PROBLEMS 7

(1) Momentum will be conserved in two dimensions. The fuel was ejected in the y direction as seen from the ground, and so the fuel had no x-component of velocity.

$$p_x: \quad m_{\text{rocket}}v_0 = (m_{\text{rocket}} - m_{\text{fuel}})v'_x + m_{\text{fuel}}0 = \tfrac{2}{3}m_{\text{rocket}}v'_x \quad \rightarrow \quad \boxed{v'_x = \tfrac{3}{2}v_0}$$

$$p_y: \quad 0 = m_{\text{fuel}}v_{\text{fuel}} + (m_{\text{rocket}} - m_{\text{fuel}})v'_y = \tfrac{1}{3}m_{\text{rocket}}(2v_0) + \tfrac{2}{3}m_{\text{rocket}}v'_y \quad \rightarrow \quad \boxed{v'_y = -v_0}$$

(2) We find the diameter of the spot from

$$\theta = \frac{\text{diameter}}{r_{\text{Earth-Moon}}} \quad \rightarrow \quad \text{diameter} = \theta r_{\text{Earth-Moon}} = (1.4\times10^{-5}\,\text{rad})(3.8\times10^{8}\,\text{m}) = \boxed{5.3\times10^{3}\,\text{m}}$$

(3)

(*a*) 1. The angular acceleration is related to the initial and final angular velocities:

$$\omega = \omega_0 + \alpha t = 0 + \alpha t$$

2. Solve for α:

$$\alpha = \frac{\omega}{t} = \frac{500\,\text{rev/min}}{5.5\,\text{s}} \times \frac{2\pi\,\text{rad}}{1\,\text{rev}} \times \frac{1\,\text{min}}{60\,\text{s}} = 9.52\,\text{rad/s}^2 = \boxed{9.5\,\text{rad/s}^2}$$

(*b*) 1. The angular displacement is related to the time by Equation 9-6:

$$\theta - \theta_0 = \omega_0 t + \tfrac{1}{2}\alpha t^2 = 0 + \tfrac{1}{2}(9.52\,\text{rad/s}^2)(5.5\,\text{s})^2 = 144\,\text{rad}$$

2. Convert radians to revolutions:

$$144\,\text{rad} \times \frac{1\,\text{rev}}{2\pi\,\text{rad}} = 22.9\,\text{rev} = \boxed{23\,\text{rev}}$$

(4) The interaction between the planet and the spacecraft is elastic, because the force of gravity is conservative. Thus kinetic energy is conserved in the interaction. Consider the problem a 1-dimensional collision, with A representing the spacecraft and B representing Saturn. Because the mass of Saturn is so much bigger than the mass of the spacecraft, Saturn's speed is not changed appreciably during the interaction. Use Eq. 7-7, with $v_A = 10.4\,\text{km/s}$ and $v_B = v'_B = -9.6\,\text{km/s}$.

$$v_A - v_B = -v'_A + v'_B \quad \rightarrow \quad v'_A = 2v_B - v_A = 2(-9.6\,\text{km/s}) - 10.4\,\text{km/s} = \boxed{-29.6\,\text{km/s}}$$

Thus there is almost a threefold increase in the spacecraft's speed.

5

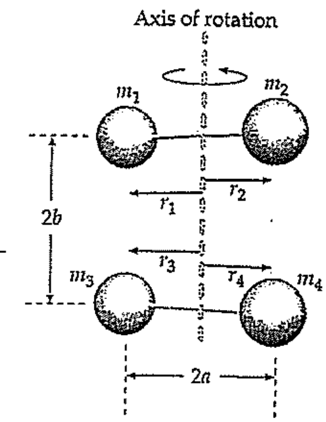


FIGURE 9-3

(*a*) 1. Apply the definition of moment of inertia $I = \Sigma m_i r_i^2$ (Equation 9-11), where r_i is the radial distance from the rotation axis to the particle of mass m_i:
$$I = \sum_i m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + m_4 r_4^2$$

2. The masses m_i and the distances r_i are given:
$$m_1 = m_2 = m_3 = m_4 = m$$
$$r_1 = r_2 = r_3 = r_4 = a$$

3. Substitution gives the moment of inertia:
$$I = ma^2 + ma^2 + ma^2 + ma^2 = 4ma^2$$

4. Using Equation 9-12, solve for the kinetic energy:
$$K = \tfrac{1}{2} I\omega^2 = \tfrac{1}{2} 4ma^2\omega^2 = \boxed{2ma^2\omega^2}$$

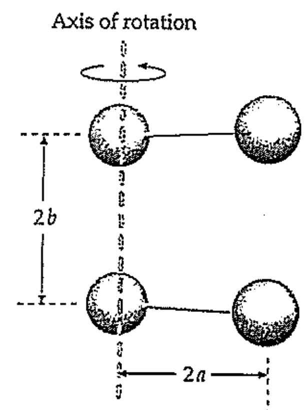
(*b*) 1. To find the kinetic energy of the *i*th particle, we must first find its speed:
$$K_i = \tfrac{1}{2} m_i v_i^2$$

2. The particles are all moving in circles of radius *a*. Find the speed of each particle:
$$v_i = r_i\omega = a\omega \quad (i = 1, \ldots, 4)$$

3. Substitute into the Part-(*b*) step-1 result:
$$K_i = \tfrac{1}{2} m_i v_i^2 = \tfrac{1}{2} ma^2\omega^2$$

4. Each particle has the same kinetic energy. Sum the kinetic energies to get the total:
$$K = \sum_{i=1}^{4} K_i = \tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2 + \tfrac{1}{2} m_3 v_3^2 + \tfrac{1}{2} m_4 v_4^2$$
$$= 4(\tfrac{1}{2} ma^2\omega^2) = 2ma^2\omega^2$$

5. Compare with the Part-(*a*) result:
$$\boxed{\text{The two calculations give the same result.}}$$



6

$$I = \sum_i m_i r_i^2 = 2mR^2$$

with $R = 2a \Rightarrow I = 8ma^2$

7

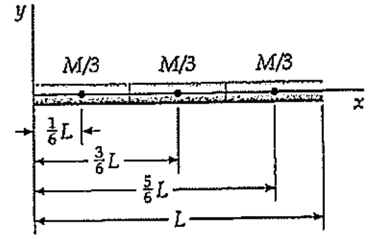


FIGURE 9-5

- Sketch the rod divided into three segments and superpose the point-particle constructs at the center of each segment (Figure 9-5):
- Apply the equation $I = \Sigma m_i r_i^2$ to the approximate system (the point-particle constructs):

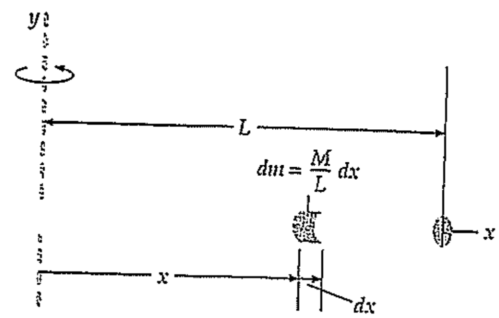
$$I = \Sigma m_i r_i^2 \approx m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2$$

- The mass of each particle is $\frac{1}{3}M$, and the distances of the particles from the axis are $\frac{1}{6}L$, $\frac{3}{6}L$, and $\frac{5}{6}L$:

$$I \approx (\tfrac{1}{3}M)(\tfrac{1}{6}L)^2 + (\tfrac{1}{3}M)(\tfrac{3}{6}L)^2 + (\tfrac{1}{3}M)(\tfrac{5}{6}L)^2$$

$$= \frac{1}{3}M\left(\frac{1 + 3^2 + 5^2}{6^2}\right)L^2 = \boxed{\frac{35}{108}ML^2}$$

8



1. Draw a sketch (Figure 9-6) showing the rod along the $+x$ axis with its end at the origin. To calculate I about the y axis, we choose a mass element dm at a distance x from the axis:
2. The moment of inertia about the y axis is given by the integral:

$$I = \int x^2\, dm$$

FIGURE 9-6

3. To compute the integral, we first relate dm to dx. Express dm in terms of the linear mass density λ and dx:

$$dm = \lambda\, dx = \frac{M}{L}dx$$

4. Substitute and perform the integration. We choose integration limits so that we integrate through the mass distribution in the direction of increasing x:

$$I = \int x^2\, dm = \int_0^L x^2 \frac{M}{L}dx = \frac{M}{L}\int_0^L x^2\, dx$$

$$= \frac{M}{L}\frac{1}{3}x^3\Big|_0^L = \frac{M}{L}\frac{L^3}{3} = \boxed{\frac{1}{3}ML^2}$$

9

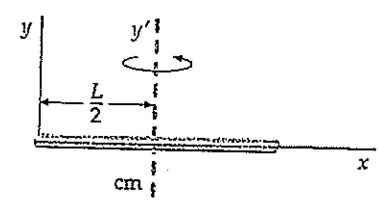


FIGURE 9-11

Cover the column to the right and try these on your own before looking at the answers.

Steps	Answers
1. Apply the parallel-axis theorem to write I about the end in terms of I_{cm}.	$I = I_{cm} + Mh^2$ $I_y = I_{y'} + M(\frac{1}{2}L)^2$
2. Substitute, using $\frac{1}{3}ML^2$ for I_y, I_{cm} for $I_{y'}$, and solve for I_{cm}.	$I_{cm} = I_y - Mh^2 = \frac{1}{3}ML^2 - M(\frac{1}{2}L)^2 = \boxed{\frac{1}{12}ML^2}$

(10)

The firing force of the rockets will create a net torque, but no net force. Since each rocket fires tangentially, each force has a lever arm equal to the radius of the satellite, and each force is perpendicular to the lever arm. Thus $\tau_{net} = 4FR$. This torque will cause an angular acceleration according to $\tau = I\alpha$, where $I = \frac{1}{2}MR^2$ for a cylinder. The angular acceleration can be found from the kinematics by $\alpha = \frac{\Delta\omega}{\Delta t}$. Equating the two expressions for the torque and substituting enables us to solve for the force.

$$4FR = I\alpha = \tfrac{1}{2}MR^2\frac{\Delta\omega}{\Delta\tau}$$

$$F = \frac{MR\Delta\omega}{8\Delta t} = \frac{(3600\text{ kg})(4.0\text{ m})(32\text{ rev/min})(2\pi\text{ rad/rev})(1\text{ min/60 s})}{8(5.0\text{ min})(60\text{ s/min})}$$

$$= 20.11\text{ N} \approx \boxed{2.0\times10^{1}\text{ N}}$$

(11)

(*a*) The moment of inertia of a thin rod, rotating about its end, is given in Figure 8-21(g). There are three blades to add.

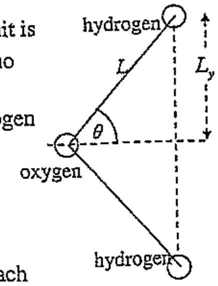
$$I_{total} = 3\left(\tfrac{1}{3}ML^2\right) = ML^2 = (160\text{ kg})(3.75\text{ m})^2 = 2250\text{ kg}\cdot\text{m}^2 \approx \boxed{2.3\times10^{3}\text{ kg}\cdot\text{m}^2}$$

(*b*) The torque required is the rotational inertia times the angular acceleration, assumed constant.

$$\tau = I_{total}\alpha = I_{total}\frac{\omega-\omega_0}{t} = \left(2250\text{ kg}\cdot\text{m}^2\right)\frac{(5.0\text{ rev/sec})(2\pi\text{ rad/rev})}{8.0\text{ s}} = \boxed{8.8\times10^{3}\text{ m}\cdot\text{N}}$$

12

The mass of a hydrogen atom is 1.01 atomic mass units. The atomic mass unit is $1.66\times10^{-27}\,\text{kg}$. Since the axis passes through the oxygen atom, it will have no rotational inertia.



(*a*) If the axis is perpendicular to the plane of the molecule, then each hydrogen atom is a distance L from the axis of rotation.

$$I_{perp} = 2m_H L^2 = 2(1.01)(1.66\times10^{-27}\,\text{kg})(0.96\times10^{-9}\,\text{m})^2$$
$$= \boxed{3.1\times10^{-45}\,\text{kg}\cdot\text{m}^2}$$

(*b*) If the axis is in the plane of the molecule, bisecting the H-O-H bonds, each hydrogen atom is a distance of $L_y = L\sin\theta = (9.6\times10^{-10}\,\text{m})\sin 52^\circ$ $= 7.564\times10^{-10}\,\text{m}$. Thus the moment of inertia is

$$I_{plane} = 2m_H L_y^2 = 2(1.01)(1.66\times10^{-27}\,\text{kg})(7.564\times10^{-10}\,\text{m})^2 = \boxed{1.9\times10^{-45}\,\text{kg}\cdot\text{m}^2}$$

13

The original horizontal distance can be found from the range formula ~~from Example 3-8~~.

$$R = v_0^2 \sin 2\theta_0 / g = (25\,\text{m/s})^2(\sin 60^\circ)/(9.8\,\text{m/s}^2) = 55.2\ \text{m}$$

The height at which the objects collide can be found ~~in Eq. 2-11c~~ for the vertical motion, with $v_y = 0$ at the top of the path. Take up to be positive.

$$v_y^2 = v_{y0}^2 + 2a(y-y_0) \rightarrow (y-y_0) = \frac{v_y^2 - v_{y0}^2}{2a} = \frac{0-\left[(25\,\text{m/s})\sin 30^\circ\right]^2}{2(-9.8\,\text{m/s}^2)} = 7.97\ \text{m}$$

Let m represent the bullet and M the skeet. When the objects collide, the skeet is moving horizontally at $v_0\cos\theta = (25\,\text{m/s})\cos 30^\circ = 21.65\,\text{m/s} = v_x$, and the bullet is moving vertically at $v_y = 200\,\text{m/s}$. Write momentum conservation in both directions to find the velocities after the totally inelastic collision.

$$p_x:\ Mv_x = (M+m)v_x' \rightarrow v_x' = \frac{Mv_x}{M+m} = \frac{(0.25\ \text{kg})(21.65\,\text{m/s})}{(0.25+0.015)\,\text{kg}} = 20.42\,\text{m/s}$$

$$p_y:\ mv_y = (M+m)v_y' \rightarrow v_y' = \frac{mv_y}{M+m} = \frac{(0.015\ \text{kg})(200\,\text{m/s})}{(0.25+0.015)\,\text{kg}} = 11.32\,\text{m/s}$$

(*a*) The speed v_y' can be used as the starting vertical speed ~~in Eq. 2-11c~~ to find the height that the skeet-bullet combination rises above the point of collision.

$$v_y^2 = v_{y0}^2 + 2a(y-y_0)_{extra} \rightarrow (y-y_0)_{extra} = \frac{v_y^2 - v_{y0}^2}{2a} = \frac{0-(11.32\,\text{m/s})^2}{2(-9.8\,\text{m/s}^2)} = \boxed{6.5\ \text{m}}$$

(*b*) From ~~Eq. 2-11b applied to~~ the vertical motion after the collision, we can find the time for the skeet-bullet combination to reach the ground.

$$y = y_0 + v_y't + \tfrac{1}{2}at^2 \rightarrow 0 = 7.97\ \text{m} + (11.32\,\text{m/s})t + \tfrac{1}{2}(-9.8\,\text{m/s}^2)t^2 \rightarrow$$
$$4.9t^2 - 11.32t - 7.97 = 0 \rightarrow t = 2.88\ \text{s},\ -0.565\ \text{s}$$

The positive time root is used to find the horizontal distance traveled by the combination after the collision.

$$x_{after} = v_x't = (20.42\,\text{m/s})(2.88\ \text{s}) = 58.7\ \text{m}$$

If the collision would not have happened, the skeet would have gone $\tfrac{1}{2}R$ horizontally.

$$\Delta x = x_{after} - \tfrac{1}{2}R = 58.7\ \text{m} - \tfrac{1}{2}(55.2\ \text{m}) = 31.1\ \text{m} \approx \boxed{31\ \text{m}}$$

14 See Slides