PROBLEMS 12

(1) See slides

(2)

The bug moves in SHM as the wave passes. The maximum KE of a particle in SHM is the total energy, which is given by $E_{\text{total}} = \frac{1}{2}kA^2$. Compare the two KE maxima.

$$\frac{KE_2}{KE_1} = \frac{\frac{1}{2}kA_2^2}{\frac{1}{2}kA_1^2} = \left(\frac{A_2}{A_1}\right)^2 = \left(\frac{2.25\text{ cm}}{3.0\text{ cm}}\right)^2 = \boxed{0.56}$$

(3)

The period of the jumper's motion is $T = \dfrac{38.0\text{ s}}{8\text{ cycles}} = 4.75\text{ s}$. The spring constant can then be found from the period and the jumper's mass.

$$T = 2\pi\sqrt{\frac{m}{k}} \quad \rightarrow \quad k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2(65.0\text{ kg})}{(4.75\text{ s})^2} = 113.73\text{ N/m} \approx \boxed{114\text{ N/m}}$$

The stretch of the bungee cord needs to provide a force equal to the weight of the jumper when he is at the equilibrium point.

$$k\Delta x = mg \quad \rightarrow \quad \Delta x = \frac{mg}{k} = \frac{(65.0\text{ kg})(9.80\text{ m/s}^2)}{113.73\text{ N/m}} = 5.60\text{ m}$$

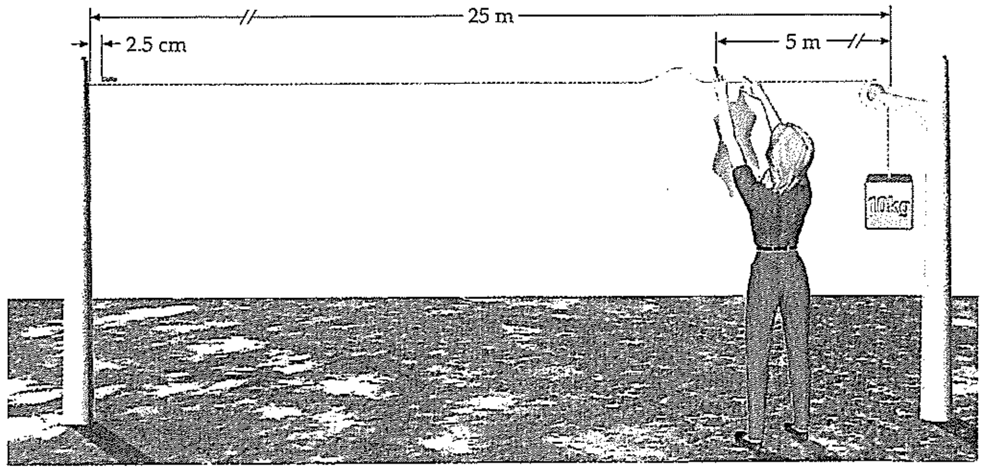
Thus the unstretched bungee cord must be $25.0\text{ m} - 5.60\text{ m} = \boxed{19.4\text{ m}}$

(4)

The unusual decrease of water corresponds to a trough ~~in Figure 11-24~~. The crest or peak of the wave is then one-half wavelength distant. The peak is 125 km away, traveling at 750 km/hr.

$$\Delta x = vt \quad \rightarrow \quad t = \frac{\Delta x}{v} = \frac{125\text{ km}}{750\text{ km/hr}}\left(\frac{60\text{ min}}{1\text{ hr}}\right) = \boxed{10\text{ min}}$$

(5)



1. The speed of the pulse is related to the tension F_T and mass density μ: $v = \sqrt{\frac{F_T}{\mu}}$

2. Express the mass density and tension in terms of the given parameters: $\mu = \frac{m_s}{L}$ and $F_T = mg$

3. Substitute these values to calculate the speed: $v = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{mgL}{m_s}} = \sqrt{\frac{(10\text{ kg})(9.81\text{ m/s}^2)(25\text{ m})}{1.0\text{ kg}}}$ $= 49.5\text{ m/s}$

4. Use this speed to find the time for the pulse to travel the 20 m to the far end: $\Delta t = \frac{\Delta x}{v} = \frac{20\text{ m}}{49.5\text{ s}} = 0.40\text{ s}$

5. Find the time it takes Inchy to travel the 2.5 cm to the end traveling at 1.0 in/s: $\Delta t' = \frac{\Delta x'}{v'} = \frac{2.5\text{ cm}}{1\text{ in/s}} \times \frac{1\text{ in}}{2.54\text{ cm}} = 0.98\text{ s}$

$\Delta t' > \Delta t$ Inchy does not beat the pulse.

(6)

We assume that the comparison is to be made from the frame of reference of the stationary tuba. The stationary observers would observe a frequency from the moving tuba of

$$f_{obs} = \frac{f_{source}}{\left(1-\frac{v_{source}}{v_{snd}}\right)} = \frac{75\text{ Hz}}{\left(1-\frac{10.0\text{ m/s}}{343\text{ m/s}}\right)} = 77\text{ Hz} \qquad f_{beat} = 77\text{ Hz} - 75\text{ Hz} = \boxed{2\text{ Hz}}.$$

(7) See Slides

(8)

The Doppler effect occurs only when there is relative motion of the source and the observer along the line connecting them. In the first four parts of this problem, the whistle and the observer are not moving relative to each other and so there is no Doppler shift. The wind speed increases (or decreases) the velocity of the waves in the direction of the wind, and the wavelength of the waves by the same factor, while the frequency is unchanged.

(*a*), (*b*), (*c*), (*d*) $\boxed{f' = f = 570\text{ Hz}}$

(*e*) The wind makes an effective speed of sound in air of 343 + 12.0 = 355 m/s, and the observer is moving towards a stationary source with a speed of 15.0 m/s.

$$f' = f\left(1 + \frac{v_{obs}}{v_{sns}}\right) = (570\text{ Hz})\left(1 + \frac{15.0\text{ m/s}}{355\text{ m/s}}\right) = \boxed{594\text{ Hz}}$$

(*f*) Since the wind is not changing the speed of the sound waves moving towards the cyclist, the speed of sound is 343 m/s. The observer is moving towards a stationary source with a speed of 15.0 m/s.

$$f' = f\left(1 + \frac{v_{obs}}{v_{sns}}\right) = (570\text{ Hz})\left(1 + \frac{15.0\text{ m/s}}{343\text{ m/s}}\right) = \boxed{595\text{ Hz}}$$

(9)

. both source and observer are in motion. There will be two Doppler shifts in this problem – first for the emitted sound with the bat as the source and the moth as the observer, and then the reflected sound with the moth as the source and the bat as the observer.

$$f'_{moth} = f_{bat}\frac{(v_{snd} + v_{moth})}{(v_{snd} - v_{bat})} \qquad f''_{bat} = f'_{moth}\frac{(v_{snd} + v_{bat})}{(v_{snd} - v_{moth})} = f_{bat}\frac{(v_{snd} + v_{moth})}{(v_{snd} - v_{bat})}\frac{(v_{snd} + v_{bat})}{(v_{snd} - v_{moth})}$$

$$= (51.35\text{ kHz})\frac{(343 + 5.0)}{(343 - 6.5)}\frac{(343 + 6.5)}{(343 - 5.0)} = \boxed{54.9\text{ kHz}}$$

(10)

It is 70.0 ms from the start of one chirp to the start of the next. Since the chirp itself is 3.0 ms long, it is 67.0 ms from the end of a chirp to the start of the next. Thus the time for the pulse to travel to the moth and back again is 67.0 ms. The distance to the moth is half the distance that the sound can travel in 67.0 ms, since the sound must reach the moth and return during the 67.0 ms.

$$d = v_{snd}t = (343\text{ m/s})\tfrac{1}{2}(67.0\times10^{-3}\text{ s}) = \boxed{11.5\text{ m}}$$

(11)

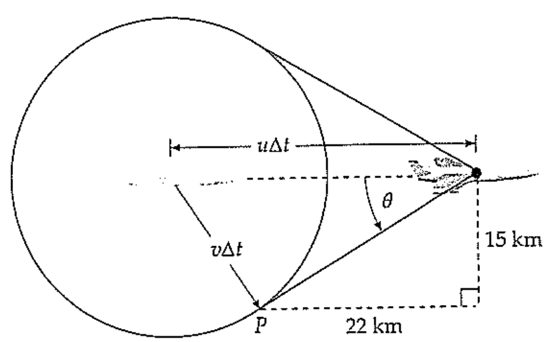


FIGURE 15-29 In the time that the plane moves distance $u\,\Delta t$, the sound moves distance $v\,\Delta t$.

Answers

1. Sketch the position of the plane (Figure 15-29) both at the instant the sonic boom is heard at point P and at the instant that sound was produced. Label the distance the sound travels $v\,\Delta t$, and the distance the plane travels $u\,\Delta t$.

2. From your sketch and Equation 15-44, calculate u:

$$\tan\theta = \frac{15\text{ km}}{22\text{ km}} \quad \text{so} \quad \theta = 34.3°$$

$$\sin\theta = \frac{v\,\Delta t}{u\,\Delta t} = \frac{v}{u} \quad \text{so} \quad u = \frac{v}{\sin\theta} = 609\text{ m/s} = \boxed{610\text{ m/s}}$$

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(a) 1. The given wave function is of the form $y(x,t) = A\sin(kx - \omega t)$. Using $\omega = kv$ (Equation 15-16), write the wave function as a function of $x - vt$. Then, use Equations 15-1 and 15-2 to find the direction of travel:

$y(x,t) = A\sin(kx - \omega t)$ and $\omega = kv$

so $y(x,t) = A\sin(kx - kvt) = A\sin[k(x - vt)]$

The wave travels in the $\boxed{+x \text{ direction.}}$

2. Because the form is $y = A\sin(kx - \omega t)$, we know A as well as both ω and k. Use these to calculate the speed:

$$v = \frac{\lambda}{T} = \frac{\lambda}{2\pi}\frac{2\pi}{T} = \frac{\omega}{k} = \frac{3.5\ \text{s}^{-1}}{2.2\ \text{m}^{-1}} = 1.59\ \text{m/s}$$

$$= \boxed{1.6\ \text{m/s}}$$

(b) The wavelength λ is related to the wave number k, and the period T and frequency f are related to ω:

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{2.2\ \text{m}^{-1}} = 2.86\ \text{m} = \boxed{2.9\ \text{m}}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{3.5\ \text{s}^{-1}} = 1.80\ \text{s} = \boxed{1.8\ \text{s}}$$

$$f = \frac{1}{T} = \frac{1}{1.80\ \text{s}} = 0.557\ \text{Hz} = \boxed{0.56\ \text{Hz}}$$

(c) The maximum displacement of a string segment is the amplitude A:

$$A = \boxed{0.030\ \text{m}}$$

(d) 1. Compute $\partial y/\partial t$ to find the velocity of a point on the string:

$$v_y = \frac{\partial y}{\partial t} = (0.030\ \text{m})\frac{\partial[\sin(2.2\ \text{m}^{-1}x - 3.5\ \text{s}^{-1}t)]}{\partial t}$$

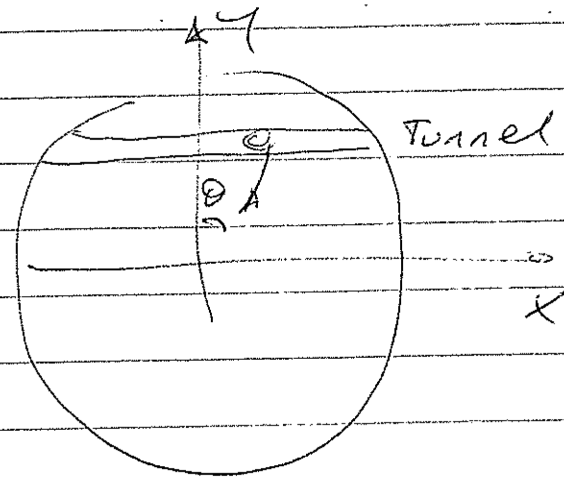
$$= (0.030\ \text{m})(-3.5\ \text{s}^{-1})\cos(2.2\ \text{m}^{-1}x - 3.5\ \text{s}^{-1}t)$$

$$= -(0.105\ \text{m/s})\cos(2.2\ \text{m}^{-1}x - 3.5\ \text{s}^{-1}t)$$

2. The maximum transverse speed occurs when the cosine function has the value of ± 1:

$$v_{y,\text{max}} = 0.105\ \text{m/s} = \boxed{0.11\ \text{m/s}}$$

(B)



The net force acting on the particle as it moves in the tunnel is the x component of the gravitational force acting on it. We can find the period of the particle from the angular frequency of its motion. We can apply Newton's second law to the particle in order to express ω in terms of the radius of the Earth and the acceleration due to gravity at the surface of the Earth

(a) From the figure we see that

$$F_x = F_r \sin\theta = -\frac{G M_\oplus m}{R_\oplus^3} r \frac{x}{r}$$

$$= -\frac{G m M_\oplus x}{R_\oplus^3}$$

Because this force is a linear restoring force, the motion of the particle is a simple harmonic motion.

(b) Express the period of the particle as a function of its angular frequency

$$T = \frac{2\pi}{\omega}$$

Apply $\sum F_x = m a_x$ to the particle

Solve for a

$$a = -\frac{GM_\oplus x}{R_\oplus^3} = -\omega^2 x$$

where

$$\omega = \sqrt{\frac{GM_\oplus}{R_\oplus^3}}$$

Use $GM_\oplus = g R_\oplus^2$ to simplify ω

$$\omega = \sqrt{\frac{g R_\oplus^2}{R_\oplus^3}} = \sqrt{\frac{g}{R_\oplus}}.$$

Substitute in eq (1) to obtain

$$T = \frac{2\pi}{\sqrt{\frac{g}{R_\oplus}}} = 2\pi \sqrt{\frac{R_\oplus}{g}} = 5.06 \times 10^3 \, s \approx 84.4 \, \text{min}$$

$T \approx 84.4$ min