat. More energy leaves the engine to do work against friction forces and air resistance.
59. (a) 1.53 J at 6.00 cm, 0 J at 0 (b) 1.75 m/s (c) 1.51 m/s (d) The answer to part (c) is not half the answer to part (b) because the equation for the speed of an oscillator is not linear in position
61. (a) 100 J (b) 0.410 m (c) 2.84 m/s (d) 9.80 mm (e) 2.85 m/s
63. 0.328
65. (a) 0.400 m (b) 4.10 m/s (c) The block stays on the track.
67. 33.4 kW
69.

71. 2.92 m/s
75. (b) 0.342
77. (a) 14.1 m/s (b) 800 N (c) 771 N (d) 1.57 kN up
79. (a) $-\mu_k gx/L$ (b) $(\mu_k gL)^{1/2}$
81. (a) 6.15 m/s (b) 9.87 m/s
83. less dangerous
85. (a) 25.8 m (b) 27.1 m/s^2

Chapter 9

Answers to Quick Quizzes

1. (d)
2. (b), (c), (a)
3. **(i)** (c), (e) **(ii)** (b), (d)
4. (a) All three are the same. (b) dashboard, seat belt, air bag
5. (a)
6. (b)
7. (b)
8. **(i)** (a) **(ii)** (b)

Answers to Odd-Numbered Problems

1. (b) $p = \sqrt{2mK}$
3. 7.00 N
5. $\vec{\mathbf{F}}_{\text{on bat}} = (+3.26\hat{\mathbf{i}} - 3.99\hat{\mathbf{j}})$ kN
7. (a) $\vec{\mathbf{v}}_{pi} = -\left(\dfrac{m_g}{m_g + m_p}\right)v_{gp}\hat{\mathbf{i}}$ (b) $\vec{\mathbf{v}}_{gi} = \left(\dfrac{m_p}{m_g + m_p}\right)v_{gp}\hat{\mathbf{i}}$
9. 40.5 g
11. (a) $-6.00\hat{\mathbf{i}}$ m/s (b) 8.40 J (c) The original energy is in the spring. (d) A force had to be exerted over a displacement to compress the spring, transferring energy into it by work. The cord exerts force, but over no displacement. (e) System momentum is conserved with the value zero. (f) The forces on the two blocks are internal forces, which cannot change the momentum of the system; the system is isolated. (g) Even though there is motion afterward, the final momenta are of equal magnitude in opposite directions, so the final momentum of the system is still zero.
13. (a) 13.5 N · s (b) 9.00 kN
15. (c) no difference
17. (a) 9.60×10^{-2} s (b) 3.65×10^5 N (c) $26.6g$
19. (a) $12.0\hat{\mathbf{i}}$ N · s (b) $4.80\hat{\mathbf{i}}$ m/s (c) $2.80\hat{\mathbf{i}}$ m/s (d) $2.40\hat{\mathbf{i}}$ N
21. 16.5 N
23. 301 m/s
25. (a) 2.50 m/s (b) 37.5 kJ
27. (a) 0.284 (b) 1.15×10^{-13} J and 4.54×10^{-14} J
29. (a) 4.85 m/s (b) 8.41 m
31. 91.2 m/s
33. 0.556 m
35. (a) 1.07 m/s at −29.7° (b) $\dfrac{\Delta K}{K_i} = -0.318$
37. $(3.00\hat{\mathbf{i}} - 1.20\hat{\mathbf{j}})$ m/s
39. $v_O = v_i \cos\theta$, $v_Y = v_i \sin\theta$
41. 2.50 m/s at −60.0°
43. (a) $(-9.33\hat{\mathbf{i}} - 8.33\hat{\mathbf{j}})$ Mm/s (b) 439 fJ
45. $\vec{\mathbf{r}}_{\text{CM}} = (0\hat{\mathbf{i}} + 1.00\hat{\mathbf{j}})$ m
47. 3.57×10^8 J
49. (a) 15.9 g (b) 0.153 m
51. (a) $(1.40\hat{\mathbf{i}} + 2.40\hat{\mathbf{j}})$ m/s (b) $(7.00\hat{\mathbf{i}} + 12.0\hat{\mathbf{j}})$ kg · m/s
53. 0.700 m
55. (a) $\vec{\mathbf{v}}_{1f} = -0.780\hat{\mathbf{i}}$ m/s, $\vec{\mathbf{v}}_{2f} = 1.12\hat{\mathbf{i}}$ m/s (b) $\vec{\mathbf{v}}_{\text{CM}} = 0.360\hat{\mathbf{i}}$ m/s before and after the collision
57. (b) The bumper continues to exert a force to the left until the particle has swung down to its lowest point.
59. (a) $\sqrt{\dfrac{F(2d - \ell)}{2m}}$ (b) $\dfrac{F\ell}{2}$
61. 15.0 N in the direction of the initial velocity of the exiting water stream.
63. (a) 442 metric tons (b) 19.2 metric tons (c) It is much less than the suggested value of 442/2.50. Mathematically, the logarithm in the rocket propulsion equation is not a linear function. Physically, a higher exhaust speed has an extra-large cumulative effect on the rocket body's final speed by counting again and again in the speed the body attains second after second during its burn.
65. (a) zero (b) $\dfrac{mv_i}{\sqrt{2}}$ upward
67. 260 N normal to the wall
69. (a) $1.33\hat{\mathbf{i}}$ m/s (b) $-235\hat{\mathbf{i}}$ N (c) 0.680 s (d) $-160\hat{\mathbf{i}}$ N · s and $+160\hat{\mathbf{i}}$ N · s (e) 1.81 m (f) 0.454 m (g) −427 J (h) +107 J (i) The change in kinetic energy of one member of the system, according to Equation 8.2, will be equal to the negative of the change in internal energy for that member: $\Delta K = -\Delta E_{\text{int}}$. The change in internal energy, in turn, is the product of the friction force and the distance through which the member moves. Equal friction forces act on the person and the cart, but the forces move through different distances, as we see in parts (e) and (f). Therefore, there are different changes in internal energy for the person and the cart and, in turn, different changes in kinetic energy. The total change in kinetic energy of the system, −320 J, becomes +320 J of extra internal energy in the entire system in this perfectly inelastic collision.
71. (a) Momentum of the bullet–block system is conserved in the collision, so you can relate the speed of the block and bullet immediately after the collision to the initial speed of the bullet. Then, you can use conservation of mechanical energy for the bullet–block–Earth system to relate the speed after the collision to the maximum height. (b) 521 m/s upward
73. $2v_i$ for the particle with mass m and 0 for the particle with mass $3m$.
75. (a) $\dfrac{m_1v_1 + m_2v_2}{m_1 + m_2}$ (b) $(v_1 - v_2)\sqrt{\dfrac{m_1m_2}{k(m_1 + m_2)}}$
(c) $v_{1f} = \dfrac{(m_1 - m_2)v_1 + 2m_2v_2}{m_1 + m_2}$, $v_{2f} = \dfrac{2m_1v_1 + (m_2 - m_1)v_2}{m_1 + m_2}$
77. m_1: 13.9 m m_2: 0.556 m
79. 0.960 m
81. 143 m/s
83. (a) 0; inelastic (b) $(-0.250\hat{\mathbf{i}} + 0.75\hat{\mathbf{j}} - 2.00\hat{\mathbf{k}})$ m/s; perfectly inelastic (c) either $a = -6.74$ with $\vec{\mathbf{v}} = -0.419\hat{\mathbf{k}}$ m/s or $a = 2.74$ with $\vec{\mathbf{v}} = -3.58\hat{\mathbf{k}}$ m/s
85. 0.403
87. (a) $-0.256\hat{\mathbf{i}}$ m/s and $0.128\hat{\mathbf{i}}$ m/s (b) $-0.064\,2\hat{\mathbf{i}}$ m/s and 0 (c) 0 and 0
89. (a) 100 m/s (b) 374 J

91. (a) 2.67 m/s (incident particle), 10.7 m/s (target particle) (b) −5.33 m/s (incident particle), 2.67 m/s (target particle) (c) 7.11×10^{-3} J in case (a) and 2.84×10^{-2} J in case (b). The incident particle loses more kinetic energy in case (a), in which the target mass is 1.00 g.
93. (a) particle of mass m: $\sqrt{2}v_i$; particle of mass $3m$: $\sqrt{\frac{2}{3}}v_i$ (b) 35.3°
95. (a) $v_{CM} = \sqrt{\frac{F}{2m}(x_1 + x_2)}$
(b) $\theta = \cos^{-1}\left[1 - \frac{F}{2mgL}(x_1 - x_2)\right]$

Chapter 10

Answers to Quick Quizzes

1. **(i)** (c) **(ii)** (b)
2. (b)
3. **(i)** (b) **(ii)** (a)
4. **(i)** (b) **(ii)** (a)
5. (b)
6. (a)
7. (b)

Answers to Odd-Numbered Problems

1. (a) 7.27×10^{-5} rad/s (b) Because of its angular speed, the Earth bulges at the equator.
3. (a) 5.00 rad, 10.0 rad/s, 4.00 rad/s^2 (b) 53.0 rad, 22.0 rad/s, 4.00 rad/s^2
5. (a) 4.00 rad/s^2 (b) 18.0 rad
7. (a) 5.24 s (b) 27.4 rad
9. (a) 8.21×10^2 rad/s^2 (b) 4.21×10^3 rad
11. 13.7 rad/s^2
13. 3.10 rad/s
15. (a) 0.180 rad/s (b) 8.10 m/s^2 radially inward
17. (a) 25.0 rad/s (b) 39.8 rad/s^2 (c) 0.628 s
19. (a) 8.00 rad/s (b) 8.00 m/s (c) 64.1 m/s^2 at an angle 3.58° from the radial line to point P (d) 9.00 rad
21. (a) 126 rad/s (b) 3.77 m/s (c) 1.26 km/s^2 (d) 20.1 m
23. 0.572
25. (a) 3.47 rad/s (b) 1.74 m/s (c) 2.78 s (d) 1.02 rotations
27. −3.55 N · m
29. 21.5 N
31. 177 N
33. (a) 24.0 N · m (b) 0.035 6 rad/s^2 (c) 1.07 m/s^2
35. (a) 21.6 kg · m^2 (b) 3.60 N · m (c) 52.5 rev
37. 0.312
39. (a) 5.80 kg · m^2 (b) Yes, knowing the height of the door is unnecessary.
41. 1.28 kg · m^2
43. $\frac{11}{12}mL^2$
45. (a) 143 kg · m^2 (b) 2.57 kJ
47. (a) 24.5 m/s (b) no (c) no (d) no (e) no (f) yes
49. 1.03×10^{-3} J
51. 149 rad/s
53. (a) 1.59 m/s (b) 53.1 rad/s
55. (a) 11.4 N (b) 7.57 m/s^2 (c) 9.53 m/s (d) 9.53 m/s
57. (a) $2(Rg/3)^{1/2}$ (b) $4(Rg/3)^{1/2}$ (c) $(Rg)^{1/2}$
59. (a) 500 J (b) 250 J (c) 750 J
61. (a) $\frac{2}{3}g\sin\theta$ (b) The acceleration of $\frac{1}{2}g\sin\theta$ for the hoop is smaller than that for the disk. (c) $\frac{1}{3}\tan\theta$
63. (a) The disk (b) disk: $\sqrt{\frac{4}{3}gh}$; hoop: $\sqrt{gh}$
65. (a) 1.21×10^{-4} kg · m^2 (b) Knowing the height of the can is unnecessary. (c) The mass is not uniformly distributed; the density of the metal can is larger than that of the soup.
67. (a) 4.00 J (b) 1.60 s (c) 0.80 m
69. (a) 12.5 rad/s (b) 128 rad
71. (a) 0.496 W (b) 413 W
73. (a) $(3g/L)^{1/2}$ (b) $3g/2L$ (c) $-\frac{3}{2}g\,\hat{\mathbf{i}} - \frac{3}{4}g\,\hat{\mathbf{j}}$ (d) $-\frac{3}{2}Mg\,\hat{\mathbf{i}} + \frac{1}{4}Mg\,\hat{\mathbf{j}}$
75. $\dfrac{g(h_2 - h_1)}{2\pi R^2}$
77. (a) Particle under a net force (b) Rigid object under a net torque (c) 118 N (d) 156 N (e) $\dfrac{r^2}{a}(T_2 - T_1)$ (f) 1.17 kg · m^2
79. $\omega = \sqrt{\dfrac{2mgd\sin\theta + kd^2}{I + mR^2}}$
81. $\sqrt{\dfrac{10}{7}\left[\dfrac{g(R - r)(1 - \cos\theta)}{r^2}\right]}$
83. (a) $2.70R$ (b) $F_x = -20mg/7$, $F_y = -mg$
85. (a) $\sqrt{\frac{3}{4}gh}$ (b) $\sqrt{\frac{3}{4}gh}$
87. (a) 0.800 m/s^2 (b) 0.400 m/s^2 (c) 0.600 N, 0.200 N forward
89. (a) $\sigma = 0.060\,2\ \text{s}^{-1}$, $\omega_0 = 3.50$ rad/s (b) $\alpha = -0.176$ rad/s^2 (c) 1.29 rev (d) 9.26 rev
91. (b) to the left
93. (a) 2.88 s (b) 12.8 s

Chapter 11

Answers to Quick Quizzes

1. (d)
2. **(i)** (a) **(ii)** (c)
3. (b)
4. (a)

Answers to Odd-Numbered Problems

1. $\hat{\mathbf{i}} + 8.00\,\hat{\mathbf{j}} + 22.0\,\hat{\mathbf{k}}$
3. (a) $7.00\,\hat{\mathbf{k}}$ (b) 60.3°
5. (a) 30 N · m (counterclockwise) (b) 36 N · m (counterclockwise)
7. 45.0°
9. (a) $F_3 = F_1 + F_2$ (b) no
11. $17.5\,\hat{\mathbf{k}}$ kg · m^2/s
13. $m(xv_y - yv_x)\,\hat{\mathbf{k}}$
15. (a) zero (b) $(-mv_i^3\sin^2\theta\cos\theta/2g)\,\hat{\mathbf{k}}$ (c) $(-2mv_i^3\sin^2\theta\cos\theta/g)\,\hat{\mathbf{k}}$ (d) The downward gravitational force exerts a torque on the projectile in the negative z direction.
17. $mvR[\cos(vt/R) + 1]\,\hat{\mathbf{k}}$
19. $60.0\hat{\mathbf{k}}$ kg · m^2/s
21. (a) $-m\ell gt\cos\theta\,\hat{\mathbf{k}}$ (b) The Earth exerts a gravitational torque on the ball. (c) $-mg\ell\cos\theta\,\hat{\mathbf{k}}$
23. 1.20 kg · m^2/s
25. (a) 0.360 kg · m^2/s (b) 0.540 kg · m^2/s
27. (a) 0.433 kg · m^2/s (b) 1.73 kg · m^2/s
29. (a) 1.57×10^8 kg · m^2/s (b) 6.26×10^3 s = 1.74 h
31. 7.14 rev/min

33. (a) The mechanical energy of the system is not constant. Some chemical energy is converted into mechanical energy. (b) The momentum of the system is not constant. The turntable bearing exerts an external northward force on the axle. (c) The angular momentum of the system is constant. (d) 0.360 rad/s counterclockwise (e) 99.9 J

35. (a) 11.1 rad/s counterclockwise (b) No; 507 J is trans formed into internal energy. (c) No; the turntable bear ing promptly imparts impulse 44.9 kg m/s north into the turntable–clay system and thereafter keeps changing the system momentum.

37. (a) down (b) /(

39. (a) (b) No; some mechanical energy of the system changes into internal energy. (c) The momentum of the system is not constant. The axle exerts a backward force on the cylinder when the clay strikes.

41. (a) yes (b) 4.50 kg /s (c) No. In the perfectly inelastic collision, kinetic energy is transformed to internal energy. (d) 0.749 rad/s (e) The total energy of the system *must* be the same before and after the collision, assuming we ignore the energy leaving by mechanical waves (sound) and heat (from the newly-warmer door to the cooler air). The kinetic energies are as follows: 2.50 J; 1.69 J. Most of the initial kinetic energy is transformed to internal energy in the collision.

43. 5.46

45. 0.910 km/s

47. 7.50

49. (a) 7 /3 (b) *mgd* (c) 3 counterclockwise (d) 2 /7 upward (e) *mgd* (f) (g) 14*gd* (h) *gd* 21

51. (a) isolated system (angular momentum) (b) /2 (c) 12 (d) 12 (e) *mv* (f) *mv* (g) (h)

53. (a) (b) ((c) *mv*

55. (a) 3 750 kg m /s (b) 1.88 kJ (c) 3 750 kg m /s (d) 10.0 m/s (e) 7.50 kJ (f) 5.62 kJ

57. (a) 2 (b) 2 /3 (c) 4 /3 (d) 4 (e) (f) 26 /27 (g) No horizontal forces act on the bola from outside after release, so the horizontal momentum stays constant. Its center of mass moves steadily with the horizontal velocity it had at release. No torques about its axis of rotation act on the bola, so the angular momen tum stays constant. Internal forces cannot affect momen tum conservation and angular momentum conservation, but they can affect mechanical energy.

59. an increase of 6.368 % or 0.550 s, which is not significant

61. (a) (b) (c) (d) 18

63. *ga*

Chapter 12

Answers to Quick Quizzes

(a)

2. (b)

3. (b)

4. **(i)** (b) **(ii)** (a) **(iii)** (c)

Answers to Odd-Numbered Problems

0, 0, cos sin 0.5 cos

3. (3.85 cm, 6.85 cm)

5. 0.750 m

(2.54 m, 4.75 m)

9. 177 kg

11. Sam exerts an upward force of 176 N, and Joe exerts an upward force of 274 N.

13. (a) 268 N, 1 300 N (b) 0.324

15. (a) 29.9 N (b) 22.2 N

17. (a) 1.04 kN at 60.0° upward and to the right (b) 370 910 N

19. (a) 27.7 kN (b) 11.5 kN (c) 4.19 kN

21. (a) 859 N (b) 1.04 kN at 36.9° to the left and upward

23. 2.81 m

25. 501 N, 672 N, 384 N

27. (a) 0.053 (b) 1.09 kg/m (c) With only a 5% change in volume in this extreme case, liquid water is indeed nearly incompressible in bio logical and student laboratory situations.

29. 23.8

31. (a) 3.14 N (b) 6.28

33. 4.90 mm

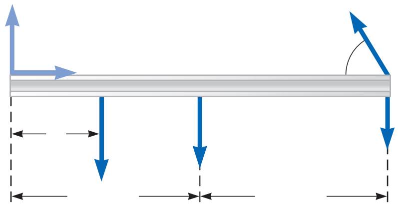
35. 0.029 2 mm

37. 5.98 N, 4.80

39. 0.896 m

41. 724 N, 716 N

43. (a)



(b) 343 N, 171 N to the right, 683 N up (c) 5.14 m

45. (a))/[sin (2)] (b)) cot /(2); /(2

47. 6.47 10 1.27 10 N, 6.47 10 N

49. (a) 5.08 kN (b) 4.77 kN (c) 8.26 kN

51. (a) sin θ − cos / cos θ − μ sin (b) + μ (c) + μ

53. (a) 9.28 kN (b) The moment arm of the force is no longer 70 cm from the shoulder joint but only 49.5 cm, therefore reducing to 6.56 kN.

55. (a) 66.7 N (b) increasing at 0.125 N/s

57. (a) 15 *mgd* (b) *mg* *mgd* (c) 15 *mgd* *mgd* (to the right and down ward on the right half of the ladder)

59. (a) 1.67 N, 3.33 N (b) 2.36 N

61. 5.73 rad/s
63. (a) 443 N (b) 221 N (to the right), 217 N (upward)
65. 9.00 ft
67. $3F_g/8$

Chapter 13

Answers to Quick Quizzes

1. (e)
2. (c)
3. (a)
4. (a) Perihelion (b) Aphelion (c) Perihelion (d) All points

Answers to Odd-Numbered Problems

1. 7.41×10^{-10} N
3. (a) 2.50×10^{-7} N toward the 500-kg object (b) between the objects and 2.45 m from the 500-kg object
5. 2.67×10^{-7} m/s^2
7. 2.97 nN
9. 2.00 kg and 3.00 kg
11. 0.614 m/s^2, toward Earth
13. (a) 7.61 cm/s^2 (b) 363 s (c) 3.08 km (d) 28.9 m/s at 72.9° below the horizontal
15. $\frac{GM}{\ell^2}(\frac{1}{2} + \sqrt{2})$ at 45° to the positive x axis
17. 1.50 h or 90.0 min
19. (a) 0.71 yr (b) The departure must be timed so that the spacecraft arrives at the aphelion when the target planet is there.
21. 1.26×10^{32} kg
23. 35.1 AU
25. 4.99 days
27. 8.92×10^{7} m
29. (a) yes (b) 3.93 yr
31. 2.82×10^{9} J
33. (a) 1.84×10^{9} kg/m^3 (b) 3.27×10^{6} m/s^2 (c) -2.08×10^{13} J
35. (a) -1.67×10^{-14} J (b) The particles collide at the center of the triangle.
37. 1.58×10^{10} J
39. (a) 4.69×10^{8} J (b) -4.69×10^{8} J (c) 9.38×10^{8} J
41. 1.78×10^{3} m
43. (a) 850 MJ (b) 2.71×10^{9} J
45. (a) 5.30×10^{3} s (b) 7.79 km/s (c) 6.43×10^{9} J
47. (a) same size force (b) 15.6 km/s
49. 2.52×10^{7} m
51. $\omega = 0.057\,2$ rad/s or 1 rev in 110 s
53. (a) 2.43 h (b) 6.59 km/s (c) 4.74 m/s^2 toward the Earth
55. 2.25×10^{-7}
57. (a) 1.00×10^{7} m (b) 1.00×10^{4} m/s
59. (a) 15.3 km (b) 1.66×10^{16} kg (c) 1.13×10^{4} s (d) No; its mass is so large compared with yours that you would have a negligible effect on its rotation.
61. (a) $v_1 = m_2\sqrt{\dfrac{2G}{d(m_1 + m_2)}}$, $v_2 = m_1\sqrt{\dfrac{2G}{d(m_1 + m_2)}}$, $v_{\text{rel}} = \sqrt{\dfrac{2G(m_1 + m_2)}{d}}$ (b) 1.07×10^{32} J and 2.67×10^{31} J
63. (a) -7.04×10^{4} J (b) -1.57×10^{5} J (c) 13.2 m/s
65. 7.79×10^{14} kg
67. (a) 2×10^{8} yr (b) ~ 10^{41} kg (c) 10^{11}
69. (a) 2.93×10^{4} m/s (b) $K = 2.74 \times 10^{33}$ J, $U = -5.39 \times 10^{33}$ J (c) $K = 2.56 \times 10^{33}$ J, $U = -5.21 \times 10^{33}$ J (d) Yes; $E = -2.65 \times 10^{33}$ J at both aphelion and perihelion.
71. 119 km
73. $\sqrt{\dfrac{GM}{4R_E}}$
75. $(800 + 1.73 \times 10^{-4})\hat{\mathbf{i}}$ m/s and $(800 - 1.73 \times 10^{-4})\hat{\mathbf{i}}$ m/s
77. 18.2 ms
79. (a) -3.67×10^{7} J (b) 9.24×10^{10} kg · m^2/s (c) $v = 5.58$ km/s, $r = 1.04 \times 10^{7}$ m (d) 8.69×10^{6} m (e) 134 min

Chapter 14

Answers to Quick Quizzes

1. (a)
2. (a)
3. (c)
4. (b) or (c)
5. (a)

Answers to Odd-Numbered Problems

1. 2.96×10^{6} Pa
3. (a) 6.24 MPa (b) Yes; this pressure could puncture the vinyl flooring.
5. 24.8 kg
7. 8.46 m
9. 7.74×10^{-3} m^2
11. (a) 3.71×10^{5} Pa (b) 3.57×10^{4} N
13. 2.71×10^{5} N
15. (a) 2.94×10^{4} N (b) 1.63×10^{4} N · m
17. 2.31 lb
19. 98.6 kPa
21. (a) 10.5 m (b) No. The vacuum is not as good because some alcohol and water will evaporate. The equilibrium vapor pressures of alcohol and water are higher than the vapor pressure of mercury.
23. (a) 116 kPa (b) 52.0 Pa
25. 0.258 N down
27. (a) 4.9 N down, 16.7 N up (b) 86.2 N (c) By either method of evaluation, the buoyant force is 11.8 N up.
29. (a) 7.00 cm (b) 2.80 kg
31. (a) 1 250 kg/m^3 (b) 500 kg/m^3
33. (a) 408 kg/m^3 (b) When m is less than 0.310 kg, the wooden block will be only partially submerged in the water. (c) When m is greater than 0.310 kg, the wooden block and steel object will sink.
35. (a) 3.82×10^{3} N (b) 1.04×10^{3} N; the balloon rises because the net force is positive: the upward buoyant force is greater than the downward gravitational force. (c) 106 kg
37. (a) 11.6 cm (b) 0.963 g/cm^3 (c) No; the density ρ is not linear in h.
39. 1.52×10^{3} m^3
41. (a) 17.7 m/s (b) 1.73 mm
43. 0.247 cm
45. (a) 2.28 N toward Holland (b) 1.74×10^{6} s
47. (a) 15.1 MPa (b) 2.95 m/s

49. (a) 1.91 m/s (b) 8.65×10^{-4} m^3/s
51. 347 m/s
53. (a) 4.43 m/s (b) 10.1 m
55. 12.6 m/s
57. (a) 1.02×10^7 Pa (b) 6.61×10^5 N
59. (a) 6.70 cm (b) 5.74 cm
61. 2.25 m
63. 455 kPa
65. 0.556 m
67. 160 kg/m^3
69. (a) 8.01 km (b) yes
71. upper scale: 17.3 N; lower scale: 31.7 N
73. 91.64%
75. 27 N · m
77. 758 Pa
79. 4.43 m/s
81. (a) 1.25 cm (b) 14.3 m/s
85. (a) 18.3 mm (b) 14.3 mm (c) 8.56 mm

Chapter 15

Answers to Quick Quizzes

1. (d)
2. (f)
3. (a)
4. (b)
5. (c)
6. **(i)** (a) **(ii)** (a)

Answers to Odd-Numbered Problems

1. (a) 17 N to the left (b) 28 m/s^2 to the left
3. 0.63 s
5. (a) 1.50 Hz (b) 0.667 s (c) 4.00 m (d) π rad (e) 2.83 m
7. 0.628 m/s
9. 40.9 N/m
11. 12.0 Hz
13. (a) −2.34 m (b) −1.30 m/s (c) −0.076 3 m (d) 0.315 m/s
15. (a) $x = 2.00 \cos(3.00\pi t - 90°)$ or $x = 2.00 \sin(3.00\pi t)$ where x is in centimeters and t is in seconds (b) 18.8 cm/s (c) 0.333 s (d) 178 cm/s^2 (e) 0.500 s (f) 12.0 cm
17. (a) 20 cm (b) 94.2 cm/s as the particle passes through equilibrium (c) ±17.8 m/s^2 at maximum excursion from equilibrium
19. (a) 40.0 cm/s (b) 160 cm/s^2 (c) 32.0 cm/s (d) −96.0 cm/s^2 (e) 0.232 s
21. 2.23 m/s
23. (a) 0.542 kg (b) 1.81 s (c) 1.20 m/s^2
25. 2.60 cm and −2.60 cm
27. (a) 28.0 mJ (b) 1.02 m/s (c) 12.2 mJ (d) 15.8 mJ
29. (a) $\frac{8}{9}E$ (b) $\frac{1}{9}E$ (c) $x = \pm\sqrt{\frac{2}{3}}A$
(d) No; the maximum potential energy is equal to the total energy of the system. Because the total energy must remain constant, the kinetic energy can never be greater than the maximum potential energy.
31. (a) 4.58 N (b) 0.125 J (c) 18.3 m/s^2 (d) 1.00 m/s (e) smaller (f) the coefficient of kinetic friction between the block and surface (g) 0.934
33. (b) 0.628 s
35. (a) 1.50 s (b) 0.559 m
37. 0.944 kg · m^2
39. 1.42 s, 0.499 m
41. (a) 0.820 m/s (b) 2.57 rad/s^2 (c) 0.641 N (d) $v_{max} = 0.817$ m/s, $\alpha_{max} = 2.54$ rad/s^2, $F_{max} = 0.634$ N (e) The answers are close but not exactly the same. The answers computed from conservation of energy and from Newton's second law are more precise.
43. (a) 3.65 s (b) 6.41 s (c) 4.24 s
45. (a) 5.00×10^{-7} kg · m^2 (b) 3.16×10^{-4} N · m/rad
47. (a) 7.00 Hz (b) 2.00% (c) 10.6 s
51. 11.0 cm
53. (a) 3.16 s^{-1} (b) 6.28 s^{-1} (c) 5.09 cm
55. 0.641 Hz or 1.31 Hz
57. (a) 2.09 s (b) 0.477 Hz (c) 36.0 cm/s (d) $E = 0.064\,8m$, where E is in joules and m is in kilograms (e) $k = 9.00m$, where k is in newtons/meter and m is in kilograms (f) Period, frequency, and maximum speed are all independent of mass in this situation. The energy and the force constant are directly proportional to mass.
59. (a) $2Mg$ (b) $Mg\left(1 + \frac{y}{L}\right)$ (c) $\frac{4\pi}{3}\sqrt{\frac{2L}{g}}$ (d) 2.68 s
61. 1.56×10^{-2} m
63. (a) $L_{Earth} = 25$ cm (b) $L_{Mars} = 9.4$ cm (c) $m_{Earth} = 0.25$ kg (d) $m_{Mars} = 0.25$ kg
65. 6.62 cm
67. $\frac{1}{2\pi L}\sqrt{gL + \frac{kh^2}{M}}$
69. 7.75 s^{-1}
71. (a) 1.26 m (b) 1.58 (c) The energy decreases by 120 J. (d) Mechanical energy is transformed into internal energy in the perfectly inelastic collision.
73. (a) $\omega = \sqrt{\frac{200}{0.400 + M}}$, where ω is in s^{-1} and M is in kilograms (b) 22.4 s^{-1} (c) 22.4 s^{-1}
75. (a) 3.00 s (b) 14.3 J (c) $\theta = 25.5°$
77. (b) 1.46 s
79. (a) $x = 2\cos\left(10t + \frac{\pi}{2}\right)$ (b) ± 1.73 m (c) 0.105 s = 105 ms (d) 0.098 0 m
81. (b) $T = \frac{2}{r}\sqrt{\frac{\pi M}{\rho g}}$
83. 9.12×10^{-5} s
85. (a) 0.500 m/s (b) 8.56 cm
87. (a) $\frac{1}{2}(M + \frac{1}{3}m)v^2$ (b) $2\pi\sqrt{\frac{M + \frac{1}{3}m}{k}}$
89. (a) $\frac{2\pi}{\sqrt{g}}\sqrt{L_i + \frac{1}{2\rho a^2}\left(\frac{dM}{dt}\right)t}$ (b) $2\pi\sqrt{\frac{L_i}{g}}$

Chapter 16

Answers to Quick Quizzes

1. **(i)** (b) **(ii)** (a)
2. **(i)** (c) **(ii)** (b) **(iii)** (d)
3. (c)
4. (f) and (h)
5. (d)

Answers to Odd-Numbered Problems

1. 184 km

3. $y = \dfrac{6.00}{(x - 4.50t)^2 + 3.00}$ where x and y are in meters and t is in seconds

5. (a) 2.00 cm (b) 2.98 m (c) 0.576 Hz (d) 1.72 m/s

7. 0.319 m

9. (a) $3.33\hat{\mathbf{i}}$ m/s (b) −5.48 cm (c) 0.667 m (d) 5.00 Hz (e) 11.0 m/s

11. (a) 31.4 rad/s (b) 1.57 rad/m (c) $y = 0.120 \sin (1.57x - 31.4t)$, where x and y are in meters and t is in seconds (d) 3.77 m/s (e) 118 m/s^2

13. (a) 0.500 Hz (b) 3.14 rad/s (c) 3.14 rad/m (d) $0.100 \sin (\pi x - \pi t)$ (e) $0.100 \sin (-\pi t)$ (f) $0.100 \sin (4.71 - \pi t)$ (g) 0.314 m/s

15. (a) −1.51 m/s (b) 0 (c) 16.0 m (d) 0.500 s (e) 32.0 m/s

17. (a) 0.250 m (b) 40.0 rad/s (c) 0.300 rad/m (d) 20.9 m (e) 133 m/s (f) positive x direction

19. (a) $y = 0.080\,0 \sin (2.5\pi x + 6\pi t)$ (b) $y = 0.080\,0 \sin (2.5\pi x + 6\pi t - 0.25\pi)$

21. 185 m/s

23. 13.5 N

25. 80.0 N

27. 0.329 s

29. (a) 0.051 0 kg/m (b) 19.6 m/s

31. 631 N

33. (a) 1 (b) 1 (c) 1 (d) increased by a factor of 4

35. (a) 62.5 m/s (b) 7.85 m (c) 7.96 Hz (d) 21.1 W

37. (a) $y = 0.075 \sin (4.19x - 314t)$, where x and y are in meters and t is in seconds (b) 625 W

39. (a) 15.1 W (b) 3.02 J

45. 0.456 m/s

47. 14.7 kg

49. (a) 39.2 N (b) 0.892 m (c) 83.6 m/s

51. (a) 21.0 ms (b) 1.68 m

53. $\sqrt{\dfrac{mL}{Mg \sin \theta}}$

55. 0.084 3 rad

57. $\dfrac{1}{\omega}\sqrt{\dfrac{m}{M}}$

59. (a) $v = \sqrt{\dfrac{T}{\rho(1.00 \times 10^{-5}\,x + 1.00 \times 10^{-6})}}$, where v is in meters per second, T is in newtons, ρ is in kilograms per meter cubed, and x is in meters (b) $v(0) = 94.3$ m/s, $v(10.0\text{ m}) = 9.38$ m/s

61. (a) $\dfrac{\mu\omega^3}{2k} A_0^2 e^{-2bx}$ (b) $\dfrac{\mu\omega^3}{2k} A_0^2$ (c) e^{-2bx}

63. 3.86×10^{-4}

65. (a) $(0.707)(2\sqrt{L/g})$ (b) $L/4$

67. (a) μv_0^2 (b) v_0 (c) clockwise: 4π; counterclockwise: 0

Chapter 17

Answers to Quick Quizzes

1. (c)

2. (b)

3. (b)

4. (e)

5. (e)

6. (b)

Answers to Odd-Numbered Problems

1. (a) 2.00 μm (b) 40.0 cm (c) 54.6 m/s (d) −0.433 μm (e) 1.72 mm/s

3. $\Delta P = 0.200 \sin (20\pi x - 6\,860\pi t)$ where ΔP is in pascals, x is in meters, and t is in seconds

5. 0.103 Pa

7. 0.196 s

9. (a) 0.625 mm (b) 1.50 mm to 75.0 μm

11. (a) 5.56 km (b) No. The speed of light is much greater than the speed of sound, so the time interval required for the light to reach you is negligible compared to the time interval for the sound.

13. 7.82 m

15. (a) 27.2 s (b) 25.7 s; the time interval in part (a) is longer.

17. (a) the pulse that travels through the rail (b) 23.4 ms

19. 66.0 dB

21. (a) 3.75 W/m^2 (b) 0.600 W/m^2

23. 3.0×10^{-8} W/m^2

25. (a) 0.691 m (b) 691 km

27. (a) 1.3×10^2 W (b) 96 dB

29. (a) 2.34 m (b) 0.390 m (c) 0.161 Pa (d) 0.161 Pa (e) 4.25×10^{-7} m (f) 7.09×10^{-8} m

31. (a) 1.32×10^{-4} W/m^2 (b) 81.2 dB

33. 68.3 dB

35. (a) 30.0 m (b) 9.49×10^5 m

37. (a) 475 Hz (b) 430 Hz

39. (a) 3.04 kHz (b) 2.08 kHz (c) 2.62 kHz; 2.40 kHz

41. (a) 441 Hz (b) 439 Hz (c) 54.0 dB

43. (a) 0.021 7 m/s (b) 28.9 Hz (c) 57.8 Hz

45. 26.4 m/s

47. (a) 56.3 s (b) 56.6 km farther along

49. 0.883 cm

51. (a) 0.515 trucks per minute (b) 0.614 trucks per minute

53. 67.0 dB

55. (a) 4.16 m (b) 0.455 μs (c) 0.157 mm

57. It is unreasonable, implying a sound level of 123 dB. Nearly all the decrease in mechanical energy becomes internal energy in the latch.

59. (a) 5.04×10^3 m/s (b) 1.59×10^{-4} s (c) 1.90×10^{-3} m (d) 2.38×10^{-3} (e) 4.76×10^8 N/m^2 (f) $\dfrac{\sigma_y}{\sqrt{\rho Y}}$

61. (a) 55.8 m/s (b) 2 500 Hz

63. (a) 3.29 m/s (b) The bat will be able to catch the insect because the bat is traveling at a higher speed in the same direction as the insect.

65. (a) 0.343 m (b) 0.303 m (c) 0.383 m (d) 1.03 kHz

67. (a) 0.983° (b) 4.40°

69. 1.34×10^4 N

71. (a) 531 Hz (b) 466 Hz to 539 Hz (c) 568 Hz

Chapter 18

Answers to Quick Quizzes

1. (c)

2. **(i)** (a) **(ii)** (d)

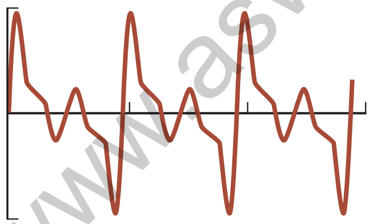
3. (d)

4. (b)

5. (c)

Answers to Odd-Numbered Problems

5.66 cm
3. (a) 1.65 cm (b) 6.02 cm (c) 1.15 cm
5. 91.3°
(a) : positive direction; : negative direction
(b) 0.750 s (c) 1.00 m
9. (a) 9.24 m (b) 600 Hz
11. (a) 156° (b) 0.058 4 cm
13. (c) Yes; the limiting form of the path is two straight lines through the origin with slope 0.75.
15. (a) 15.7 m (b) 31.8 Hz (c) 500 m/s
17. (a) 4.24 cm (b) 6.00 cm (c) 6.00 cm (d) 0.500 cm, 1.50 cm, 2.50 cm
19. at 0.089 1 m, 0.303 m, 0.518 m, 0.732 m, 0.947 m, and 1.16 m from one speaker
21. 19.6 Hz
23. (a) 163 N (b) 660 Hz
25. (a) second harmonic (b) 74.0 cm (c) 3
27. (a) 350 Hz (b) 400 kg
29. 1.86 g
31. (a) 3.8 cm (b) 3.85%
33. (a) three loops (b) 16.7 Hz (c) one loop
35. (a) 3.66 m/s (b) 0.200 Hz
37. 57.9 Hz
39. (a) 0.357 m (b) 0.715 m
41. (a) 0.656 m (b) 1.64 m
43. (a) 349 m/s (b) 1.14 m
45. (a) 0.195 m (b) 841 Hz
47. (0.252 m) with 1, 2, 3, . . .
49. 158 s
51. (a) 50.0 Hz (b) 1.72 m
53. (a) 21.5 m (b) seven
55. (a) 1.59 kHz (b) odd-numbered harmonics (c) 1.11 kHz
57. 5.64 beats/s
59. (a) 1.99 beats/s (b) 3.38 m/s
61. The following coefficients are approximate: 100, 156, 62, 104, 52, 29, 25.



63. 31.1 N
65. 800 m
67. 1.27 cm
69. 262 kHz
71. (a) 45.0 or 55.0 Hz (b) 162 or 242 N
73. (a) 0.078 2 — (b) 3 (c) 0.078 2 m
(d) The sphere floats on the water.
75. (a) 34.8 m/s (b) 0.986 m
77. 3.85 m/s away from the station or 3.77 m/s toward the station
79. 283 Hz
81. 407 cycles
83. (b) 11.2 m, 63.4°
85. (a) 78.9 N (b) 211 Hz
87. $\sqrt{15}Mg$

Chapter 19

Answers to Quick Quizzes

(c)
2. (c)
3. (c)
4. (c)
5. (a)
6. (b)

Answers to Odd-Numbered Problems

(a) 106.7°F (b) Yes; the normal body temperature is 98.6°F, so the patient has a high fever and needs immediate attention.
3. (a) 109°F, 195 K (b) 98.6°F, 310 K
5. (a) 320°F (b) 77.3 K
(a) 270°C (b) 1.27 atm, 1.74 atm
9. (a) 0.176 mm (b) 8.78 m (c) 0.093 0 cm
11. 3.27 cm
13. 1.54 km. The pipeline can be supported on rollers. -shaped loops can be built between straight sections. They bend as the steel changes length.
15. (a) 0.109 cm (b) increase
17. (a) 437°C (b) 2.1 °C (c) No; aluminum melts at 660°C (Table 20.2). Also, although it is not in Table 20.2, Internet research shows that brass (an alloy of copper and zinc) melts at about 900°C.
19. (a) 99.8 mL (b) It lies below the mark. The acetone has reduced in volume, and the flask has increased in volume.
21. (a) 99.4 mL (b) 2.01 L (c) 0.998 cm
23. (a) 11.2 kg/m (b) 20.0 kg
25. 1.02 gallons
27. 4.28 atm
29. (a) 2.99 mol (b) 1.80 molecules
31. 1.50 molecules
33. (a) 41.6 mol (b) 1.20 kg (c) This value is in agreement with the tabulated density.
35. 3.55 L
37. (a) 3.95 atm 400 kPa (b) 4.43 atm 449 kPa
39. 473 K
41. 3.68 cm
43. 1.89 MPa
45. 6.57
47. (a) 2.542 cm (b) 300°C
49. 1.12 atm
51. 3.37 cm
53. 0.094 2 Hz
55. (a) 94.97 cm (b) 95.03 cm
57. (b) As the temperature increases, the density decreases (assuming is positive). (c) 5 (°C)
(d) 2.5 (°C)
59. (a) 9.5 s (b) It loses 57.5 s.
61. (b) It assumes is much less than 1.
63. (a) yes, as long as the coefficients of expansion remain constant (b) The lengths and at 0°C need to satisfy 17 . Then the steel rod must be longer. With 5.00 cm, the only possibility is 14.2 cm and 9.17 cm.

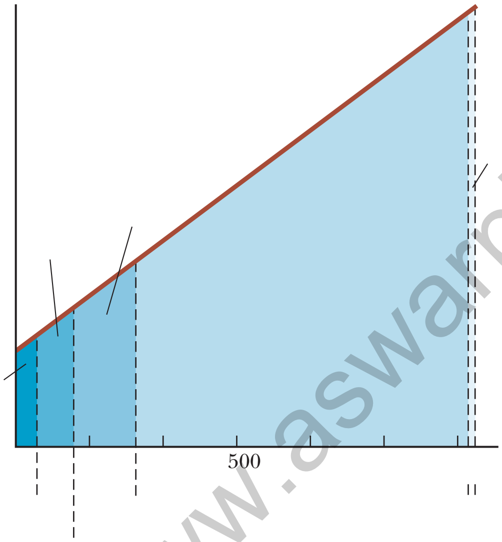
65. (a) 0.34% (b) 0.48% (c) All the moments of inertia have the same mathematical form: the product of a con stant, the mass, and a length squared.
67. 2.74 m
69. (a) $\frac{gP}{+ \rho\ gh}$ (b) decrease (c) —— 10.3 m
73. (a) 6.17 kg/m (b) 632 N (c) 580 N (d) 192 Hz
75. No; steel would need to be 2.30 times stronger.
77. (a) (b) (2.00)% (c) 59.4% (d) With this approach, 102 mL of turpentine spills, 2.01 L remains in the cylinder at 80.0°C, and the turpentine level at 20.0°C is 0.969 cm below the cylinder's rim.
79. 4.54 m

Chapter 20

Answers to Quick Quizzes

(i) iron, glass, water **(ii)** water, glass, iron

2. The figure below shows a graphical representation of the internal energy of the system as a function of energy added. Notice that this graph looks quite different from Figure 20.3 in that it doesn't have the flat portions dur ing the phase changes. Regardless of how the tempera ture is varying in Figure 20.3, the internal energy of the system simply increases linearly with energy input; the line in the graph below has a slope of 1.



3.

Situation	System	*Q*	*W*	int
(a) Rapidly pumping up a bicycle tire	Air in the pump	0	+	+
(b) Pan of room-temperature water sitting on a hot stove	Water in the pan			
(c) Air quickly leaking out of a balloon	Air originally in the balloon		–	–

4. Path A is isovolumetric, path B is adiabatic, path C is iso thermal, and path D is isobaric.
5. (b)

Answers to Odd-Numbered Problems

(a) 2.26 J (b) 2.80 steps (c) 6.99 steps
3. 23.6°C
5. 0.845 kg
1.78
9. 88.2 W
11. 29.6°C
13. (a) 1 822 J/kg °C (b) We cannot make a definite iden tification. It might be beryllium. (c) The material might be an unknown alloy or a material not listed in the table.
15. (a) 380 K (b) 2.04 atm
17. 2.27 km
19. 16.3°C
21. (a) 10.0 g of ice melts, 40.4°C
(b) 8.04 g of ice melts, 0°C
23. (a) 0°C (b) 114 g
25. 466 J
27. (a) (b) According to *nRV* , it is pro portional to the square of the volume.
29. 1.18 MJ
31.

Process	int

33. 720 J
35. (a) 0.041 0 m (b) 5.48 kJ (c) 5.48 kJ
37. (a) 7.50 kJ (b) 900 K
39. (a) 0.048 6 J (b) 16.2 kJ (c) 16.2 kJ
41. (a) 9.08 kJ (b) 9.08 kJ
43. (a) 6.45 W (b) 5.57
45. 74.8 kJ
47. 3.49
49. (a) 1.19 (b) a factor of 1.19
51. 8.99 cm
53. (a) 1.85 ft °F h/Btu (b) a factor of 1.78
55. 51.2°C
57. (a) W (b) K/s
59. (a) 6.08 J (b) 4.56
61. (a) 17.2 L (b) 0.351 L/s
63. 1.90 J/kg
65. (a) 9.31 J (b) 8.47 J (c) 8.38
67. (a) 13.0°C (b) 0.532°C/s
69. (a) 2 000 W (b) 4.46°C
71. 2.35 kg
73. (5.87)°C
75. (a) 3.16 W (b) 3.17
(c) It is 0.408% larger. (d) 5.78
77. 3.76 m/s
79. 1.44 kg
81. (a) 4.19 mm/s (b) 12.6 mm/s
83. 3.66 10.2 h

Chapter 21

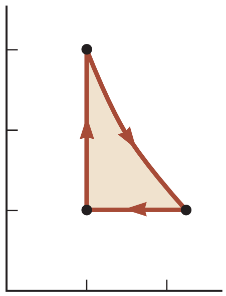
Answers to Quick Quizzes

(i) (b) **(ii)** (a)
2. **(i)** (a) **(ii)** (c)
3. (d)
4. (c)

Answers to Odd-Numbered Problems

(a) 3.54 atoms (b) 6.07 ^{21}J (c) 1.35 km/s
3. (a) 0.943 N (b) 1.57 Pa
5. 3.32 mol

5.05 [21]
9. (a) 4.00 u 6.64 [27] kg (b) 55.9 u 9.28 kg
(c) 207 u 3.44
11. (a) 2.28 kJ (b) 6.21 [−21]
13. 17.4 kPa
15. 13.5
17. (a) 3.46 kJ (b) 2.45 kJ (c) 1.01 kJ
19. 74.8 J
21. (a) 5.66 J (b) 1.12 kg
23. 2.32 [21]
25. (a) 41.6 J/K (b) 58.2 J/K (c) 58.2 J/K, 74.8 J/K
27. (a) a factor of 0.118 (b) a factor of 2.35 (c) 0 (d) 135 J
(e) 135 J
29. 227 K
31. 25.0 kW
33. (a)



(b) 8.77 L (c) 900 K (d) 300 K (e) 336 J
35. 132 m/s
37. (a) 2.00 [163] 0 atoms (b) 2.70 atoms
39. (a) 2.37 K (b) 1.06
41. (b) 0.278
43. (b) 8.31 km
45. (a) 1.69 h (b) 1.00
47. (a) 367 K (b) The rms speed of nitrogen would be higher because the molar mass of nitrogen is less than that of oxygen. (c) 572 m/s
49. 5.74 Pa 56.6 atm
51. **(i)** (a) 100 kPa (b) 66.5 L (c) 400 K (d) 5.82 kJ (e) 7.48 kJ (f) 1.66 kJ; **(ii)** (a) 133 kPa (b) 49.9 L (c) 400 K (d) 5.82 kJ (e) 5.82 kJ (f) 0; **(iii)** (a) 120 kPa (b) 41.6 L (c) 300 K (d) 0 (e) 909 J (f) 909 J; **(iv)** (a) 120 kPa (b) 43.3 L (c) 312 K (d) 722 J (e) 0 (f) 722 J
53. 0.623
55. (a) 0.514 m (b) 2.06 m (c) 2.38 10 K (d) 480 kJ
(e) 2.28 MJ
57. (a) 3.65 (b) 3.99 (c) 3.00 (d) $\frac{106}{\quad}$ (e) 7.98
59. (a) 300 K (b) 1.00 atm
61. (a) $_{rms}$ (18 $^{1/2}$ (4.81 $^{3/2}$, where $_{rms}$ is in meters per second and is in meters (b) (2.08 10 $^{5/2}$, where is in seconds and is in meters (c) 0.926 mm/s and 3.24 ms (d) 1.32 m/s and 3.88
63. 0.480°C
65. (a) 0.203 mol (b) 900 K (c) 900 K (d) 15.0 L (e) : Lock the piston in place and put the cylinder into an oven at 900 K. : Keep the gas in the oven while gradually letting the gas expand to lift the piston as far as it can. : Move the cylinder from the oven back to the 300-K room and let the gas cool and contract.

(f, g)

		,	int
	1.52		1.52
	1.67	1.67	
	2.53	1.01	1.52
ABCA	0.656	0.656	

67. (a) 1.09 (b) 2.69 (c) 0.529 (d) 1.00
(e) 0.199 (f) 1.01 (g) 1.25 [1 082]
71. (a) 3.34 molecules (b) during the 27th day
(c) 2.53
73. (a) 0.510 m/s (b) 20 ms
75. 510 K and 290 K

Chapter 22

Answers to Quick Quizzes

(i) (c) **(ii)** (b)
2. (d)
3. C, B, A
4. (a) one (b) six
5. (a)
6. false (The adiabatic process must be *reversible* for the entropy change to be equal to zero.)

Answers to Odd-Numbered Problems

(a) 10.7 kJ (b) 0.533 s
3. (a) 6.94% (b) 335 J
5. (a) 0.294 (or 29.4%) (b) 500 J (c) 1.67 kW
55.4%
9. (a) 75.0 kJ (b) 7.33
11. 77.8 W
13. (a) 4.51 J (b) 2.84 J (c) 68.1 kg
15. (a) 67.2% (b) 58.8 kW
17. (a) 8.70 J (b) 3.30
19. 9.00
21. 11.8
23. 1.86
25. (a) 564°C (b) No; a real engine will always have an effi ciency *less* than the Carnot efficiency because it operates in an irreversible manner.
27. (a) 741 J (b) 459 J
29. (a) 9.10 kW (b) 11.9 kJ
31. (a) 564 K (b) 212 kW (c) 47.5%
33. (a) — 1.40 $\frac{0.5 \quad 383}{383}$ where is in megawatts and is in kelvins (b) The exhaust power decreases as the firebox temperature increases. (c) 1.87 MW (d) 3.84 K (e) No answer exists. The energy exhaust cannot be that small.
35. 1.17
37. (a) 244 kPa (b) 192 J
39. (a)

Macrostate	Microstates	Number of ways to draw
All R	RRR	
2 R, 1 G	GRR, RGR, RRG	
1 R, 2 G	GGR, GRG, RGG	
All G	GGG	

(b)

Macrostate	Microstates	Number of ways to draw
All R	RRRR	
4R, 1G	GRRRR, RGRRR, RRGRR, RRRGR, RRRRG	
3R, 2G	GGRRR, GRGRR, GRRGR, GRRRG, RGGRR, RGRGR, RGRRG, RRGGR, RRGRG, RRRGG	10
2R, 3G	RRGGG, RGRGG, RGGRG, RGGGR, GRRGG, GRGRG, GRGGR, GGRRG, GGRGR, GGGRR	10
1R, 4G	RGGGG, GRGGG, GGRGG, GGGRG, GGGGR	
All G		

41. (a) one (b) six
43. 143 J/K
45. 1.02 kJ/K
47. 57.2 J/K
49. 0.507 J/K
51. 195 J/K
53. (a) 3.45 J/K (b) 8.06 J/K (c) 4.62 J/K
55. 3.28 J/K
57. 32.9 kJ
59. (a) – (b) –
61. 0.440 44.0%
63. (a) 5.00 kW (b) 763 W
65. (a) 0.390 (b) 0.545
67. (a) $3nRT$ (b) $3nRT$ ln 2 (c) nRT (d) nRT ln 2 (e) $3nRT$ (1 ln 2) (f) 2 nRT ln 2 (g) 0.273
69. (a) 39.4 J (b) 65.4 rad/s 625 rev/min (c) 293 rad/s 2.79 rev/min
71. 5.97 kg/s
73. (a) 4.10 J (b) 1.42 J (c) 1.01 J (d) 28.8% (e) Because 80.0%, the efficiency of the cycle is much lower than that of a Carnot engine operating between the same temperature extremes.
75. (a) 0.476 J/K (b) 417 J
77. ln 3
79. (b) yes (c) No; the second law refers to an engine operating in a cycle, whereas this problem involves only a single process.
81. (a) 25.0 atm, 1.97 4.13 atm, 1.19 10 m 1.00 atm, 3.28 10 m 6.05 atm, 5.43 (b) 2.99

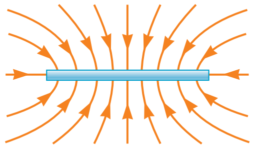
Chapter 23

Answers to Quick Quizzes

(a), (c), (e)
2. (e)
3. (b)
4. (a)
5.

Answers to Odd-Numbered Problems

(a) 1.60 C, 1.67 ^{27} kg
(b) 1.60 C, 3.82 kg
(c) 1.60 C, 5.89 kg
(d) 3.20 C, 6.65 kg
(e) 4.80 C, 2.33 kg
(f) 6.40 C, 2.33 kg
(g) 1.12 C, 2.33 kg
(h) 1.60 C, 2.99
3. 57.5 N
5. 3.60 N downward
2.25 N/m
9. (a) 8.74 N (b) repulsive
11. (a) 1.38 N (b) 77.5° below the negative axis
13. (a) 0.951 m (b) yes, if the third bead has positive charge
15. 0.872 N at 330°
17. (a) 8.24 N (b) 2.19 m/s
19.
21. (a) 2.16 N toward the other
(b) 8.99 N away from the other
23. (a) 5.58 10 ^{11} N (b) 1.02 10 N
25. (a) 3.06 5.06 (b) 3.06 5.06
27. (a) [(
(b) [(
29. 1.82 m to the left of the 2.50- C charge
31. (a) 1.80 N/C to the right
(b) 8.98 N to the left
33. 5.25
35. (a) 0.599 2.70 kN (b) 3.00 13.5
37. (a) 1.59 N/C (b) toward the rod
39. (a) 6.64 N/C away from the center of the ring
(b) 2.41 N/C away from the center of the ring
(c) 6.39 N/C away from the center of the ring
(d) 6.64 N/C away from the center of the ring
41. (a) 9.35 N/C (b) 1.04 N/C (about 11% higher)
(c) 5.15 N/C (d) 5.19 N/C (about 0.7% higher)
43. (a) (b) to the left
45. (a) 2.16 N/C (b) to the left
47.



49. (a) – (b) is negative, and is positive.
51. (a) 6.13 m/s (b) 1.96 s (c) 11.7 m
(d) 1.20
53. 4.38 m/s for the electron; 2.39 m/s for the proton
55. (a) $\frac{}{ed}$ (b) in the direction of the velocity of the electron
57. (a) 111 ns (b) 5.68 mm (c) 450 102 km/s
59.

61. (a) $\frac{mg}{}$ sin (b) 3.19 N/C down the incline

63. ——

65. (a) 2.18 m (b) 2.43 cm
67. (a) 1.09 C (b) 5.44
69. (a) 24.2 N (b) 4.21 8.42
71. 0.706
73. 25.9 cm
75. 1.67
77. 1.98
79. 1.14 C on one sphere and 5.69 C on the other
81. (a)
83. (a) 0.307 s (b) Yes; the downward gravitational force is not negligible in this situation, so the tension in the string depends on both the gravitational force and the electric force.

85. (a) 1.90 — (b) 3.29 —
(c) away from the origin
89. 1.36 1.96 kN
91. (a) $\frac{935}{0.062\,5}$ where is in newtons per coulomb and is in meters (b) 4.00 kN
(c) 0.016 8 m and 0.916 m
(d) nowhere is the field as large as 16 000 N/C

Chapter 24

Answers to Quick Quizzes

(e)
2. (b) and (d)
3. (a)

Answers to Odd-Numbered Problems

(a) 1.98 /C (b) 0
3. 4.14 MN/C
5. (a) 858 N /C (b) 0 (c) 657 N
28.2 N
9. (a) 6.89 MN /C (b) less than
11. for ; 0 for for ; 0 for
13. 1.77 C/m ; positive
15. (a) 339 N m /C (b) No. The electric field is not uniform on this surface. Gauss's law is only practical to use when all portions of the surface satisfy one or more of the conditions listed in Section 24.3.

17. (a) 0 (b) —

19. 18.8 kN

21. (a) — (b) —

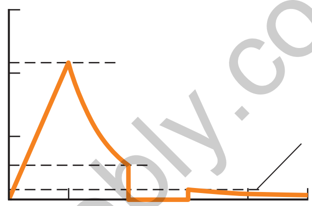
23. 3.50 kN
25. 2.48 C/m
27. 508 kN/C up
29. (a) 0 (b) 7.19 MN/C away from the center
31. (a) 51.4 kN/C outward (b) 645 N
33. = ρ away from the axis
35. (a) 0 (b) 3.65 N/C (c) 1.46 N/C
(d) 6.49 N/C

37. (a) 0 (b) 5.39 N/C outward (c) 539 N/C outward
39. —

41. glass
43. 2.00 N
45. (a) (b) (c) — radially outward

47. (a) 0 (b) 7.99 N/C (outward)
(c) 0 (d) 7.34 N/C (outward)
49. 0.438 N
51. 8.27
53. (a) — (b) — (c) —

55.



57. (a) 4.01 nC (b) 9.57 nC (c) 4.01 nC (d) 5.56 nC
59. — radially outward

61. (a) $\frac{Cd}{24}$ to the right for /2 and to the left for /2 (b) $\frac{Cx}{}$

63. (a) 0.269 N /C (b) 2.38
65. — radially outward

67. (a) — — (b) —— ——

Chapter 25

Answers to Quick Quizzes

(i) (b) **(ii)** (a)
2. to to to to
3. **(i)** (c) **(ii)** (a)
4. **(i)** (a) **(ii)** (a)

Answers to Odd-Numbered Problems

(a) 1.13 N/C (b) 1.80 $^{-14}$ N (c) 4.37 $^{-17}$
3. (a) 1.52 m/s (b) 6.49 m/s
5. 260 V
(a) 38.9 V (b) the origin
9. 0.300 m/s
11. (a) 0.400 m/s (b) It is the same. Each bit of the rod feels a force of the same size as before.
13. (a) 2.12 V (b) 1.21

15. 6.93 —

17. (a) 45.0 V (b) 34.6 km/s
19. (a) 0 (b) 0 (c) 44.9 kV
21. (a) — — (b) — $\frac{qQ}{}$

23. (a) 4.83 m (b) 0.667 m and 2.00 m
25. (a) 32.2 kV (b) 0.096 5 J

27. 8.94 J

29. —

31. (a) 10.8 m/s and 1.55 m/s (b) They would be greater. The conducting spheres will polarize each other, with most of the positive charge of one and the negative charge of the other on their inside faces. Immediately before the spheres collide, their centers of charge will be closer than their geometric centers, so they will have less electric potential energy and more kinetic energy.

33. 22.8 —

35. 2.74 27.4 fm

37. (a) 10.0 V, 11.0 V, 32.0 V
(b) 7.00 N/C in the positive direction

39. (a) xy yz
(b) 7.07 N/C

41. (a) 0 (b) —

43. 0.553 —

45. (a) — (b) ln —)]

47. 2 ln 3)

49. 1.56

51. (a) 1.35 V (b) larger sphere: 2.25 V/m (away from the center); smaller sphere: 6.74 V/m (away from the center)

53. Because is not an integer, this is not possible. Therefore, the energy given cannot be possible for an allowed state of the atom.

55. (a) 6.00 m/s (b) 3.64 m (c) 9.00 m/s (d) 12.0 m/s

57. 253 MeV

59. (a) 30.0 cm (b) 6.67 nC (c) 29.1 cm or 3.44 cm
(d) 6.79 nC or 804 pC
(e) No; two answers exist for each part.

61. 702 J

63. 4.00 nC at (1.00 m, 0) and 5.01 nC at (0, 2.00 m)

65. —— ln —

67. ln ————

69. (a) 4.07 kV/m (b) 488 V (c) 7.82 J (d) 306 km/s
(e) 3.89 m/s toward the negative plate
(f) 6.51 N toward the negative plate
(g) 4.07 kV/m (h) They are the same.

71. (b) cos —— sin —— (c) yes (d) no
(e) py ——
(f) pxy —— ——

73. ln ——)]

75. (a) — ln ——
(b) — dh ln ——)]

Chapter 26

Answers to Quick Quizzes

(d)
2. (a)
3. (a)
4. (b)
5. (a)

Answers to Odd-Numbered Problems

(a) 9.00 V (b) 12.0 V

3. (a) 48.0 C (b) 6.00

5. (a) 2.69 nF (b) 3.02 kV

4.43

9. (a) 11.1 kV/m toward the negative plate (b) 98.4 nC/m (c) 3.74 pF (d) 74.8 pC

11. (a) 1.33 C/m (b) 13.4 pF

13. (a) 17.0 F (b) 9.00 V (c) 45.0 C on 5 F, 108 C on 12

15. (a) 2.81 F (b) 12.7

17. (a) in series (b) 398 F (c) in parallel; 2.20

19. (a) 3.33 F (b) 180 C on the 3.00- F and 6.00- capacitors; 120 C on the 2.00- F and 4.00- F capacitors (c) 60.0 V across the 3.00- F and 2.00- F capacitors; 30.0 V across the 6.00- F and 4.00- F capacitors

21. ten

23. (a) 5.96 F (b) 89.5 C on 20 F, 63.2 C on 6 F, and 26.3 C on 15 F and 3

25. 12.9

27. 6.00 pF and 3.00 pF

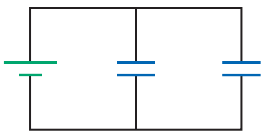
29. 19.8

31. 3.24

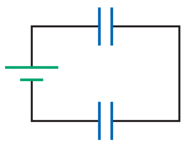
33. (a) 1.50 C (b) 1.83 kV

35. (a) 2.50 J (b) 66.7 V (c) 3.33 J (d) Positive work is done by the agent pulling the plates apart.

37. (a)



(b) 0.150 J (c) 268 V
(d)



39. 9.79 kg

41. (a) 400 C (b) 2.5 kN/m

43. (a) 13.3 nC (b) 272 nC

45. (a) 81.3 pF (b) 2.40 kV

47. (a) 369 pC (b) 1.2 F, 3.1 V (c) 45.5 nJ

49. (a) 40.0 J (b) 500 V

51. 9.43 10 N

55. (a) 11.2 pF (b) 134 pC (c) 16.7 pF (d) 67.0 pC
57. 2.51×10^{-3} m^3 = 2.51 L
59. 0.188 m^2
61. (a) volume 9.09×10^{-16} m^3, area 4.54×10^{-10} m^2 (b) 2.01×10^{-13} F (c) 2.01×10^{-14} C; 1.26×10^5 electronic charges
63. 23.3 V across the 5.00-μF capacitor, 26.7 V across the 10.0-μF capacitor
65. (a) $\dfrac{Q_0^2 d(\ell - x)}{2\epsilon_0 \ell^3}$ (b) $\dfrac{Q_0^2 d}{2\epsilon_0 \ell^3}$ to the right (c) $\dfrac{Q_0^2}{2\epsilon_0 \ell^4}$ (d) $\dfrac{Q_0^2}{2\epsilon_0 \ell^4}$ (e) They are precisely the same.
67. 4.29 μF
69. 750 μC on C_1, 250 μC on C_2
71. (a) One capacitor cannot be used by itself—it would burn out. The technician can use two capacitors in series, connected in parallel to another two capacitors in series. Another possibility is two capacitors in parallel, connected in series to another two capacitors in parallel. In either case, one capacitor will be left over: upper and lower (b) Each of the four capacitors will be exposed to a maximum voltage of 45 V.
73. $\dfrac{C_0}{2}(\sqrt{3} - 1)$
75. $\frac{4}{3}C$
77. 3.00 μF

Chapter 27

Answers to Quick Quizzes

1. (a) > (b) = (c) > (d)
2. (b)
3. (b)
4. (a)
5. $I_a = I_b > I_c = I_d > I_e = I_f$

Answers to Odd-Numbered Problems

1. 27.0 yr
3. 0.129 mm/s
5. 1.79×10^{16} protons
7. (a) $0.632 I_0 \tau$ (b) $0.999\ 95 I_0 \tau$ (c) $I_0 \tau$
9. (a) 17.0 A (b) 85.0 kA/m^2
11. (a) 2.55 A/m^2 (b) 5.30×10^{10} m^{-3} (c) 1.21×10^{10} s
13. 3.64 h
15. silver ($\rho = 1.59 \times 10^{-8}\ \Omega \cdot$ m)
17. 8.89 Ω
19. (a) 1.82 m (b) 280 μm
21. (a) 13.0 Ω (b) 255 m
23. $6.00 \times 10^{-15} (\Omega \cdot \text{m})^{-1}$
25. 0.18 V/m
27. 0.12
29. 6.32 Ω
31. (a) 3.0 A (b) 2.9 A
33. (a) 31.5 n$\Omega \cdot$ m (b) 6.35 MA/m^2 (c) 49.9 mA (d) 658 μm/s (e) 0.400 V
35. 227°C
37. 448 A
39. (a) 8.33 A (b) 14.4 Ω
41. 2.1 W
43. 36.1%
45. (a) 0.660 kWh (b) \$0.072 6
47. \$0.494/day
49. (a) 3.98 V/m (b) 49.7 W (c) 44.1 W
51. (a) 4.75 m (b) 340 W
53. (a) 184 W (b) 461°C
55. 672 s
57. 1.1 km
59. 15.0 h
61. 50.0 MW
63. (a) $\dfrac{Q}{4C}$ (b) $\dfrac{Q}{4}$ on C, $\dfrac{3Q}{4}$ on $3C$ (c) $\dfrac{Q^2}{32C}$ in C, $\dfrac{3Q^2}{32C}$ in $3C$ (d) $\dfrac{3Q^2}{8C}$
65. 0.478 kg/s
67. (a) 8.00 V/m in the positive x direction (b) 0.637 Ω (c) 6.28 A in the positive x direction (d) 200 MA/m^2
69. (a) 116 V (b) 12.8 kW (c) 436 W
71. (a) $\dfrac{\rho}{2\pi L} \ln\left(\dfrac{r_b}{r_a}\right)$ (b) $\dfrac{2\pi L\, \Delta V}{I \ln (r_b/r_a)}$
73. 4.1×10^{-3} (°C)$^{-1}$
75. 1.418 Ω
77. (a) $\dfrac{\epsilon_0 \ell}{2d}(\ell + 2x + \kappa\ell - 2\kappa x)$ (b) $\dfrac{\epsilon_0 \ell v\, \Delta V(\kappa - 1)}{d}$ clockwise
79. 2.71 MΩ
81. (2.02×10^3)°C

Chapter 28

Answers to Quick Quizzes

1. (a)
2. (b)
3. (a)
4. **(i)** (b) **(ii)** (a) **(iii)** (a) **(iv)** (b)
5. **(i)** (c) **(ii)** (d)

Answers to Odd-Numbered Problems

1. (a) 6.73 Ω (b) 1.97 Ω
3. (a) 12.4 V (b) 9.65 V
5. (a) 75.0 V (b) 25.0 W, 6.25 W, and 6.25 W (c) 37.5 W
7. $\frac{7}{3}R$
9. (a) 227 mA (b) 5.68 V
11. (a) 1.00 kΩ (b) 2.00 kΩ (c) 3.00 kΩ
13. (a) 17.1 Ω (b) 1.99 A for 4.00 Ω and 9.00 Ω, 1.17 A for 7.00 Ω, 0.818 A for 10.0 Ω
15. 470 Ω and 220 Ω
17. (a) 11.7 Ω (b) 1.00 A in the 12.0-Ω and 8.00-Ω resistors, 2.00 A in the 6.00-Ω and 4.00-Ω resistors, 3.00 A in the 5.00-Ω resistor
19. 14.2 W to 2.00 Ω, 28.4 W to 4.00 Ω, 1.33 W to 3.00 Ω, 4.00 W to 1.00 Ω
21. (a) 4.12 V (b) 1.38 A
23. (a) 0.846 A down in the 8.00-Ω resistor, 0.462 A down in the middle branch, 1.31 A up in the right-hand branch (b) −222 J by the 4.00-V battery, 1.88 kJ by the 12.0-V battery (c) 687 J to 8.00 Ω, 128 J to 5.00 Ω, 25.6 J to the 1.00-Ω resistor in the center branch, 616 J to 3.00 Ω, 205 J to the 1.00-Ω resistor in the right branch

(d) Chemical energy in the 12.0-V battery is transformed into internal energy in the resistors. The 4.00-V battery is being charged, so its chemical potential energy is increasing at the expense of some of the chemical potential energy in the 12.0-V battery. (e) 1.66 kJ

25. (a) 0.395 A (b) 1.50 V

27. 50.0 mA from to

29. (a) 0.714 A (b) 1.29 A (c) 12.6 V

31. (a) 0.385 mA, 3.08 mA, 2.69 mA
(b) 69.2 V, with at the higher potential

33. (a) 0.492 A; 0.148 A; 0.639 A
(b) $_{28.0}$ 6.77 W, $_{12.0}$ 0.261 W, $_{16.0}$ 6.54 W

35. 3.05 V, 4.57 V, 7.38 V, 1.62 V

37. (a) 2.00 ms (b) 1.80 C (c) 1.14

39. (a) 61.6 mA (b) 0.235 C (c) 1.96 A

41. (a) 1.50 s (b) 1.00 s (c) 200 100 , where is in microamperes and is in seconds

43. (a) 6.00 V (b) 8.29

45. (a) 0.432 s (b) 6.00

47. (a) 6.25 A (b) 750 W

49. (a) — (b) — (c) parallel

51. 2.22 h

53. (a) 1.02 A down (b) 0.364 A down (c) 1.38 A up (d) 0
(e) 66.0

55. (a) 2.00 k (b) 15.0 V (c) 9.00 V

57. (a) 4.00 V (b) Point is at the higher potential.

59. 87.3%

61. 6.00 , 3.00

63. (a) 24.1 C (b) 16.1 C (c) 16.1 mA

65. (a) 240(1
(b) 360(1), where in both answers, is in microcoulombs and is in milliseconds

67. (a) 9.93 C (b) 33.7 nA (c) 335 nW (d) 337 nW

69. (a) 470 W (b) 1.60 mm or more (c) 2.93 mm or more

71. (a) 222 C (b) 444

73. (a) 5.00 (b) 2.40 A

75. (a) 0 in 3 k , 333 A in 12 k and 15 k (b) 50.0
(c) 278 $^{/0.180}$, where is in microamperes and is in seconds (d) 290 ms

77. (a) – (b) No; 2.75 , so the station is inadequately grounded.

79. (a) – (b) 3

81. (a) 3.91 s (b) 782

83. 20.0 or 98.1

Chapter 29

Answers to Quick Quizzes

(e)

2. **(i)** (b) **(ii)** (a)

3. (c)

4. **(i)** (c), (b), (a) **(ii)** (a) (b) (c)

Answers to Odd-Numbered Problems

Gravitational force: 8.93 N down, electric force: 1.60 N up, and magnetic force: 4.80 down.

3. (a) into the page (b) toward the right
(c) toward the bottom of the page

5. (a) the negative direction (b) the positive direction
(c) The magnetic force is zero in this case.

(a) 7.91 N (b) zero

9. (a) 1.25 N (b) 7.50 m/s

11. 20.9

13. (a) 4.27 cm (b) 1.79

15. (a) (b)

17. 115 keV

19. (a) 5.00 cm (b) 8.79 m/s

21. 7.88

23. 8.00

25. 0.278 m

27. (a) 7.66 (b) 2.68 m/s (c) 3.75 MeV
(d) 3.13 revolutions (e) 2.57

29. 244 kV/m

31. 70.0 mT

33. (a) 8.00 T (b) in the positive direction

35. 2.88

37. 1.07 m/s

39. (a) east (b) 0.245 T

41. (a) 5.78 N (b) toward the west (into the page)

43. 2.98 N west

45. (a) 4.0 m (b) 6.9

47. (a) north at 48.0° below the horizontal
(b) south at 48.0° above the horizontal (c) 1.07

49. 9.05 m, tending to make the left-hand side of the loop move toward you and the right-hand side move away.

51. (a) 9.98 N m (b) clockwise as seen looking down from a position on the positive axis

53. (a) 118 m (b) 118 118

55. 43.2

57. (a) 9.27 (b) away from observer

59. (a) 3.52 1.60 10 ^{18} N (b) 24.4°

61. 0.588 T

63.

65. 39.2 mT

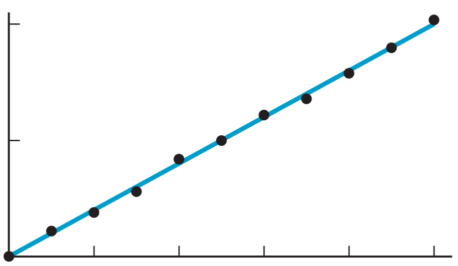
67. (a) the positive direction (b) 0.696 m (c) 1.09 m
(d) 54.7 ns

69. (a) 0.713 A counterclockwise as seen from above

71. (a) *mg/NIw* (b) The magnetic field exerts forces of equal magnitude and opposite directions on the two sides of the coils, so the forces cancel each other and do not affect the balance of the system. Hence, the vertical dimension of the coil is not needed. (c) 0.261 T

73. (a) 1.04 m (b) 1.89

75. (a) (1.00 , where is in volts and is in teslas



(b) 0.125 mm

77. 3.71

79. (a) 0.128 T (b) 78.7° below the horizontal

Chapter 30

Answers to Quick Quizzes

1. $B > C > A$
2. (a)
3. $c > a > d > b$
4. $a = c = d > b = 0$
5. (c)

Answers to Odd-Numbered Problems

1. (a) 21.5 mA (b) 4.51 V (c) 96.7 mW
3. 1.60×10^{-6} T
5. (a) 28.3 μT into the page (b) 24.7 μT into the page
7. 5.52 μT into the page
9. (a) $2I_1$ out of the page (b) $6I_1$ into the page
11. $\dfrac{\mu_0 I}{2r}\left(\dfrac{1}{\pi} + \dfrac{1}{4}\right)$
13. 262 nT into the page
15. (a) 53.3 μT toward the bottom of the page (b) 20.0 μT toward the bottom of the page (c) zero
17. $\dfrac{\mu_0 I}{2\pi a d}(\sqrt{d^2 + a^2} - d)$ into the page
19. (a) 40.0 μT into the page (b) 5.00 μT out of the page (c) 1.67 μT out of the page
21. (a) 10 μT (b) 80 μN toward the other wire (c) 16 μT (d) 80 μN toward the other wire
23. (a) 3.00×10^{-5} N/m (b) attractive
25. $-27.0\hat{\mathbf{i}}$ μN
27. 0.333 m
29. (a) opposite directions (b) 67.8 A (c) It would be smaller. A smaller gravitational force would be pulling down on the wires, requiring less magnetic force to raise the wires to the same angle and therefore less current.
31. (a) 200 μT toward the top of the page (b) 133 μT toward the bottom of the page
33. 5.40 cm
35. (a) 4.00 m (b) 7.50 nT (c) 1.26 m (d) zero
37. (a) zero (b) $\dfrac{\mu_0 I}{2\pi R}$ tangent to the wall (c) $\dfrac{\mu_0 I^2}{(2\pi R)^2}$ inward
39. 20.0 μT toward the bottom of the page
41. 31.8 mA
43. (a) 226 μN away from the center of the loop (b) zero
45. (a) 920 turns (b) 12 cm
47. (a) 3.13 mWb (b) 0
49. (a) 8.63×10^{45} electrons (b) 4.01×10^{20} kg
51. 3.18 A
53. (a) $\sim 10^{-5}$ T (b) It is $\sim 10^{-1}$ as large as the Earth's magnetic field.
55. 143 pT
57. $\dfrac{\mu_0 I}{2\pi w} \ln\left(1 + \dfrac{w}{b}\right)\hat{\mathbf{k}}$
59. (a) $\mu_0 \sigma v$ into the page (b) zero (c) $\frac{1}{2}\mu_0\sigma^2 v^2$ up toward the top of the page (d) $\dfrac{1}{\sqrt{\mu_0 \epsilon_0}}$; we will find out in Chapter 34 that this speed is the speed of light. We will also find out in Chapter 39 that this speed is not possible for the capacitor plates.
61. 1.80 mT
63. 3.89 μT parallel to the xy plane and at 59.0° clockwise from the positive x direction
65. (b) 3.20×10^{-13} T (c) 1.03×10^{-24} N (d) 2.31×10^{-22} N
67. $B = 4.36 \times 10^{-4}\, I$, where B is in teslas and I is in amperes
69. (a) $\dfrac{\mu_0 I N}{2\ell}\left[\dfrac{\ell - x}{\sqrt{(\ell - x)^2 + a^2}} + \dfrac{x}{\sqrt{x^2 + a^2}}\right]$
71. $-0.012\,0\,\hat{\mathbf{k}}$ N
73. (b) $\dfrac{\mu_0 I}{4\pi}(1 - e^{-2\pi})$ out of the page
75. (a) $\dfrac{\mu_0 I(2r^2 - a^2)}{\pi r(4r^2 - a^2)}$ to the left (b) $\dfrac{\mu_0 I(2r^2 + a^2)}{\pi r(4r^2 + a^2)}$ toward the top of the page
77. (b) 5.92×10^{-8} N

Chapter 31

Answers to Quick Quizzes

1. (c)
2. (c)
3. (b)
4. (a)
5. (b)

Answers to Odd-Numbered Problems

1. 0.800 mA
3. (a) 101 μV tending to produce clockwise current as seen from above (b) It is twice as large in magnitude and in the opposite sense.
5. 33.9 mV
7. 10.2 μV
9. 61.8 mV
11. (a) 1.60 A counterclockwise (b) 20.1 μT (c) left
13. (a) $\dfrac{\mu_0 I L}{2\pi} \ln\left(1 + \dfrac{w}{h}\right)$ (b) 4.80 μV (c) counterclockwise
15. (a) 1.88×10^{-7} T · m² (b) 6.28×10^{-8} V
17. 272 m
19. $\boldsymbol{\mathcal{E}} = 0.422 \cos 120\pi t$, where $\boldsymbol{\mathcal{E}}$ is in volts and t is in seconds
21. 2.83 mV
23. 13.1 mV
25. (a) 39.9 μV (b) The west end is positive.
27. (a) 3.00 N to the right (b) 6.00 W
29. (a) 0.500 A (b) 2.00 W (c) 2.00 W
31. 2.80 m/s
33. 24.1 V with the outer contact negative
35. (a) 233 Hz (b) 1.98 mV
37. 145 μA upward in the picture
39. (a) 8.01×10^{-21} N (b) clockwise (c) $t = 0$ or $t = 1.33$ s
41. (a) $E = 9.87 \cos 100\pi t$, where E is in millivolts per meter and t is in seconds (b) clockwise
43. 13.3 V
45. (a) $\boldsymbol{\mathcal{E}} = 19.6 \sin 100\pi t$, where $\boldsymbol{\mathcal{E}}$ is in volts and t is in seconds (b) 19.6 V
47. $\boldsymbol{\mathcal{E}} = 28.6 \sin 4.00\pi t$, where $\boldsymbol{\mathcal{E}}$ is in millivolts and t is in seconds
49. (a) $\Phi_B = 8.00 \times 10^{-3} \cos 120\pi t$, where Φ_B is in T · m² and t is in seconds (b) $\boldsymbol{\mathcal{E}} = 3.02 \sin 120\pi t$, where $\boldsymbol{\mathcal{E}}$ is in volts and t is in seconds (c) $I = 3.02 \sin 120\pi t$, where I is in amperes and t is in seconds (d) $P = 9.10 \sin^2 120\pi t$, where P is in watts and t is in seconds (e) $\tau = 0.024\,1 \sin^2 120\pi t$, where τ is in newton meters and t is in seconds
51. (a) 113 V (b) 300 V/m

53. 8.80 A
55. 3.79 mV
57. (a) 43.8 A (b) 38.3 W
59. 7.22 cos 1 046 , where is in millivolts and is in seconds
61. 283 A upward
63. (a) 3.50 A up in 2.00 and 1.40 A up in 5.00 (b) 34.3 W (c) 4.29 N
65. 2.29
67. (a) 0.125 V clockwise (b) 0.020 0 A clockwise
69. (a) 97.4 nV (b) clockwise
71. (a) 36.0 V (b) 0.600 Wb/s (c) 35.9 V (d) 4.32 N · m
73. (a) NB (b) $\frac{NB}{\;}$ (c) (d) (e) clockwise (f) directed to the left.
75. 6.00 A
77. 87.1 cos (200), where is in millivolts and is in seconds
79. 0.062 3 A in 6.00 , 0.860 A in 5.00 , and 0.923 A in 3.00
81. $\frac{\;}{Bd}$ mR
83. $\frac{MgR}{\;}$

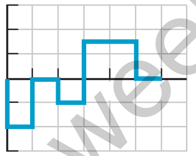
Chapter 32

Answers to Quick Quizzes

(c), (f)
2. **(i)** (b) **(ii)** (a)
3. (a), (d)
4. (a)
5. **(i)** (b) **(ii)** (c)

Answers to Odd-Numbered Problems

19.5 mV
3. 100 V
5. 19.2
4.00 mH
9. (a) 360 mV (b) 180 mV (c) 3.00 s
11. $\frac{\;}{Lk}$
13. 18.8 cos 120 , where is in volts and is in seconds
15. (a) 0.469 mH (b) 0.188 ms
17. (a) 1.00 k (b) 3.00 ms
19. (a) 1.29 k (b) 72.0 mA
21. (a) 20.0% (b) 4.00%
23. 92.8 V
25. (a) 0.500(1 10.0), where is in amperes and is in seconds (b) 1.50 0.250 10.0 , where is in amperes and is in seconds
27. (a) 0.800 (b) 0
29. (a) 6.67 A/s (b) 0.332 A/s
31. (a) 5.66 ms (b) 1.22 A (c) 58.1 ms
33. 2.44
35. (a) 44.3 nJ/m (b) 995 J/m
37. (a) 18.0 J (b) 7.20 J
39. (a) 8.06 MJ/m (b) 6.32 kJ
41. 1.00 V
43. (a) 18.0 mH (b) 34.3 mH (c) 9.00 mV
45. 781 pH
47. 281 mH
49. 400 mA
51. 20.0 V
53. (a) 503 Hz (b) 12.0 C (c) 37.9 mA (d) 72.0
55. (a) 135 Hz (b) 119 C (c) 114 mA
57. (a) 2.51 kHz (b) 69.9
59. (a) 0.693 (b) 0.347
61. (a) 20.0 mV (b) 10.0 , where is in mega volts and is in seconds (c) 63.2
63.
65. (a) 4.00 H (b) 3.50
67. (a) (b) 10 H (c) 10
69.



71. 91.2
73. (a) 6.25 J (b) 2.00 N/m
75. (a) 50.0 mT (b) 20.0 mT (c) 2.29 MJ (d) 318 Pa
79. (a) (b) 2.70
81. 300
83.

Chapter 33

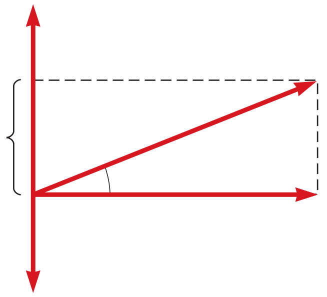
Answers to Quick Quizzes

(i) (c) **(ii)** (b)
2. (b)
3. (a)
4. (b)
5. (a) (b) (c)
6. (c)
(c)

Answers to Odd-Numbered Problems

(a) 96.0 V (b) 136 V (c) 11.3 A (d) 768 W
3. (a) 2.95 A (b) 70.7 V
5. 14.6 Hz
3.38 W
9. 3.14 A
11. 5.60 A
13. (a) 12.6 (b) 6.21 A (c) 8.78 A
15. 0.450 Wb
17. 32.0 A
19. (a) 41.3 Hz (b) 87.5
21. 100 mA
23. (a) 141 mA (b) 235 mA

25.



27. (a) 47.1 (b) 637 (c) 2.40 k (d) 2.33 k (e) 14.2°
29. (a) 17.4° (b) the voltage
31. (a) 194 V (b) The current leads by 49.9°.
33.
35. 353 W
37. 88.0 W
39. (a) 16.0 (b) 12.0
41. $\frac{11 \quad \text{rms}}{14}$
43. 1.82 pF
45. 242 mJ
47. (a) 0.633 pF (b) 8.46 mm (c) 25.1
49. 687 V
51. 87.5
53. 0.756
55. (a) 34% (b) 5.3 W (c) \$3.9
57. (a) 1.60 turns (b) 30.0 A (c) 25.3 A
59. (a) 22.4 V (b) 26.6° (c) 0.267 A (d) 83.9 (e) 47.2 (f) 0.249 H (g) 2.67 W
61. 2.6 cm
63. (a) could be 53.8 or it could be 1.35 k Ω (b) capacitive reactance is 53.8 (c) must be 1.43 k
65. (b) 31.6
67. (a) 19.7 cm at 35.0° (b) 19.7 cm at 35.0° (c) The answers are identical. (d) 9.36 cm at 169°
69. (a) Tension and separation must be related by 274 , where is in newtons and is in meters. (b) One possibility is 10.9 N and 0.200 m.
71. (a) 0.225 A (b) 0.450 A
73. (a) 78.5 (b) 1.59 k (c) 1.52 k (d) 138 mA (e) 84.3° (f) 0.098 7 (g) 1.43 W
75. 56.7 W
77. (a) 580 H (b) 54.6 F (c) 1.00 (d) 894 Hz (e) At 200 Hz, 60.0° ($_{out}$ leads); at $_{out}$ is in phase with); and at 4.00 Hz, 60.0° $_{out}$ lags). (f) At 200 Hz and at 4.00 Hz, 1.56 W; and at 6.25 W. (g) 0.408
79. (a) 224 s (b) 500 W (c) 221 s and 226 s
81. 58.7 Hz or 35.9 Hz. The circuit can be either above or below resonance.

Chapter 34

Answers to Quick Quizzes

(i) (b) **(ii)** (c)
2. (c)
3. (c)
4. (b)
5. (a)
6. (c)
(a)

Answers to Odd-Numbered Problems

(a) out of the page (b) 1.85
3. (a) 11.3 GV m/s (b) 0.100 A
5. 2.87 5.75 10 m
(a) 0.690 wavelengths (b) 58.9 wavelengths
9. (a) 681 yr (b) 8.32 min (c) 2.56 s
11. 74.9 MHz
13. 2.25 m/s
15. (a) 6.00 MHz (b) 73.4 nT (c) = −73.4 cos 0.126 3.77 10 , where is in nT, is in meters, and is in seconds
17. 2.9 m/s
19. (a) 0.333 T (b) 0.628 m (c) 4.77
21. 3.34 J/m
23. 3.33
25. (a) 1.19 W/m (b) 2.35
27. (a) 2.33 mT (b) 650 MW/m (c) 511 W
29. 307 W/m
31. 49.5 mV
33. (a) 332 kW/m radially inward (b) 1.88 kV/m and 222
35. 5.31 N/m
37. (a) 1.90 kN/C (b) 50.0 pJ (c) 1.67 kg m/s
39. 4.09°
41. (a) 1.60 kg each second (b) 1.60 (c) The answers are the same. Force is the time rate of momentum transfer.
43. (a) 5.48 N (b) 913 m/s away from the Sun (c) 10.6 days
45. (a) 134 m (b) 46.8 m
47. 56.2 m
49. (a) away along the perpendicular bisector of the line segment joining the antennas (b) along the extensions of the line segment joining the antennas
51. (a) 6.00 pm (b) 7.49 cm
53. (a) 4.16 m to 4.54 m (b) 3.41 m to 3.66 m (c) 1.61 m to 1.67 m
55. (a) 3.85 W (b) 1.02 kV/m and 3.39
57. 5.50
59. (a) 3.21 W (b) 0.639 W/m (c) 0.513% of that from the noon Sun in January
61.
63. 378 nm
65. (a) 6.67 T (b) 5.31 W/m (c) 1.67 W (d) 5.56
67. (a) 625 kW/m (b) 21.7 kV/m (c) 72.4 T (d) 17.8 min
69. (a) 388 K (b) 363 K
71. (a) 3.92 W/m (b) 308 W
73. (a) 0.161 m (b) 0.163 m (c) 76.8 W (d) 470 W/m (e) 595 V/m (f) 1.98 T (g) 119 W
75. (a) The projected area is , where is the radius of the planet. (b) The radiating area is 4 . (c) 1.61
77. (a) 584 nT (b) 419 m (c) 1.26 (d) vibrates in the plane. (e) 40.6 (f) 271 nPa (g) 407 nm
79. (a) 22.6 h (b) 30.6 s

Chapter 35

Answers to Quick Quizzes

1. (d)
2. Beams ② and ④ are reflected; beams ③ and ⑤ are refracted.
3. (c)
4. (c)
5. **(i)** (b) **(ii)** (b)

Answers to Odd-Numbered Problems

1. (a) 2.07×10^3 eV (b) 4.14 eV
3. 114 rad/s
5. (a) 4.74×10^{14} Hz (b) 422 nm (c) 2.00×10^8 m/s
7. 22.5°
9. (a) 1.81×10^8 m/s (b) 2.25×10^8 m/s (c) 1.36×10^8 m/s
11. (a) 29.0° (b) 25.8° (c) 32.0°
13. 86.8°
15. 158 Mm/s
17. (a) $\theta_{1i} = 30°$, $\theta_{1r} = 19°$, $\theta_{2i} = 41°$, $\theta_{2r} = 77°$ (b) First surface: $\theta_{\text{reflection}} = 30°$; second surface: $\theta_{\text{reflection}} = 41°$
19. $\sim 10^{-11}$ s, $\sim 10^3$ wavelengths
21. (a) 1.94 m (b) 50.0° above the horizontal
23. 27.1 ns
25. (a) 2.0×10^8 m/s (b) 4.74×10^{14} Hz (c) 4.2×10^{-7} m
27. 3.39 m
29. (a) 41.5° (b) 18.5° (c) 27.5° (d) 42.5°
31. 23.1°
33. 1.22
35. $\tan^{-1}(n_g)$
37. 0.314°
39. 4.61°
41. 62.5°
43. 27.9°
45. 67.1°
47. 1.000 07
49. (a) $\dfrac{nd}{n-1}$ (b) $R_{\min} \to 0$. Yes; for very small d, the light strikes the interface at very large angles of incidence. (c) $R_{\min}$ decreases. Yes; as n increases, the critical angle becomes smaller. (d) $R_{\min} \to \infty$. Yes; as $n \to 1$, the critical angle becomes close to 90° and any bend will allow the light to escape. (e) 350 μm
51. 48.5°
53. 2.27 m
55. 25.7°
57. (a) 0.042 6 or 4.26% (b) no difference
59. (a) 334 μs (b) 0.014 6%
61. 77.5°
63. 2.00 m
65. 27.5°
67. 3.79 m
69. 7.93°
71. $\sin^{-1}\left[\dfrac{L}{R^2}\left(\sqrt{n^2R^2 - L^2} - \sqrt{R^2 - L^2}\right)\right]$ or $\sin^{-1}\left[n \sin\left(\sin^{-1}\dfrac{L}{R} - \sin^{-1}\dfrac{L}{nR}\right)\right]$
73. (a) 38.5° (b) 1.44
75. (a) 53.1° (b) $\theta_1 \geq 38.7°$
77. (a) 1.20 (b) 3.40 ns
79. (a) 0.172 mm/s (b) 0.345 mm/s (c) and (d) northward and downward at 50.0° below the horizontal.
81. 62.2%
83. (a) $\left(\dfrac{4x^2 + L^2}{L}\right)\omega$ (b) 0 (c) $L\omega$ (d) $2L\omega$ (e) $\dfrac{\pi}{8\omega}$
87. 70.6%

Chapter 36

Answers to Quick Quizzes

1. false
2. (b)
3. (b)
4. (d)
5. (a)
6. (b)
7. (a)
8. (c)

Answers to Odd-Numbered Problems

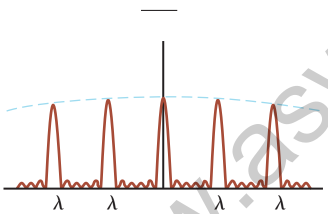
1. 89.0 cm
3. (a) younger (b) $\sim 10^{-9}$ s younger
5. (a) $p_1 + h$, behind the lower mirror (b) virtual (c) upright (d) 1.00 (e) no
7. (a) 1.00 m behind the nearest mirror (b) the palm (c) 5.00 m behind the nearest mirror (d) the back of her hand (e) 7.00 m behind the nearest mirror (f) the palm (g) All are virtual images.
9. **(i)** (a) 13.3 cm (b) real (c) inverted (d) −0.333 **(ii)** (a) 20.0 cm (b) real (c) inverted (d) −1.00 **(iii)** (a) ∞ (b) no image formed (c) no image formed (d) no image formed
11. (a) −12.0 cm; 0.400 (b) −15.0 cm; 0.250 (c) both upright
13. (a) −7.50 cm (b) upright (c) 0.500 cm
15. 3.33 m from the deepest point in the niche
17. 0.790 cm
19. (a) 0.160 m (b) −0.400 m
21. (a) convex (b) at the 30.0-cm mark (c) −20.0 cm
23. (a) 15.0 cm (b) 60.0 cm
25. (a) concave (b) 2.08 m (c) 1.25 m from the object
27. (a) 25.6 m (b) 0.058 7 rad (c) 2.51 m (d) 0.023 9 rad (e) 62.8 m
29. (a) 45.1 cm (b) −89.6 cm (c) −6.00 cm
31. (a) 1.50 m (b) 1.75 m
33. 4.82 cm
35. 8.57 cm
37. 1.50 cm/s
39. (a) 6.40 cm (b) −0.250 (c) converging
41. (a) 39.0 mm (b) 39.5 mm
43. 20.0 cm
45. (a) 20.0 cm from the lens on the front side (b) 12.5 cm from the lens on the front side (c) 6.67 cm from the lens on the front side (d) 8.33 cm from the lens on the front side
47. 2.84 cm
49. (a) 16.4 cm (b) 16.4 cm
51. (a) 1.16 mm/s (b) toward the lens
53. 7.47 cm in front of the second lens, 1.07 cm, virtual, upright
55. 21.3 cm

57. 2.18 mm away from the CCD
59. (a) 42.9 cm (b) 2.33 diopters
61. 23.2 cm
63. (a) −0.67 diopters (b) 0.67 diopters
65. (a) Yes, if the lenses are bifocal.
(b) 56.3 cm, 1.78 diopters (c) 1.18 diopters
67. 575
69. 3.38 min
71. (a) 267 cm (b) 79.0 cm
73. 40.0 cm
75. (a) 1.50 (b) 1.90
77. (a) 160 cm to the left of the lens (b) 0.800 (c) inverted
79. (a) 32.1 cm to the right of the second surface (b) real
81. (a) 25.3 cm to the right of the mirror (b) virtual (c) upright (d) 8.05
83. (a) 1.40 kW/m (b) 6.91 mW/m (c) 0.164 cm (d) 58.1 W/m
87. 8.00 cm
89. 11.7 cm
91. (a) 1.50 m in front of the mirror (b) 1.40 cm
(a) 0.334 m or larger (b) 0.025 5 or larger
95. (a) 1.99 (b) 10.0 cm to the left of the lens (c) 2.50 (d) inverted
97. and

Chapter 37

Answers to Quick Quizzes

(c)
2. The graph is shown here. The width of the primary maxima is slightly narrower than the 5 primary width but wider than the 10 primary width. Because 6, the secondary maxima are $\frac{}{36}$ as intense as the primary maxima.



3. (a)

Answers to Odd-Numbered Problems

641
3. 632 nm
5. 1.54 mm
2.40
9. (a) 2.62 mm (b) 2.62 mm
11. Maxima at 0°, 29.1°, and 76.3°; minima at 14.1° and 46.8°
13. (a) 55.7 m (b) 124 m
15. 0.318 m/s
17. 148 m
21. (a) 1.93 m (b) 3.00
(c) It corresponds to a maximum. The path difference is an integer multiple of the wavelength.
23. 0.968
25. 48.0
27. (a) 1.29 rad (b) 99.6 nm
29. (a) 7.95 rad (b) 0.453
31. 512 nm
33. 0.500 cm
35. 290 nm
37. 8.70
39. 1.31
41. 1.20 mm
43. 1.001
45. 1.25 m
47. 1.62 cm
49. 78.4
51. $\frac{}{48}$ 650, where and are in nanometers and 0, 1, 1, 2, 2, 3, 3, .
53. ————
55. 5.00 5.00 km
57. 2.50 mm
59. 113
61. (a) 72.0 m (b) 36.0 m
63. (a) 70.6 m (b) 136 m
65. (a) 14.7 m (b) 1.53 cm (c) 16.0 m
67. 0.505 mm
69. 3.58°
71. 115 nm
73. (a) $\frac{}{\;-\lambda}$ (b) 266 nm
75. 0.498 mm

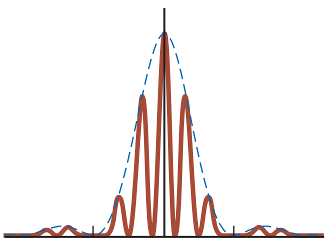
Chapter 38

Answers to Quick Quizzes

(a)
2. (i)
3. (b)
4. (a)
5. (c)
6. (b)
(c)

Answers to Odd-Numbered Problems

(a) 1.1 m (b) 1.7 mm
3. (a) four (b) 28.7°, 73.6°
5. 91.2 cm
2.30
9.



11. 1.62
13. 462 nm
15. 2.10 m
17. 0.284 m
19. 30.5 m
21. 0.40 rad
23. 16.4 m

25. 1.81
27. (a) three (b) 0°, 45.2°, 45.2°
29. 74.2 grooves/mm
31.
33. 514 nm
35. (a) 3.53 rulings/cm (b) 11
37. (a) 5.23 m (b) 4.58
39. 0.093 4 nm
41. (a) 0.109 nm (b) four
43. (a) 54.7° (b) 63.4° (c) 71.6°
45. 0.375
47. (a) six (b) 7.50°
49. 60.5°
51. 6.89 units
53. (a) 0.045 0 (b) 0.016 2
55. 5.51 m, 2.76 m, 1.84 m
57. 632.8 nm
59. (a) 7.26 rad, 1.50 arc seconds (b) 0.189 ly (c) 50.8 rad (d) 1.52 mm
61. (a) 25.6° (b) 18.9°
63. 545 nm
65. 13.7°
67. 15.4
69. (b) 3.77 nm/cm
71. (a) 4.49 compared with the prediction from the approximation of 1.5 4.71 (b) 7.73 compared with the prediction from the approximation of 2.5 7.85
73. (b) 0.001 90 rad 0.109°
75. (b) 15.3
77. (a) 41.8° (b) 0.592 (c) 0.262 m

Chapter 39

Answers to Quick Quizzes

(c)
2. (d)
3. (d)
4. (a)
5. (a)
6. (c)
(d)
8. **(i)** (c) **(ii)** (a)
9. (a) (b) (c)

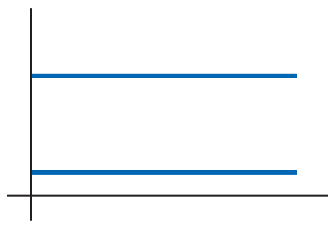
Answers to Odd-Numbered Problems

10.0 m/s toward the left in Figure P39.1
3. 5.70 degrees or 9.94 rad
5. 0.917
0.866
9. 0.866
11. 0.220
13. 5.00 s
15. The trackside observer measures the length to be 31.2 m, so the supertrain is measured to fit in the tunnel, with 18.8 m to spare.
17. (a) 25.0 yr (b) 15.0 yr (c) 12.0 ly
19. 0.800
21. (b) 0.050 4
23. (c) 2.00 kHz (d) 0.075 m/s 0.17 mi/h
25. 1.55 ns
27. (a) 2.50 m/s (b) 4.98 m (c) 1.33
29. (a) 17.4 m (b) 3.30°
31. Event B occurs first, 444 ns earlier than A
33. 0.357
35. 0.998 toward the right
37. (a) 0.943 2.83 m/s
(b) The result would be the same.
39. (a) 929 MeV/ (b) 6.58 MeV/ (c) No
41. 4.51
43. 0.285
45. (a) 3.07 MeV (b) 0.986
47. (a) 938 MeV (b) 3.00 GeV (c) 2.07 GeV
49. (a) 5.37 335 MeV
(b) 1.33 8.31 GeV
51. 1.63 MeV/
53. (a) smaller (b) 3.18 kg
(c) It is too small a fraction of 9.00 g to be measured.
55. 4.28 kg/s
57. (a) 8.63 J (b) 9.61
59. (a) 0.979 (b) 0.065 2 (c) 15.0
(d) 0.999 999 97 ; 0.948 ; 1.06
61. (a) 4.08 MeV (b) 29.6 MeV
63. 2.97
65. (a) 2.66 m (b) 3.87 km/s (c) 8.35
(d) 5.29 (e) 4.46
67. 0.712%
69. (a) 13.4 m/s toward the station and 13.4 m/s away from the station. (b) 0.056 7 rad/s
71. (a) 1.12 (b) 6.00 27
(c) $2.17
73. (a) 21.0 yr (b) 14.7 ly (c) 10.5 ly (d) 35.7 yr
75. (a) 6.67 (b) 1.97 h
77. (a) or 10 s (b)
79. (a) 0.905 MeV (b) 0.394 MeV
(c) 0.747 MeV/ 3.99 kg m/s (d) 65.4°
81. (b) 1.48 km
83. (a) 0.946 (b) 0.160 ly (c) 0.114 yr (d) 7.49
85. (a) 229 s (b) 174 s
87. 1.83
91. (a) 0.800 (b) 7.51 s (c) 1.44 m (d) 0.385
(e) 4.88

Chapter 40

Answers to Quick Quizzes

(b)
2. Sodium light, microwaves, FM radio, AM radio.
3. (c)
4. The classical expectation (which did not match the experiment) yields a graph like the following drawing:



5. (d)
6. (c)

(b)
8. (a)

Answers to Odd-Numbered Problems

6.85 m, which is in the infrared region of the spectrum
3. (a) lightning: m; explosion: m (b) lightning: ultraviolet; explosion: x-ray and gamma ray
5. 5.71 photons/s
(a) 2.99 K (b) 2.00
9. 5.18
11. 1.30 photons/s
13. (a) 0.263 kg (b) 1.81 W (c) 0.015 3°C/s 0.919°C/min (d) 9.89 m (e) 2.01 J (f) 8.99 photon/s
15. 1.34 31
17. (a) 295 nm, 1.02 PHz (b) 2.69 V
19. (a) 1.89 eV (b) 0.216 V
21. (a) 1.38 eV (b) 3.34
23. 8.34
25. 1.04
27. 22.1 keV/ = 478 eV
29. 70.0°
31. (a) 43.0° (b) 0.601 MeV; 0.601 MeV/ 3.21 kg m/s (c) 0.279 MeV; 0.279 MeV/ 3.21 kg m/s
33. (a) 4.89 nm (b) 268 keV (c) 31.8 keV
35. (a) 0.101 nm (b) 80.8°
37. To have photon energy 10 eV or greater, according to this definition, ionizing radiation is the ultraviolet light, x-rays, and rays with wavelength shorter than 124 nm; that is, with frequency higher than 2.42 Hz.
39. (a) 1.66 27 kg m/s (b) 1.82 km/s
41. (a) 14.8 keV or, ignoring relativistic correction, 15.1 keV (b) 124 keV
43. 0.218 nm
45. (a) 3.91 10 (b) 20.0 GeV/ 1.07 10 kg m/s (c) 6.20 m (d) The wavelength is two orders of magnitude smaller than the size of the nucleus.
47. (a) γ where $\gamma =$ (b) 1.60
(c) no change (d) 2.00 (e) 1 (f)
49. (a) phase
(b) This is different from the speed at which the particle transports mass, energy, and momentum.
51. (a) 989 nm (b) 4.94 mm (c) No; there is no way to identify the slit through which the neutron passed. Even if one neutron at a time is incident on the pair of slits, an interference pattern still develops on the detector array. Therefore, each neutron in effect passes through both slits.
53. 105 V
55. within 1.16 mm for the electron, 5.28 m for the bullet
57.
61. 1.36 eV
63. (a) 19.8 m (b) 0.333 m
65. (a) 1.7 eV (b) 4.2 s (c) 7.3
67. (a) 2.82 m (b) 1.06 J (c) 2.87
69. (a) 8.72 10^{16} $\frac{\text{electrons}}{\text{cm}}$ (b) 14.0 mA/cm
(c) The actual current will be lower than that in part (b).
71. (a) 0.143 nm (b) This is the same order of magnitude as the spacing between atoms in a crystal
(c) Because the wavelength is about the same as the spacing, diffraction effects should occur.
73. (a) The Doppler shift increases the apparent frequency of the incident light. (b) 3.86 eV (c) 8.76 eV

Chapter 41

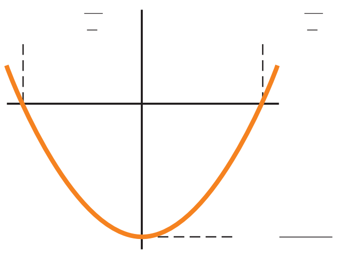
Answers to Quick Quizzes

(d)
2. **(i)** (a) **(ii)** (d)
3. (c)
4. (a), (c), (f)

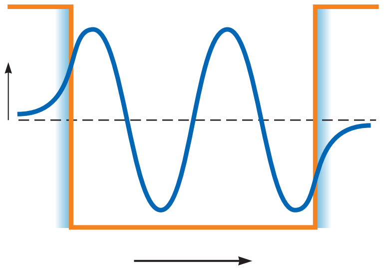
Answers to Odd-Numbered Problems

(a) 126 pm (b) 5.27 kg m/s (c) 95.3 eV
3. (a) (b) 0.037 0 (c) 0.750
5. (a) 0.511 MeV, 2.05 MeV, 4.60 MeV
(b) They do; the MeV is the natural unit for energy radiated by an atomic nucleus.
(a)

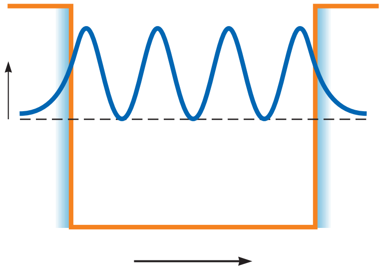
(b) 2.20 nm, 2.75 nm, 4.12 nm, 4.71 nm, 6.59 nm, 11.0 nm
9. 0.795 nm
11. (a) 6.14 MeV (b) 202 fm (c) gamma ray
13. (a) 0.434 nm (b) 6.00 eV
15. (a) (15 1/2 (b) 1.25
17. (a) (b) 0.409
19. (a) (b) 5.26 (c) 3.99
(d) In the 2 graph in the text's Figure 41.4b, it is more probable to find the particle either near /4 or /4 than at the center, where the probability density is zero. Nevertheless, the symmetry of the distribution means that the average position is /2.
21. (a) 0.196 (b) The classical probability is 0.333, which is significantly larger.
(c) 0.333 for both classical and quantum models
23. (a) 0.196 (b) 0.609
25. (b)
27. (a) mL
(b)



29. (a)



(b)



31. (a) 0.010 3 (b) 0.990
33. 85.9
35. 3.92%
37. 600 nm

39. (a) (b)

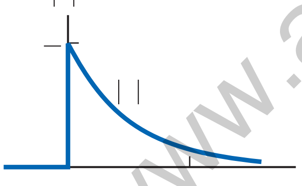
43. (a) 2.00 m (b) 3.31 kg m/s (c) 0.171 eV
45. 0.250
47. (a) 0.903 (b) 0.359 (c) 0.417 (d) 10 $^{6.59}$
49. (a) 435 THz (b) 689 nm (c) 165 peV or more

51. (a) (b)

53. (a) nhc mc mc (b) 4.68

(c) 28.6% larger

55. (a)



(b) 0 (d) 0.865

57. (a) 0 (b) 0 (c)

59. (b) 0.092 0 (c) 0.908
61. (a) 0.200 (b) 0.351 (c) 0.376 eV (d) 1.50 eV

63. (a) (b) 0 (c) = ± (d)

(e) 0 (f)

Chapter 42

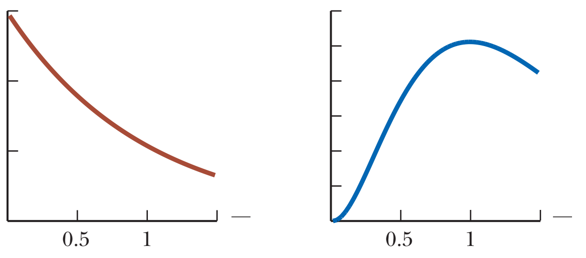
Answers to Quick Quizzes

(c)
2. (a)
3. (b)
4. (a) five (b) nine
5. (c)
6. true

Answers to Odd-Numbered Problems

(a) 121.5 nm, 102.5 nm, 97.20 nm (b) ultraviolet
3. 1.94
5. (a) 5 (b) no (c) no
(a) 5.69 m (b) 11.3 N
9. (a) 13.6 eV (b) 1.51 eV
11. (a) 0.968 eV (b) 1.28 m (c) 2.34
13. (a) 2.19 m/s (b) 13.6 eV (c) 27.2 eV
15. (a) 2.89 kg · m/s (b) 2.74 (c) 7.30
17. (a) 0.476 nm (b) 0.997 nm
19. (a) 3 (b) 520 km/s
21. (a) 54.4 eV/ for 1, 2, 3, .

(b) 54.4 eV
23. (b) 0.179 nm
25.



27.
29. 797
31.
33. (a) 2.58 10 ^{34} J
(b) 3.65 10 ^{34} J

35.
37. 2.58 10 ^{34} J
39. 3; 2; 2, 1, 0, 1, or 2; 1; 1, 0, or 1, for a total of 15 states
41. (a) 1
(b) ℓ ℓ
43. aluminum
45. (a) 30 (b) 36
47. 18.4 T
49. 17.7 kV
51. (a) 14 keV (b) 8.8
53. (a) If 2, then 2, 1, 0, 1, 2; if 1, then 1, 0, 1; if 0, then 0. (b) 6.05 eV
55. 0.068 nm
57. gallium
59. (a) 28.3 THz (b) 10.6 m (c) infrared
61. 3.49 photons
63. (a) 4.24 W/m (b) 1.20
65. (a) 3.40 eV (b) 0.136 eV
67. (a) 1.57 $^{3/2}$ (b) 2.47 ^{28}
(c) 8.69
69. 9.80 GHz
71. between 10 K and 10 K; use Equation 21.19 and set the kinetic energy equal to typical ionization energies
73. —, no
75. (a) 609 eV (b) 6.9 eV (c) 147 GHz (d) 2.04 mm
77. 0.866
79. (a) 486 nm (b) 0.815 m/s
81. (a)
(b)
(c) 0, , and (d)
(e) where 0.191
83. (a) 4.20 mm (b) 1.05 photons
(c) 8.84
85. mL
87. 0.125
89. (a) 0.106 , where is in nanometers and 1, 2, 3, . . (b) $= -\frac{6.80}{}$ where is in electron volts and 1, 2, 3, .
91. The classical frequency is 4

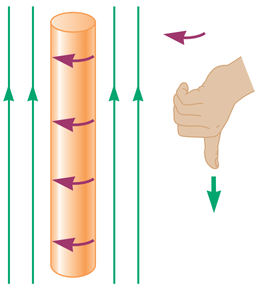
Chapter 43

Answers to Quick Quizzes

(a) van der Waals (b) ionic (c) hydrogen (d) covalent
2. (c)
3. (a)
4. A: semiconductor; B: conductor; C: insulator

Answers to Odd-Numbered Problems

10 K
3. 4.3 eV
5. (a) 74.2 pm (b) 4.46 eV
(a) 1.46 10 ^{46} kg m (b) The results are the same, suggesting that the molecule's bond length does not change measurably between the two transitions.
9. 9.77 rad/s
11. (a) 0.014 7 eV (b) 84.1
13. (a) 12.0 pm (b) 9.22 pm
15. (a) 2.32 kg (b) 1.82 kg (c) 1.62 cm
17. (a) 0, 3.62 eV, 1.09 eV
(b) 0.097 9 eV, 0.294 eV, 0.490 eV
19. (a) 472 m (b) 473 m (c) 0.715
21. (a) 4.60 kg (b) 1.32 Hz (c) 0.074 1 nm
23. 6.25
25. 7.83 eV
27. 5.28 eV
29.
31. (a) 4.23 eV (b) 3.27
33. (a) 2.54 ^{28} (b) 3.15 eV
35. 0.939
41. (a) 276 THz (b) 1.09
43. 1.91 eV
45. 227 nm
47. (a) 59.5 mV (b) 59.5 mV
49. 4.18 mA
51. (a)



(b) 10.7 kA
53. 203 A to produce a magnetic field in the direction of the original field
55.

57. 5.24 J/g
61. (a) 0.350 nm (b) 7.02 eV (c) 1.20
63. (a) 6.15 Hz (b) 1.59 ^{46} kg
(c) 4.78 m or 4.96

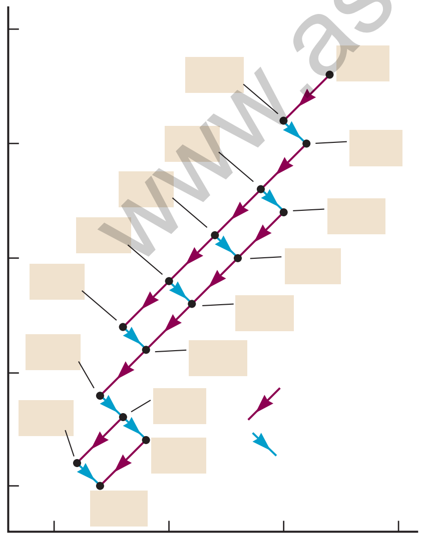
Chapter 44

Answers to Quick Quizzes

(i) (b) **(ii)** (a) **(iii)** (c)
2. (e)
3. (b)
4. (c)

Answers to Odd-Numbered Problems

(a) 1.5 fm (b) 4.7 fm (c) 7.0 fm (d) 7.4 fm
3. (a) 455 fm (b) 6.05 m/s
5. (a) 4.8 fm (b) 4.7 (c) 2.3 kg/m
16 km
9. 8.21 cm
11. (a) 27.6 N (b) 4.16 ^{27} m/s (c) 1.73 MeV
13. 6.1 N toward each other
15. (a) 1.11 MeV (b) 7.07 MeV (c) 8.79 MeV (d) 7.57 MeV
17. greater for N by 3.54 MeV
19. (a) ^{139}Cs (b) ^{139}La (c) ^{139}
21. 7.93 MeV
23. (a) 491 MeV (b) term 1: 179%; term 2: 53.0%; term 3: 24.6%; term 4: 1.37%
25. 86.4 h
27. 1.16
29. 9.47 nuclei
31. (a) 0.086 2 d 3.59 9.98
(b) 2.37 nuclei (c) 0.200 mCi
33. 1.41
35. (a) cannot occur (b) cannot occur (c) can occur
37. 0.156 MeV
39. 4.27 MeV
41. (a) e (b) 2.75 MeV
43. (a) 148 Bq/m (b) 7.05 atoms/m (c) 2.17
45.



47. 1.02 MeV
49. (a) ^{21}Ne (b) ^{144}Xe (c) e
51. 8.005 3 u; 10.013 5 u
53. (a) 29.2 MHz (b) 42.6 MHz (c) 2.13 kHz
55. 46.5 d
57. (a) 2.7 fm (b) 1.5 N (c) 2.6 MeV
(d) 7.4 fm; 3.8 N; 18 MeV
59. 2.20
61. (a) smaller (b) 1.46 u (c) 1.45 % (d) no
63. (a) 2.52 (b) 2.29 Bq (c) 1.07
65. 5.94 Gyr
67. (b) 1.95
69. 0.401%
71. (a) Mo (b) electron capture: all levels; e emission: only 2.03 MeV, 1.48 MeV, and 1.35 MeV
73. (b) 1.16 u
75. 2.66 d

Chapter 45

Answers to Quick Quizzes

(b)
2. (a), (b)
3. (a)
4. (d)

Answers to Odd-Numbered Problems

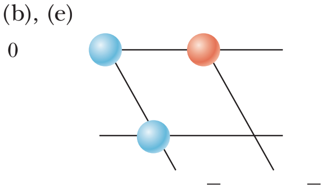
1.1 fissions
3. ^{144}Xe, ^{143}Xe, and ^{142}
5. ^{232}Th Th; Th Pa
Pa
126 MeV
9. 184 MeV
11. 5.58
13. 2.68
15. 26 MeV
17. (a) 3.08 g (b) 1.31 mol (c) 7.89 ^{31} nuclei
(d) 2.53 ^{21} J (e) 5.34 yr (f) Fission is not sufficient to supply the entire world with energy at a price of $130 or less per kilogram of uranium.
19. 1.01 g
21. (a) Be (b) C (c) 7.27 MeV
23. 5.49 MeV
25. (a) 31.9 g/h (b) 123 g/h
27. (a) 2.61 ^{31} J (b) 5.50
29. (a) 2.23 m/s (b)
31. (a) 10 (b) 1.2 J/m (c) 1.8 T
33. (a) 0.436 cm (b) 5.79 cm
35. (a) 10.0 h (b) 3.16 m
37. 2.39 °C, which is negligible
39. 1.66
41. (a) 421 MBq (b) 153 ng
43. (a) 0.963 mm (b) It increases by 7.47%.
45. (a) atoms (b)
47. 1.01 MeV
49. (a) 1.5 nuclei (b) 0.6 kg
51. (a) 3.12 (b) 3.12 electrons
53. (a) 1.94 MeV, 1.20 MeV, 7.55 MeV, 7.30 MeV, 1.73 MeV, 4.97 MeV (b) 1.02 MeV (c) 26.7 MeV
(d) Most of the neutrinos leave the star directly after their creation, without interacting with any other particles.
55. 69.0 W
57. 2.57
59. (b) 26.7 MeV

61. (a) 5.67 K (b) 120 kJ
63. 14.0 MeV or, ignoring relativistic correction, 14.1 MeV
65. (a) 3.4 Ci, 16 Ci, 3.1 Ci (b) 50%, 2.3%, 47% (c) It is dangerous, notably if the material is inhaled as a powder. With precautions to minimize human contact, however, microcurie sources are routinely used in laboratories.
67. (a) 8 eV (b) 4.62 MeV and 13.9 MeV (c) 1.03 kWh
69. (a) 4.92 kg/h → 4.92 /h (b) 0.141 kg/h
71. 4.44 kg/h
73. (a) 10 electrons (b) 10 (c) 10

Chapter 46

Answers to Quick Quizzes

(a)
2. **(i)** (c), (d) **(ii)** (a)
3. (b), (e), (f)
4. (b), (e)
5.



6. false

Answers to Odd-Numbered Problems

(a) 5.57 J (b) $1.70
3. (a) 4.54 Hz (b) 6.61
5. 118 MeV
(b) The range is inversely proportional to the mass of the field particle. (c)
9. (a) 67.5 MeV (b) 67.5 MeV/ (c) 1.63
11. (a) muon lepton number and electron lepton number (b) charge (c) angular momentum and baryon number (d) charge (e) electron lepton number
13. (a) ⁻ (b) (c) ⁻ (d) (e) (f) ⁻ + ν
15. (a) It cannot occur because it violates baryon number conservation. (b) It can occur. (c) It cannot occur because it violates baryon number conservation. (d) It can occur. (e) It can occur. (f) It cannot occur because it violates baryon number conservation, muon lepton number conservation, and energy conservation.
17. 0.828
19. (a) 37.7 MeV (b) 37.7 MeV (c) 0 (d) No. The mass of the meson is much less than that of the proton, so it carries much more kinetic energy. The correct analysis using relativistic energy conservation shows that the kinetic energy of the proton is 5.35 MeV, while that of the meson is 32.3 MeV.
21. (a) It is not allowed because neither baryon number nor angular momentum is conserved. (b) strong interaction (c) weak interaction (d) weak interaction (e) electromagnetic interaction
23. (a) K (scattering event) (b) (c)
25. (a) Strangeness is not conserved. (b) Strangeness is conserved. (c) Strangeness is conserved. (d) Strangeness is not conserved. (e) Strangeness is not conserved. (f) Strangeness is not conserved.
27. 9.25 cm
33. (a) (b) 0 (c) antiproton; antineutron
35. The unknown particle is a neutron, udd.
39. (a) 1.06 mm (b) microwave
41. (a) K (b)
43. 7.73
45. (a) 0.160 (b) 2.18
47. (a) 590.09 nm (b) 599 nm (c) 684 nm
49. 6.00
51. (a) Charge is not conserved. (b) Energy, muon lepton number, and electron lepton number are not conserved. (c) Baryon number is not conserved.
53. 0.407%
55.
59. 1.12 GeV/
61. (a) electron–positron annihilation; e (b) A neutrino collides with a neutron, producing a proton and a muon; W
63.
65. neutron
67. 5.35 MeV and 32.3 MeV
69. (a) 0.782 MeV (b) 0.919 382 km/s (c) The electron is relativistic; the proton is not.
71. (b) 9.08 Gyr
73. (a) 2*Nmc* (b) ⁻*Nmc* (c) method (a)

Index

Locator note: **boldface** indicates a definition; *italics* indicates a figure; *t* indicates a table

Conversions

Length

1 in. = 2.54 cm (exact)
1 m = 39.37 in. = 3.281 ft
1 ft = 0.304 8 m
12 in. = 1 ft
3 ft = 1 yd
1 yd = 0.914 4 m
1 km = 0.621 mi
1 mi = 1.609 km
1 mi = 5 280 ft
1 μm = 10^{-6} m = 10^3 nm
1 light-year = 9.461×10^{15} m

Area

1 m^2 = 10^4 cm^2 = 10.76 ft^2
1 ft^2 = 0.092 9 m^2 = 144 $in.^2$
1 $in.^2$ = 6.452 cm^2

Volume

1 m^3 = 10^6 cm^3 = 6.102×10^4 $in.^3$
1 ft^3 = 1 728 $in.^3$ = 2.83×10^{-2} m^3
1 L = 1 000 cm^3 = 1.057 6 qt = 0.035 3 ft^3
1 ft^3 = 7.481 gal = 28.32 L = 2.832×10^{-2} m^3
1 gal = 3.786 L = 231 $in.^3$

Mass

1 000 kg = 1 t (metric ton)
1 slug = 14.59 kg
1 u = 1.66×10^{-27} kg = 931.5 MeV/c^2

Force

1 N = 0.224 8 lb
1 lb = 4.448 N

Velocity

1 mi/h = 1.47 ft/s = 0.447 m/s = 1.61 km/h
1 m/s = 100 cm/s = 3.281 ft/s
1 mi/min = 60 mi/h = 88 ft/s

Acceleration

1 m/s^2 = 3.28 ft/s^2 = 100 cm/s^2
1 ft/s^2 = 0.304 8 m/s^2 = 30.48 cm/s^2

Pressure

1 bar = 10^5 N/m^2 = 14.50 lb/$in.^2$
1 atm = 760 mm Hg = 76.0 cm Hg
1 atm = 14.7 lb/$in.^2$ = 1.013×10^5 N/m^2
1 Pa = 1 N/m^2 = 1.45×10^{-4} lb/$in.^2$

Time

1 yr = 365 days = 3.16×10^7 s
1 day = 24 h = 1.44×10^3 min = 8.64×10^4 s

Energy

1 J = 0.738 ft · lb
1 cal = 4.186 J
1 Btu = 252 cal = 1.054×10^3 J
1 eV = 1.602×10^{-19} J
1 kWh = 3.60×10^6 J

Power

1 hp = 550 ft · lb/s = 0.746 kW
1 W = 1 J/s = 0.738 ft · lb/s
1 Btu/h = 0.293 W

Some Approximations Useful for Estimation Problems

1 m ≈ 1 yd
1 kg ≈ 2 lb
1 N ≈ $\frac{1}{4}$ lb
1 L ≈ $\frac{1}{4}$ gal
1 m/s ≈ 2 mi/h
1 yr ≈ $\pi \times 10^7$ s
60 mi/h ≈ 100 ft/s
1 km ≈ $\frac{1}{2}$ mi

Note: See Table A.1 of Appendix A for a more complete list.

The Greek Alphabet

Alpha	Α	α	Iota	Ι	ι	Rho	Ρ	ρ
Beta	Β	β	Kappa	Κ	κ	Sigma	Σ	σ
Gamma	Γ	γ	Lambda	Λ	λ	Tau	Τ	τ
Delta	Δ	δ	Mu	Μ	μ	Upsilon	Υ	υ
Epsilon	Ε	ϵ	Nu	Ν	ν	Phi	Φ	ϕ
Zeta	Ζ	ζ	Xi	Ξ	ξ	Chi	Χ	χ
Eta	Η	η	Omicron	Ο	o	Psi	Ψ	ψ
Theta	Θ	θ	Pi	Π	π	Omega	Ω	ω

Standard Abbreviations and Symbols for Units

Symbol	Unit	Symbol	Unit
A	ampere	K	kelvin
u	atomic mass unit	kg	kilogram
atm	atmosphere	kmol	kilomole
Btu	British thermal unit	L	liter
C	coulomb	lb	pound
°C	degree Celsius	ly	light-year
cal	calorie	m	meter
d	day	min	minute
eV	electron volt	mol	mole
°F	degree Fahrenheit	N	newton
F	farad	Pa	pascal
ft	foot	rad	radian
G	gauss	rev	revolution
g	gram	s	second
H	henry	T	tesla
h	hour	V	volt
hp	horsepower	W	watt
Hz	hertz	Wb	weber
in.	inch	yr	year
J	joule	Ω	ohm

Mathematical Symbols Used in the Text and Their Meaning

Symbol	Meaning
$=$	is equal to
$\equiv$	is defined as
$\neq$	is not equal to
$\propto$	is proportional to
$\sim$	is on the order of
$>$	is greater than
$<$	is less than
$\gg (\ll)$	is much greater (less) than
$\approx$	is approximately equal to
Δx	the change in x
$\sum_{i=1}^{N} x_i$	the sum of all quantities x_i from $i = 1$ to $i = N$
$\|x\|$	the absolute value of x (always a nonnegative quantity)
$\Delta x \rightarrow 0$	Δx approaches zero
$\frac{dx}{dt}$	the derivative of x with respect to t
$\frac{\partial x}{\partial t}$	the partial derivative of x with respect to t
$\int$	integral