## Solutions of problems set \# 1

Physics 168

1. By measuring several people's arms we can estimate that a cubit is about a half of a meter. Hence, the dimension of Noah's ark would be 150 m long, 25 m wide, 15 m high. The volume is the about $5.6 \times 10^{4} \mathrm{~m}^{3}$.
2. See slides.
3. See slides.
4. By definition the instantaneous velocity is $v_{x}(t)=\lim _{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t}=\lim _{\Delta t \rightarrow 0} \frac{x(t+\Delta t)-x(t)}{\Delta t}$. Now, $x(t)=5 t^{2}$ and $x(t+\Delta t)=5(t+\Delta t)^{2}=5 t^{2}+10 t \Delta t+5 \Delta t^{2}$, so $\Delta x=10 t \Delta t+5 \Delta t^{2}$. Since $\left\langle v_{x}\right\rangle=\frac{\Delta x}{\Delta t}=10 t+5 \Delta t$, we obtain $v_{x}=\lim _{\Delta t \rightarrow 0}\left\langle v_{x}\right\rangle=10 t$.
5. Choose downward to be the positive direction and $y_{0}=0$ to be at the top of the Empire State building. The initial velocity is $v_{0}=0$ and the acceleration is $g$. (a) $y=\frac{1}{2} g t^{2}$ and so $t=\sqrt{2 y / g} \approx 8.8 \mathrm{~s}$. (b) $v=g t=86 \mathrm{~m} / \mathrm{s}$.
6. See slides.
7. See slides.
8. See slides.
9. Choose downward to be the positive direction and $y_{0}=0$ to be the height from which the stone is dropped. Call the location at the top of the window $y_{\mathrm{w}}$ and the time for the stone to fall from release to the top of the window $t_{\mathrm{w}}$. Since the stone is dropped from rest we have $y_{\mathrm{w}}=y_{0}+v_{0} t_{\mathrm{w}}+\frac{1}{2} a t_{\mathrm{w}}^{2}=\frac{1}{2} g t_{\mathrm{w}}^{2}$. The location of the bottom of the window is $y_{w}+2.2 \mathrm{~m}$ and the time for the stone to fall from release to the bottom of the window is $t_{w}+0.28 \mathrm{~s}$. Since the stone is dropped from ret we have $y_{w}+2.2 \mathrm{~m}=\frac{1}{2} g\left(t_{\mathrm{w}}+0.28 \mathrm{~s}\right)^{2}$. Substituting the first expression for $y_{w}$ into the second one we have $\frac{1}{2} g t_{\mathrm{w}}^{2}+2.2 \mathrm{~m}=\frac{1}{2} g\left(t_{\mathrm{w}}+0.28 \mathrm{~s}\right)^{2}$, which leads to $t_{\mathrm{w}}=0.662 \mathrm{~s}$. Using this time in the first equation we have $y_{\mathrm{w}}=\frac{1}{2} g t_{\mathrm{w}}^{2}=2.1 \mathrm{~m}$.
10. For the falling rock, choose downward to be the positive direction, and $y_{0}=0$ to be the height from which the stone is dropped. The initial velocity is zero, the acceleration is $a=g$, the displacement is $y=H$, and the time of fall is $t_{1}$. Then $H=y_{0}+v_{0} t_{1}+\frac{1}{2} a t_{1}^{2}=\frac{1}{2} g t_{1}^{2}$. For the sound wave use the constant speed equation $v_{s}=H /\left(T-t_{1}\right)$, which can be rearranged to give $t_{1}=T-H / v_{s}$, where $T=3.2 \mathrm{~s}$ is the total time elapsed from dropping the rock to hearing the sound. Insert this expression for $t_{1}$ into the equation for $H$ and solve for $H$. It follows that $H=\frac{1}{2} g\left(T-H / v_{s}\right)^{2}$, yiedling $\frac{g}{2 v_{s}^{2}} H^{2}-\left(\frac{g T}{v_{s}}+1\right) H+\frac{1}{2} g T^{2}=0$. Solving for $H$ we have $H=46 \mathrm{~m}$ and/or $H=2.57 \times 10^{4} \mathrm{~m}$. If the larger number is used in $t_{1}=T-H / v_{s}$, we obtain a negative
time of fall, and so the physically correct answer is $H=46 \mathrm{~m}$.
11. For the free falling part of the motion, choose downwards to be the positive direction and $y_{0}$ to be the height from which the person jumped. The initial velocity is $v_{0}=0$, the acceleration is $a=g=9.8 \mathrm{~m} / \mathrm{s}^{2}$ and the location of the net is $y=15 \mathrm{~m}$. Find the speed upon reaching the net $v= \pm \sqrt{2 a y}=17.1 \mathrm{~m} / \mathrm{s}$. The positive root is selected since the person is moving downard. For the net strecthing part of the motion choose downward to be the positive direction, and $y_{0}=15 \mathrm{~m}$ to be the height at which the person first contact the net. The initial velocity is $v_{0}=17.1 \mathrm{~m} / \mathrm{s}$, the final velocity is $v=0$, and the location of the strectched position is $y=16.0 \mathrm{~m}$. Then we have a system of two equations: $y=y_{0}+v_{0} t+\frac{1}{2} a t^{2}$ and $v=v_{0}+a t$. From the second expression we first get $t=-v_{0} / a$ and then substituting this relation for $t$ in the first equation we obtain $a=-v_{0}^{2} /\left[2\left(y-y_{0}\right)\right]=-150 \mathrm{~m} / \mathrm{s}^{2}$.
12. Choose downward to be the positive direction, and $y_{0}=0$ to be the height of the bridge. 007 has an initial velocity $v_{0}=0$, an acceleration of $g$, and when reaching the truck will have a displacement of $y=12 \mathrm{~m}-1.5 \mathrm{~m}=10.5 \mathrm{~m}$. You can now find the time of fall using $y=y_{0}+v_{0} t+\frac{1}{2} a t^{2}$ which leads to $t=\sqrt{2 y / a}=1.47 \mathrm{~s}$. If the truck is approaching at $v_{\mathrm{tr}}=25 \mathrm{~m} / \mathrm{s}$ then he needs to jump when the truck is a distance away given by $d_{\mathrm{tr}}=v_{\mathrm{tr}} t=36.75 \mathrm{~m}$. Convert this distance into poles $d_{\mathrm{tr}}=(36.75 \mathrm{~m})(1$ pole $/ 25 \mathrm{~m})=1.5$ poles.
13. Choose downward to be the positive direction and $y_{0}=0$ to be at the start of the pelican's dive. The pelican has an initial velocity $v_{0}=0$ and an acceleration $a=g$. The final location of the pelican will be at $y=16.0 \mathrm{~m}$. Find the total time of the pelican's dive using $y=y_{0}+v_{0} t+\frac{1}{2} a t^{2}$. This yields $t_{\text {dive }}=\sqrt{2 y / a}=1.81 \mathrm{~s}$. The fish can take evasive action if he sees the pelican at a time $1.81 \mathrm{~s}-0.20 \mathrm{~s}=1.61 \mathrm{~s}$ into the dive. Now, you can find the position of the pelican at that time: $y=y_{0}+v_{0} t+\frac{1}{2} a t^{2}=12.7 \mathrm{~m}$. Therefore, the fish must spot the pelican at a minimum height from the surface of the water of $\Delta y=16 \mathrm{~m}-12.7 \mathrm{~m}=3.3 \mathrm{~m}$.
14. See slides.
15. See slides.
