PROBLEMS # 13

(1) $T = t_c + 273.15$

So $92 = t_c + 273.15 \Rightarrow t_c = -181.15°C$

$t_c = \frac{5}{9}(t_F - 32°)$

So $-181.15°C = \frac{5}{9}(t_F - 32°)$

$\Rightarrow t_F = -294°F$

(2) Take the 300 m height to be the height in January. Then the increase in the height of the tower is

$\Delta L = \alpha L_0 \Delta T = 12 \times 10^{-6} \frac{1}{°C} \, 300\,m \,(25°C - 2°C)$

$= 8 \times 10^{-2}\,m$

(3) There are three forces to consider: the buoyant force upwards (which is the weight of the cold air displaced by the volume of the balloon), the downward weight of the hot air in the balloon, and the downward weight of the passengers and equipment. For the balloon to rise at constant speed, the buoyant force must equal the two weights

$$F_{buoyant} = m_{hot}\, g + 2700\ N$$

$$\Rightarrow \quad V \rho_{cold}\, g = V \rho_{hot}\, g + 2700\ N$$

The ideal gas law can be written in terms of the gas density ρ and the molecular mass M as follows:

$$PV = nRT = \frac{m}{M}RT \Rightarrow \frac{PM}{R} = \frac{m}{V}T = \rho T$$

The gas outside and inside the balloon is air, and so M is the same outside and inside. Also, since the balloon is open to the atmosphere, the pressure inside the balloon is the same as the pressure outside the balloon. Thus the ideal gas law reduces to

$$\rho T = \text{constant} = (\rho T)_{cold} = (\rho T)_{hot}$$

$$V \rho_{cold}\, g = V \rho_{hot}\, g + 2700\,N = V \rho_{cold} \frac{T_{cold}}{T_{hot}} g + 2700\,N$$

$$T_{hot} = \frac{V \rho_{cold} T_{cold}\, g}{(V \rho_{cold}\, g - 2700\,N)} = 309.8\,K \approx 37^{o}C$$

One factor limiting the maximum altitude would be that as the balloon rises, the density of the air decreases, and thus the temperature required gets higher. Eventually the air would be too hot and the balloon fabric might be damaged.

(4) The final volume of both gases is 30L. The initial temperatures of both gases are equal. Thus, we can use Boyle's Law to find the partial pressure of each gas in the mixture. Then, we use the law of partial pressures to find the pressure of the mixture: the pressure of the mixture is the sum of the partial

pressure of the two gases

$$P = P_{O_2} + P_{N_2}$$

$$P_i V_i = P_f V_f \Rightarrow P_f = \frac{V_i}{V_f} P_i$$

$$P_{O_2} = \frac{V_i}{V_f} P_i = \frac{20L}{30L} 0.3 \, P_{at} = 0.2 \, P_{at}$$

$$P_{N_2} = \frac{V_i}{V_f} P_i = \frac{30L}{30L} 0.6 \, P_{at} = 0.6 \, P_{at}$$

$$P = P_{O_2} + P_{N_2} = 0.2 \, P_{at} + 0.6 \, P_{at} = 0.8 \, P_{at}$$

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We can find the volume using the ideal gas law with $T = 273 \, K$

$$V = \frac{nRT}{P} = 22.4 \, L$$

(6) $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$

$P_2 = \frac{T_2 V_1}{T_1 V_2} P_1$

$T_1 = 273.15 + 30.0 = 303.15\,K$

$T_2 = 273.15 + 60.0 = 333.15\,K$

$P_2 = 1.47$ atm

(7) The average mass of a hydrogen atom is the molar mass divided by Avogadro's number

$m = \frac{M}{N_A} = \frac{1.008 \text{ g/mol}}{6.022 \times 10^{23} \text{ atoms/mol}} = 1.674 \times 10^{-24} \text{ g/atom}$

(8) Assume the helium is an ideal gas. Since the amount of gas is ~~[illegible]~~ constant, the value

$\frac{PV}{T}$ is constant. We assume that since the outside air pressure decreases by 30%, the air pressure inside the balloon will also decrease 30%.

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \Rightarrow \frac{V_2}{V_1} = \frac{P_1 T_2}{P_2 T_1} = 1.4$$

So it is 1.4 times the original volume.

(9) We assume that the last breath Galileo took has been spread uniformly through the atmosphere since his death. Calculate the number of molecules in Galileo last breath, and divide it by the volume of the atmosphere, to get "Galileo molecules/m^3".

Multiply that factor times the size of a breath to find the number of Galileo molecules in one of our breath.

$PV = NkT \Rightarrow N = \frac{PV}{kT} = 4.9 \times 10^{22}$ molecules

Atmospheric volume $= 4\pi R_{Earth}^2 h = 5.1 \times 10^{18}\ m^3$

$\frac{\text{Galileo molecules}}{m^3} = 9.6 \times 10^{3}$ molecules$/m^3$

$\frac{\text{\# Galileo molecules}}{\text{breath}} = 9.6 \times 10^{3} \frac{\text{molecules}}{m^3} \left(\frac{2 \times 10^{-3}\ m^3}{1\ \text{breath}} \right)$

$= 19 \frac{\text{molecules}}{\text{breath}}$

10 (a) $v_{RMS} = \sqrt{3kT/m} = 2.9 \times 10^{2}\ m/s$

(b) $v_{RMS} = \sqrt{3kT/m'} = 12\ m/s$

(11)

$$\frac{(v_{RMS})_{235_{UF_6}}}{(v_{RMS})_{238_{UF_6}}} = \frac{\sqrt{3RT/M_{235_{UF_6}}}}{\sqrt{3RT/M_{238_{UF_6}}}}$$

$$= \sqrt{\frac{M_{238_{UF_6}}}{M_{235_{UF_6}}}} = 1.00\cancel{6}$$

(12) (a)

$$v_{es} = \sqrt{2gR_\oplus}$$

$$v_{RMS},\, O_2 = \sqrt{3RT/M_{O_2}}$$

$$\Rightarrow \quad v_{RMS} = 0.15\, v_{es}$$

$$3RT/M_{O_2} = (0.15)^2\, 2gR_\oplus$$

$$T = \frac{(0.15)^2\, 2\, g\, R_\oplus\, M_{O_2}}{3R} \sim 3600\ K$$

$$R_\oplus = 6.378\ 10^6\ m$$

(b)
$$T = \frac{(0.15)^2 \, 2 \, g \, R_\oplus \, M_{H_2}}{3R} \simeq 225 \text{ K}$$

(C) The $v_{RMS} \gg v_{escape}$ at $T = 1000$ K

$$T_{O_2} = (0.15)^2 \frac{2}{6} \, g \frac{R_{moon} \, M_{O_2}}{3R} = 164 \text{ K}$$

$$T_{H_2} = (0.15)^2 \frac{2}{6} \, g \frac{R_{moon}}{3R} H_2 \approx 10 \text{ K}$$

The mean surface temperature of the moon is 107 °C during the day and −153 °C during night, so all molecules of O_2 and H_2 escape.

(13)

$$\lambda = \frac{1}{\sqrt{2}\, n_V \pi d^2}$$

$$n_V = \frac{N}{V} = \frac{P}{kT} = 2.451 \times 10^{25} \ \frac{\text{molecules}}{m^3}$$

$$\lambda = 6.53 \times 10^{-8} \, m$$

$$\tau = \frac{\lambda}{v_{av}}$$

$$v_{rms} = \sqrt{\frac{3RT}{M}} = 517.0 \, m/s$$

$$\tau = \frac{\lambda}{v_{av}} \simeq \frac{\lambda}{v_{rms}} = 1.26 \times 10^{-10} \, s$$

(14)

$$\sum \vec{F} = ma$$

$$\vec{F}_T + \vec{F}_B + \vec{F}_g = 0 \Rightarrow F_B = F_T + F_g$$

$$F_B = \rho_1 V g$$

where ρ_1 is the density of air near the balloon

$$F_g = \rho_2 V g$$

where ρ_2 is the density of air within the balloon

$$F_T = F_B - F_g$$

$$F_T = \rho_1 V g - \rho_2 V g = (\rho_1 - \rho_2) V g$$

The mass of a sample of air in the balloon is the number of moles n times the molar mass M of air

$$\rho_1 = \frac{m_1}{V} = \frac{n_1 M}{V}$$

$$\rho_2 = \frac{m_2}{V} = \frac{n_2 M}{\cancel{M}}$$

$$F_T = \left(\frac{n_1 M}{V} - \frac{n_2 M}{V}\right) V g \Rightarrow \frac{F_T}{Mg} = n_1 - n_2$$

$$\frac{F_T}{Mg} = \frac{P_1 V}{R T_1} - \frac{P_2 V}{R T_2}$$

$$P_2 = \left(\frac{P_1}{T_1} - \frac{F_T R}{M g V}\right) T_2$$

$$P_2 = 1.02 \times 10^5 \text{ Pa} = 1.01 \text{ atm}$$

(15)

$$v_{RMS} = \sqrt{3RT/M}$$

$\Rightarrow$

$$v_{RMS,H_2} = \sqrt{\frac{3\,(8.314\ J/mol\,K)\ (123\,K)}{2\times10^{-3}\ kg/mol}}$$

$= 1.24\ km/s$

$$v_{RMS,O_2} = \sqrt{\frac{3\,(8.314\ J/mol\,K)\ (123\,K)}{32\times10^{-3}\ kg/mol}}$$

$= 310\ m/s$

$$v_{RMS,CO_2} = \sqrt{\frac{3\,(8.314\ J/mol\,K)\ 123\,K}{44\times10^{-3}\ kg/mol}}$$

$= 246\ m/s$

20% of escape velocity

$$v = \frac{1}{5} v_{esc} = \frac{1}{5}\left(60\ km/s\right) = 12\ km/s$$

(16)

$v_{rms}(H_2) = 1.8$ km/s

$v_{rms}(O_2) = 461$ m/s

$v_{rms}(CO_2) = 393$ m/s

$v_{20\%} = 1$ km/s

Then O_2 and CO_2 should be found in the atmosphere of Mars, the molecules of H_2 will escape.

(17) The rms speed is

$$v_{RMS} = \sqrt{3kT/m}.$$

The temperature can be found from the ideal gas law

$PV = NkT \Rightarrow kT = PV/N$. The mass of the gas is the mass of a molecule times the number of molecules $M = Nm$, and the density of the gas is the mass per unit volume

$$\rho = \frac{M}{V}$$

Combining these relationships

$$v_{RMS} = \sqrt{3kT/m} = \sqrt{\frac{3PV}{Nm}} = \sqrt{\frac{3PV}{M}} = \sqrt{\frac{3P}{\rho}}$$

(18) The rms speed is

$$v_{RMS} = \sqrt{3kT/m}$$

$$\frac{(v_{RMS})_2}{(v_{RMS})_1} = \frac{\sqrt{3kT/m_2}}{\sqrt{3kT/m_1}} = \sqrt{\frac{m_1}{m_2}}$$

(19) The pressure can be stated in terms of the ideal gas law, $P = NkT/V$. Substitute for the temperature from the expression for the rms speed $v_{rms} = \sqrt{3kT/m} \Rightarrow T = m v_{rms}^2 / 3k$. The mass of the gas is the mass of a molecule times the number of molecules: $M = Nm$, and the density of the gas is the mass per unit volume, $\rho = M/V$. Combining these relationships

$$P = \frac{NkT}{V} = \frac{Nk}{V}\frac{m v_{rms}^2}{3k} = \frac{1}{3}\frac{Nm}{V} v_{rms}^2$$

$$= \frac{1}{3}\frac{M}{V} v_{rms}^2 = \frac{1}{3}\rho v_{rms}^2 \Rightarrow$$

$$P = \frac{1}{3}\rho v_{rms}^2$$