# PHY 166 Recitation 1 — Solutions

## Chapters 1, 2 and 3.

**1.** Answer the following: (a) Represent the number 314024 in scientific notation. (b) Multiply the following three numbers and express your answer with the appropriate number of significant figures: $1.45 \times 10^5$, $6.2391 \times 10^{-2}$ and $4.10 \times 10^3$. (c) Add the following numbers together and express your answer with the appropriate number of significant figures: $2.35 \times 10^5$ and $6.78 \times 10^4$. (d) Add the following distances together and express your answer with the appropriate number of significant figures: 5.2 m, 310.75 cm, and $4.0 \times 10^4$ mm.

*Solution*: a) $314024 = 3.14024 \times 10^5$. The decimal point moves 5 positions to the left, so that there is one cypher on the left of it, the 3. To compensate this change, the number is multiplied by $10^5$. The general rule is thus that the pre-factor (here 3) has to be between 1 and 10.

However, this I not the only way of writing numbers in the scientific nutation. For instance, the electron mass is $9.10938356 \times 10^{-31}$ kg. I prefer writing it as $0.910938356 \times 10^{-30}$ kg because here the pre-factor is close to 1. Course rounding it up gives simply $10^{-30}$ kg that is easier to memorize than the usually quoted value. Thus, I prefer to keep the pre-factor in the range between 0.3 and 3, so that it can be dropped for a coarse estimation.

b) Use MSWord's equation editor

$$1.45 \times 10^5 \times 6.2391 \times 10^{-2} \times 4.10 \times 10^3 = 37.0914 \times 10^{5-2+3} = 37.1 \times 10^6 = 3.71 \times 10^7.$$

Here, we multiplied all pre-factors and summed all exponents, and then the resulting pre-factor has been rounded up to three significant figures, as the minimal number of significant figures in the inputs is three.

c) To add numbers in the scientific notation, they have to be expressed in the form with the same exponent. After that the pre-factors can be added:

$$2.35 \times 10^5 + 6.78 \times 10^4 = 2.35 \times 10^5 + 0.678 \times 10^5 = 3.028 \times 10^5 \Longrightarrow 3.03 \times 10^5,$$

where the final result is rounded up to three significant figures.

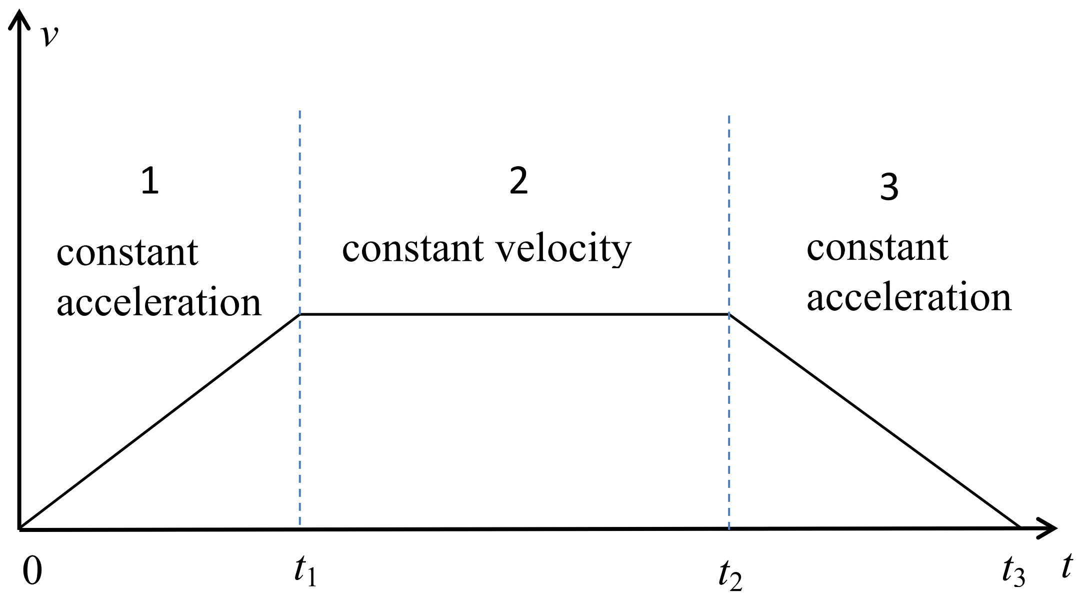
d) To sum quantities with units, one has to put them into the standard form first. Convert everything to meters and then add:

$$5.2\ \mathrm{m} + 310.75\ \mathrm{cm} + 4.0 \times 10^4\ \mathrm{mm} = 5.2\ \mathrm{m} + 3.1075\ \mathrm{m} + 40\ \mathrm{m} = 48\ \mathrm{m}.$$

We keep two significant figures, according to the accuracy of the input.

**2.** A car travels along a straight road and covers a distance in three segments: Segment 1: The car starts at rest and accelerates at 3.00 m/s$^2$ until it reaches its top velocity of 15.0 m/s. Segment 2: It then cruises at that constant velocity for 15.0 seconds. Segment 3: Right after that, it brakes and comes to a stop 3.00 seconds later. (a) Sketch a plot of velocity versus time for the entire trip. (b) How long did it take for the car to reach its top velocity in segment 1? (c) What was the acceleration in segment 3? (d) What was the total distance traveled for the entire trip?

*Solution*: a) It is always good to start with a sketch and introduce notations to get connection to formulas that have to be used. After the sketch, we write down the inputs in the mathematical form used in physics.




Segment 1. Time from 0 to $t_1$ (unknown), acceleration $a_1$ = 3 m/s$^2$, final velocity $v_1$ = 15 m/s.

Segment 2. Time from $t_1$ to $t_2$, while the duration is $t_2 - t_1 = 15\ s$. Constant velocity $v_2 = v_1$ = 15 m/s.

Segment 3. Time from $t_2$ to $t_3$, while the duration is $t_3 - t_2 = 3\ s$. Final velocity $v_3$ = 0.

b) In segment 1 there is a motion with constant (positive) acceleration, the velocity depends on time as

$$v(t) = v_0 + at \Longrightarrow a_1 t,$$

as the initial velocity is zero. From this equation, one finds $t_1$ substituting it instead of $t$ and noticing that $v(t) = v_1$. This yields

$$v_1 = a_1 t_1 \implies t_1 = \frac{v_1}{a_1} = \frac{15}{3} = 5\ s.$$

c) In segment 3 there is also a motion with a constant acceleration. As this acceleration is negative, one speaks of *deceleration*. Using the same velocity formula as above, one obtains

$$v(t) = v_2 + a_3 t \quad \implies \quad 0 = v_2 + a_3(t_3 - t_2).$$

From here one obtains

$$a_3 = -\frac{v_2}{t_3 - t_2} = -\frac{15}{3} = -5\frac{m}{s^2}.$$

d) For the distance traveled we use the general formula

$$x = x_0 + v_0 t + \frac{1}{2}at^2.$$

In our case, the total distance consists of three parts:

$$x_{total} = x_1 + x_2 + x_3.$$

Here, using the result for $t_1$, one obtains

$$x_1 = \frac{1}{2}a_1 t_1^2 = \frac{1}{2}a_1\left(\frac{v_1}{a_1}\right)^2 = \frac{v_1^2}{2a_1}.$$

(There is such as formula in the textbook but it is better to derive it on the fly). Also,

$$x_2 = v_2(t_2 - t_1).$$

Then, using the result for $a_3$, one obtains

$$x_3 = v_2(t_3 - t_2) + \frac{1}{2}a_3(t_3 - t_2)^2 = v_2(t_3 - t_2) - \frac{1}{2}\frac{v_2}{t_3 - t_2}(t_3 - t_2)^2 =$$
$$v_2(t_3 - t_2) - \frac{1}{2}v_2(t_3 - t_2) = \frac{1}{2}v_2(t_3 - t_2).$$

Collecting all terms of $x_{\text{total}}$, one obtains

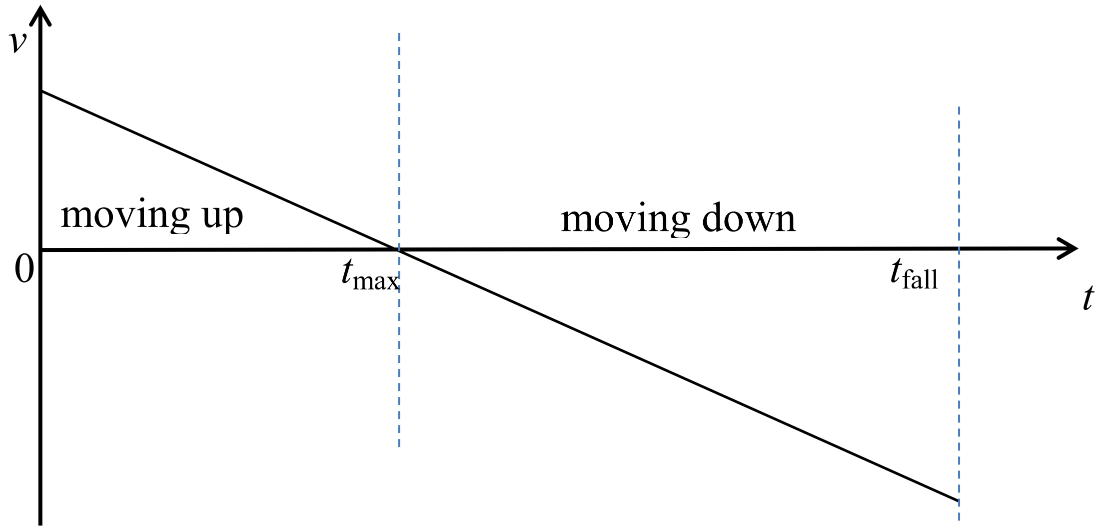
$$x_{total} = \frac{v_1^2}{2a_1} + v_2(t_2 - t_1) + \frac{1}{2}v_2(t_3 - t_2).$$

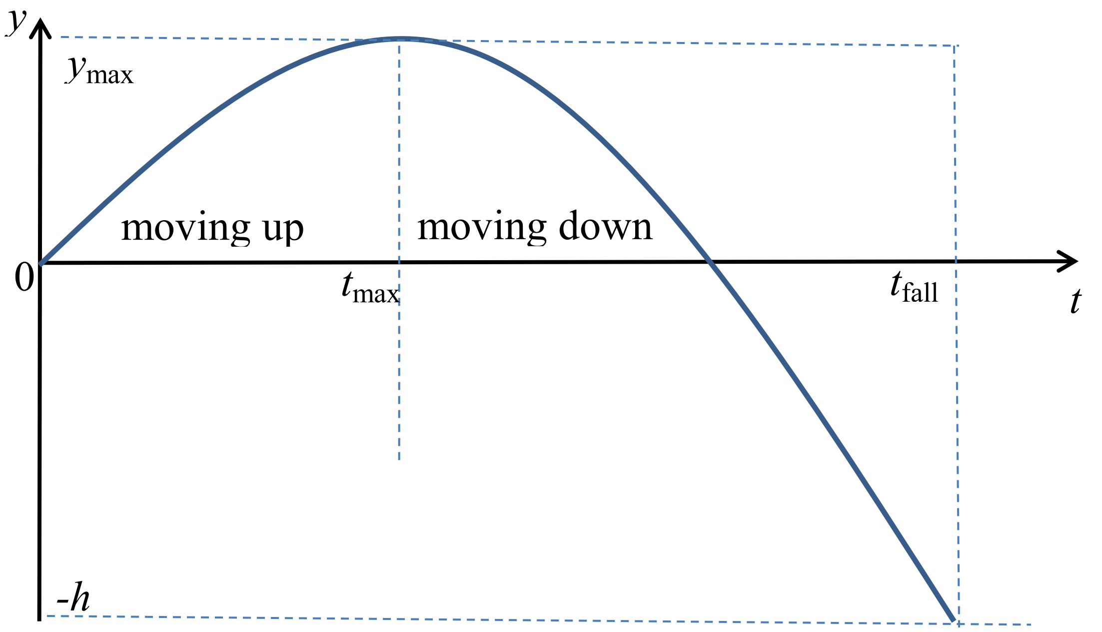
This is the analytical result expressing the searched value directly through the input values. Substituting numbers, one obtains

$$x_{total} = \frac{15^2}{2 \times 3} + 15 \times 15 + \frac{1}{2}15 \times 3 = 37.5 + 225 + 22.5 = 285\ m.$$

**3.** A person standing on the edge of a cliff throws a rock straight upwards with an initial speed of 44 m/s. (a) Sketch a plot of velocity versus time and position versus time for the motion of the rock. (b) What will be the maximum height the rock reaches? (c) If the cliff stands at a height of 105 meters from the bottom of the ravine, how long will it take to reach the ground? (d) How fast will it be traveling when it reaches the ground?

*Solution*: (a) Making a sketch is a guarantee of success in problem solving.




b) The time dependences of the velocity and displacement in the motion with constant acceleration are given by the formulas

$$v(t) = v_0 + at; \qquad y(t) = y_0 + v_0 t + \frac{1}{2}at^2.$$

Here $y_0 = 0$ by the choice of the reference level. For the free fall, the acceleration is directed down, *a* = -*g*, so that

$$v(t) = v_0 - gt; \qquad y(t) = v_0 t - \frac{1}{2}gt^2 \qquad (1).$$

Finding the maximum of $y(t)$ directly from the second formula requires using the calculus. However, one can use the physical argument and point out that when the height reaches its maximum, the vertical velocity must vanish. Thus, from the first equation one obtains

$$0 = v_0 - gt_{max} \quad \Longrightarrow \quad t_{max} = \frac{v_0}{g} = \frac{44}{9.8} = 4.5\ s.$$

(The value $v_0 = 44\ m/s$ is totally unrealistic for a human!). After that, one finds the maximal height from the height formula substituting $t \Rightarrow t_{max}$, that is,

$$y_{max} \equiv y(t_{max}) = v_0 t_{max} - \frac{1}{2}gt_{max}^2 = v_0\frac{v_0}{g} - \frac{1}{2}g\left(\frac{v_0}{g}\right)^2 = \frac{v_0^2}{2g}.$$

Substituting numbers, one obtains

$$y_{max} = \frac{44^2}{2 \times 9.8} = 99\ m$$

(It was not a human throwing this rock but, possibly, a terminator).

c) The time to reach the ground, that is, the fall time $t_{fall}$, can be found from the height equation (1) substituting $y \Rightarrow -h$:

$$-h = v_0 t_{fall} - \frac{1}{2}gt_{fall}^2.$$

This is a quadratic equation that can be rewritten into the canonical corm

$$gt_{fall}^2 - 2v_0 t_{fall} - 2h = 0.$$

The solution of this equation is

$$t_{fall} = \frac{1}{g}\left(v_0 \pm \sqrt{v_0^2 + gh}\right).$$

One can see that one of these solutions is positive and another is negative. The negative solution should be discarded on physical grounds. Substituting numbers, one obtains

$$t_{fall} = \frac{1}{9.8}\left(44 + \sqrt{44^2 + 9.8 \times 105}\right) = 15.8\ s.$$

d) The velocity at the end of the fall can be obtained from the velocity equation (1) as

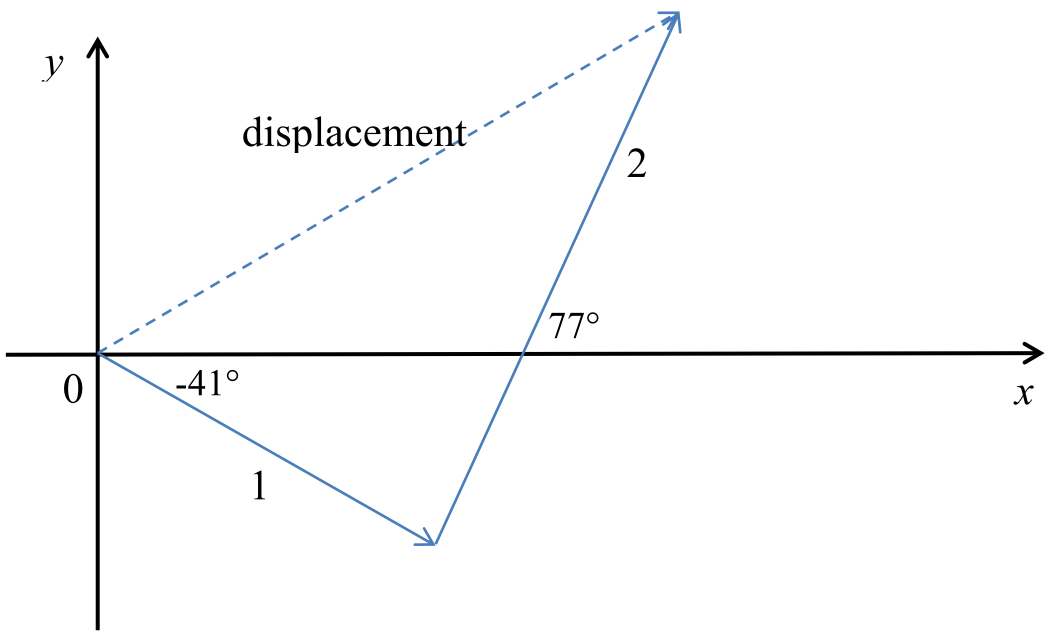
$$v(t_{fall}) = v_0 - g\frac{1}{g}\left(v_0 + \sqrt{v_0^2 + gh}\right) = -\sqrt{v_0^2 + gh}.$$

This velocity is negative as it is directed down. The value given by the square root can be found from the energy conservation law in a shorter way. Substituting numbers, one obtains

$$v(t_{fall}) = -\sqrt{44^2 + 9.8 \times 105} = -111\frac{m}{s}.$$

**4.** A police officer leaves the station to begin her patrol. She drives a distance of 6 miles in a direction 41 degrees south of east. She then follows a route traveling 12 miles in a direction 77 degrees north of east. (a) What is the officer's displacement from the police station? What is the total distance she traveled? Sketch a figure of her patrol. (b) If the first leg of her patrol was made with the speed 20 mi/hr and the second was done with the speed 30 mi/hr, what is the average speed and average velocity of her trip?

Solution: a) Again, we start with a sketch.




Using vector notations, for the displacements in trips 1 and 2 one obtains

$$\Delta\mathbf{r}_1 = (6\cos 41°, 6\sin(-41°))\ mi, \quad \Delta\mathbf{r}_2 = (12\cos 77°, 12\sin 77°)\ mi.$$

The total displacement vector is given by

$$\Delta\mathbf{r} = \Delta\mathbf{r}_1 + \Delta\mathbf{r}_2 = (6\cos 41° + 12\cos 77°, -6\sin 41° + 12\sin 77°) = (7.228, 7.756)\ mi.$$

The absolute value of the displacement (the magnitude of the displacement vector) is

$$|\Delta\mathbf{r}| = \sqrt{7.228^2 + 7.756^2} = 10.6\ mi.$$

The total traveled distance is the sum of the two distances:

$$d_{tot} = d_1 + d_2 \equiv |\Delta\mathbf{r}_1| + |\Delta\mathbf{r}_2| = 6 + 12 = 18\ mi.$$

Clearly, the condition $|\Delta\mathbf{r}| \leq d_{tot}$ is satisfied here.

b) The total travel time is

$$t_{tot} = t_1 + t_2 = \frac{d_1}{v_1} + \frac{d_2}{v_2}.$$

Now the average speed is

$$s_{avr} = \frac{d_{tot}}{t_{tot}} = \frac{d_1 + d_2}{\frac{d_1}{v_1} + \frac{d_2}{v_2}} = \frac{v_1 v_2 (d_1 + d_2)}{v_1 d_2 + v_2 d_1}.$$

Substituting numbers, one obtains

$$s_{avr} = \frac{6 + 12}{\frac{6}{20} + \frac{12}{30}} = \frac{18}{0.3 + 0.4} = 25.7 \frac{mi}{hr}.$$

The average velocity is defined as

$$v_{avr} = \frac{|\Delta \mathbf{r}|}{t_{tot}} = \frac{10.6}{\frac{6}{20} + \frac{12}{30}} = \frac{10.6}{0.3 + 0.4} = 15.1 \frac{mi}{hr}.$$

Of course, $v_{avr} \leq s_{avr}$ is satisfied.

Note that here we use the natural units, miles and hours, and do not convert them to SI.

**5.** An archer shoots an arrow at an angle 30 degrees from the height 1.4 meters. It takes 1.6 seconds for the arrow to hit a 1.4-meter tall target. (a) What is the arrow's initial speed? (b) How far away is the target? (c) What is the maximum height, from the ground, the arrow reaches?

*Solution*: a) The equations of motion along the *x*- and *y*-axes are as follows

$$v_x = v_{x,0} = v_0 \cos\theta\,, \qquad x = v_{x,0}t = v_0 \cos\theta\, t$$

$$v_y = v_{y,0} - gt = v_0 \sin\theta - gt, \qquad y = y_0 + v_{y,0}t - \frac{1}{2}gt^2 = y_0 + v_0 \sin\theta\, t - \frac{1}{2}gt^2.$$

Here $\theta = 30°$, $v_0$ is the initial speed (unknown) , and $y_0 = 1.4\ m$ is the initial height. Time $t_f = 1.6\ s$ is the traveling time of the arrow. Let us write down the event of the arrow hitting the target:

$$y_0 = y_0 + v_0 \sin\theta\, t_f - \frac{1}{2}gt_f^2.$$

One can see that $y_0$ cancels. From this equation one obtains

$$v_0 = \frac{gt_f}{2\sin\theta} = \frac{9.8 \times 1.6}{2 \times 0.5} = 15.7\frac{m}{s}.$$

b) Now the distance to the target can be obtained from the *x*-equation above:

$$d \equiv x(t_f) = v_0 \cos\theta\, t_f = \boxed{\frac{gt_f^2}{2}\cot\theta = d.}$$

It is always good to frame your results to show to the grader that you understand what your result is. For this, you can write the quantity you have found once again. Substituting the numbers one obtains

$$d = \frac{9.8 \times 1.6^2}{2}\cot 30° = 21.7\ m.$$

c) The maximum height will be achieved at the time $t_{max} = t_f/2$. This is clear since the trajectory is a parabola that is symmetric with respect to its top. However, the result for $t_{max}$ can be obtained independently. At this time, the vertical component of the velocity is zero, $v_y = 0$. From the $v_y$-equation one obtains

$$0 = v_0 \sin\theta - gt_{max} \quad \Longrightarrow \quad t_{max} = \frac{v_0 \sin\theta}{g} = \frac{t_f}{2}.$$

Here we used the result for $v_0$ above. Now, the maximal height can be obtained from the *y*-equation above as

$$y_{max} \equiv y(t_{max}) = y_0 + v_0 \sin\theta\, t_{max} - \frac{1}{2}gt_{max}^2.$$

Substituting here the results for $v_0$ and $t_{max}$, one obtains

$$y_{max} = y_0 + \frac{g t_f}{2 \sin \theta} \sin \theta \frac{t_f}{2} - \frac{1}{2} g \left(\frac{t_f}{2}\right)^2 = y_0 + \frac{1}{8} g t_f^2.$$

Substituting the numbers, one obtains

$$y_{max} = 1.4 + \frac{1}{8} 9.8 \times 1.6^2 = 4.5\ m.$$