



Laboratory Manual

Physics 166, 167, 168, 169

Department of Physics and Astronomy
HERBERT LEHMAN COLLEGE

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Writing a laboratory report

OBJECTIVES

The main way to communicate scientific information today is through articles and reports in scientific journals. Traditionally these were distributed in print, but can now be read in digital format as well, as shown in table 1.

Table 1. Online resources for digitally distributed scientific publications

Resource	Topics	URL
arXiv	Physics, Mathematics, Computer Science	http://arxiv.org/
NASA Scientific and Technical Information (STI)	Astronomy, Aerospace Engineering	www.sti.nasa.gov/STI-public-homepage.html
SOA/NASA Astrophysics Data System	Astronomy, physics	adswww.harvard.edu/

In college physics, you write a laboratory report for each experiment that contains the essential information about the experiment. For scientific information, a consistent format is helpful to the reader (and your lab instructor). Each laboratory report you turn in contains a subset of the sections found in a professional scientific publication for experimental topics, as shown in table 2.

Table 2. A comparison of the sections of a laboratory report and a professional scientific publication

Laboratory Report	Professional Publication
1. Name, date and title of the experiment	1. Cover page: name, date, and title
2. Abstract	2. Abstract
	3. Introduction
	4. Methods and procedure
3. Data	5. Raw data and graphs
4. Calculations and analysis	6. Calculations and analysis
	7. Results
5. Conclusion	8. Discussion and conclusion

The content to include in each section is detailed below. Your lab instructor requires all five sections to evaluate your work, so be sure to include every section in every report.

A laboratory report must be typed. Your laboratory instructor can tell you whether your laboratory report must be printed or can be delivered in a digital format such as email.

ABSTRACT

Describe in your own words what you did in the experiment and why. Your abstract should include one or two sentences each for Purpose, Methods and Conclusions.

- **Purpose:** What physical principle or law does this experiment test?
- **Methods:** What apparatus did you use? How did you analyze the data?
- **Conclusions:** Do your results support the physical law or principle? You should describe any significant experimental errors or uncertainties.

Note that the abstract should be no more than 5 or 6 sentences long, and should not include too much detail. The goal of the abstract is to sum up the experiment quickly and succinctly.

DATA

The data section includes all the raw data you collected in the laboratory without any calculation or interpretation. At a minimum, include the following information:

- A copy of the data table with all fields and rows filled with measurements.
- Any drawings or sketches you were required to make in the laboratory. You must deliver drawings with a printed lab report. You can take a digital photograph of your drawings and import it to a document as needed.

CALCULATIONS AND ANALYSIS

In the calculations and analysis section, you write out all of your calculations and results as explained in the instructions for the experiment. Be sure to answer all of the questions in the lab manual.

Include the following information as instructed:

- The equations you used to make all calculations
- Tables of calculated values
- Graphs of the raw data or calculated values
- Average values, uncertainty, and % uncertainty calculations
- Error and % error calculations

CONCLUSION

In the conclusion section, interpret the results you obtained by analyzing the data. Include the following information:

- Do your data and calculations support the physical principle or law being tested?
- What are the important sources of experimental error and uncertainty?
- Are there ways you could have improved your experimental results?
- Also answer any specific questions posed by your lab instructor.

Introduction: Measurement and Uncertainty

No physical measurement is ever completely precise. All measurements are subject to some uncertainty, and the determination of this uncertainty is an essential part of the analysis of the experiment.

Experimental data include three components: 1) the value measured, 2) the uncertainty, and 3) the units. For example a possible result for measuring a length is $3.6 \pm 0.2 \text{ m}$. Here 3.6 is the measured value, ± 0.2 specifies the uncertainty, and m gives the units (meters).

ERRORS AND UNCERTAINTIES

The accuracy of any measurement is limited by experimental errors and uncertainties. An *error* is a discrepancy between the measured value of some quantity and its true value. Errors in measurements arise from different sources:

- a) A common type of error is blunders due to carelessness in making a measurement, such as in an incorrect reading of an instrument. Of course these kinds of mistakes should be avoided.
- b) Errors also arise from defective or uncalibrated instruments. For example, if a balance does not read zero when there is no mass on it, then all of its readings will be in error, and we must either recalibrate it, or be careful to subtract the empty reading from all subsequent measurements.
- c) Even after we have made every effort to eliminate this kind of error, the accuracy of our measurements is still limited, due to experimental uncertainties. Uncertainties reflect unpredictable random variations in the measurement process: variations in the experimental system, in the measuring apparatus, and in our own perception! Since these variations are random, they will tend to cancel out if we average over a set of repeated measurements. To measure a quantity in the laboratory, one should repeat the measurement many times. The average of all the results is the best estimate of the value of the quantity.
- d) Besides the uncertainty introduced in a measurement due to random fluctuations, vibrations, etc., there is the so-called systematic or reading uncertainty which has to do with the limited accuracy of the measuring instruments we use. For example, if we use a meter stick to measure a length, we can, at best, estimate the length to about 1/10 of a millimeter. Beyond that we have no knowledge. It is important to realize that this kind of uncertainty persists, even if we obtain identical readings on repeated trials.

To summarize, all measurements have an uncertainty (both random and systematic). Often we refer to this uncertainty as an error, but it should be emphasized that a true error reflects the difference between our results and the actual value of what we want to measure.

CALCULATING AVERAGES

There are several important steps we will follow to help us quantify and control the errors and uncertainties in our laboratory measurements.

Most importantly, in order to minimize the effect of random errors, one should always perform several independent measurements of the same quantity and take an average of all these readings. In taking the average the random fluctuations tend to cancel out. In fact, the larger the number of measurements taken, the more likely it is that random errors will cancel out.

When we have a set of n measurements x_1, x_2, \dots, x_n of a quantity x , our *best estimate* for the value of x is the average value \bar{x} , is defined as follows.

$$\text{Average value:} \quad \bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n} \quad (0.1)$$

The average value is also known as the mean value. Note that when making repeated measurements of a quantity, one should pay attention to the consistency of the results. If one of the numbers is substantially different from the others, it is likely that a blunder has been made, and this number should be excluded when analyzing the results.

REPORTING ERRORS

Quite often in these labs one has to compare a value obtained by measurement with a standard or generally accepted value. To quantify this one can compute the *percent error*, defined as follows.

$$\text{Percent error:} \quad \% \text{ Error} = \frac{\text{average value} - \text{accepted value}}{\text{accepted value}} \times 100 \quad (0.2)$$

Sometimes one has to report an error when the accepted value is zero. You'll encounter this situation in experiment 3. The procedure to follow is described at the end of that experiment.

CALCULATING UNCERTAINTIES

To estimate the uncertainty associated with our best estimate of x , we begin by examining scatter of the measurements about the mean \bar{x} . Specifically, we start by determining the absolute value of the deviation of each measurement from the mean:

$$\text{Deviation:} \quad \Delta x_i = |x_i - \bar{x}| \quad (0.3)$$

Next we have to compare the deviation to the systematic or reading uncertainty due to limited accuracy of the instrument used. If this systematic uncertainty R is bigger than the deviation Δx_i , then the result of our measurements can be written as

$$\bar{x} \pm R \quad (0.4)$$

If, however, the deviation is larger than the systematic or reading error, then we must determine how big the random uncertainty in our measurements is. This is given by the standard deviation, defined as follows.

$$\text{Standard deviation: } \sigma = \sqrt{\frac{(\Delta x_1)^2 + (\Delta x_2)^2 + \dots + (\Delta x_n)^2}{n-1}} \quad (0.5)$$

The standard deviation has the following meaning: if we were to make one single additional measurement of the quantity x , there is 68% probability of obtaining a value which lies between $\bar{x} - \sigma$ and $\bar{x} + \sigma$. The uncertainty in the average value \bar{x} is smaller (that's the whole point of taking an average!). In fact the uncertainty in \bar{x} is the standard deviation divided by the square root of the number of measurements:

$$\text{Uncertainty in } \bar{x}: \quad U = \frac{\sigma}{\sqrt{n}} \quad (0.6)$$

Sometimes it is useful to express this as a percent uncertainty, defined as one hundred times the uncertainty divided by the average value.

$$\% \text{ uncertainty in } \bar{x}: \quad \% \text{ uncertainty} = \frac{U}{\bar{x}} \times 100 \quad (0.7)$$

EXAMPLE

To illustrate the calculation of \bar{x} and the associated uncertainty U , suppose we are measuring the length of a stick and have obtained, in five separate measurements, the results tabulated below.

Length x (cm)	Deviation Δx (cm)	$(\Delta x)^2$
54.84	0.43	0.1849
53.92	0.49	0.2401
54.46	0.05	0.0025
54.55	0.14	0.0196
54.30	0.11	0.0121
sum: 272.07		sum: 0.4592

From this information we can calculate

$$\text{Average: } \quad \bar{x} = \frac{272.07 \text{ cm}}{5} = 54.41 \text{ cm}$$

$$\text{Standard deviation: } \quad \sigma = \sqrt{\frac{0.4592}{4}} = 0.3388 \text{ cm}$$

$$\text{Uncertainty: } \quad U = \frac{\sigma}{\sqrt{n}} = \frac{0.3388}{\sqrt{5}} = 0.1515 \text{ cm}$$

Thus our final result for the length is $54.41 \pm 0.15 \text{ cm}$.

SIGNIFICANT FIGURES

A number expressing the result of a measurement, or of computations based on measurements, should be written with the proper number of *significant figures*, which just means the number of reliably known digits in a number. The number of significant figures is independent of the position of the decimal point, for example 2.163 cm, 21.63 mm and 0.02163 m all have the same number of significant figures (four).

In doing calculations, all digits which are not significant can be dropped. (It is better to round off rather than truncate). A result obtained by multiplying or dividing two numbers has the same number of significant figures as the input number with the fewest significant figures.

EXAMPLE

Suppose that we want to calculate the area of a rectangular plate whose measured length is 11.3 cm and measured width is 6.8 cm. The area is found to be

$$\text{Area} = 11.3 \text{ cm} \times 6.8 \text{ cm} = 76.84 \text{ cm}^2$$

But since the width only has two significant figures we can round to two figures and report that the area is 77 cm^2 .

GRAPHING, SLOPE AND INTERCEPTS

In almost every laboratory exercise, you plot a graph based on the data measured or calculated. A graph lets you visualize the relation between two physical quantities. In plotting a graph, use the following steps:

1. Arrange the data into a table with two columns listing the values for the two measured or calculated quantities. For example, the first column could list the values for time and the second column could list the values for the average velocity.
2. Decide which of the two quantities to plot along each axis. Graphs have two perpendicular axes, the x-axis and the y-axis and by convention you plot the independent quantity along the x-axis and the dependent quantity along the y-axis.
3. Choose the scale for each axis to cover the range of variation of each quantity. You should choose the scale so that the final curve spans the largest area possible on the graph paper.
4. Label each axis with the quantity plotted on that axis and the units used.
5. Mark the main divisions along each axis.
6. Mark each data point on the graph using the values in each row of the data table. Data points must align with the value of each quantity on their respective axes. Make each data point clearly visible on the graph.
7. Fit and draw a smooth curve through the data points so that the curve comes as close as possible to most of data points. Do not force the curve to go exactly through all the points or through the origin of the coordinate system. The fact that not all points lie along the fitted curve just indicates that measurements are subject to some uncertainty.

In many cases the fitted curve is a straight line. The best straight line fit has nearly the same number of data points above and below the line. The equation for a straight line is given by

Straight Line $y = mx + b$ (0.8)

The quantity b is the intercept: it is the value of y when $x = 0$. The quantity m is the slope of the curve. Given two points on the straight line, $\{y_1 = mx_1 + b, y_2 = mx_2 + b\}$, called basis points, the slope is defined as the ratio of the change in y to the change in x between these points, as shown in equation 0.9.

Slope $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$ (0.9)

Basis points are NOT experimental points. They should be chosen as far from each other as possible to increase the precision of m , as shown in figure 0.1.

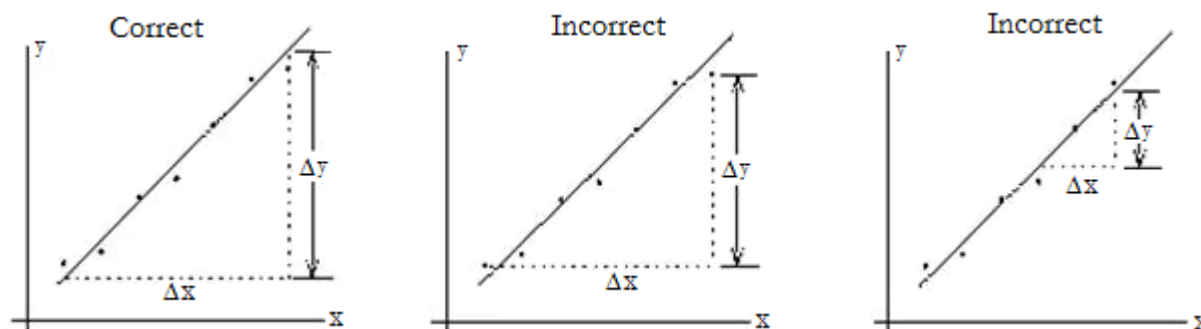


Figure 0.1 Choosing the correct basis points to calculate the slope

PRACTICE CALCULATIONS

1. The accepted value of the acceleration due to gravity on Earth is $g = 980 \text{ cm/s}^2$. When trying to measure this quantity, we performed an experiment and got the following five values for g .

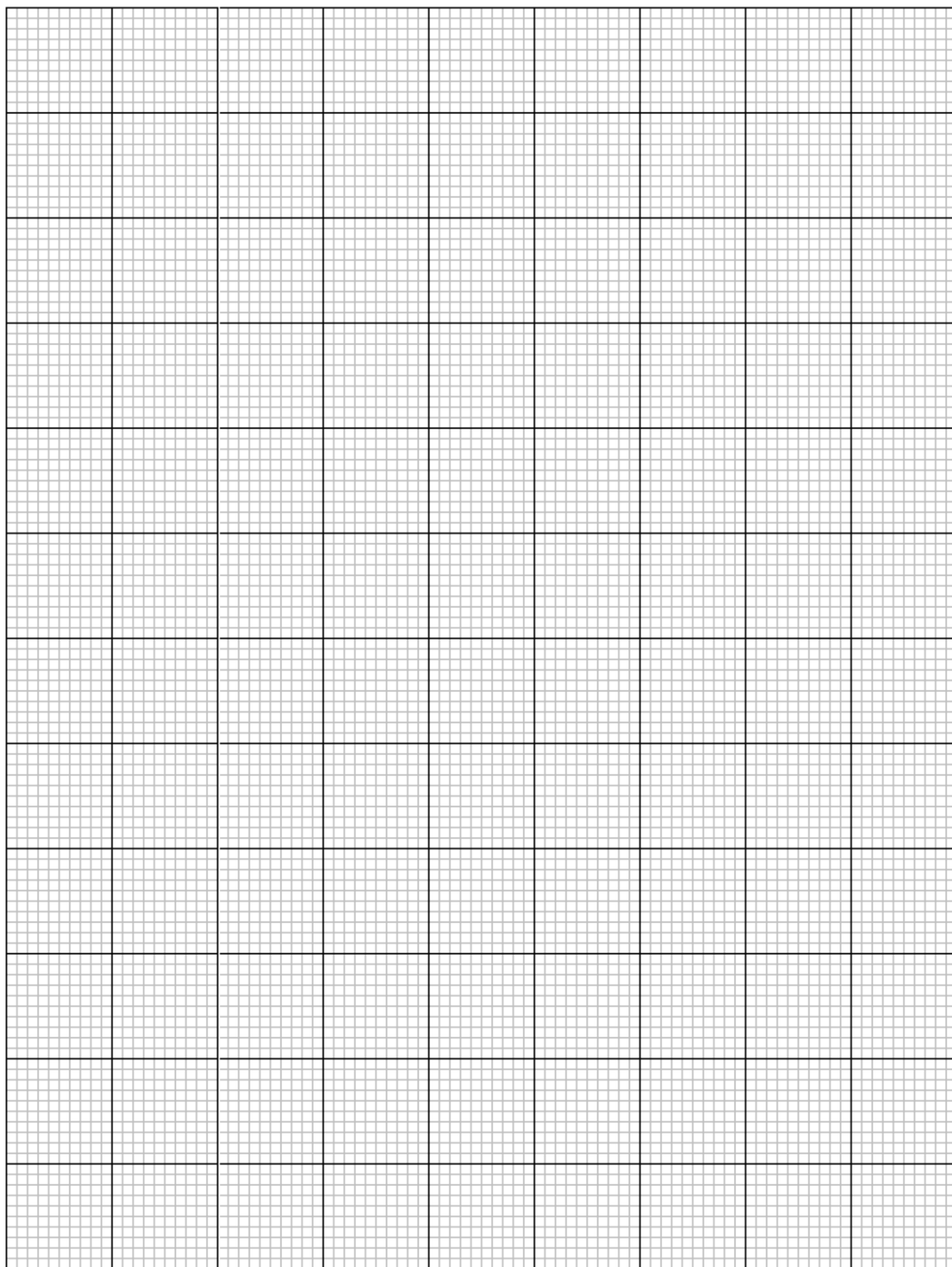
Trial	$g \text{ (cm/s}^2\text{)}$
1	1004
2	992
3	978
4	985
5	982

- Find the average value and standard deviation of our measurements of g .
- Find the uncertainty in our average value for g .
- What is the percent error in our measurement of g ?

2. A box is moving along a frictionless inclined plane. Experimental measurements of velocity at various times are given below.

time $t \text{ (s)}$	velocity $v \text{ (m/s)}$
0	0
1	5.2
2	10.1
3	14.8
4	19.9
5	25

- Plot a graph of v versus t . Can the data be represented by a straight line? (You can use the graph paper on the next page.)
- Calculate the slope.
- What physical quantity does this slope represent?
- From your estimate of the slope, what would the velocity be at $t = 10 \text{ s}$?



Experiment 1: Density

OBJECTIVES

Density describes how much matter is distributed within any given region of space. Quantitatively, it is the amount of mass contained in a unit of volume. In this experiment, you measure the mass and spatial dimensions of a specimen of an unknown metal, calculate its density, and use the result to identify the metal. The objectives of this experiment are as follows:

1. To identify the limits of precision for different measuring devices and calculate the uncertainty for measurements made with them
2. To measure the mass and spatial dimensions of a specimen
3. To calculate the volume and density of a metal specimen

THEORY

The density of a substance is constant for any pure sample at a constant temperature and pressure. For example, the density of a sample of pure Iron has the same density as any other sample of pure Iron at the same temperature and pressure. The density of any substance is its mass per unit volume, calculated using equation 1.1.

$$\text{Density} \quad \rho = \frac{m}{V} \quad (1.1)$$

Where

- m = mass in grams (g), which you measure with a balance.
- V = volume in cubic centimeters (cm^3). The specimens in this lab are rectangular solids, so you calculate volume as the product of the specimen's length, width, and height, each of which you measure using a different instrument.
- ρ = density in grams per cubic centimeter (g/cm^3), which you calculate using equation 1.1.

ACCEPTED VALUES

Metals in their solid state do not change significantly in density with minor changes in temperature and pressure. The accepted values for the density at 20°C of a pure sample of the metals analyzed in this lab are as follows:

- Aluminum: $2.6989 \text{ g}/\text{cm}^3$
- Copper: $8.96 \text{ g}/\text{cm}^3$
- Iron: $7.874 \text{ g}/\text{cm}^3$

APPARATUS

- meter stick
- Vernier calipers
- micrometer calipers
- metal specimens
- platform balance

THE PLATFORM BALANCE

The platform balance can measure the mass of a specimen to the nearest 0.05 grams. The balance measures the mass of specimens on the left platform against the known masses of weights placed on the right platform and the sliders suspended from the two beams, as shown in Figure 1.1.

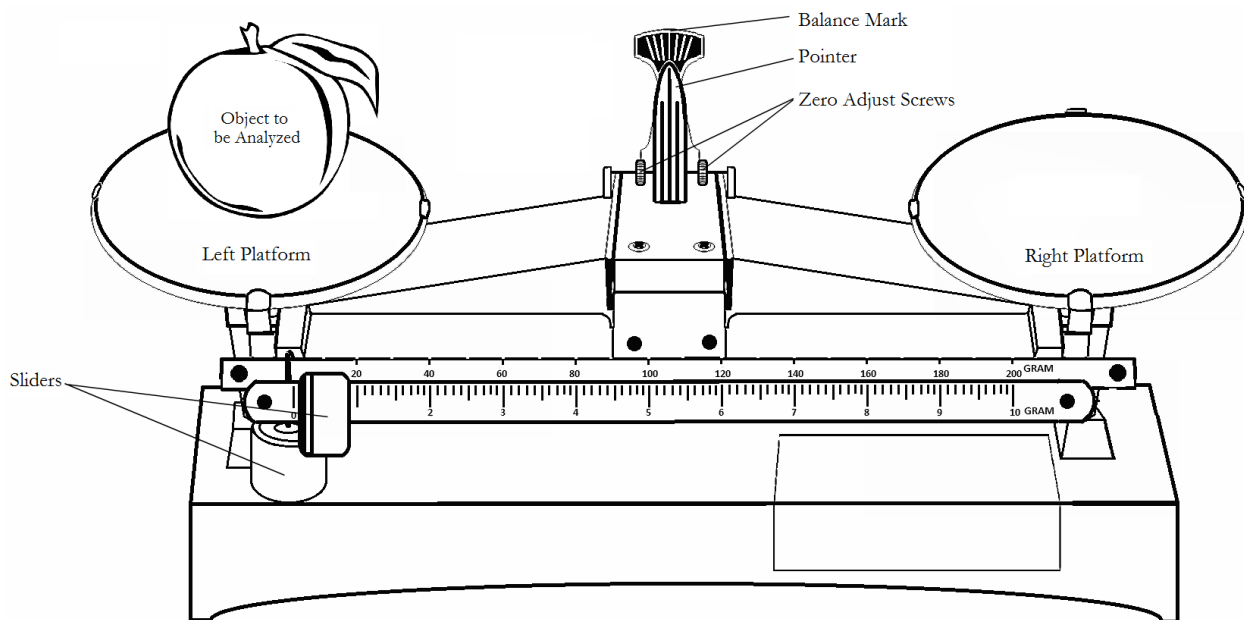


Figure 1.1 The platform balance

To zero the balance, move both sliders to their leftmost positions so that the balance reads zero. If the pointer does not align with the balance mark, then calibrate the balance by turning the zero adjust screws behind the pointer until the pointer aligns with the balance mark.

To measure the mass of a specimen, place the specimen on the left platform. Move the slider along the rear beam, letting it stop at each notch, until the pointer moves to the right of the balance mark. Then, move the slider on the rear beam to the notch immediately to the left of its current position, which measures the mass of the specimen to within 10 grams. Move the slider on the front beam to the right until the pointer aligns with the balance mark. Measure the grams from the front beam using the boldface numbers and tenths of a gram using hash marks between the boldface numbers. If the slider rests between hash marks, add an additional 0.05 grams. If the slide rests on a hash mark, add an additional trailing zero to the right of the decimal point. For example, if you measure 5.6 grams on the front beam and the slider rests exactly on the 5.6 gram hash mark, the precise measurement is 5.60 grams. Add the mass measured on the front beam to the mass measured on the rear beam to measure the precise mass of the specimen.

THE METER STICK

The meter stick can measure distance to the nearest half a scale division, 0.5 millimeters (0.05 cm). The ends of the specimen being measured seldom line up exactly with a mark on the scale, so estimate any fractional part of the smallest division at both ends.

THE VERNIER CALIPERS

The Vernier calipers can measure distance to the nearest tenth of a millimeter (0.01 cm). This instrument is shown in Figure 1.2. The Vernier scale on the slide measures a fractional part of the

main scale. To take a measurement, place the specimen between the large jaws of the instrument. The hash mark on the main scale that aligns with the zero mark on the Vernier scale indicates the length of the specimen to within 1 millimeter (0.1 cm). The zero mark of the Vernier scale is the long line at the left end.

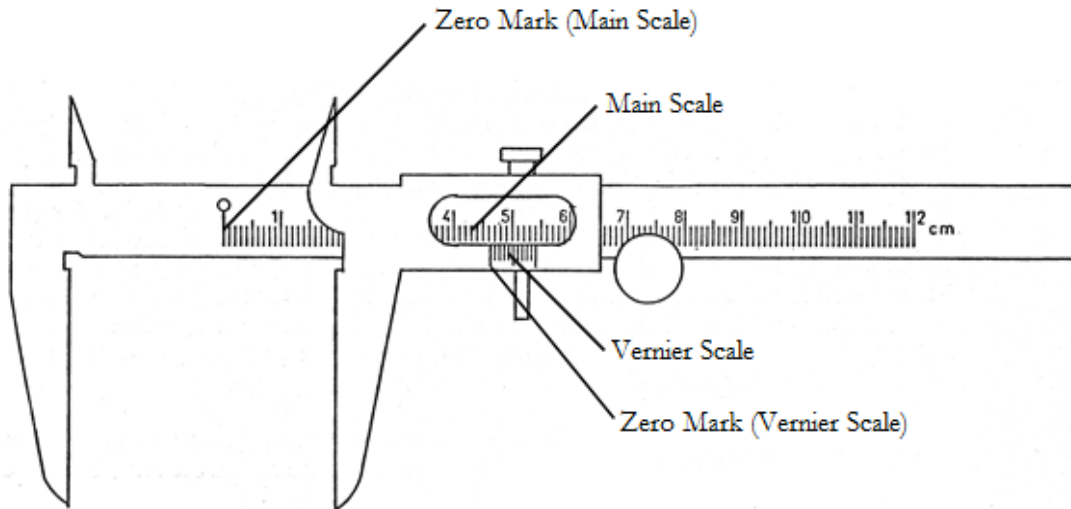


Figure 1.2 The Vernier calipers

On the Vernier scale, the 10 divisions have the length of only 9 divisions on the main scale. Figure 1.3 shows the relation between the two scales when the calipers are closed.

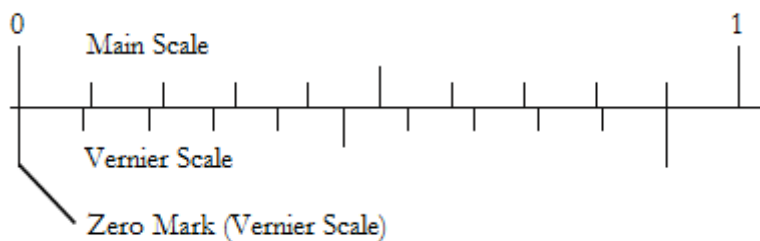


Figure 1.3 The initial positions of the main scale and Vernier scale

The main scale is divided into 0.1 cm intervals and the Vernier scale is 0.09 cm long, which provides an additional level of precision to one hundredth of a centimeter. In Figure 1.4, the zero line of the Vernier scale lies between the 2.3 cm and 2.4 cm marks of the main scale and the fourth mark on the Vernier scale aligns with a main scale line.

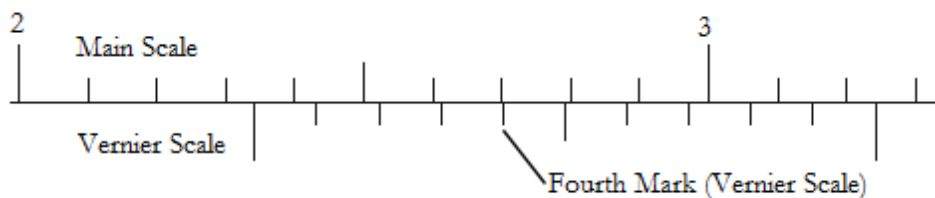


Figure 1.4 An example of the Vernier scale, which reads 0.04 cm past 2.3 cm or 2.34 cm

The difference between the size of a main scale division and a Vernier division is 0.01 cm. Therefore, the difference between the 2.3 main scale mark and the Vernier zero mark is 0.04 cm and the precise measurement is 2.34 cm. In general, if the n th line of the Vernier scale coincides with a main scale division, the Vernier zero mark is at a distance ($n \times 0.01$ cm) beyond the main scale division immediately to the left of the Vernier zero mark. To find the length of the specimen, add this distance to the main scale measurement. For example, if line 1 on the Vernier scale coincides with a main scale line, the Vernier zero mark is 0.01 cm beyond the mark on the main scale. If line 2 on the Vernier scale coincides with a main scale line, the Vernier zero mark is 0.02 cm beyond the mark on the main scale, and so on.

THE MICROMETER CALIPERS

The micrometer calipers can measure distance to the nearest hundredth of a millimeter (0.001 cm). The construction of this instrument is shown in figure 1.5. To measure a specimen, place it between the jaws (A and B). The spindle (B) moves by turning a precision screw connected to the thimble (D). Turning the thimble opens or closes the jaws. The distance between the jaws is given by the scale on the sleeve (C), which is ruled in millimeters. There are 50 divisions on the circular scale of the thimble. It takes two turns of the thimble to advance the spindle 1 mm, so each division on the circular scale of the thimble corresponds to an advancement of one hundredth of a millimeter.

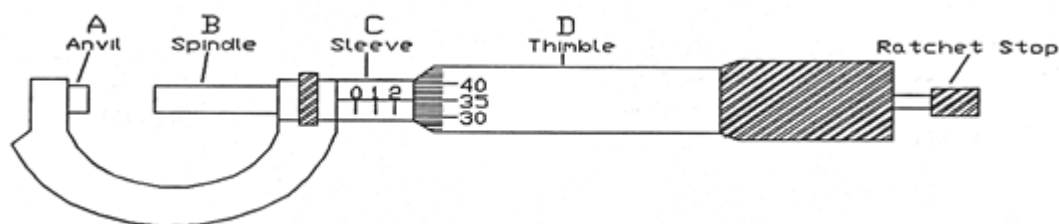


Figure 1.5 The micrometer calipers

To measure the length of a specimen, add the highest number of millimeters visible on the sleeve (C) to the hash mark on the thimble (D) that aligns with the horizontal axis on the sleeve. For example, if the edge of thimble (D) lies between 2.0 and 2.5 mm on the sleeve, as shown in figure 1.5, and the thirty-fifth mark of the circular scale is aligned with the horizontal mark of the sleeve scale, the distance between the jaws is 2.350 mm (0.2350 cm). If the thimble were rotated until the measurement on the circular scale was again 0.350 mm, then the sleeve scale measurement is between 2.5 mm 3.0 mm, and the total distance between the jaws is 2.850 mm (0.2850 cm).

Before measuring lengths with either the Vernier calipers or the micrometer calipers, check the measurement when the jaws are completely closed to observe the systemic uncertainty caused by a consistent error in the measuring instruments. Record the measurement when the jaws are closed as the **offset**. If the **offset** is not zero, correct all other measurements by that amount. For example, if the micrometer measurement is 0.02 mm when the jaws are closed, then correct all measurements made by the micrometer by subtracting 0.02 mm from the micrometer measurement. This avoids a systematic error due to irregularities in the measuring instrument.

PROCEDURE

1. Zero the balance so that it reads zero when the platform is empty. Do not wait for the pointer to come to rest. Adjust the zero adjust screws behind the pointer until the pointer swings equally to the left and to the right in successive swings.
2. Measure the mass of the specimen, recording the value to the nearest 0.05 grams. Take three measurements.
3. Measure the length of the specimen using the meter stick, recording three measurements across the same dimension but at different intervals. Estimate to the nearest 0.05 cm. Avoid using the ends of a wooden meter stick, as they may be worn.
4. Record the **offset** of the Vernier calipers when the jaws are closed. Subtract the offset from all subsequent measurements made with this instrument to obtain the actual measurement. If the offset is negative, then subtracting the negative number results in a net addition to the measurement to obtain the actual measurement.
5. Follow your instructor's instructions for correct use and reading of the Vernier calipers. Measure the width of the specimen and record three measurements on different parts of the specimen. Estimate to the nearest 0.01 cm.
6. Record the **offset** of the micrometer calipers when the jaws are completely closed. Do not force the calipers closed. They close with a modest effort.
7. Follow your instructor's instructions for correct use and reading of the micrometer calipers. Measure the height of the specimen and record three measurements at different points on the specimen. Estimate to the nearest 0.001 cm.

DATA

1.1 Mass, Length, Width, and Height

Measurement	mass (g)	length (cm)	width (cm)	height (cm)
offset				
1				
2				
3				

CALCULATION AND ANALYSIS

When analyzing repeated measurements, pay attention to the consistency of the results. If one of the numbers is substantially different from the others, it is likely that a procedural error has been made. Exclude a highly deviated value when analyzing the results.

1. Calculate the average measurements for the mass, length, width and height of your specimen.
2. Calculate the uncertainty of the average measurements for the mass, length, width and height of your specimen.
3. Calculate the volume of the specimen using the average measurements for length, width, and height.
4. How many significant figures does your result for the volume have?
5. Calculate the density of the specimen.
6. How many significant figures does your result for the density have?
7. Identify the metal from which your specimen is made using the standard density values from the theory section of this lab. Assume that the specimen is a pure metal.
8. Calculate the % error of the calculated density of the specimen using the standard density as the accepted value.
9. Suggest at least three hypotheses that could explain the % error and how you could change the experiment to eliminate that error.

Experiment 2: Acceleration of a Freely Falling Object

OBJECTIVES

Acceleration is the rate at which the velocity of an object changes over time. An object's acceleration is the result of the sum of all the forces acting on the object, as described by Newton's second law. Under ideal circumstances, gravity is the only force acting on a freely falling object. In this lab, you measure the displacement of a freely falling object, calculate the average velocity of a falling object at set time intervals, and calculate the object's acceleration due to gravity. The objectives of this experiment are as follows:

1. To measure the displacement of a freely falling object
2. To test the hypothesis that the acceleration of a freely falling object is uniform
3. To calculate the uniform acceleration of a falling object due to gravity, g

THEORY

The average acceleration of an object measures how quickly its velocity is changing. If the velocity changes by an amount Δv during a time Δt , the average acceleration is calculated as shown in equation 2.1.

$$\text{Average Acceleration} \quad \bar{a} = \frac{\Delta v}{\Delta t} \quad (2.1)$$

In this experiment we study the motion of an object falling vertically down, that is, one-dimensional motion. Because the acceleration is constant, the average acceleration is equal to g . If the velocity of an object at $t = 0$ is v_0 , the velocity v at time t of an object moving in one dimension with constant acceleration g is shown in equation 2.2.

$$\text{Velocity} \quad v = v_0 + gt \quad (2.2)$$

If the position of an object at $t = 0$ is y_0 , then the position y at time t of an object moving in one dimension with constant acceleration g and an initial velocity of v_0 is shown in equation 2.3.

$$\text{Position} \quad y = y_0 + v_0 t + \frac{1}{2} gt^2 \quad (2.3)$$

Because the velocity of an accelerating object constantly changes, it is not possible to calculate the velocity at an exact time algebraically from measuring positions over time. However, you can approximate the velocity at the midpoint of any time interval by calculating the average velocity over that time interval. The average velocity \bar{v} of an object in one dimension as it moves along the y -axis is defined as the change in its position, Δy , over time, Δt , as shown in equation 2.4.

$$\text{Average Velocity} \quad \bar{v} = \frac{\Delta y}{\Delta t} \quad (2.4)$$

The change in position, or displacement, of an object in one dimension as it moves along the y -axis is defined as the change in its position, Δy , from an initial position y_i to a new position y_{i+1} , as shown in equation 2.5.

$$\text{Displacement} \quad \Delta y = y_{i+1} - y_i \quad (2.5)$$

ACCEPTED VALUES

The acceleration due to gravity varies slightly, depending on the latitude and the height above the earth's surface. In this experiment the change in height of the falling object is negligible and can be approximated as 0 km for its entire descent. The acceleration due to gravity at $40^\circ 52' 21''$ N latitude (the latitude of Lehman College) and 0 km altitude is

$$g = 980.2 \text{ cm} / \text{s}^2$$

APPARATUS

- synchronized sparking device
- meter stick
- earth's gravity

THE SYNCHRONIZED SPARKING DEVICE

The synchronized sparking device records the position of a falling object at regular time intervals. The sparking device consists of a pair of parallel vertical wires with a high electrical potential between them, a strip of heat sensitive recording paper along the rear wire, and an object made of conducting material that falls between the wires, as showing in figure 2.1.

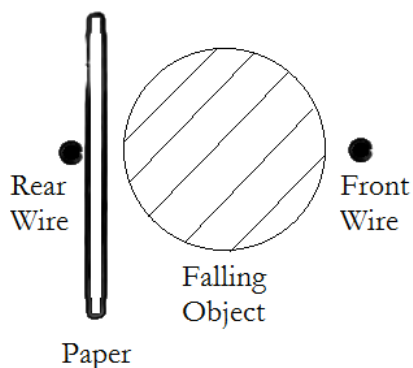


Figure 2.1 A horizontal cross-section of the sparking device and falling object

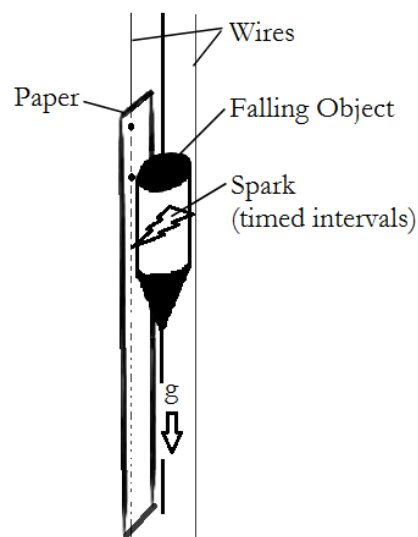


Figure 2.2 Sparks mark the position of the falling object every $1/60$ of a second

The conductive object, held initially by an electromagnet, falls between the wires along the recording paper so that sparks pass through the object and the paper from one wire to the other wire every $1/60$ of a second, as shown in figure 2.2. Sparks leave a mark on the recording paper to mark the object's position at the time each spark is generated without changing the motion of the object as it falls.

To quantify the data from the recording paper, remove it from the device and measure the distance between the marks using a meter stick to determine the displacement of the object between the marks. To minimize errors due to the initial release of the object and the fact that the sparks do not always travel exactly horizontally, ignore the first two data points. To measure intervals for acceleration calculations, record only every third mark, such that the interval between two successive recorded marks corresponds to a time interval, Δt , of $3/60$ of a second or 0.05 second.

PROCEDURE

1. Recording paper tapes are prepared before the lab begins. The instructor can explain and demonstrate the method used in marking the recording paper.
2. Lay your group's recording paper on your table. Have one person hold each end flat and be sure not to stretch it.
3. Skip the first two spark marks and circle the third spark mark on the recording paper. Then, circle every third spark mark after that.
4. Place a meter stick on its edge over the recording paper so that the millimeter and centimeter hash marks lie next to the spark marks on the recording paper, as shown in Figure 2.3.
5. Record the distance of each of the circled spark marks from the starting position in the distance y (cm) column of your data table.
6. Record the difference in distance between each pair of successive circled spark marks in the displacement Δy (cm) column of your data table.
7. Have the instructor check your recording paper and measurements and rerecord any values that are incorrect.

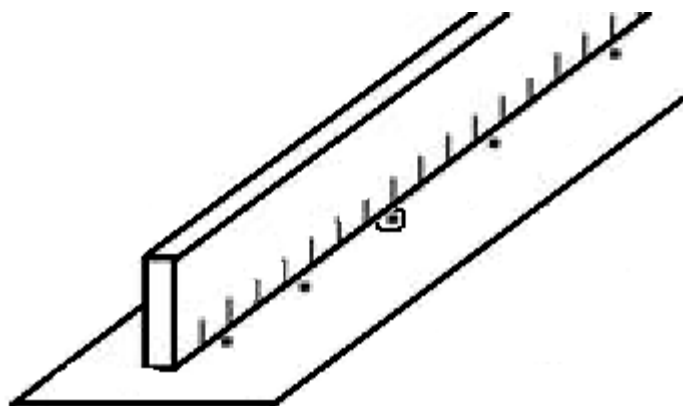


Figure 2.3 Placing a meter stick over the recording paper

DATA

2.1 Position of a falling object

time t (s)	distance y (cm)	displacement Δy (cm)
0.050		
0.100		
0.150		
0.200		
0.250		
0.300		
0.350		
0.400		
0.450		
0.500		

2.2 Average Velocity

time t at the middle of an interval (s)	average velocity \bar{v} (cm/s)
0.075	
0.125	
0.175	
0.225	
0.275	
0.325	
0.375	
0.425	
0.475	

CALCULATION AND ANALYSIS

- Using plotting software, create a table with two columns and label the first column “time t at the middle of an interval (s)” and the second column “average velocity \bar{v} (cm/s)” as shown in table 2.2. If you do not have access to plotting software, use table 2.2 instead.
- Calculate the midpoint of each time interval and record the results in the first column “time t at the middle of an interval (s).”

3. Calculate the average velocity \bar{v} for each time interval Δt and record the results in the second column “average velocity \bar{v} (cm/s).” Remember that Δt for each interval is the same, $3/60$ of a second or 0.05 seconds, and that the velocity should be increasing over time.
4. Using plotting software or good plotting paper, plot a graph of the average velocity \bar{v} versus time t at the middle of an interval. Be sure to observe the following rules:
 - a. Time t at the middle of an interval is the independent variable and goes on the horizontal x -axis.
 - b. Start both axes at zero and choose scales large enough so that your data points fill most of the plot
 - c. Label each axis with the variable and units of the axis
5. Fit the plotted data points to a straight line using a straight edge or graphing software. An ideal straight line fit has nearly the same number of points above and below the line. The experimental points should lie around a straight line that is nearly identical to the line generated by graphing equation 2.2, which is $v = gt + v_0$ when arranged in y-intercept form with g as the slope and v_0 as the y-intercept.
6. Find the slope of your graph g by plugging two points on the line into equation (0.9). Observe the following rules which apply to all future labs:
 - a. Find the slope from the best fit line, not from data points which lie off the line. The line averages out some of the random errors in individual points.
 - b. Find the slope from points near opposite ends of the line, not points near each other. If small numbers are off a little, there is a greater magnitude of error than if large numbers are off a little.
7. Calculate the % error for your value for g , as defined in equation (0.2).
8. What value do you get for the y-intercept and x-intercept of your best straight line? Give units for these intercepts. What is the meaning of these numbers?

Experiment 3: Static Equilibrium

OBJECTIVES

When all the external forces acting on object do not accelerate the object, the object is in a state of mechanical equilibrium. If the object is also at rest, the object is in a state of static equilibrium. In this experiment, you arrange sets of forces to put an object into static equilibrium, measure the vector quantities of these forces, and calculate the net force acting upon an object in equilibrium. The objectives of this experiment are as follows:

1. To measure vector quantities for forces using the force table
2. To calculate the net force on an object using vector addition
3. To test the hypothesis that an object in equilibrium has no net force acting upon it

THEORY

According to Newton's second law of motion, an object accelerates in direct proportion to the net force acting on it. An object in static equilibrium is not moving, so has an acceleration of zero, and the net force on the object is also zero. Therefore, the necessary condition for equilibrium is that the vector sum of all external forces acting on the object is zero, as shown in equation 3.1.

$$\text{Equilibrium} \quad F_{total} = \sum_i F_i = 0 \quad (3.1)$$

In this experiment, you apply forces to an object in two dimensions until it is in static equilibrium, measure the vector forces, and calculate the vector sum. All forces are applied in the plane (two dimensions), so one can project the forces on the x - and y -axes as shown in equations 3.2.

$$\begin{aligned} \text{2-D Equilibrium} \quad F_{total,x} &= \sum_i F_{i,x} = 0 \\ F_{total,y} &= \sum_i F_{i,y} = 0 \end{aligned} \quad (3.2)$$

To decompose a force vector into its x and y components, it is convenient to choose the x -axis along the direction $\phi = 0^\circ$ and the y -axis along the direction $\phi = 90^\circ$. Then the components of the forces are shown in the pair of equations 3.3.

$$\text{Component Forces} \quad F_{i,x} = F_i \cos \phi_i, \quad F_{i,y} = F_i \sin \phi_i \quad (3.3)$$

ACCEPTED VALUES

The accepted value for the sum of the forces on an object in equilibrium is 0.

APPARATUS

- Horizontal force table
- four pulleys
- one metal ring
- four cords
- four weight hangers
- a degree scale
- Assortment of known weights.

THE FORCE TABLE

A force table consists of a circular platform supported by a heavy tripod base. The circular platform has a graduated degree scale around its rim and a small peg located directly in the center. Four cords are attached to a metal ring placed over a peg in the center of the platform and the cords are connected over pulleys to weight hangers, as shown in Figure 3.1.

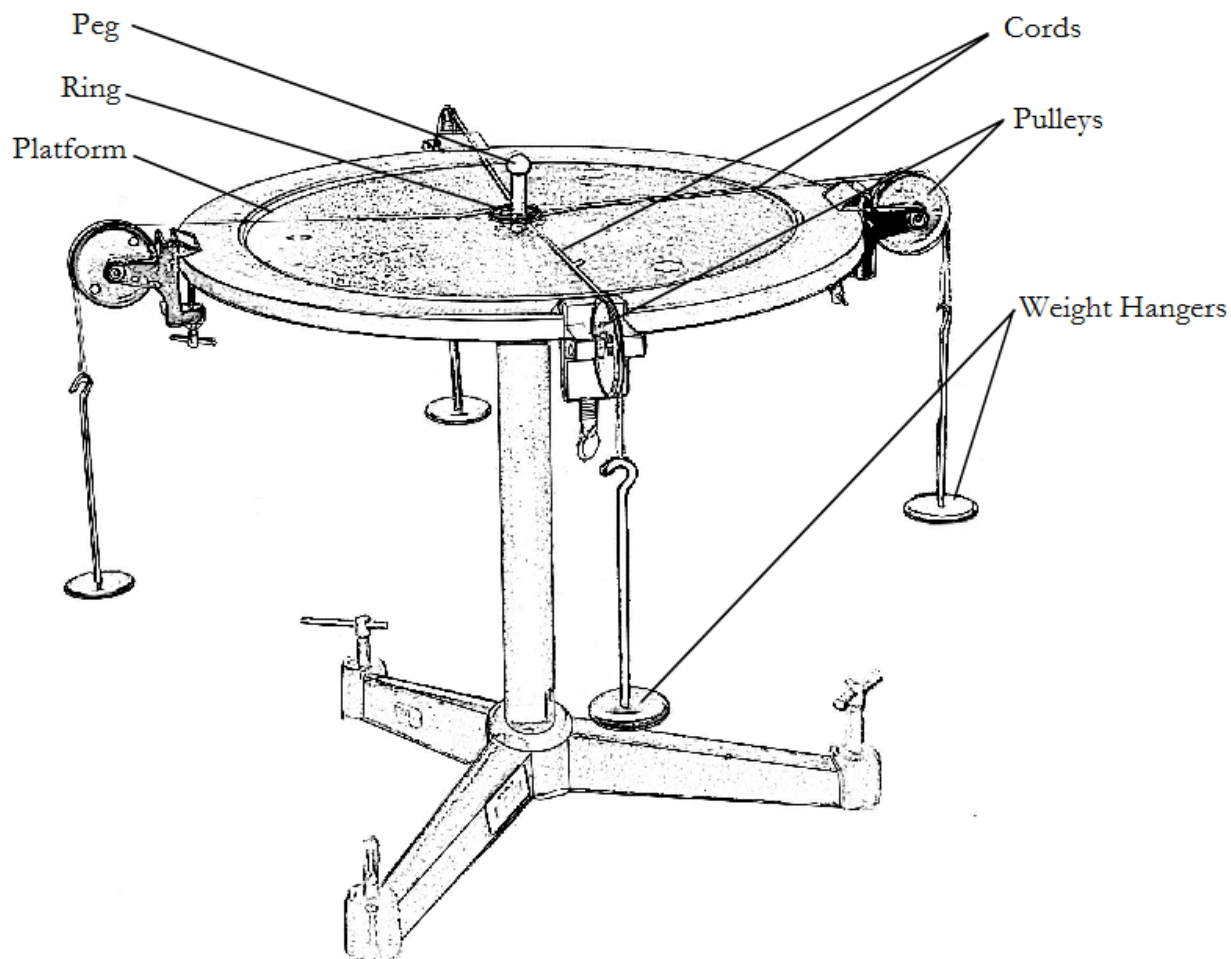


Figure 3.1 An assembled force table

Tension forces are applied to the ring by varying the total mass on each weight hanger and moving the pulleys to change the direction in which each force acts. The ring is in a state of static equilibrium when it is over the peg but not touching the peg, as shown in figure 3.2.

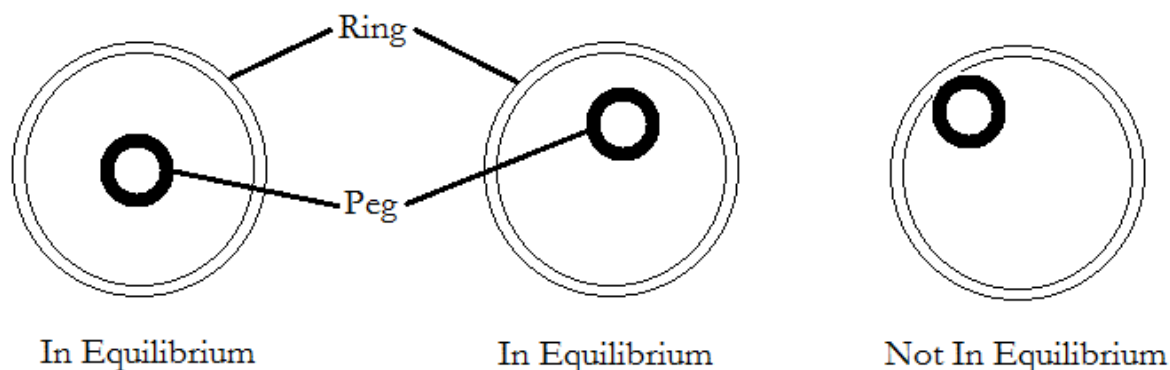


Figure 3.2 Overhead view of ring and peg positions for system in and not in equilibrium

PROCEDURE

1. Mount a pulley at 0° and attach 250 g to the cord running over it. Remember that the holder is part of the mass.
2. Mount a second pulley at 60° with a load of 350 g.
3. Holding a third cord in your hand, find the direction in which a third force should act in order to balance the system. Set the cord on a pulley in the proper position and add weights to the holder until the system is in static equilibrium, as shown in figure 3.2. It may be necessary to adjust the position of the weight holder to achieve equilibrium.
4. Record masses, angles and forces in the data sheet labeled Trial 1.
5. Repeat step 1 through 4 using a 45° angle between the two loads. Record your results in the data sheet labeled Trial 2.
6. Set up four pulleys and suspend unequal loads on the cords running over them. Arrange the system so that it is in equilibrium, and record the masses and angles. Do not have any two cords form an angle of 180° . Record your results in the data sheet labeled Trial 3.
7. Suppose you place a mass, $m = 300$ g, at $\phi = 210^\circ$ mark. Compute the masses m_a and m_b you would place at 0° and 90° to balance mass m . Try it, and see if your solution is correct. Report what masses you had to place at 0° and 90° to balance the mass at 210° .

DATA

Trial 1

i	m_i (grams)	$F_i = m_i g$ (Newtons)	ϕ_i ($^\circ$)
1			0.0
2			60.0
3			

Trial 2

i	m_i (grams)	$F_i = m_i g$ (Newtons)	ϕ_i ($^\circ$)
1			0.0
2			45.0
3			

Trial 3

i	m_i (grams)	$F_i = m_i g$ (Newtons)	ϕ_i ($^\circ$)
1			
2			
3			
4			

CALCULATION AND ANALYSIS

1. Draw a picture showing the three forces for Trial 1. (This is sometimes called a free body diagram.) Be sure to label each vector's direction and magnitude.
2. Calculate the component forces for each force vector, F_{ix} and F_{iy} , using equation 3.3.
3. Calculate $|F_{total}|$ and $\sum_i |F_i|$ using equations 3.5 and 3.6, given below.

4. Calculate the % discrepancy of the force calculations using equation 3.4.
5. Repeat steps 2 through 4 using data from trials 2 and 3.
6. In procedure 7, was the system in equilibrium? If not, suggest some possible reasons.
7. What are the sources of experimental error in this experiment? Do any of these factors help the ring achieve static equilibrium?

REPORTING % ERROR WHEN THE ACCEPTED VALUE IS ZERO

In this experiment, the accepted value of the total force is zero. If you could measure all the forces on the ring with perfect precision, you would find that the net force vanishes. If zero is inserted into our initial % Error equation (0.2), the result is undefined because the denominator is zero. Due to experimental errors and measurement uncertainties, the calculated net force isn't zero. A useful way to characterize the accuracy of our measurements is to divide the magnitude of net force, $|F_{total}|$, by the sum of the magnitudes of all the component forces, $\sum_i |F_i|$, multiplied by 100% as shown in equation 3.4.

$$\% \text{ Discrepancy} = \frac{|F_{total}|}{\sum_i |F_i|} \times 100\% \quad (3.4)$$

The quantities appearing in this formula are

$$\text{Magnitude of Net Force, 2-D} \quad |F_{total}| = \sqrt{F_{x,total}^2 + F_{y,total}^2} \approx 0 \quad (3.5)$$

$$\begin{aligned} \text{Component Magnitude Sum} \quad \sum_i |F_i| &= \sum_i \sqrt{F_{x,i}^2 + F_{y,i}^2} \neq 0 \quad (3.6) \\ &= \sqrt{F_{x,1}^2 + F_{y,1}^2} + \sqrt{F_{x,2}^2 + F_{y,2}^2} + \dots + \sqrt{F_{x,n}^2 + F_{y,n}^2} \end{aligned}$$

Experiment 4: Newton's Second Law

OBJECTIVES

Newton's second law predicts that acceleration is a function of force and mass. To test this mathematical relationship, a good experiment must isolate each contributing component and vary it independently of the others. In this experiment, you measure the acceleration of an object by varying the force acting upon the object without changing its mass and by varying the object's mass without changing the force. The objectives of this experiment are as follows:

1. To measure the linear acceleration of objects acted on by external forces.
2. To predict the acceleration of an object by applying Newton's Second law.
3. To test the predictions using calculations and graphical methods.

THEORY

Newton's second law in vector form is shown in equation 4.1.

$$\text{Newton's second law} \quad \vec{F} = m\vec{a} \quad (4.1)$$

Here \vec{F} is the net force acting on an object, m is the mass of the object, and \vec{a} is its acceleration. If the force is constant, as, for instance, the force of gravity, the object moves with constant acceleration. Newton's second law also applies to systems of bodies considered as a whole, like two masses connected by a cord. Each of the objects in this experiment moves along a straight line. Thus it is sufficient to consider projections of the vectors on the direction of motion and we can remove the vector notation: $F = ma$.

In this experiment we measure the acceleration of a system consisting of a glider moving along a nearly frictionless air track, and a falling/hanging mass tied to the glider via a cord. The net force on the system is exerted by the gravitational force acting on the hanging mass over a low-friction pulley. If you ignore friction, the acceleration of the system according to Newton's second law is shown in equation 4.2.

$$\text{Acceleration of the glider} \quad a = g \frac{m}{M} \quad (4.2)$$

Here m is the hanging mass, M is the total moving mass, and g is the free-fall acceleration due to gravity.

The acceleration of an object is the rate of change in its velocity. If the velocity changes by an amount Δv during a time Δt , the average acceleration is shown in equation 4.3.

$$\text{Average Acceleration} \quad \bar{a} = \frac{\Delta v}{\Delta t} \quad (4.3)$$

Here $\Delta t = t_2 - t_1$ and $\Delta v = v_2 - v_1 \equiv v(t_2) - v(t_1)$. If Δt becomes very small, equation 4.3 gives the instantaneous acceleration at $t_2 \approx t_1$. For the motion with constant acceleration that we study in this experiment, the average and instantaneous accelerations are the same.

In the experiment, the glider goes through two photo gates, G_1 and G_2 , its flag interrupting the light path in the gates. The computerized system measures three time intervals, T_1 , T' , and T_2 , as depicted in figure 4.1.

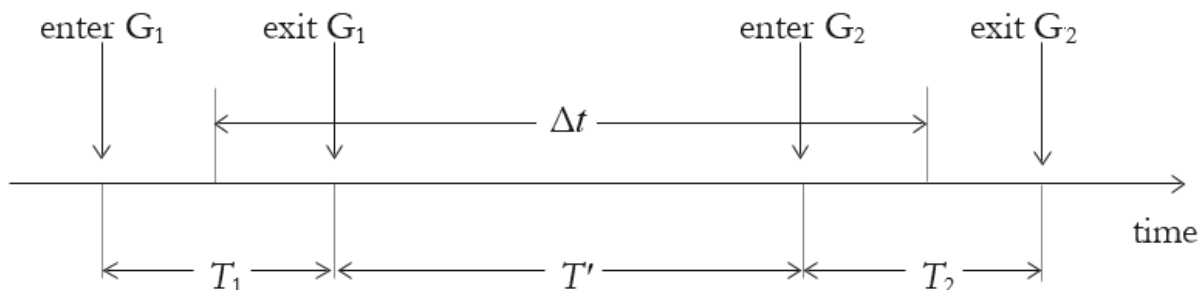


Figure 4.1 The motion of the glider flag through photo gates G_1 and G_2

Time interval T_1 begins when the flag enters the first gate, G_1 . As the flag clears this gate, T_1 ends and the second interval, T' , begins. Then, as the flag enters the second gate, G_2 , T' ends and the third interval, T_2 , begins. It ends when the flag clears gate G_2 .

From the length of the flag L measured in this experiment, you can calculate the average velocities of the glider crossing gates 1 and 2, as shown in equation 4.4.

$$\text{Average velocities} \quad \bar{v}_1 = \frac{L}{T_1}, \quad \bar{v}_2 = \frac{L}{T_2} \quad (4.4)$$

For motion with constant acceleration the velocity changes linearly with time, so these average velocities coincide with instantaneous velocities in the middle of the time intervals T_1 and T_2 . Thus the time interval Δt corresponding to the velocity change $\Delta v = v_2 - v_1$ is shown in equation 4.5.

$$\text{Elapsed time} \quad \Delta t = T' + \frac{1}{2}T_1 + \frac{1}{2}T_2 \quad (4.5)$$

Therefore, the formula for experimentally determining the acceleration is the difference of the average velocities divided by the elapsed time, as shown in equation 4.6.

$$\text{Acceleration (experimental)} \quad a = \frac{\Delta v}{\Delta t} = \frac{\left(\frac{L}{T_2} - \frac{L}{T_1} \right)}{\left(T' + \frac{1}{2}T_1 + \frac{1}{2}T_2 \right)} \quad (4.6)$$

ACCEPTED VALUES

The expected value of the acceleration for all of the mass distributions in this experiment is the result of equation 4.2. For Part A, you change the masses of the falling/hanging object and the total moving mass, so obtain a different expected value for each setup.

For Part B, the same force is acting in each case, and therefore Newton's second law predicts that the product of the total moving mass M_1 and its acceleration a_1 is equal to the product of any other mass and its acceleration under the same conditions, as shown in equation 4.7.

Newton's second law

$$M_1 a_1 = M_2 a_2 = F = mg$$

(4.7)

APPARATUS

- glider
- air track
- photo gates
- computer with PASCO interface.
- cord
- pulley
- flags
- earth's gravity

THE AIR TRACK AND GLIDERS

In this experiment you measure the acceleration of a system consisting of a glider moving along a nearly frictionless air track, and a falling/hanging mass tied to the glider via a string, as shown in figure 4.2.

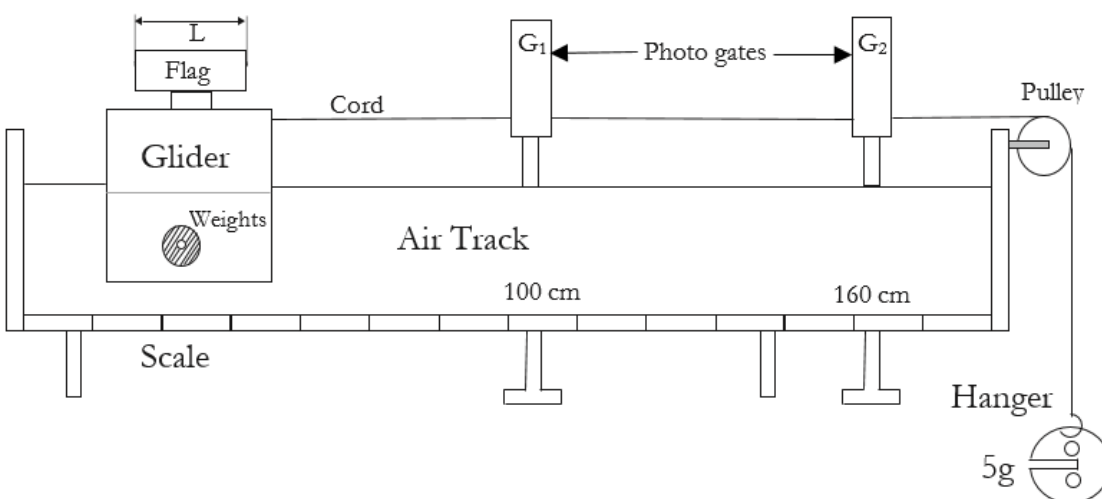


Figure 4.2 System consisting of a glider on a nearly-frictionless air track connected to a hanging weight by a cord over a low-friction pulley

The net force on the system is exerted by the gravitational force acting on the hanging mass. The glider is supported by a cushion of air coming out of the holes in the horizontal frame, so that friction is almost eliminated and can be neglected.

The glider has two rods on which weights may be set. You should have four fifty gram weights (shiny cylinders) and five 5 gram weights (flat slotted disks). The slotted disks stay firmly on the glider if the fifty-gram weights are placed on top of them.

Your experimental station also includes a pair of photo gates, connected to a computer, which function as an electric stopwatch.

PROCEDURE

PART A: ACCELERATION AS A FUNCTION OF FORCE WITH A CONSTANT MASS

1. Measure the length of the flag, using the scale on the air track and record this as length of the flag, L .

2. Measure the mass of the weight hanger and record this value as the mass of the weight hanger, m_{hanger} .
3. Put two 50 gram weights, five 5 gram weights, the flag, the cord, and the weight hanger on the glider, and weigh everything together. Record this as the total moving mass, M , in the data table for Part A.
4. Place Photo gate G_1 at 110 cm, and photo gate G_2 at 160 cm. Make sure that G_1 is connected to “DIGITAL CHANNELS” input 1 of the interface, and G_2 to input 2, respectively.
5. Place the glider on the air track. Turn on the blower and run the glider slowly through the gates. Make sure that the flag blocks the light but does not hit the gates. Then turn off the blower.
6. Turn on the PC with Interface turned ON and log in as “student” using the password provided by the lab instructor.
7. From desktop click the icon “Newton Second Law”.
8. Move the glider to 100 cm on the air track.
9. Place two 5g weights and one 50 g weight onto the rods on each side of the glider.
10. Let the cord fall over the pulley at the end of the track and attach a 5 gram weight to the hanger, as shown in figure 4.2.
11. Measure the motion of the glider with a 5 gram hanging/falling weight by performing the following steps:
 - A. Click “Start” on the PC monitor: The system is ready to collect the data.
 - B. Hold the glider at 100 cm and turn on the air blower, wait for the pitch of the blower to reach a stationary level.
 - C. Let the glider go. The times T_1 , T' , and T_2 , respectively, fill the first row of the table with columns labeled “Timer 1”, “Timer2”, and “Timer 3”. Be careful not to let it bounce back into the second gate!
 - D. Remove the glider from the air track and replace it at 100 cm without disturbing the photo gates. Do not stop the PC data acquisition after the glider crosses the two gates and do not switch off the air blower.
 - E. Repeat runs described by steps C and D four times. Each time a new row in the data table is added. Record the values of T_1 , T' , and T_2 of each run into your data sheet for the rows with the hanging/falling mass. If one or more of the runs yields times substantially different from the results of other runs, an error occurred. In this case clear the data sheets and repeat the experiment.
 - F. To clear all entries, click “Stop”, and from “Experiment” menu click “Clear ALL Data Runs”.
12. Repeat step 11 by increasing the hanging mass by transferring 5 gram weights from the glider to the hanger. For each value of the mass on the hanger $m = 5, 10, 15, 20,$ and 25 grams plus the mass of the hanger m_{hanger} , make five runs and record your data for T_1 , T' , and T_2 of each run in the data sheet for Part A. Always transfer weights from the glider to the hanger so the total mass of the system remains constant. When removing weights from the glider, ensure that the weights on each side of the glider are approximately equal, otherwise the glider can slip off the air cushion and add friction to your measurements.

PART B: ACCELERATION AS A FUNCTION OF MASS WITH CONSTANT FORCE

1. Put two 50 gram weights, one 5 gram weight, the flag, the cord, and the weight hanger on the glider, and weigh everything together. Record this as the total moving mass, M_1 , in the data Section for Part B. Place one 50 g weight onto the rods on each side of the glider.

- Place the glider on the track. Let the cord fall over the pulley at the end of the track and attach a 5 gram weight to the hanger, as shown in figure 4.2.
- Measure the motion of the glider attached to a 5 gram hanging/falling weight using the same procedure from Part A to measure T_1 , T' , and T_2 , for each run and record them in the first five rows of the data table for Part B.
- Remove or add two 50 gram weights to the glider. Calculate the resulting mass M_2 using the value of M_1 and enter the mass of M_2 in the sixth row of the data table for Part B.
- Measure the motion of the glider attached to a 5 gram hanging/falling weight using the same procedure from Part A to measure T_1 , T' , and T_2 , for each run and record them in the second five rows of the data table for Part B.

DATA

Part A: Acceleration as a function of force with a constant mass

Total Moving Mass $M =$

Mass of the hanger $m_{\text{hanger}} =$

Length of the Flag $L =$

m (g)	run	T_1 (s)	T' (s)	T_2 (s)
$5 + m_{\text{hanger}}$	1			
	2			
	3			
	4			
	5			
$10 + m_{\text{hanger}}$	1			
	2			
	3			
	4			
	5			
$15 + m_{\text{hanger}}$	1			
	2			
	3			
	4			

	5			
$20 + m_{banger}$	1			
	2			
	3			
	4			
	5			
$25 + m_{banger}$	1			
	2			
	3			
	4			
	5			

Part B: Acceleration as a function of mass with a constant force

mass of the falling/hanging object $m = 5 \text{ g} + m_{banger}$

$M(\text{g})$	run	$T_1(\text{s})$	$T'(\text{s})$	$T_2(\text{s})$
$M_1 = \underline{\hspace{2cm}}$	1			
	2			
	3			
	4			
	5			
$M_2 = \underline{\hspace{2cm}}$	1			
	2			
	3			
	4			
	5			

CALCULATION AND ANALYSIS

1. Calculate the acceleration for each of your twenty-five runs from Part A using equation 4.6 and the values for T_1 , T' , and T_2 recorded by the computer. Calculate the average acceleration for each of your five m values by averaging the accelerations of five runs for each mass.
2. Calculate and record the accelerating force, $F = mg$, for each of your five m values. Use $g=9.80$ m/s².
3. Draw a graph of the accelerating force F versus the acceleration a , with a on the x -axis. Draw a best fit straight line through the five points on your graph.
4. Find the slope of the straight line you fit to the points and compare it with the total mass of the system M . By what percent does the slope differ from M ?
5. Calculate the acceleration for each of your 10 runs from Part B using equation 4.6 and the values for T_1 , T' , and T_2 recorded by the computer. Calculate the average acceleration for each M value by averaging the accelerations of five runs for each mass.
6. Using the average acceleration a_1 of the system with mass M_1 , and the average acceleration a_2 for mass M_2 from Part B, calculate and compare M_1a_1 and M_2a_2 . By what percent do the two values differ?

Experiment 5: Conservation Laws in Collisions

OBJECTIVES

The conservation laws for linear momentum and energy state that the total momentum and energy of an isolated system remain constant. This is true at all times in the system, even if some momentum or energy is transferred from one component of the system to another. In this experiment, you measure the motion and mass of a system comprised of colliding objects and calculate the energy and momentum of the system before and after the collision. The objectives of this experiment are as follows:

1. To measure the motion of objects that undergo elastic and inelastic collisions
2. To calculate changes in energy and momentum in elastic and inelastic collisions
3. To test the conservation laws for linear momentum and energy

THEORY

A conservation law states that a measurable property of an isolated physical system does not change with time. Two conservation laws are particularly important: conservation of linear momentum and conservation of energy

CONSERVATION OF LINEAR MOMENTUM

The law of conservation of linear momentum states that in a system where the sum of external forces is zero, the total momentum of a system does not change. In a system composed of n objects, the total momentum is given by the vector sum shown in equation 5.1.

$$\text{Total Momentum} \quad p = \sum_{i=1}^n m_i \vec{v}_i = m_1 \vec{v}_1 + m_2 \vec{v}_2 + \dots + m_n \vec{v}_n \quad (5.1)$$

Here, m_i and \vec{v}_i are the mass and velocity of object number i , respectively. As the objects interact with one another, the individual velocities may change, but the total momentum \mathbf{p} remains constant. In this experiment, you study collisions between two objects. Before the collision suppose one object has mass m_1 and is moving at velocity \vec{v}_{1i} and the other object has mass m_2 and is moving at velocity \vec{v}_{2i} . After the collision their velocities are \vec{v}_{1f} and \vec{v}_{2f} . The law of conservation of momentum predicts that the total momentum is the same before and after the collision, as shown in equation 5.2.

$$\text{Conservation of Momentum} \quad m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f} \quad (5.2)$$

Velocities are vector quantities, with direction as well as magnitude. In this experiment they act along a straight line so they have components only along one axis. However velocity in one direction (e.g. to the right) must be taken as positive while a velocity in the opposite direction must be taken as negative.

In an inelastic collision, two objects stick together and become one combined object. Momentum is still conserved, but the calculation changes to meet the new condition as shown in equation 5.3.

$$p \text{ for Inelastic Collisions} \quad m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = (m_1 + m_2)\vec{v}_f \quad (5.3)$$

Here, before the collision one object has mass m_1 and is moving with velocity \vec{v}_{1i} and the other object has mass m_2 and is moving with velocity \vec{v}_{2i} . After the collision the combined mass is $(m_1 + m_2)$ and the velocity is \vec{v}_f .

CONSERVATION OF ENERGY

The kinetic energy of an object of mass m moving at speed $|\vec{v}|$ is defined as $\frac{1}{2}m|\vec{v}|^2$. Speed is the magnitude of the velocity vector; both speed and kinetic energy are scalar quantities. The law of conservation of energy states that the total energy of an isolated system is constant, but this does not imply that the kinetic energy is constant. In a collision, the objects may be deformed or set into vibration. Then some or all of the kinetic energy is converted into heat or other forms of energy.

You study two kinds of collisions in this experiment: collisions which are nearly elastic (only a small fraction of the kinetic energy is lost), and inelastic collisions, in which a large fraction of the kinetic energy is lost. By using a rubber band so that the gliders bounce away from each other with little loss of kinetic energy, we obtain (nearly) elastic collisions. In an elastic collision the total kinetic energy does not change, as shown in equation 5.4.

$$\text{Energy in Elastic Collisions} \quad KE_i = KE_f \quad (5.4)$$

Here KE_i and KE_f are the initial and final kinetic energies, as shown in equations 5.5 and 5.6.

$$\text{Initial Kinetic Energy} \quad KE_i = \frac{1}{2}m_1|\vec{v}_{1i}|^2 + \frac{1}{2}m_2|\vec{v}_{2i}|^2 \quad (5.5)$$

$$\text{Final Kinetic Energy} \quad KE_f = \frac{1}{2}m_1|\vec{v}_{1f}|^2 + \frac{1}{2}m_2|\vec{v}_{2f}|^2 \quad (5.6)$$

If we fix a system so that the objects stick together after colliding, we obtain the maximum possible loss in kinetic energy. The initial and final kinetic energies for an inelastic collision when one object has no initial velocity ($|\vec{v}_{2i}| = 0$) are shown in equations 5.7 and 5.8.

$$\text{Initial Kinetic Energy} \quad KE_i = \frac{1}{2}m_1|\vec{v}_{1i}|^2 \quad (5.7)$$

$$\text{Final Kinetic Energy} \quad KE_f = \frac{1}{2}(m_1 + m_2)|\vec{v}_f|^2 \quad (5.8)$$

To calculate the percent kinetic energy lost in any collision, use equation 5.9.

$$\text{Percent Kinetic Energy Lost} \quad \%KE_{lost} = \frac{KE_i - KE_f}{KE_i} \quad (5.9)$$

Here, KE_i and KE_f are the energies calculated in equations 5.5 through 5.8.

ACCEPTED VALUES

In this experiment the mass and velocity values depend on the experimental conditions, so there is no single accepted value. The accepted value of the total energy in a collision is the kinetic energy of the objects before colliding KE_b , given in equation 5.5 (elastic) or equation 5.7 (inelastic).

For momentum, you measure the masses of the objects and test to what degree the velocities support the law of conservation of momentum. For an inelastic collision when one object has no initial velocity ($|\vec{v}_{2i}| = 0$), we can rearrange equation 5.3 algebraically to compare the ratio of the initial and final masses to the initial and final velocities, as shown in equation 5.10.

$$\text{Ratio for Inelastic Collisions} \quad \frac{|\vec{v}_f|}{|\vec{v}_{1i}|} = \frac{m_1}{(m_1 + m_2)} \quad (5.10)$$

Here, the ratio of the masses before and after the collision $\frac{m_1}{(m_1 + m_2)}$ is the accepted value to test the law of conservation of momentum.

For an elastic collision, we can rearrange equation 5.2 algebraically to compare the ratio of the initial and final masses to the initial and final velocities, as shown in equation 5.11.

$$\text{Ratio for Elastic Collisions} \quad \frac{v_{2f} - v_{2i}}{v_{1i} - v_{1f}} = \frac{m_1}{m_2} \quad (5.11)$$

Since v_{1i} and v_{2f} are negative we can multiply numerator and denominator by (-1) and write this as

$$\text{Ratio for Elastic Collisions} \quad \frac{|v_{2f}| + |v_{2i}|}{|v_{1i}| + |v_{1f}|} = \frac{m_1}{m_2} \quad (5.12)$$

Hence the ratio of the masses $\frac{m_1}{m_2}$ is the accepted value to test the law of conservation of momentum.

APPARATUS

- | | | |
|-------------|---------------------------------|---------------------------------|
| • Air track | • photo-gates | • 2 metal flags |
| • 2 gliders | • elastic and inelastic bumpers | • computer with PASCO interface |
| • weights | | |

COLLIDING GLIDERS ON AN AIR TRACK

The colliding objects in this experiment are gliders, mounted on an air track to minimize friction. The photo gates are used to measure the speed of the gliders, as in experiment 4. You use the inelastic bumpers for inelastic collisions, and the elastic bumpers for nearly elastic collisions.

The inelastic bumpers stick together when two gliders collide. The pin on one glider sticks in the wax-filled hole on the other, as shown in figure 5.1. The flag is used to measure the speed of a glider before and after the collision by timing its passage through the photo gates.

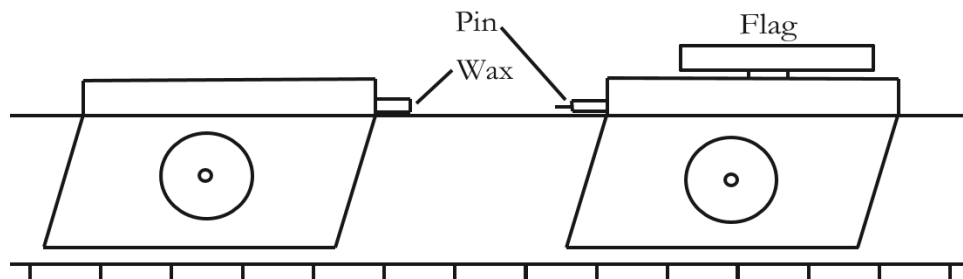


Figure 5.1 Gliders fitted with the inelastic bumpers and one flag

The elastic bumper prevents the loss of energy due to heat and vibration when two gliders collide. The bumper is attached to one glider only, as shown in figure 5.2. The flags are used to measure the speed of both gliders before and after the collision by timing their passage through the photo gates.

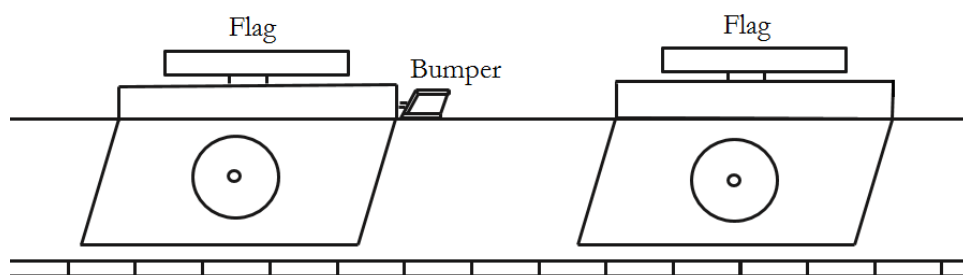


Figure 5.2 Gliders fitted with flags and one elastic bumper

PROCEDURE

PART A: INELASTIC COLLISION

1. Attach the flag to one glider and the inelastic bumpers to both gliders.
2. Weigh the two gliders individually. Record the mass of the glider with the flag as m_1 and the mass of the other glider as m_2 in the data table for Part A.
3. Measure the length of the flag and record it as L in the data table for Part A.
4. Adjust the height of the photo-gates so that the light is blocked by the flag, not by the entire body of the glider.
5. Place the photo-gates at about 70 and 130 cm.
6. Place the glider with the flag on the air track. Turn on the blower and run the glider slowly through the gates. Make sure that the flag blocks the light but does not hit the gates. Then turn off the blower.
7. Turn on your computer.
8. Double-click the icon labeled "Conservation of energy" in your desktop.
9. Set up the two gliders as shown in figure 5.3 with glider m_1 to the right or left of both photo-gates and glider m_2 between the photo-gates.

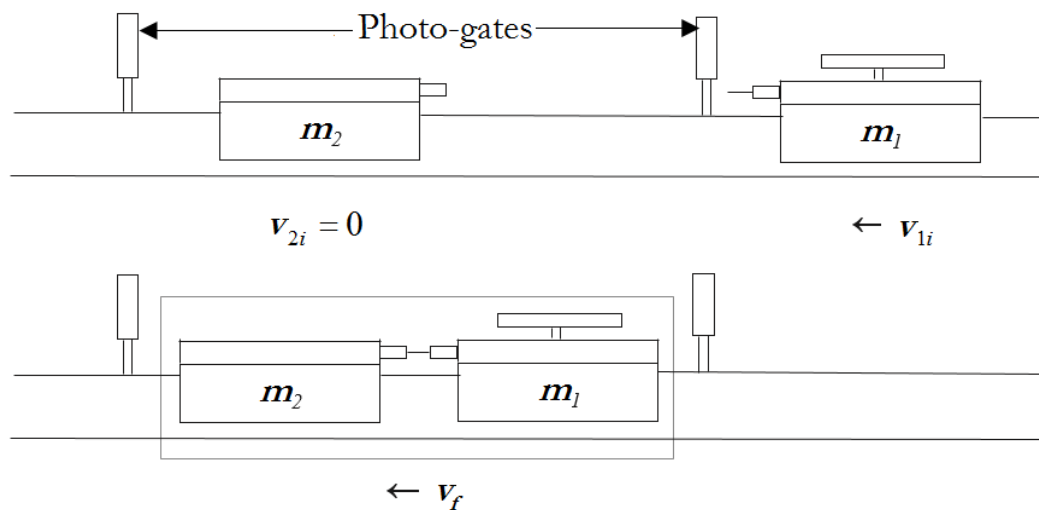


Figure 5.3 The initial and final motion of the gliders in an inelastic collision

10. Choose “Start” on the computer interface.
11. Turn on the air track blower.
12. Release glider m_2 (without moving it) and push glider m_1 towards glider m_2 at a moderate speed. Be sure you have finished pushing before the flag enters the photo-gate. Glider m_1 should strike glider m_2 and stick to it, and the two gliders together should pass through the second photo-gate. Be sure the gliders collide after the flag has passed through the first photo-gate, and that the gliders do not bounce back between the photo-gates.
13. Repeat step 12 four more times.
14. Choose “Stop” on the computer interface.
15. Record the values for T_1 and T_2 for all five trials from the computer in the data table for Part A.

PART B. ELASTIC COLLISION

1. Remove the inelastic bumpers and set them aside.
2. Attach a flag and elastic bumper to the glider without a flag.
3. Weigh each glider and record the mass of the one with the bumper as m_1 and the other as m_2 in the data table for Part B.
4. Measure the length of the flags and record them as L_1 and L_2 in the data table for Part B.
5. Place the photo-gates near 90 and 160 cm.
6. Place the gliders on the air track. Turn on the blower and run the gliders slowly through the gates. Make sure that the flag on glider m_1 blocks photo-gate 1 and the flag on glider m_2 blocks photo-gate 2. Then turn off the blower.
7. Place the gliders as in the upper diagram in figure 5.4, with glider m_1 to the right of both photo-gates and glider m_2 to the left of both photo-gates.

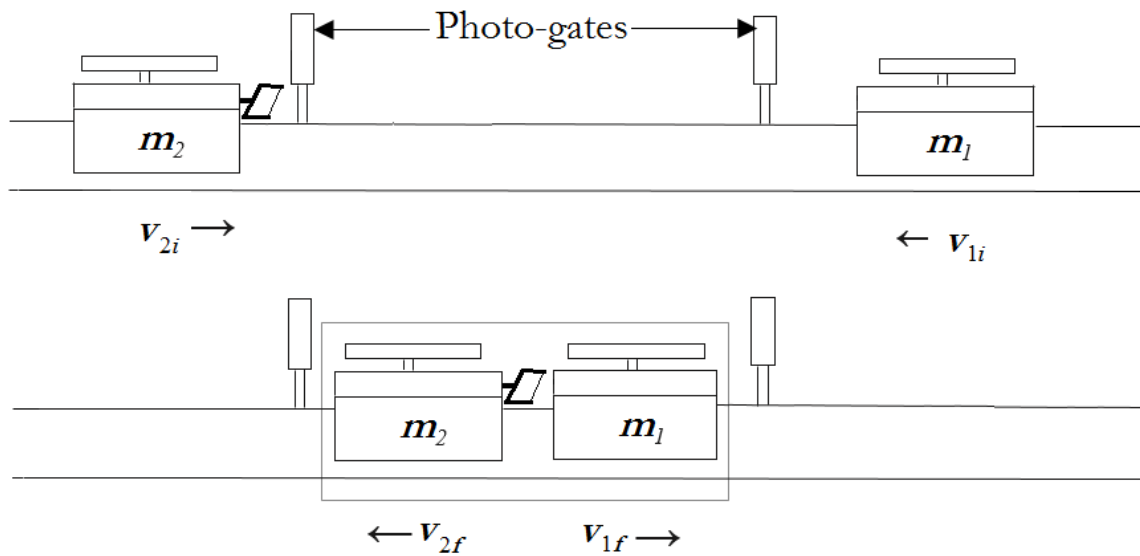


Figure 5.4 The initial and final motion of the gliders in an elastic collision

8. Turn on the blower and choose “Start” on the computer interface.
9. Push the gliders towards each other so that they collide between the gates. Be sure you have stopped pushing them before they enter the gates. The computer records the time it takes for each glider to pass through the photo-gates, both before (*i*) and after (*f*) the collision. The computer records the passage of the gliders before the collision in one row and the passage of the gliders after the collision in the next row. Record the data from the first row as T_{1i} and T_{2i} in the data table for Part B. Record the data from the second row as T_{1f} and T_{2f} .
10. Without choosing “Stop” on the computer, repeat procedure 9 four more times so that the gliders collide a total of five times, which the computer records in a total of 10 rows.
11. Choose “Stop” on the computer interface.
12. Record the data from each pair of rows on the computer into the data table for Part B. Record the data from the first row of each pair as T_{1i} and T_{2i} and the data from the second row of each pair as T_{1f} and T_{2f} .

DATA

Part A: Inelastic Collision

$$m_1(\text{g}) = \underline{\hspace{2cm}}$$

$$m_2(\text{g}) = \underline{\hspace{2cm}}$$

$$\frac{m_1}{m_1+m_2} = \underline{\hspace{2cm}}$$

$$L(\text{m}) = \underline{\hspace{2cm}}$$

i	$T_{1i}(\text{s})$	$T_{2i}(\text{s})$
1		
2		
3		
4		
5		

Part B: Elastic Collision

$$m_1(\text{g}) = \underline{\hspace{2cm}}$$

$$m_2(\text{g}) = \underline{\hspace{2cm}}$$

$$\frac{m_1}{m_2} = \underline{\hspace{2cm}}$$

$$L_1(\text{m}) = \underline{\hspace{2cm}}$$

$$L_2(\text{m}) = \underline{\hspace{2cm}}$$

i	$T_{1i}(\text{s})$	$T_{2i}(\text{s})$	$T_{1f}(\text{s})$	$T_{2f}(\text{s})$
1				
2				
3				
4				
5				

CALCULATION AND ANALYSIS

1. For inelastic collisions, calculate the values of v_{1i} and v_f by dividing the length of the flag L by the time it took for the flag to pass through the photo-gate, T_{1i} and T_f and then calculate $\frac{|v_f|}{|v_{1i}|}$ for all five of your trials.
2. Calculate the average (mean) of the five values $\frac{|v_f|}{|v_{1i}|}$, the deviation of each value, the uncertainty of the average (mean), and the percent uncertainty for the average (mean) using equations 0.1, 0.3, 0.6 and 0.7.
3. Compare your average value of $\frac{|v_f|}{|v_{1i}|}$ with the accepted value of $\frac{m_1}{(m_1+m_2)}$. Calculate the percent error of your calculation using equation 0.2. Is the % error less than or greater than the % uncertainty? Does the experimental evidence support the conservation of momentum?
4. For ONE of your collisions, calculate the kinetic energy of m_1 before the collision and the kinetic energy of m_1+m_2 after the collision. Calculate the percentage of the initial kinetic energy lost in the collision, as shown in equation 5.9.
5. For each of the five elastic collisions studied, calculate the values of v_{1i} , v_{1f} , v_{2i} , and v_{2f} by dividing the length of the flags L_1 and L_2 by the time it took for the flag to pass through the photo-gate, T_{1i} , T_{2i} , T_{1f} , and T_{2f} . Then calculate the value of $\frac{|v_{2f}|+|v_{2i}|}{|v_{1i}|+|v_{1f}|}$ for all five of your trials.
6. Calculate the average (mean) of your five values of $\frac{|v_{2f}|+|v_{2i}|}{|v_{1i}|+|v_{1f}|}$, the deviation of each value, the uncertainty of the average (mean), and the percent uncertainty for the average (mean).
7. Compare your average value of $\frac{|v_{2f}|+|v_{2i}|}{|v_{1i}|+|v_{1f}|}$ with the accepted value $\frac{m_1}{m_2}$. Calculate the percent error of your calculation using equation 0.2. Is the % error less than or greater than the % uncertainty? Does the experimental evidence support the conservation of momentum?
8. For ONE of your collisions, calculate the kinetic energy of the two masses before and after the collision, using the definition of the kinetic energy. Calculate the percentage of the initial kinetic energy lost in the collision, as shown in equation 5.9.
9. How does this compare with what happened in the inelastic collision?

Experiment 6: The Ballistic Pendulum

OBJECTIVES

An inelastic collision is a collision in which kinetic energy is not conserved. The total energy is always conserved, but in an inelastic collision some of the initial kinetic energy is converted to other forms such as heat and vibration. The ballistic pendulum provides a means to measure the gravitational potential energy converted from kinetic energy in an inelastic collision. The objectives of this experiment are as follows.

1. To observe the motion of a ballistic pendulum.
2. To calculate the initial and final velocities and the change in kinetic energy during the collision.
3. To make a comparison to the initial velocity determined by observing ballistic motion of a projectile.

THEORY

When two objects collide, they exert equal and opposite forces on each other. If there are no external forces, this implies that the total momentum is conserved, namely the total momentum before and after the collision is the same. This is true for any collision whether elastic (kinetic energy conserved) or inelastic (kinetic energy not conserved).

The ballistic pendulum is a device sometimes employed to determine the speed of a bullet. The pendulum consists of a large bob hollowed out to receive the projectile and is suspended by a light rod. When the projectile is fired into the pendulum bob it remains imbedded in it. This is an inelastic collision. Conservation of momentum implies that the total momentum of the bob and the projectile just after the collision is equal to the total momentum of the projectile just before the collision (the momentum of the bob just before the collision is zero). Momentum in such a collision is conserved, as shown in equation 6.1.

$$\vec{p} \text{ for Inelastic Collisions} \quad m_1 \vec{v}_{1i} = (m_1 + m_2) \vec{v}_f \quad (6.1)$$

Here, the projectile has mass m_1 and is moving with an initial velocity \vec{v}_{1i} and the bob has mass m_2 . After the collision the combined system (projectile and bob) has mass $(m_1 + m_2)$ and moves with a common velocity \vec{v}_f . The kinetic energies in this situation before and after the collision are shown in equations 6.2 and 6.3.

$$\text{Initial Kinetic Energy} \quad E_i = \frac{m_1 |\vec{v}_{1i}|^2}{2} \quad (6.2)$$

$$\text{Final Kinetic Energy} \quad E_f = \frac{(m_1 + m_2) |\vec{v}_f|^2}{2} \quad (6.3)$$

As a result of the collision, the pendulum with the imbedded projectile swings about its point of support and rises to a height h (see figure 6.1). Nearly all the kinetic energy of the system just after the collision is converted to potential energy at the top of the pendulum swing. The conversion of

kinetic energy to potential energy results in an equation to calculate the final velocity $|\vec{v}_f|$ of the system right after the collision as shown in equation 6.4.

$$\text{Energy Conservation} \quad E_f = \frac{(m_1 + m_2)|\vec{v}_f|^2}{2} = (m_1 + m_2)gh \therefore |\vec{v}_f| = \sqrt{2gh} \quad (6.4)$$

Therefore initial velocity $|\vec{v}_{li}|$ is calculable by substituting the value of $|\vec{v}_f|$ from equation 6.4 and the values of m_1 and m_2 into equation 6.1, as shown in equation 6.5.

$$\text{Initial Velocity} \quad |\vec{v}_{li}| = \frac{(m_1 + m_2)}{m_1} |\vec{v}_f| = \frac{(m_1 + m_2)}{m_1} \sqrt{2gh} \quad (6.5)$$

LOSS OF ENERGY IN A COLLISION

The energy lost in the collision is the difference between the initial and final kinetic energies. For the ballistic pendulum where only the projectile has an initial velocity, substituting the value of equation 6.1 into the difference in initial and final energies results in a simplified way to calculate the loss of energy shown in equation 6.6.

$$\text{Energy Lost} \quad \Delta E \equiv E_i - E_f = \frac{m_1 m_2 |\vec{v}_{li}|^2}{2(m_1 + m_2)} \quad (6.6)$$

Equation 6.6 predicts the energy lost in an inelastic collision using the initial velocity $|\vec{v}_{li}|$.

ACCEPTED VALUES

To obtain an accepted value for the initial velocity of the bullet we observe ballistic motion of the projectile without the pendulum.

You can calculate the initial velocity of the projectile $|\vec{v}_{li}|$ by firing it horizontally and allowing it to fall on the floor without striking the pendulum (see figure 6.2). In the x-direction the initial velocity is $|\vec{v}_{li}|$ and there is no acceleration (air resistance is neglected). The horizontal displacement of the projectile x is shown in equation 6.7.

$$\text{Horizontal displacement} \quad x = |\vec{v}_{li}|t \quad (6.7)$$

Here t is the time of flight of the projectile. In the y-direction the initial velocity is zero and the motion is that of a freely falling object starting from rest. Therefore the vertical displacement is calculated using equation (6.8).

$$\text{Vertical displacement} \quad y = \frac{1}{2}gt^2 \quad (6.8)$$

Here the positive direction along the y-axis is down. Eliminating t from equations (6.7) and (6.8), the initial velocity is calculated using equation (6.9).

$$\text{Initial Velocity} \quad |\vec{v}_{li}| = x \sqrt{\frac{g}{2y}} \quad (6.9)$$

The value calculated in equation (6.9) is the accepted value for the initial velocity $|\vec{v}_i|$. From this initial value, the accepted value for the energy lost in the collision is calculated using equation 6.6.

APPARATUS

- ballistic pendulum
- level
- plumb line
- balance
- meter stick
- carbon paper

BALLISTIC PENDULUM

The apparatus used in this experiment is a combination of a ballistic pendulum and a spring gun for propelling the projectile as shown in figure 6.1.

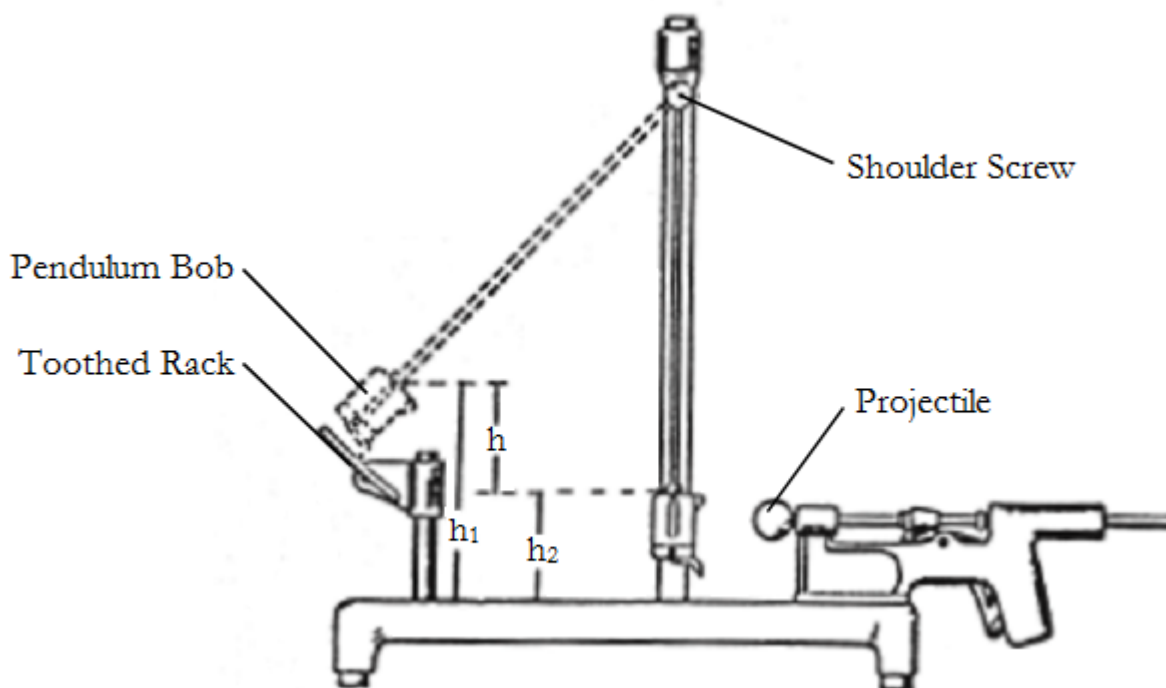


Figure 6.1 The parts of the ballistic pendulum, showing the height of pendulum bob before and after the collision with the projectile

The pendulum consists of a massive cylindrical bob hollowed out to receive the projectile and suspended by a strong, light rod. The pendulum may be removed from its supporting yoke by unscrewing the shoulder screw at the yoke. The projectile is a brass ball which, when propelled into the pendulum bob, is caught and held in such a position that its center of gravity lies on the axis of the suspension rod. A pointer on the side of the bob indicates the height of the pendulum-projectile system. When the projectile is shot into the pendulum, the pendulum swings upward and is caught at its highest point by a pawl which engages a tooth on the curved rack. The height of rise is determined by measuring the heights of the center of gravity above the table before and after the impact. This gives a direct measurement of the height, h , the ball and pendulum rise. The numbers on the toothed rack do not measure height, but the number of teeth if counted from right to left.

The projectile is launched from a spring-loaded gun that can be fired with the pendulum out of the path of the projectile. When the projectile is shot without the pendulum to catch it, the projectile

moves as a projectile with a measurable range along the x-axis when fired from a specific height on the y-axis, as shown in figure 6.2. This alternate setup is used to obtain the accepted value for the initial velocity of the projectile using equations 6.7, 6.8, and 6.9.

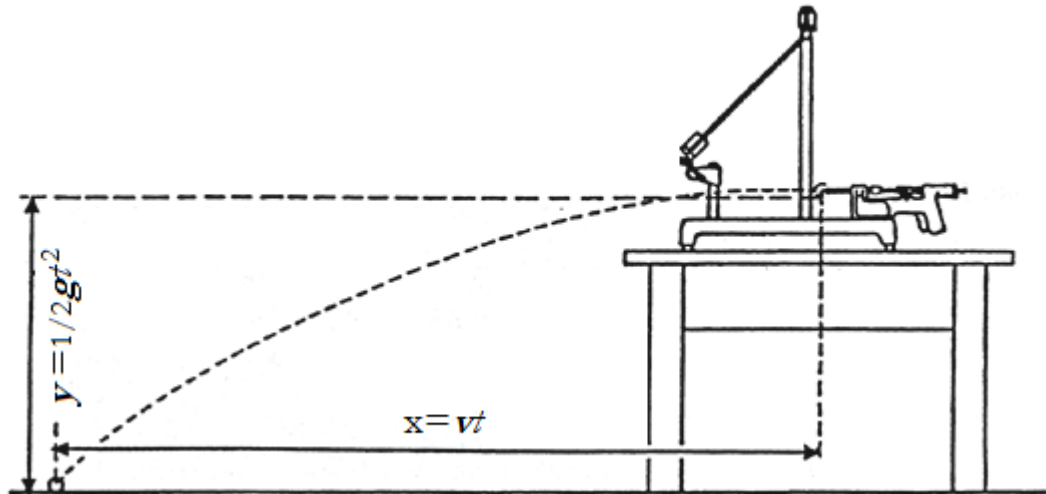


Figure 6.2 Measuring the range and fall of the projectile fired without the pendulum

PROCEDURE

PART A: INELASTIC COLLISION WITH A BALLISTIC PENDULUM

1. Place the pendulum in its central (straight down) position and allow it to swing freely. Get the gun ready for firing by compressing the spring and, when the pendulum is at rest, pull the trigger, thereby firing the projectile into the pendulum bob. This causes the pendulum with the projectile inside it to swing up along the rack where it is caught at its highest point. Record the notch on the curved scale reached by the pawl when it catches the pendulum in the data table for Part A. To remove the projectile from the pendulum, push it out with your finger or a rubber-tipped pencil while holding up the spring catch.
2. Repeat procedure 1 five more times, recording the position of the pendulum on the rack each time.
3. Calculate the average (mean) value of the highest notch reached by the pendulum and record it in the data table for Part A.
4. Raise the pendulum until its pawl is engaged in the notch corresponding most closely to the average (mean) value of the highest notch reached by the pendulum. Measure h_1 , the vertical distance from the base of the apparatus to the index point attached to the pendulum as shown in figure 6.1. This index point is at the vertical position of the center of gravity of the pendulum and projectile.
5. Release the pendulum and allow it to hang in its lowest position. Measure h_2 , the vertical distance from the base of the apparatus to the index point.
6. Calculate the difference between these two values and record it as the average height h , which is the vertical distance through which the center of gravity of the system is raised after shooting the projectile.

7. Loosen the outer shoulder screw and carefully remove the pendulum from its support. It is important that only the outer thumbscrew is loosened-not both-since this ensures proper alignment of the pendulum arm with the gun. Measure the masses of the pendulum and of the projectile and record the results in the data table for Part A. Replace the pendulum and carefully adjust the thumb screw.

PART B: BALLISTIC TRAJECTORY OF A PROJECTILE

1. Set the apparatus near one edge of a level table or stool. Swing the pendulum up onto the rack so that it does not interfere with the free flight of the projectile as shown in figure 6.2. Check that the base of the apparatus is accurately horizontal using a level; if not, place sheets of paper under the feet as required.
2. Fire the projectile out across the floor and make note of the approximate spot where it hits the floor. Place on this spot a sheet of white paper and secure it with masking tape. Cover this sheet with a sheet of carbon paper, carbon side down, so that a record is made of the exact spot where the projectile strikes. Fire the projectile six times. Note: It is important that the apparatus not move on the bench between shots.
3. Measure the range for each spot; this is the horizontal distance from the point of projection to the point of contact with the floor as shown in figure 2. Measure the vertical height from the bottom of the projectile to the floor.

DATA

Part A: Inelastic collision with a ballistic pendulum

Trial	pawl's position (notch number)
1	
2	
3	
4	
5	
6	
Average	

$$m_1(\text{g}) = \underline{\hspace{2cm}} \quad m_2(\text{g}) = \underline{\hspace{2cm}}$$

$$m_1 + m_2(\text{g}) = \underline{\hspace{2cm}}$$

$$h_1(\text{m}) = \underline{\hspace{2cm}} \quad h_2(\text{m}) = \underline{\hspace{2cm}}$$

$$h(\text{m}) = \underline{\hspace{2cm}}$$

Part B: Ballistic trajectory of a projectile

$$y(\text{m}) = \underline{\hspace{2cm}}$$

Trial	x = Range of Projectile (m)
1	
2	
3	
4	
5	
6	
Average	

CALCULATION AND ANALYSIS

1. Calculate the velocity $|\vec{v}_f|$ of the system immediately after collision and the experimental value for the initial velocity $|\vec{v}_{i1}|$ of the projectile using data from part A.
2. Calculate the accepted value for the initial velocity $|\vec{v}_{i1}|$ of the projectile using the data from Part B.
3. Calculate the percent error of the experimental value for the initial velocity $|\vec{v}_{i1}|$ of the projectile.
4. Calculate the total translational kinetic energy before (E_i) and after (E_f) the impact using the calculated velocities from Part A.
5. Calculate the experimental value for the energy lost by subtracting E_f from E_i .
6. Calculate the accepted value for the energy lost by using the accepted value for the initial velocity $|\vec{v}_{i1}|$ of the projectile in equation 6.6.
7. Calculate the percent error of the experimental value for the loss of energy.
8. Calculate the percentage of the kinetic energy converted to potential energy in this experiment.

Experiment 7: Rotational Equilibrium

OBJECTIVES

When the forces acting on an object do not make the object rotate, the object is in a state of rotational equilibrium. In this experiment you arrange forces that put an object into rotational equilibrium. You measure the vector quantities of these forces, calculate the torques exerted by these forces, and calculate the net torque acting on the object. The objectives of this experiment are as follows:

1. To measure the forces on an object in rotational equilibrium
2. To calculate the torques exerted by these forces
3. To test the hypothesis that an object in rotational equilibrium has no net torque acting on it

THEORY

An object is rigid if its shape remains unchanged when forces are applied. A rigid object is in translational equilibrium when it has no linear acceleration. This was studied in Experiment 3. A rigid object is in rotational equilibrium when it has no angular acceleration. Therefore, to be in equilibrium, two conditions must be satisfied.

1. As described by Newton's second law, the vector sum of the forces acting on the object and labeled by the index i must be zero, as shown in equation 7.1.

$$\text{Translational Equilibrium} \quad F_{total} = \sum_i F_i = 0 \quad (7.1)$$

If all the forces F_i are applied in a plane, then one can project the forces on the x and z axes as shown in equation 7.2.

$$\begin{aligned} \text{2-D Translational Equilibrium} \quad F_{total,x} &= \sum_i F_{i,x} = 0 \\ F_{total,z} &= \sum_i F_{i,z} = 0 \end{aligned} \quad (7.2)$$

Here the x-axis is horizontal and the z-axis is vertical.

2. The sum of the torques acting on the object and labeled by the index i must be zero, as shown in equation 7.3.

$$\text{Rotational Equilibrium} \quad \tau_{total} = \sum_i \tau_i = 0 \quad (7.3)$$

Here the torque due to the force F_i about a pivot point O is defined by

$$\text{Torque} \quad \tau_{i,O} = F_{i,\perp} r_i \quad (7.4)$$

In this formula r_i is the length of the lever arm, defined as the distance between the pivot point O and the application point A_i of the i th force, and $F_{i,\perp}$ is the component of F_i perpendicular to the vector r_i . This is illustrated in figure 7.1. The torque condition is true for every pivot point O . Since the choice of pivot point is arbitrary, you can use one that is convenient for calculation.

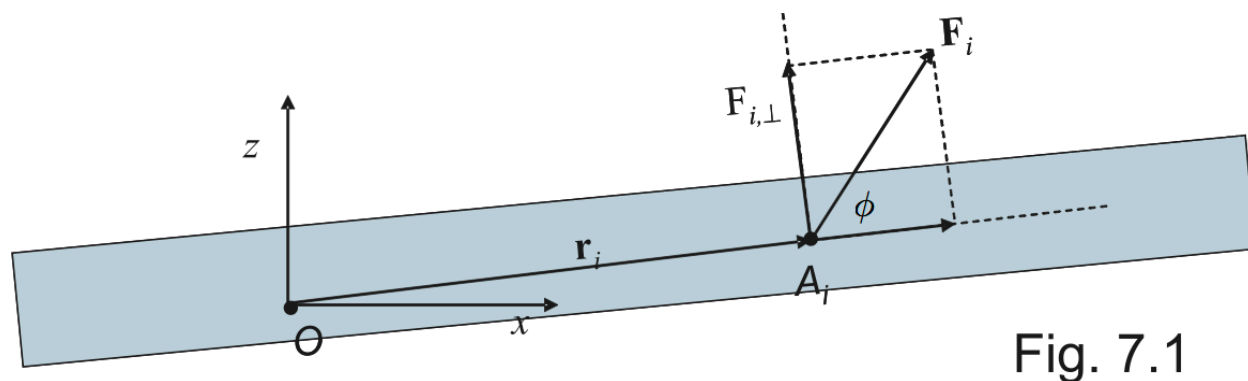


Figure 7.1 The torque due to a force \vec{F}_i about a pivot point O

If the force tends to rotate the object counterclockwise the torque is considered positive. If the force tends to rotate the object clockwise the torque is considered negative. If ϕ is the angle between the force vector \vec{F}_i and lever arm, then the component of the force in the direction perpendicular to r_i is given by equation 7.5.

$$\text{Perpendicular force} \quad F_{i,\perp} = |\vec{F}_i| \sin \phi \quad (7.5)$$

The center of gravity of an object is the point, inside or outside the object, where the net force of gravity on all the particles of which the object is composed acts. Since one of the forces always acting on an object is gravity, you must measure the mass of an object and the location of its center of gravity in order to verify the equilibrium conditions above.

ACCEPTED VALUES

As in Experiment 3, the accepted value for the sum of the torques on an object in equilibrium is 0. A useful way to characterize the accuracy of our measurement is to divide the magnitude of the net torque by the sum of the magnitudes of the individual torques multiplied by 100%, as shown in equation 7.6.

$$\% \text{ Discrepancy} = \frac{|\tau_{total}|}{\sum_i |\tau_i|} \times 100\% \quad (7.6)$$

APPARATUS

- A rigid object
- 2 spring balances (0-2000 g)
- assortment of weights (hook type)
- knife edge
- balance

The rigid object used in this experiment consists of a metal bar with four pivoted hooks mounted along the bar, two on each side, with protractors to indicate the angles the forces on the hook make with the major axis of the bar as shown in figure 7.2. We choose the pivot point O to be on the left end of the bar.

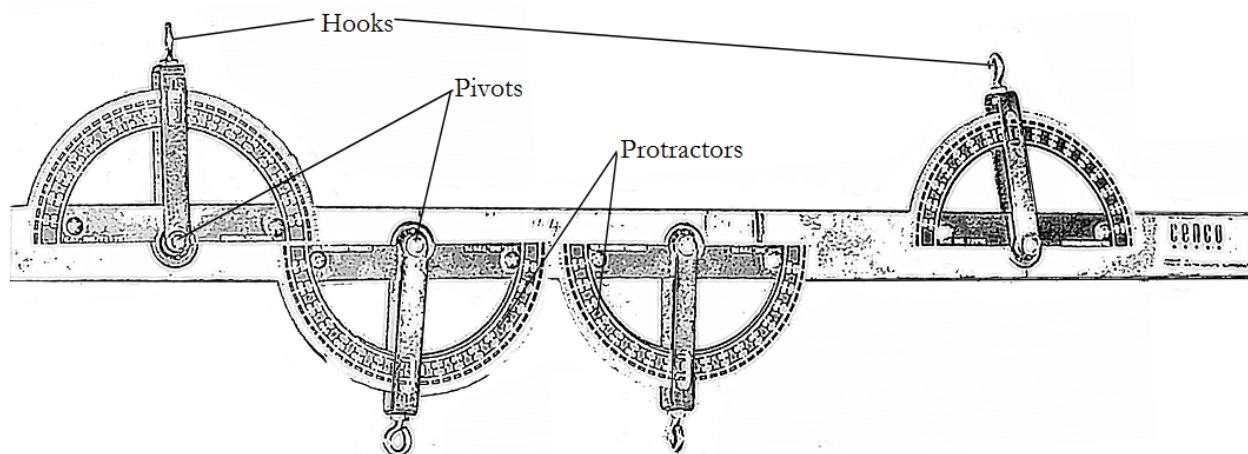


Figure 7.2 The components of the rigid bar with protractors to measure force angles

PROCEDURE

1. Balance the rigid object over a knife edge and measure the distance from the left end of the object to the knife edge. Record the result as the length lever arm r_0 .
2. Weigh the rigid object using your balance and record the result as the mass of the rigid object m_0 in the data table.
3. Check each spring balance with nothing attached. If they do not read zero, turn the adjustment screws until the spring balances all read zero.
4. Support the rigid object by the two spring balances so that each balance pulls vertically on one of the two pivoted hooks nearest the ends of the bar as shown in figure 7.3. Record the forces acting on the rigid object in the data sheet for Part A. Measure the distances from the left end which enables you to calculate the lever arms and thus the torques.

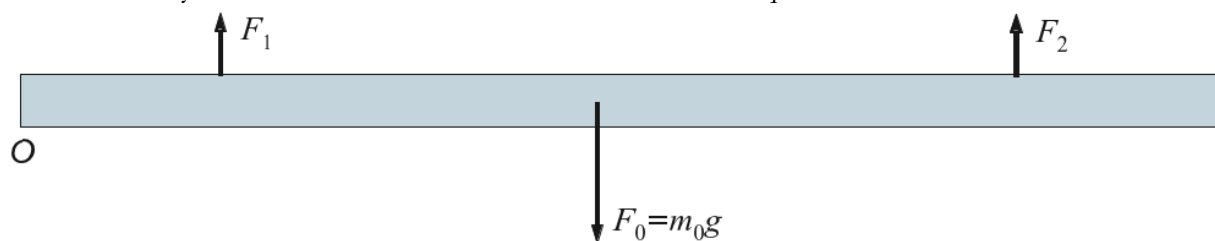


Figure 7.3 The force vectors for Part A which tests the gravitational force only

5. Support the rigid object as in step 4 and add weights to the other two pivoted hooks so that all the external forces acting are vertical and the bar horizontal as shown in figure 7.4. Attach masses that exceed 400 g with one sufficiently different from the other so that the spring balance readings differ by at least 300 g. Record all forces and distances in the data sheet for Part B.

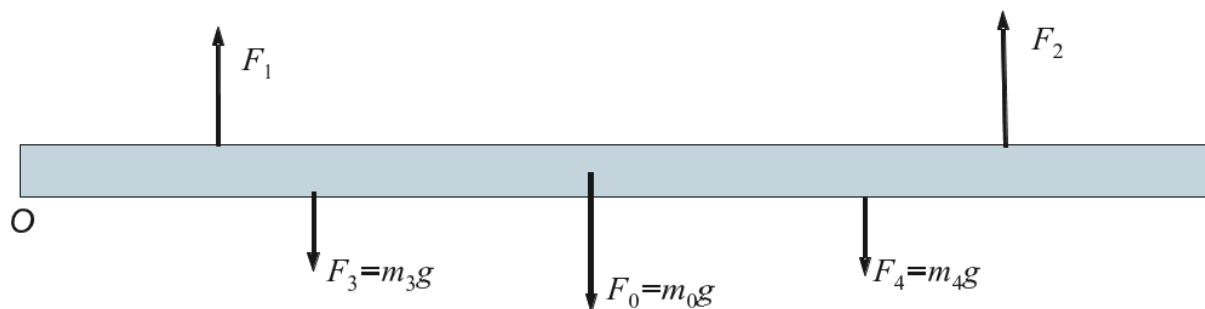


Figure 7.4 The force vectors for Part B which tests two vertically hanging weights

6. Repeat step 5 with the two upward forces pointing outward as shown in figure 7.5. Again, both hanging masses should differ by at least 300 g. Record angles between the upward forces and the bar, all forces, and all distances in the data sheet for Part C.

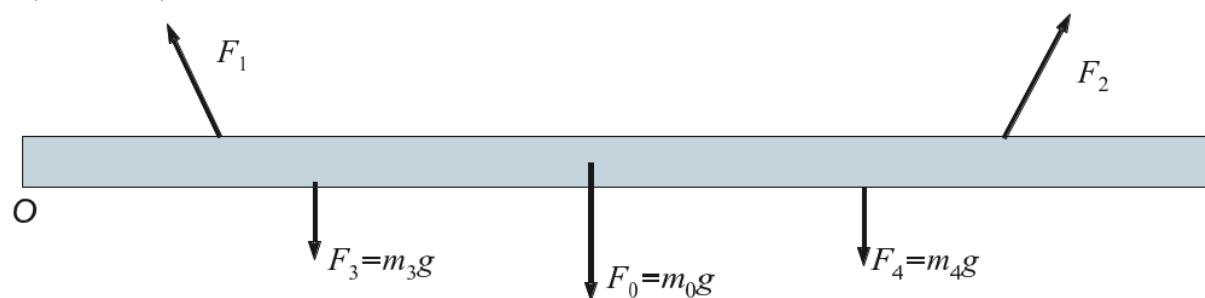


Figure 7.5 The force vectors for Experiment 3 which tests two angled upward forces

DATA

$m_0(\text{g}) =$ _____

Part A

i	m_i (g)	F_i (N)	ϕ_i ($^\circ$)	r_i (cm)
0				
1				
2				

Part B

i	m_i (g)	F_i (N)	ϕ_i ($^\circ$)	r_i (cm)
0				
1				
2				
3				
4				

Part C

i	m_i (g)	F_i (N)	ϕ_i ($^\circ$)	r_i (cm)
0				
1				
2				
3				
4				

CALCULATION AND ANALYSIS

1. Explain why the center of gravity of the object is located directly above the knife edge position found in procedure 1.
2. Calculate the sum of the torques on the rigid object for Parts A and B.
3. Calculate the % discrepancies of the torques for Part A and B using equation 7.6.
4. Calculate the sum of the horizontal and vertical forces for Part C using equations 7.2.
5. Calculate the sum of the torques on the rigid object for Part C.
6. Calculate the % discrepancies of the torques for Part C using equation 7.6.
7. Would it be possible to achieve equilibrium with only one of the two upward forces vertical and the other at an angle to the vertical? Use a diagram as part of your answer.

Experiment 8: Archimedes' Law

OBJECTIVES

Archimedes discovered that you can measure the volume of a geometrically irregular solid by measuring the displacement of a liquid in which the solid is completely submerged. When scientists quantified the force of buoyancy, they discovered that you can find the density of an irregular object by comparing the weight of the liquid displaced by a submerged object with the apparent loss in the weight of an object. In this experiment, you measure the displacement of liquids by submerged and floating objects and measure the buoyancy force of liquids on submerged objects. The objectives of this experiment are as follows:

1. To measure the liquid displaced by floating and submerged objects
2. To test Archimedes' law
3. To calculate the density of liquids, solid objects that sink, and solid objects that float

THEORY

Archimedes' law states that an object immersed in a liquid is buoyed up by a force equal to the weight of the liquid displaced by the object as shown in equation 8.1.

$$\text{Archimedes' Law} \quad F_b = \rho_L Vg \quad (8.1)$$

Here ρ_L is the density of the liquid and V is the volume of the object, so that $\rho_L V$ is the mass of the displaced liquid. This law can measure the density of an irregular solid object ρ_{obj} if the density of the liquid ρ_L is known. Weigh the object in the standard way (without the liquid) determining its weight as a function of density as shown in equation 8.2.

$$\text{Weight (standard)} \quad W = mg = \rho_{obj} Vg \quad (8.2)$$

Then weigh the object while immersed in a liquid to determine its apparent weight as shown in equation 8.3.

$$\text{Apparent Weight (in liquid)} \quad W_{app} = W - F_b = (\rho_{obj} - \rho_L)Vg \quad (8.3)$$

Dividing equation 8.3 by equation 8.2 eliminates V , which might be difficult to measure directly, and results in the relation shown in equation 8.4.

$$\text{Ratio of Apparent Weight to Standard Weight} \quad \frac{W_{app}}{W} = 1 - \frac{\rho_L}{\rho_{obj}} \quad (8.4)$$

Solving equation 8.4 for ρ_{obj} yields a formula for the density of the object, as shown in equation 8.5.

$$\text{Density of a solid} \quad \rho_{obj} = \rho_L \frac{W}{W - W_{app}} \quad (8.5)$$

Equation 8.5 can also calculate the density of the liquid, such as ethanol, once the density of the submerged object is known, as shown in equation 8.6.

$$\text{Density of a liquid} \quad \rho_L = \rho_{obj} \frac{W - W_{app}}{W} \quad (8.6)$$

In this experiment, you weight the object in two different liquids, water and ethanol, thus it is convenient to label the symbols by the indices W and E. Then equation 8.5 becomes

$$\text{Density of a solid, using water} \quad \rho_{obj} = \rho_W \frac{W}{W - W_{app,W}} \quad (8.7)$$

Substituting equation 8.7 in 8.6 gives a formula for the density of ethanol.

$$\text{Density of Ethanol} \quad \rho_E = \rho_W \frac{W - W_{app,E}}{W - W_{app,W}} \quad (8.8)$$

An object which floats in a liquid displaces a weight of the liquid equal to its own weight. If the object is elongated with length L and constant cross-sectional area S , its volume is $V = LS$ and its mass is $\rho_{obj}V$. If the object is floating in a vertical position and the length of its submerged part is L_{sub} , the volume of the displaced liquid is $V_{sub} = L_{sub}S$ and the mass of displaced liquid is $\rho_L V_{sub}$. So for a floating object $\rho_{obj}V = \rho_L V_{sub}$ which algebraically yields equation 8.10.

$$\text{Density of a solid, floating} \quad \rho_{obj} = \rho_L \frac{L_{sub}}{L} \quad (8.10)$$

In the equations above, it is convenient to measure all the weights in grams instead of Newtons because the acceleration of gravity g cancels in all cases.

ACCEPTED VALUES

The accepted value when testing Archimedes' Law in Part A is the weight of the water displaced by the submerged solid object.

The accepted values for the density of the liquids and solids analyzed in this lab at 20°C and in an atmosphere of 1 bar of pressure are as follows:

- Water: $\rho_L = 998.21 \text{ kg/m}^3$
- Ethanol: $\rho_L = 789.3 \text{ kg/m}^3$
- Aluminum: $\rho_{obj} = 2698.9 \text{ kg/m}^3$

APPARATUS

- | | | |
|---|----------------|----------------------|
| • Object | • overflow can | • thread |
| • platform balance supported above the table by a stand | • metal can | • meter stick |
| | • ethanol | • graduated cylinder |
| | • wooden dowel | • beaker |

The object in this lab is suspended by a thread so that it can be immersed in liquids without changing the displaced volume significantly. The overflow can sends displaced water through a spout into the beaker, as shown in Figure 8.1, which can be weighed on a platform balance.

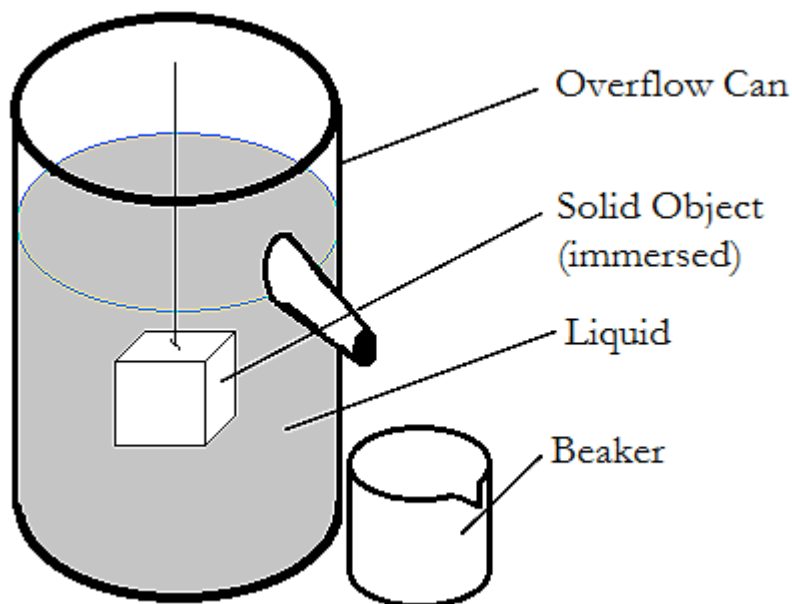


Figure 8.1 An overflow can directs displaced liquid into a beaker

To measure the weight of the object when it is immersed in a liquid, loop the thread over a hook at the end of the platform balance on a stand and make measurements as you would if the object sat upon the balance platform.

PROCEDURE

1. Hang the object from the hook under the left pan of the balance using the thread. Measure the object's weight and record the result as the object weight W in the data table for Part A and again in the data table for Part B.
2. Arrange the overflow can and the beaker so that water can flow from the spout of the overflow can into the beaker. Pour water into the can until it overflows. When the water has stopped dripping from the spout, weigh the beaker with the water in it and record the result as the initial weight of the beaker $W_{b,i}$ in the data table for Part A.
3. Place the beaker and its contents back under the spout. While keeping the object hung from the balance, lower the object by a thread into the water in the overflow can until it is completely immersed. When all the water displaced by the object has flowed into the beaker, weigh the beaker with the water in it and record the result as the final weight of the beaker $W_{b,f}$ in the data table for Part A.
4. Adjust the apparatus so that the object isn't touching the sides or bottom of the overflow can. Measure the apparent weight of the object when immersed in water $W_{app,W}$ and record the result in the data table for Part A and again in the data table for Part B. Set the overflow can aside.
5. Dry the object. Fill the metal can with ethanol. Measure the apparent weight of the object when completely immersed in ethanol, $W_{app,E}$ and record your result in the data table for Part B. Pour the ethanol back into the ethanol bottle and close the bottle.

6. Measure the length of the wooden dowel and record the result as the total length L_{total} in the data table for Part C.
7. Using calipers, measure the width W and the height H of the wooden dowel.
8. Using a balance, measure the mass, m , of the wooden dowel.
9. Fill the graduated cylinder with approximately 650 cm^3 of water. Lower the wooden dowel into the water in the graduated cylinder until it floats.
10. Measure the length of the wooden dowel below the level of the water and record the result as the submerged length L_{sub} in the data table for Part C.

DATA

Part A: Testing Archimedes' Law

W (g)	$W_{\text{app},W}$ (g)	$W_{b,i}$ (g)	$W_{b,f}$ (g)

Part B: Calculating the density of a solid object and ethanol

W (g)	$W_{\text{app},W}$ (g)	$W_{\text{app},E}$ (g)

Part C: Calculating the density of a floating object

L_{total} (cm)	W (cm)	H (cm)	m (g)	L_{sub} (cm)

CALCULATION AND ANALYSIS

1. Calculate the weight of the water displaced by the immersed object, $W_{b,f} - W_{b,i}$, using the data from Part A.
2. Calculate the apparent loss of weight of the object when completely immersed in water, $W - W_{\text{app},W}$, using the data from Part A.
3. Archimedes' Law predicts that the weight of the displaced water equals the apparent loss of weight of the object. Do your results support Archimedes' Law?
4. Calculate the density of the object ρ_{obj} using equation 8.7 and the data from Part B.
5. Calculate the % error in your result for ρ_{obj} .
6. Calculate the density of the ethanol ρ_E using equation 8.8 and the data from Part B.
7. Calculate the % error in your result for ρ_E .

8. Calculate the density of the wooden dowel $\rho_{\text{wood, float}}$ using equation 8.10 and the data from Part C.
9. Calculate the volume of the wooden dowel using your data from Part C. (Remember for a cube volume = length \times width \times height.)
10. Calculate the actual density of the wooden dowel, $\rho_{\text{wood, actual}} = \frac{m}{\text{volume}}$, using your data from Part C.
11. Using $\rho_{\text{wood, actual}}$ as the accepted value, calculate the % error in your result for $\rho_{\text{wood, float}}$.

Experiment 9: Simple Harmonic Motion

OBJECTIVES

Simple harmonic motion is the motion of an object that is subject to a force that is proportional to the object's displacement. An object attached to a spring undergoes simple harmonic motion. The quantitative relationship between the spring force and the displacement is known as Hooke's Law. In this experiment, you observe the oscillation of an object attached to a spring to test Hooke's law and calculate the spring constant for the spring. The objectives of this experiment are as follows:

1. To measure the period of oscillation of a mass-spring system
2. To test Hooke's law with a spring
3. To calculate the spring constant for a spring

THEORY

When a body is suspended from a spring, its weight causes the spring to elongate. The elongation x is directly proportional to the external force F_{spring} exerted by the spring as shown in equation 9.1.

$$\text{Hooke's Law} \quad F_{spring} = -kx \quad (9.1)$$

Here k is the force constant of the spring. This relationship is known as Hooke's law.

If the spring is oriented to resist the acceleration of gravity and an object is pulled down and then released, the object oscillates about the position of the body when the spring was stationary, known as the equilibrium position. This motion is called simple harmonic motion. The period T of an object in simple harmonic motion is a function of the spring constant k and the moving mass of the system M , as shown in equation 9.2.

$$\text{Period of spring-mass system} \quad T = 2\pi\sqrt{\frac{M}{k}} \quad (9.2)$$

Thus a plot of T^2 vs. M should be a straight line with slope $4\pi^2/k$.

NOTE: In the calculations and analysis, you measure the slope of a plot of T^2 vs. M . In this system the moving mass includes the mass of the weight hanger and some part of the spring mass. Since these weights do not change from run to run, they do not affect the slope calculation. So we can simply take M to be the mass of the suspended body.

ACCEPTED VALUES

The accepted value for the spring constant is calculated using the force of gravity opposing the spring force when the object is stationary. The relationship between the spring constant k and the force of gravity F_G is shown in equation 9.4.

$$\text{Spring constant} \quad F_G = -k\Delta x \rightarrow k = -\frac{F_G}{\Delta x} \quad (9.4)$$

Here, Δx is the displacement of the mass that stretches the spring to a new equilibrium point. Be sure to keep the axis and signs consistent. The force of gravity is a vector pointing down.

APPARATUS

- Hooke's law apparatus
- slotted weights
- stop timer
- half meter stick
- platform balance
- computer with Pasco interface

The spring is equipped with a distance scale, as shown in Figure 9.1, so that you can see the equilibrium point change as you change the weight of the object. However, you do not need the scale to measure the period.

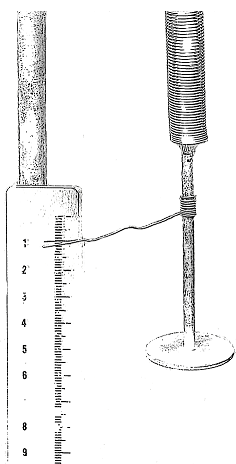


Figure 9.1 Detail of the spring and scale of the Hooke's law apparatus

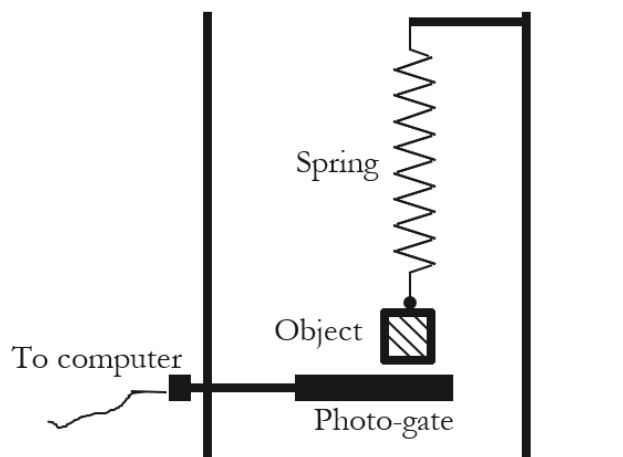


Figure 9.2 Photo-gate positioned to measure the motion of the object

The computer interface measures motion through a photo-gate. The Hooke's law apparatus hangs the object at the end of a spring, which passes through photo-gate, as shown in Figure 9.2.

PROCEDURE

1. Adjust the scale on the Hooke's law apparatus so the pointer aligns with zero.
2. Add a 25 g weight to the hanger and record the new position of the pointer as the equilibrium point x in the data table for Part A.
3. Repeat step 4 seven more times, adding 25 g and recording the equilibrium point next to the total weight added to the apparatus in the data table for Part A until the final load is 200 g.
4. Turn on the computer and photo-gate interface.
5. From the desktop choose the "Simple Harmonic Motion" icon.
6. Remove the weight holder from the spring and hook the 100 g mass onto the spring.
7. Position the photo-gate so that it is just below the bottom of the mass. Start the system oscillating by gently pulling down on the mass and releasing it. Adjust the position of the photo-gate so that when the system is oscillating, the bottom of the mass interrupts the photo-gate but does not pass completely through the gate. This way the bottom of the mass starts the timer and, when it completes one oscillation, stop the timer.
8. Stop the system oscillating and swing the photo-gate out of the way so that it does not interrupt the oscillating mass.
9. Start the system oscillating by gently pulling down on the mass and releasing it. When the system is oscillating smoothly, with little sideways drift, swing the photo-gate back so that it starts recording data.
10. Choose "Play" on the computer interface to start the computer collecting data. Choose "Stop" when the table is filled. Record the average period displayed at the bottom of the table in the data table for Part B
11. Repeat steps 10 through 12 with masses of 125 g, 150 g, 175 g and 200 g. Increase the mass by sliding the appropriate slotted masses on top of the 100 g hanging weight. For the 200 g trial you can use the 200 g hanging weight.
12. With a 200 g hanging weight, observe the oscillations for some time as the amplitude slowly decreases. Do you notice any change in the period, or does the period stay roughly constant? Record your observation on the data sheet.

DATA

Part A: Equilibrium point

Added mass (g)	x_0 (cm)
0.0	0.00
25.0	
50.0	
75.0	
100.0	
125.0	
150.0	
175.0	
200.0	

Part B: Oscillation data

Mas (g)	Average T (s)
100.0	
125.0	
150.0	
175.0	
200.0	

Does the period seem to depend on the amplitude? _____

CALCULATION AND ANALYSIS

1. Calculate the gravitational force F_G due to each of the masses in the data table in Part A, using the formula $F_G = mg$.
2. Draw a graph of the force F_G versus the stationary position x using data from the data table for Part A.
3. Calculate k graphically by taking the slope of the graph using equation 0.9.
4. Draw a graph of the period squared T^2 versus the moving mass M using the data from Part B.

5. Find the slope of the graph using equation 0.9. Calculate k using $k = 4\pi^2 / \text{slope}$.
6. Calculate the % error of k calculated from Part B using k calculated from Part A as the accepted value.
7. As the spring oscillates while you measure its period, the amplitude of the oscillation decreases. Do you see any evidence that the period of oscillation changes as the amplitude decreases?

Experiment 10: Boyle's Law

OBJECTIVES

Gases are comprised of small particles in constant random motion separated by relatively large distances. The characteristics of gases can therefore be analyzed in terms of the volume in space they occupy and the pressure their composite particles produce by constant random collisions. The quantitative relationship between an ideal gas's volume and pressure is known as Boyle's law. In this experiment, you test Boyle's law by changing the volume of air in a sealed chamber and observing the corresponding change in pressure using a barometer. The objectives of this experiment are as follows:

1. To measure the changes in pressure that correlate to changes in volume
2. To test Boyle's Law by expanding and contracting volume
3. To calculate an experimental value of a gas constant

THEORY

The volume V of a fixed mass of a gas is a function of temperature T and pressure P . If the temperature is held constant, many gases exhibit the common property that for any given amount of gas, as the pressure P changes, the volume V changes in a manner such that the product of pressure and volume remains constant, as shown in equation 10.1.

$$\text{Boyle's Law} \quad P_1V_1 = P_2V_2 = \text{const.} \quad (10.1)$$

This relation is known as Boyle's law. The value of the constant depends on the temperature and the amount of gas present. Boyle's law accurately describes the pressure-volume behavior of most common gases at moderate temperatures and pressures. Deviations from Boyle's law may become significant if the pressure is too high or the temperature is too low. These limiting values vary drastically with different gases. For example, hydrogen obeys Boyle's law at minus 200°C and 100 atmospheres; sulfur dioxide does not at 20°C and 1 atmosphere.

In this experiment, a column of air of constant cross sectional area S and length L is trapped in a Plexiglas cylinder with a graduated scale along the side. The length L is controlled by the position of the piston which can be read on the graduated scale. Because $V = SL$ and S is constant, then Boyle's law can be written in the form shown in equation 10.2.

$$\text{Boyle's Law (length)} \quad P_1L_1 = P_2L_2 = \text{const.} \quad (10.2)$$

The position of the piston and thus L is controlled by the threaded rod and the pressure is measured by a pressure gauge. By systematically changing the length of the air column and recording the pressure, you can test Boyle's law.

ACCEPTED VALUES

The accepted value for the constant resulting from equation 10.2 depends on the experimental conditions of temperature and amount of gas. However, once an amount of gas is trapped in the cylinder, the accepted value for the constant is known for all subsequent measurements until the

graduated cylinder is ventilated. Therefore the difference between all the values calculated with equation 10.2 has an accepted value of 0.

APPARATUS

- Boyle's law apparatus
- pressure gauge

The Boyle's law apparatus consists of a sealed graduated cylinder that traps gas under a piston. The piston is attached to a threaded screw that when turned changes the volume of the gas in the graduated cylinder. A pressure gauge measures the change in pressure as the volume in the graduated cylinder changes.

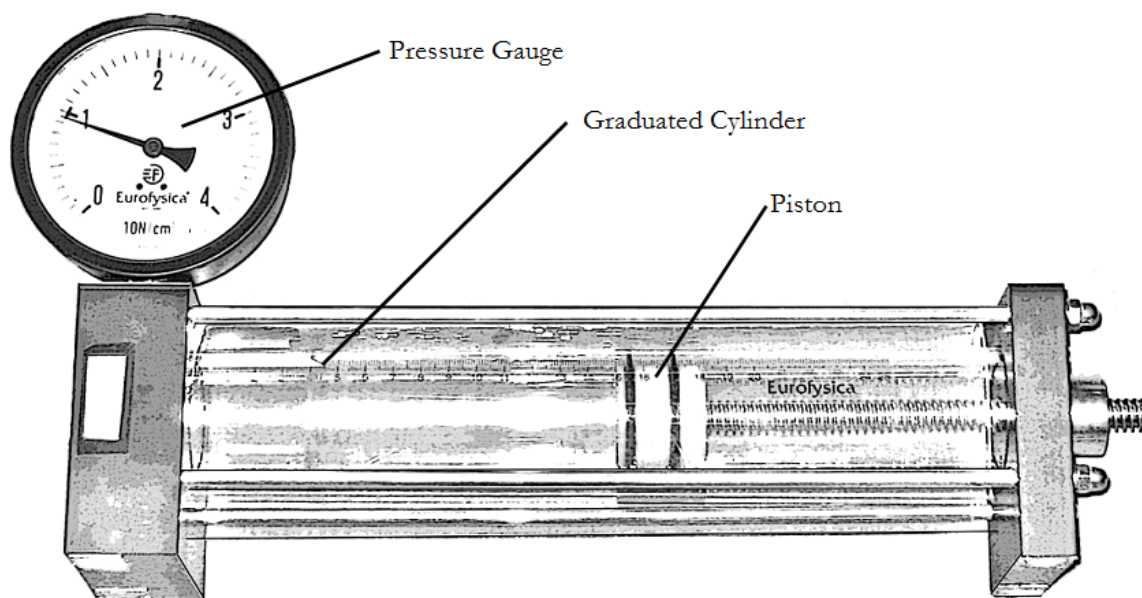


Figure 10.1 the Boyle's law apparatus

PROCEDURE

1. Ventilate the cylinder by turning the hand valve screw on the left end of the cylinder.
2. Set the piston on the 25 cm mark.
3. Close the ventilation valve. You have trapped a column of air in the cylinder at exactly the atmospheric pressure. Check that the pressure gauge reads 1 atm.
4. Before each pressure reading, tap the pressure gauge gently to make sure the needle isn't stuck in a wrong setting.
5. Turn the rotary knob to move the piston to the 24 cm mark. Read the pressure gauge and record the pressure in the data table for Part A. Repeat the procedure in 1 cm steps until the piston reaches the 7 cm mark.
6. Ventilate the cylinder by turning the hand valve screw on the left end of the cylinder.
7. Set the piston on the 7 cm mark.
8. Close the ventilation valve. You have trapped a column of air in the cylinder at exactly the atmospheric pressure. Check that the pressure gauge reads 1 atm.
9. Turn the rotary knob so that the piston moves outward to increase the volume of the trapped air.

10. Record the pressure at each 1cm increment in the data table for Part B until the piston reaches 25 cm.

DATA

Part A: Decreasing Piston Displacement (L)

L (cm)	P_g (atm)
25.0	
24.0	
23.0	
22.0	
21.0	
20.0	
19.0	
18.0	
17.0	
16.0	
15.0	
14.0	
13.0	
12.0	
11.0	
10.0	
9.0	
8.0	
7.0	

Part B: Increasing Piston Displacement (L)

L (cm)	P_g (atm)
7.0	
8.0	
9.0	
10.0	
11.0	
12.0	
13.0	
14.0	
15.0	
16.0	
17.0	
18.0	
19.0	
20.0	
21.0	
22.0	
23.0	
24.0	
25.0	

CALCULATION AND ANALYSIS

1. Calculate the product of the pressure P and length L for every row in both data tables.
2. Calculate the deviation for each value of PL .
3. Throwing out any highly deviating values, use plotting software or good plotting paper to plot the graph of the product of the pressure and length PL versus the length L using data from the data table for Part A.
4. Fit the plotted data points to a straight line. Is the best fit line horizontal?
5. Process the data from Part B using steps 3 and 4, plotting PL versus L and drawing the best fit line. Is the best fit line horizontal?
6. If the trapped gas slowly leaked out during your measurement how would the PL values behave? Do you think this occurred?

Experiment 11: Electrostatic Field

OBJECTIVES

An electrostatic field can be measured indirectly by measuring the electrostatic potential or voltage at different points in the field. In this experiment, you use a multimeter and probe to detect the voltage changes in an electrostatic field distributed across a 2-D surface. The objectives of this experiment are as follows:

1. To measure the voltage at different points in an electrostatic field
2. To find equipotential lines based on different voltage readings
3. To draw electrostatic field lines using equipotential lines

THEORY

The force between charged particles at rest is attributed to an electrostatic field E that surrounds static charges. A point with charge q in an electric field experiences an electrostatic force F_E that is proportional to q , as shown in equation 11.1.

$$\text{Electrostatic Force} \quad F_E = qE \quad (11.1)$$

Here, E is measured in V/m (volts per meter), q is measured in C (coulombs), and F_E is measured in N (newtons). The direction of the electrostatic force at any point in space coincides with the direction of the electrostatic field for a positive charge.

While it is possible to measure the force on a point charge, it is sometimes simpler to calculate an electrostatic field by measuring the work required to move charges through the field. The electrostatic potential or voltage difference V between two points is defined as the work W required to move a positive test charge q from one point to the other divided by the magnitude of q , as shown in equation 11.2.

$$\text{Voltage difference} \quad V_{BA} = V_B - V_A = \frac{W}{q} \quad (11.2)$$

Here, the work W is measured in Joules or $N \cdot m$, which is the force multiplied by distance. Because the force in this case is the electrostatic force F_E from equation 11.1, the voltage change depends on the electrostatic field E and the distance moved d , calculated by combining equations 11.1 and 11.2, as shown in equation 11.3.

$$\text{Voltage difference (E)} \quad V_{BA} = \frac{-F_E d}{q} = \frac{-qEd}{q} = -Ed \quad (11.3)$$

Here d is the component of the distance from point A to point B measured along the field lines. An equipotential line is a curve that has the same voltage everywhere. By definition if you move a charge along an equipotential line, so that the displacement is tangent to the curve, the voltage does not change. Then from equation 11.3 the electric field has no component along the displacement. This means the electric field is perpendicular to equipotential lines.

Because equipotential lines are perpendicular to electrostatic field lines, if you can find equipotential lines, you can also find electrostatic field lines. Figure 11.1 shows the equipotential lines and electrostatic field lines of an electric dipole.

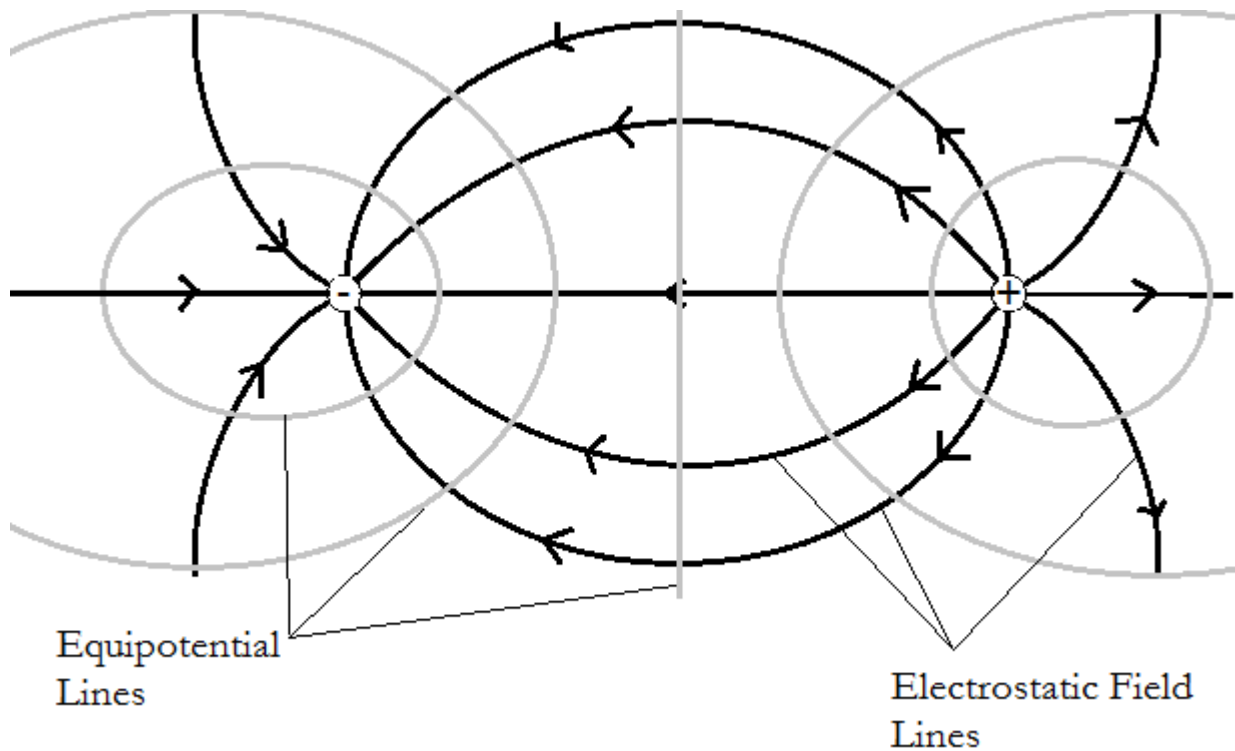


Figure 11.1 The electrostatic field lines and equipotential lines around an electric dipole

In this experiment, you measure the voltage at different points so that you can determine the equipotential lines. It is important to note that because one point cannot have two different values of V , equipotential lines never cross.

ACCEPTED VALUES

There are no accepted values for this lab except that the equipotential lines do not cross.

APPARATUS

- DC power supply
- field mapping board
- multimeter
- u-shaped probe
- wires

When the field mapping board is powered on, the board produces several static charges on a graphite plate that generate an electrostatic field on the board's surface. The terminals marked E1 through E7 across the top of the board provide a linear scale of voltages, ranging from lowest to highest. The multimeter is used to measure the voltage difference between one of the E# terminals and a point on the field mapping board. When the voltage reported by the multimeter is zero, it means the tip of the U-shaped probe is at the same electrostatic potential as the E# terminal the multimeter is connected to. So you can find one equipotential line for each E# terminal by marking points where the multimeter reads zero voltage and connecting those points with a curve.

SETTING UP THE FIELD MAPPING BOARD

The power supply provides Direct Current (DC) power to the field mapping board through two wires. The first step is setting up the board and measuring the voltage at the E terminals.

1. With the power supply OFF, plug the red wire into the positive out terminal (+) and plug a black wire into the ground terminal (\oplus), as shown in figure 11.2.

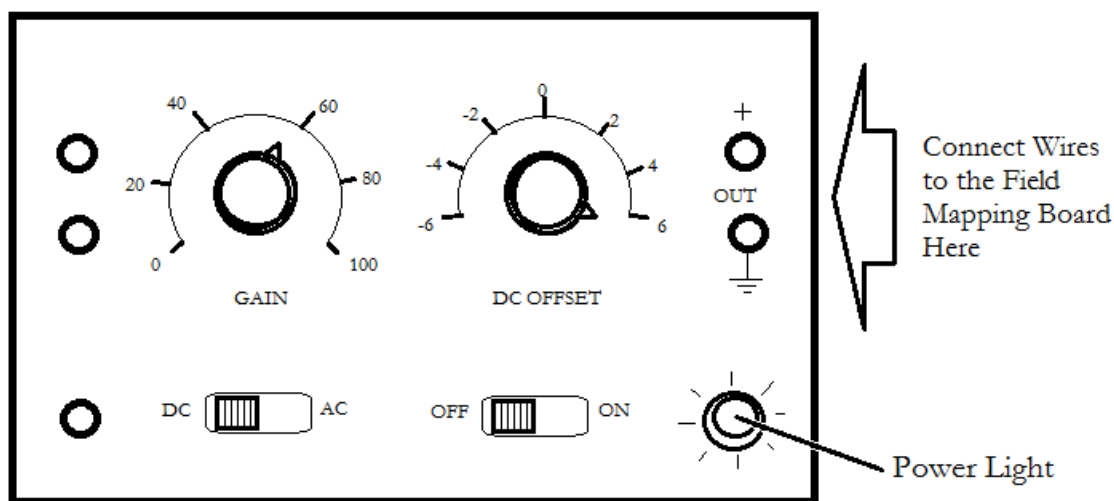


Figure 11.2 Setting up the DC power supply

2. On the power supply, set the AC/DC switch to DC. Turn the DC offset to the maximum positive value, which is past 6 in the clockwise direction. The GAIN knob does not significantly change the DC output, so leave it between 40 and 60.
3. Connect the black wire from the power supply to the left "Battery or Oscillator" terminal on the field mapping board and connect the red wire from the power supply to the right "Battery or Oscillator" terminal on the field mapping board, as shown in figure 11.3.

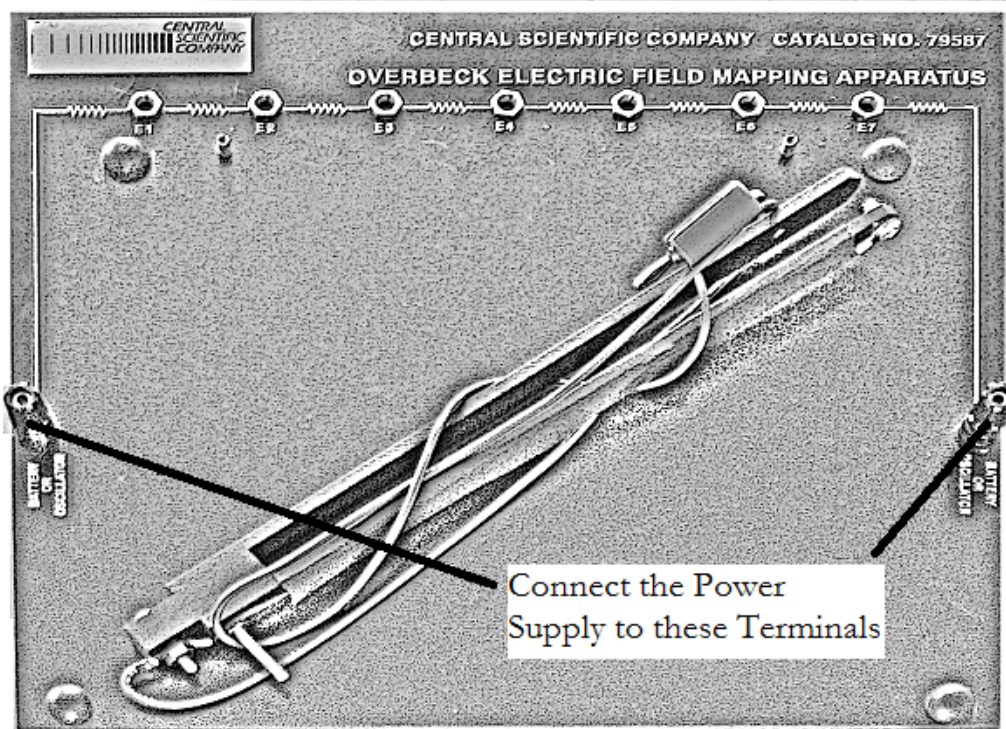


Figure 11.3 Connecting the power supply to the field mapping board

4. Connect a wire from the into the ground terminal \ominus of the power supply to the “COM” jack of the multimeter. Connect a second wire from the (+) terminal of the power supply to the “V” jack of the multimeter.
5. Switch the multimeter AC/DC switch to DC and set the multimeter dial to the 20 V DC scale in the “V” range of the dial. DO NOT turn the multimeter dial to any other settings.
6. Turn on the power supply. Adjust the DC Offset knob on the power supply until the multimeter reads 4.0 volts. This is the total voltage difference from one Battery or Oscillator jack on the field mapping board to the other.
7. Unplug the wire connecting the “V” jack of the multimeter and the (+) terminal of the power supply. Reconnect it so the “V” jack of the multimeter is connected to the “E1” plug on the field mapping board. The multimeter should read 0.5 volts. If you get a significantly different value, ask your lab instructor for help.
8. Repeat the process in step 7 for the E2 through E7 plugs. Each plug should be 0.5 volts above the preceding plug. This provides a linear scale of voltage differences from plug E1 through E7.
9. Unplug the wires from the multimeter. Plug the wire from the u-shaped probe into the “V” jack of the multimeter. Plug a wire into the “COM” jack of the multimeter and plug the other end of the wire into the terminal marked “E2” on the field mapping board, as shown in figure 11.4. Proceed to the procedure.

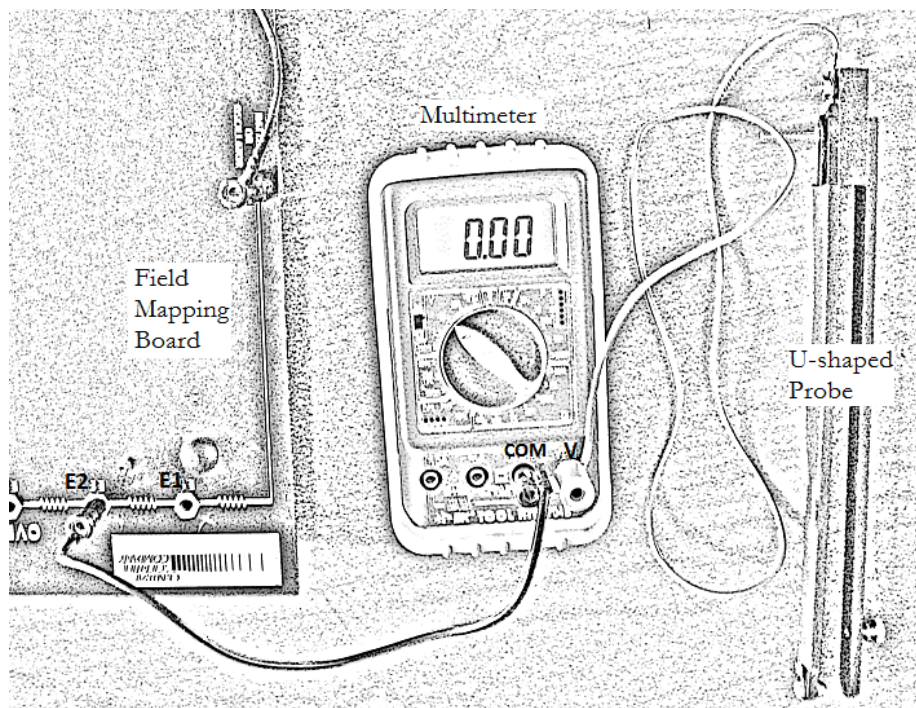


Figure 11.4 Connecting the multimeter and U-shaped probe to the field mapping board

PROCEDURE

1. Lay the field mapping board flat on the lab table with the terminals and rubber bumpers facing up.
2. Fasten a sheet of 8.5 x 11-inch paper to surface of the field mapping board. Secure the paper by slipping the corners of the paper under the four rubber bumpers.
3. Slide the U-shaped probe onto the field mapping board with the ball end facing the underside of the mapping board. Notice the screw below the probe acts as a support leg.
4. Switch the multimeter AC/DC switch to DC and set the multimeter dial to the 200mV DC scale in the “V” range of the dial. **DO NOT** turn the multimeter dial to any other settings. In particular, do not connect the multimeter to any power supply and switch the dial to a setting in the “ Ω ” scale; this damages the meter.
5. Turn on the power supply. The power light lights up when the power is on.
6. Gently glide the U-shaped probe over the paper until the reading of the multimeter is close to zero. Then, slide the U-shaped probe back and forth until you can clearly see a point where the voltage on reading of the multimeter switches from positive to negative. With a pencil, mark that point on the paper through the eye of the U-shaped probe and label the point E2 as you mark it. From the way we set up the field mapping board, we know this point is at a potential of 1.0 V.
7. Repeat step 5 at different places on the board until you have generated a series of zero-voltage points across the paper for E2, labeling each point E2.
8. Unplug the wire from terminal E2 and plug it into terminal E3. Repeat steps 5 and 6 for the new voltage, labeling each point with E3 as you mark it. Repeat this process with the wire plugged into terminals E4, E5, and E6. Be sure to label each point with the terminal code.
9. Turn the power supply off. The power light should turn off.

10. Create copies of the drawing and data points using a photocopier or digital camera. Distribute a copy of the drawing to each member of the lab group.

DATA

The data sheet in this experiment is the sheet of paper upon which you draw the equipotential lines. Experiments seldom produce perfect results, and the charges on the field mapping board typically do not produce a balanced dipole. The equipotential lines might align in unexpected ways but each line should form a closed loop or extend from one edge of the page to another, as shown in figure 11.5.

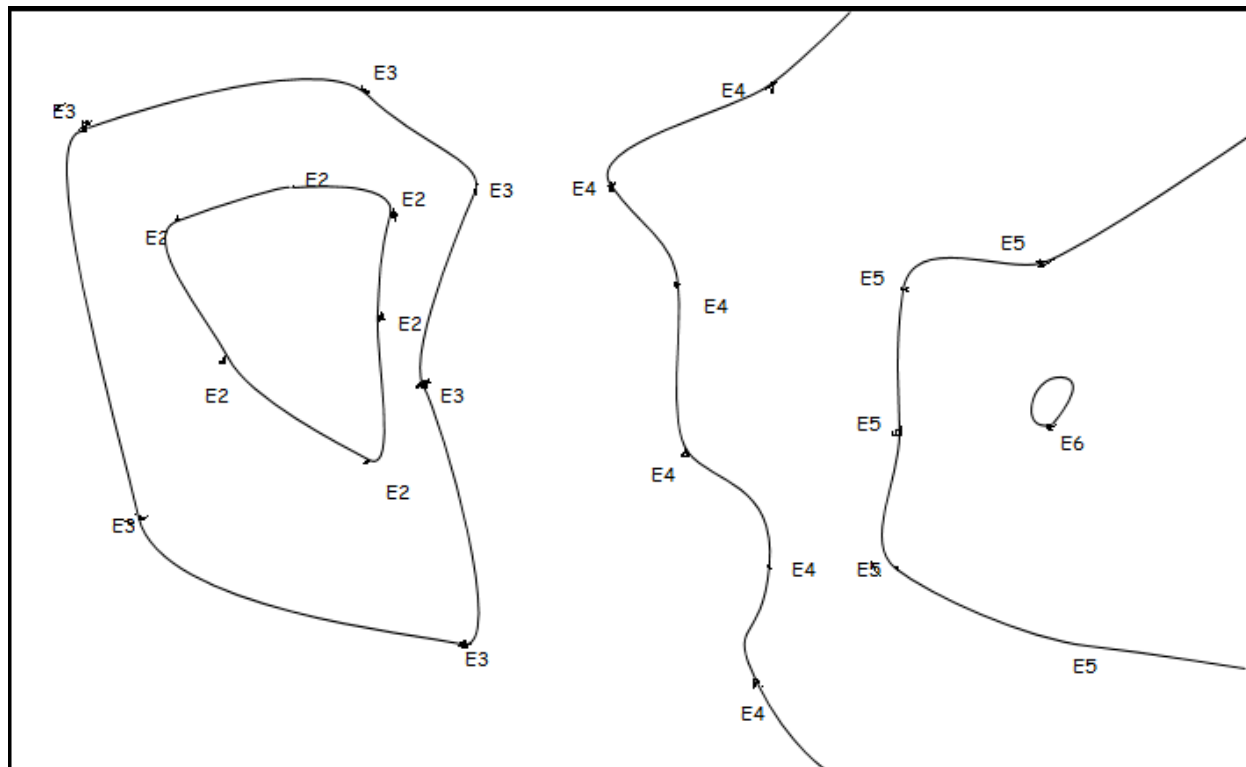


Figure 11.5 Equipotential lines form loops or proceed from one edge of the page to another

CALCULATION AND ANALYSIS

1. Draw a smooth curve that connects the dots labeled E2 on the data sheet using a pencil. This is the equipotential line for E2.
2. Repeat step 1 for each series of points E3 through E6.
3. Draw the electric field lines. Remember that the electric field is perpendicular to the equipotential lines.
4. Sketch your idea of the locations and signs of the charges on the electrostatic field board.
5. If a point charge is placed in an electric field and released from rest, will it travel along a field line?

Experiment 12: Ohm's Law

OBJECTIVES

The movement of charge in a conductor, referred to as current, is possible because electrons can move when a voltage is applied. A good conducting material transfers electrons easily and is said to have low resistance, while a poor conductor transfers electrons less readily and has high resistance. In many materials, it is found that current is directly proportional to voltage. Georg Ohm discovered this and formalized this relationship as Ohm's Law. In this experiment you measure the voltage, current, and resistance of circuits, and test Ohm's law by predicting one of these quantities based on the other two. The objectives of this experiment are as follows:

1. To measure current and voltage in a circuit
2. To calculate resistance using different methods given current and voltage measurements
3. To test Ohm's law

THEORY

When there is a potential difference V between two points in a circuit, a current flows. Experimentally it is found that for many materials if the temperature remains constant, the voltage V is proportional to the current I , as shown in equation 12.1.

$$\text{Ohm's Law} \qquad V = IR \qquad (12.1)$$

The proportionality constant R is the resistance between the two points considered.

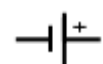
ACCEPTED VALUES

The accepted value for the resistance R is measured directly from the resistor, which you do at the end of the procedure for this experiment.

APPARATUS

- resistor
- wires
- DC power supply
- multimeters
- fuse

Simple electric circuits consist of conducting wires, resistors, power supplies, and fuses. A circuit diagram is a model of a circuit that corresponds to the arrangement of the components, denoted by the following symbols:



DC power supplies provide a potential difference that causes current to flow through a closed circuit at a constant rate.



Conducting wires provide a medium through which charge can move.



Resistors provide resistance to charge moving.



Fuses are safety devices that break a circuit when a current threshold is exceeded

A simple example of a circuit diagram is shown in figure 12.1. A fuse is always included in the circuits to protect the equipment. When too high a current flows through a fuse it interrupts the current.

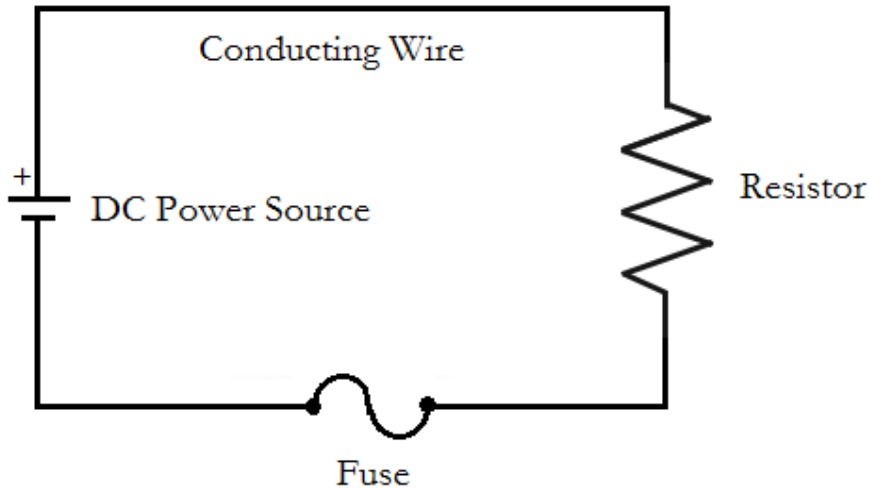


Figure 12.1 A simple circuit diagram displaying four essential components

In this experiment you measure the current and voltage at specific points on the circuit. To do this you'll use an instrument called a multimeter. You will connect two multimeters to the circuit at the same time, with one set up to measure current and the other set up to measure voltage.

Multimeters can be set for several functions.

- to measure direct (DC) or alternating (AC) current
- to measure voltage (V), current (A) or resistance (Ω)

They can also be set for the maximum value of the quantity to be measured. For an illustration see figure 12.2.

To use the multimeter to measure current (ammeter), set the meter to DC and 200 mA. Then connect the multimeter in series with the circuit, meaning that you plug the source of the current in in the mA socket and the destination of the current into the COM socket of the multimeter. For example, to measure the current before a resistor, plug the wire from the power supply into the mA socket and plug a wire connected to the resistor into the COM socket.

To use the multimeter to measure voltage (voltmeter), set the meter to DC and 20 V. Then connect the multimeter in parallel with the circuit, meaning that you plug the source of the current into the V socket and the destination of the current into the COM socket of the multimeter. For example, to measure the voltage across a resistor, plug a wire from the end of the resistor nearest the power supply into the V socket and plug a wire connected to the other end of the resistor into the COM socket.

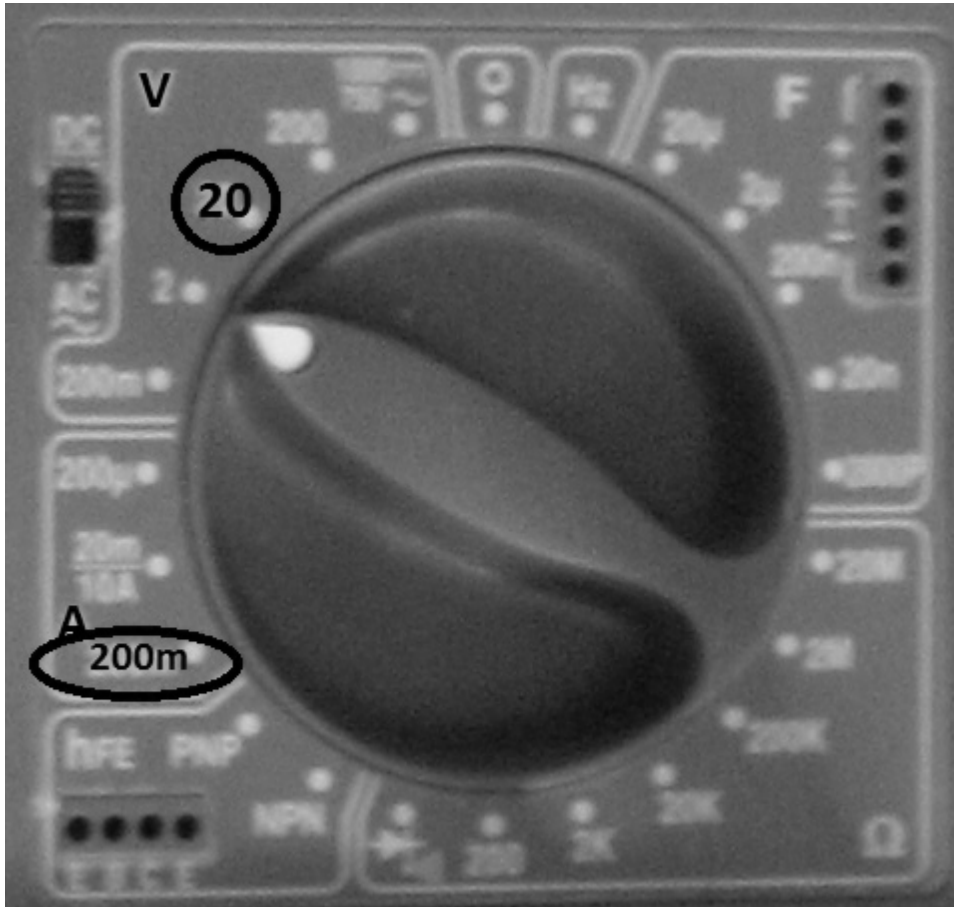


Figure 12.2 Locating the 20V and 200mA settings on the BK Toolkit 2704B multimeter.

In a circuit diagram, the voltmeter and ammeter are denoted by the following symbols:



Ammeters measure current and are setup in series



Voltmeters measure voltage and are setup in parallel

Ammeters are always connected in series so that the current to be measured flows through the instrument. Voltmeters are connected in parallel with the portion of the circuit across which the potential difference is measured. In figure 12.3, the circuit diagram for this experiment is shown with the ammeter in series with the resistor and the voltmeter in parallel with the resistor.

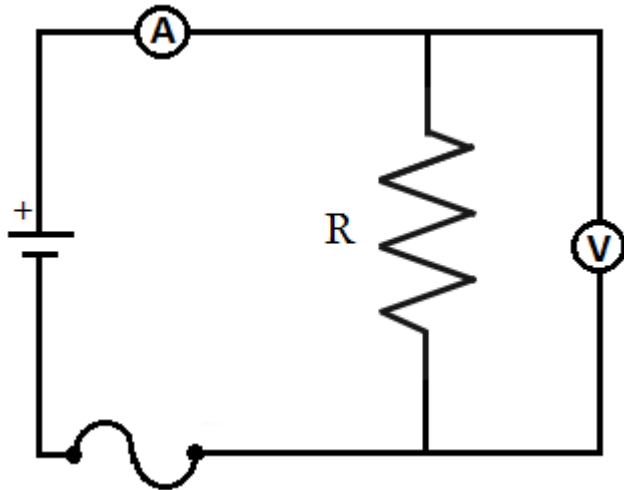


Figure 12.3 A circuit diagram with an ammeter setup in series and a voltmeter setup in parallel

Current in the circuit is supplied by the DC power supply, illustrated in figure 12.4. The current is varied by adjusting the DC OFFSET knob, which controls the voltage produced by the power supply. The zero position for this control is at twelve o'clock. Clockwise rotation increases the voltage (0 to +6V), while counterclockwise rotation yields the opposite polarity (0 to -6V). Use only positive voltages in this experiment.

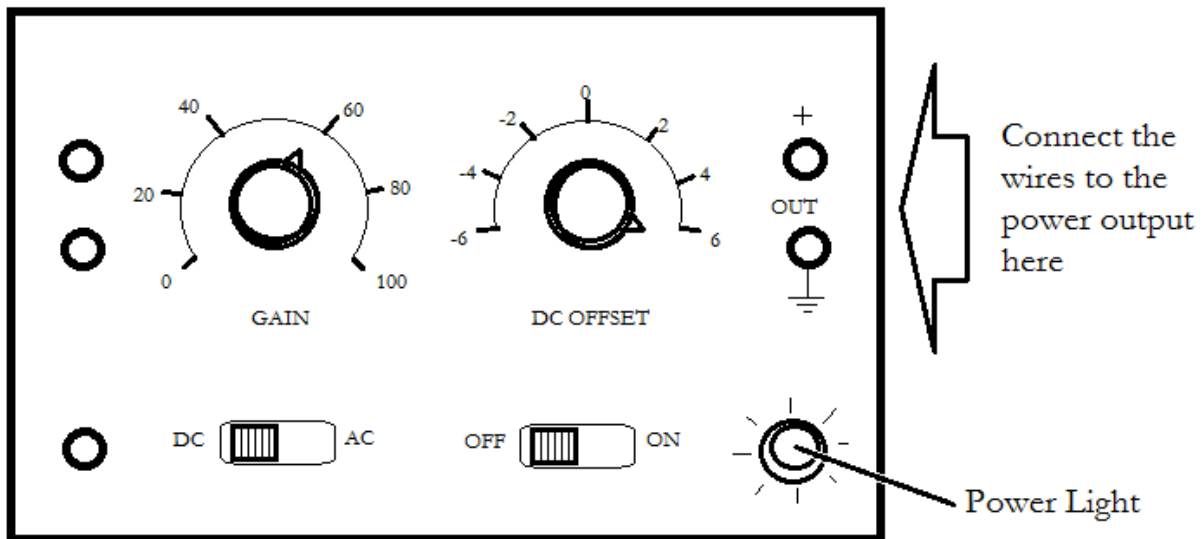


Figure 12.4 Vary current from the power supply by adjusting the DC offset knob

CALCULATION AND ANALYSIS

1. Calculate V/I for each pair of voltage current readings. Compute the average of your V/I values. This is one estimate for the resistance R .
2. Draw a graph showing the relation between voltage (y-axis) and current (x-axis). Draw the best fit straight line through the points on your graph. Calculate the slope of this line as another estimate for R .
3. Calculate the % error for the two resistance values you obtained, using the direct measurement you made in step 8 of the procedure as the accepted value. Explain whether your results support Ohm's law.

Experiment 13: Electric Circuits

OBJECTIVES

An electric circuit is a closed path of conducting material through which an electric current flows. In this experiment, you build circuits out of conductors and resistors and measure the currents through resistors in parallel and in series. The objectives of this experiment are as follows:

1. To build electric circuits with resistors in parallel and in series
2. To measure the current and voltage in a circuit
3. To calculate the current and voltage for resistors in parallel and in series

THEORY

Circuits may include combinations of several resistors. Two connected so that the same current flows through both are said to be in series, as shown in the circuit diagram in figure 13.1.

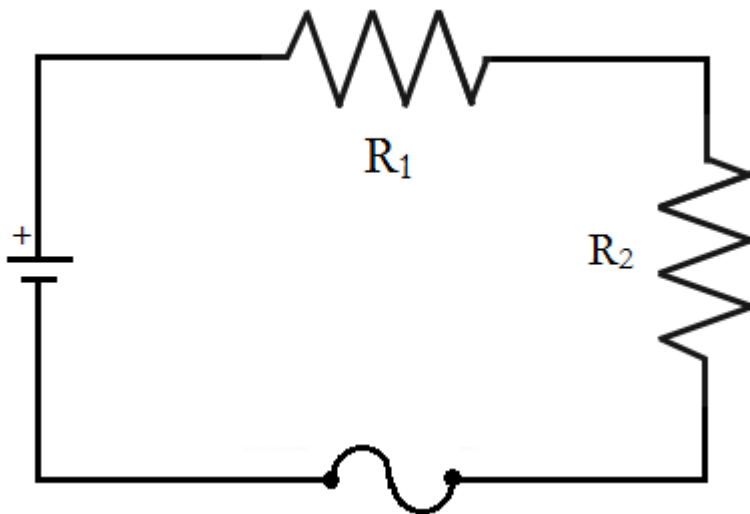


Figure 13.1 A circuit diagram showing resistors R_1 and R_2 in series

Resistors connected so that the sum of the currents through the two resistors is equal to the total current in the circuit are said to be in parallel, as shown in the circuit diagram in figure 13.2.

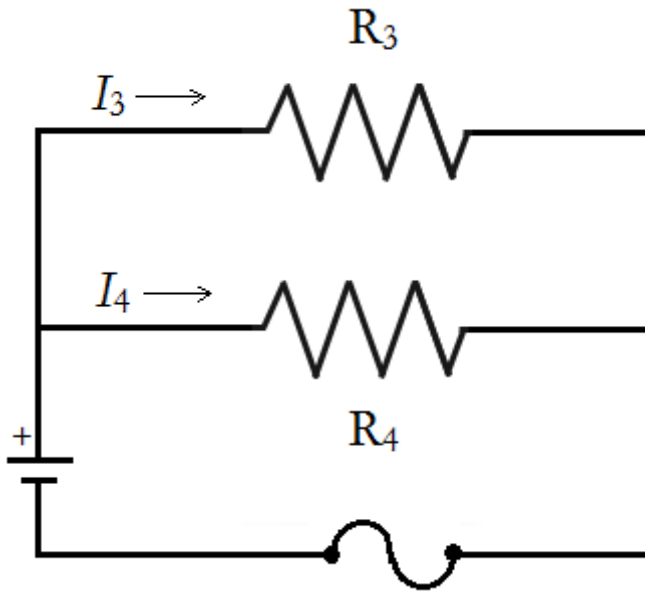


Figure 13.2 A circuit diagram showing resistors R_3 and R_4 in parallel

A series circuit showing an ammeter measuring the current before both resistors and a voltmeter measuring the voltage across resistor R_2 is shown in figure 13.3.

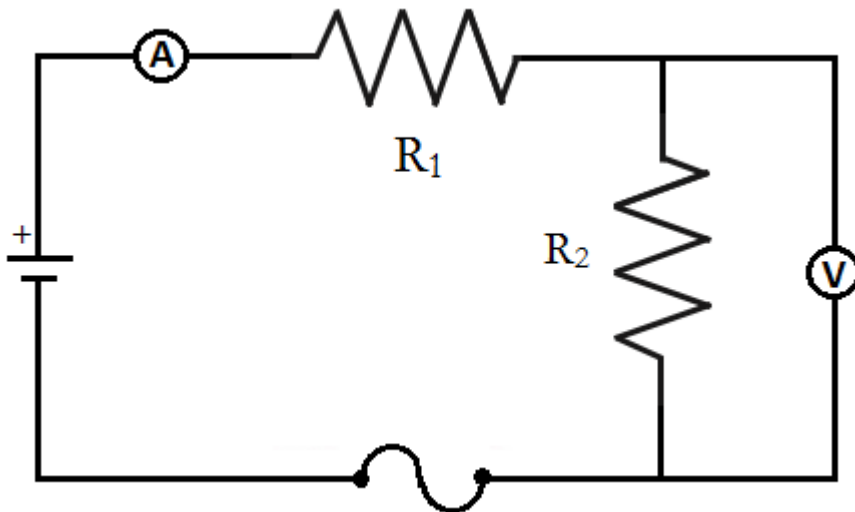


Figure 13.3 A circuit diagram with an ammeter set up in series and a voltmeter set up in parallel

The basic rules for analyzing circuits are known as Kirchhoff's laws. They give relations among currents and voltages as follows.

KIRCHHOFF'S CURRENT LAW

Kirchhoff's current law states that the current at any junction in a circuit is the same going in to the junction as going out.

$$\text{Kirchhoff's Current Law} \quad \sum I_{in} = \sum I_{out} \quad (13.1)$$

This is equivalent to conservation of charge. It tells us that we neither gain nor lose electrical charge at any junction. Consequently, in a series circuit, the same current flows through each resistor, while in a parallel circuit the sum of the currents through the resistors equals the total current in the circuit.

KIRCHHOFF'S VOLTAGE LAW

Recall that electrical potential, or voltage, is the potential energy per unit charge. It does not change along any conductor that has zero resistance. If we go around any closed loop in a circuit, we must get back to the same potential we started from. This is Kirchhoff's voltage law, which states that the sum of the voltage changes ΔV around any closed loop in a circuit is zero.

$$\text{Kirchhoff's Voltage Law} \quad \sum \Delta V = 0 \quad (13.2)$$

Across a resistor the potential decreases as we follow the circuit in the direction of the current. Consequently the total decrease (drop) in voltage in a series circuit is the sum of the voltage drops across each resistor. For a parallel circuit, the input of each resistor is connected to a common conductor. Consequently in a parallel circuit the voltage drop is the same across each resistor.

ACCEPTED VALUES

The accepted values for the circuit with resistors in series (part A) is that the current at the end of the circuit is equal to the current at the beginning ($I_{initial} = I_1 = I_2$) and that the total voltage on the circuit is equal to the sum of the voltages across the resistors ($V_{total} = V_1 + V_2$).

The accepted values for the circuit with resistors in parallel (part B) is that the initial voltage equal to the voltages across either resistor ($V_{initial} = V_3 = V_4$) and that the total current on the circuit is equal to the sum of the currents through each resistor ($I_{total} = I_3 + I_4$).

APPARATUS

- power supply
- fuse
- multimeters
- resistors
- conducting wires

PROCEDURE

PART A. RESISTORS IN SERIES

1. Connect a simple series circuit as shown in figure 13.1. Have your lab instructor check the circuit before turning on the power supply. The voltmeter should be set to DC and 20 V, and the ammeter to DC and 200 mA.
2. Measure the voltage across resistor R_1 , across resistor R_2 , and across the series combination of R_1 and R_2 . Recall that the voltmeter is connected across (or in parallel with) resistors. Be sure the voltmeter is set for direct current voltage (DCV) on the 20 Volt scale. Record the voltages in the data table for Part A.
2. Measure the current entering resistor R_1 , between resistors R_1 and R_2 , and leaving resistor R_2 . Ammeters are connected in series with the resistor whose current is being measured. Be sure your ammeter is set for direct current amps (DCA) and the maximum current is set for 200 mA. Record the currents in the data table for Part A.

PART B. RESISTORS IN PARALLEL

1. Connect resistors R_3 and R_4 in parallel as shown in figure 13.2. Have your lab instructor check the circuit before turning on the power supply. Set the voltmeter and ammeter as before.
2. Measure the voltage across resistor R_3 , across resistor R_4 , and across the whole circuit. Record the voltages in the data table for Part B. (Note that these three measurements involve essentially the same connection to the circuit.)
3. Measure the current through the circuit from the power supply, through resistor R_3 , and through resistor R_4 . Again be sure your ammeter is in series with the circuit to be measured and that the meter is correctly set to DC. Record the currents in the data table for Part B.

DATA

Part A. Resistors in series

Position	Voltage (Volts)	Current (mA)
across R_1		
across R_2		
across R_1+R_2		
entering R_1		
between R_1 and R_2		
leaving R_2		

Part B. Resistors in parallel

Position	Voltage (Volts)	Current (mA)
across R_3		
across R_4		
across R_3+R_4		
entire circuit		
through R_3		
through R_4		

CALCULATION AND ANALYSIS**PART A. RESISTORS IN SERIES**

1. From your data for the series circuit, compare the three voltages you measured: across R_1 , R_2 , and the combination of R_1 and R_2 together. What simple relationship do you find?
2. Calculate the percent error for the sum of the voltages through R_1 and R_2 using the measured voltage across both resistors as the accepted value
3. Compare the three currents you measured through each resistor and through the entire circuit. What simple relation do you find? How would you interpret this result in terms of the Law of Conservation of Charge?
4. Calculate the percent error for the currents between R_1 and R_2 and leaving R_2 using the measured current entering R_1 as the accepted value.
5. The manufacturer claims that the multimeter has 0.5% accuracy. Compare the manufacturer's stated accuracy with the calculated % error and explain whether either of the results verify Kirchhoff's laws.

PART B. RESISTORS IN PARALLEL

6. Compare the voltage across the whole circuit with the voltage across each resistor individually. What relation do you find
7. What relationship do you find between the current through the whole circuit and the current through each resistor? How do you interpret this in terms of the law of conservation of charge?
8. Calculate the percent error for the sum of the currents through R_3 and R_4 using the measured current through the entire circuit as the accepted value
9. The manufacturer claims that the multimeter has 0.5% accuracy. Compare the manufacturer's stated accuracy with the calculated % error and explain whether either of the results verify Kirchhoff's laws.

Experiment 14: The Oscilloscope

OBJECTIVES

While a voltmeter measures voltage at a specific time, an oscilloscope measures how voltage changes over time. This makes it useful for studying electrical signals with alternating current. In this lab, you learn how to use an oscilloscope and measure the signal on an RC circuit. The objectives of this experiment are as follows:

1. To measure voltage changes in a circuit using an oscilloscope
2. To test an RC circuit using an oscilloscope
3. To estimate the time constant for an RC circuit using an oscilloscope

THEORY

An oscilloscope measures voltage over time and displays a graph of voltage (vertical axis) versus time (horizontal axis). This provides a way to analyze circuits carrying alternating current. You can think of the central vertical line on the CRT display as the y-axis (voltage) and the central horizontal line on the CRT display as the x-axis (time) of the graph.

RC CIRCUIT

An RC circuit is comprised of a capacitor C in series with a resistor R as shown in figure 14.1. In this circuit diagram, there are several specific components denoted by the following symbols:



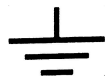
AC power supplies provide a potential difference that causes current to flow through a closed circuit at a varying rate.



Capacitors store electrical energy in an electrostatic field, which blocks direct current (DC) but charges and discharges in a regular pattern when connected to an alternating current (AC).



Resistors provide resistance to charge moving.



Grounds are large conducting bodies (such as the earth) used as a common return path for an electric current.

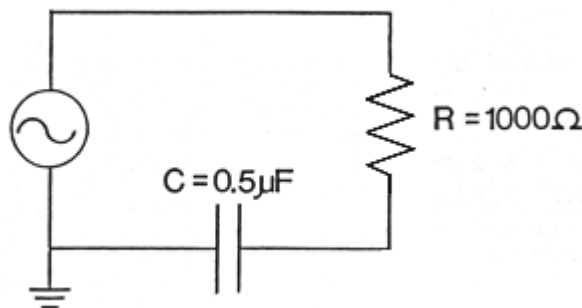


Figure 14.1 An RC circuit with an alternating current (AC) power supply

For the RC circuit modeled in figure 14.1, the AC power supply produces a positive voltage for some period of time, and then switches to a negative voltage for an equal period of time. Depending on the specific function of the signal, it may be referred to as a sinusoidal, square, or triangular wave form. In this experiment, after calibration, we are interested in the square wave, which charges the plates of the capacitor when the voltage is positive, and discharges the plates of the capacitor when the voltage is negative. As the current passes through the capacitor, the CRT display displays a specific pattern of charging and discharging voltage over time, as shown in figure 14.2.

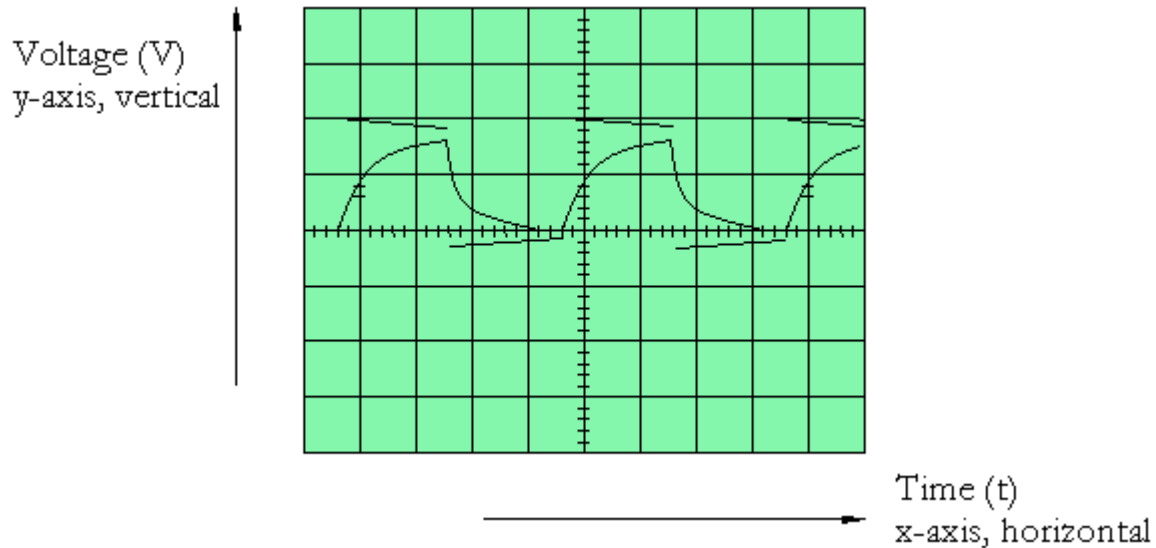


Figure 14.2 charging and discharging capacitor pattern on the CRT display of an oscilloscope

The almost horizontal lines in figure 14.2 are a trace of the input voltages from the square wave. The continuous wave function is the trace of the resulting voltage across the capacitor.

Assume the initial charge on a capacitor is zero. When the positive voltage from the power supply is applied, the voltage across the capacitor increases. While the capacitor is charging, the voltage across the capacitor is a function of the supply voltage, the resistance of the circuit, the capacitance of the capacitor, and the time since the supply voltage began, as shown in equation 14.1.

$$\text{Capacitor Voltage (charging)} \quad V_c = V_0(1 - e^{-t/RC}) \quad (14.1)$$

Where

- V_c is the voltage across the capacitor
- V_0 is the supply voltage, which is also the maximum voltage across the capacitor
- t is the elapsed time since the application of the supply voltage
- RC is the time constant of the RC charging circuit

Assuming that the capacitor starts fully charged, when the negative voltage from the power supply is applied, the voltage of the capacitor decreases. While the capacitor is discharging, the voltage across the capacitor is a function of the maximum voltage across the charged capacitor, the resistance of the circuit, the capacitance of the capacitor, and the time since the negative voltage began, as shown in equation 14.2.

$$\text{Capacitor Voltage (discharging)} \quad V_c = V_0(e^{-t/RC}) \quad (14.2)$$

Where

- V_c is the voltage across the capacitor
- V_0 is the maximum (initial) voltage across the capacitor
- t is the elapsed time since the application of the supply voltage
- RC is the time constant of the RC charging circuit

The product of resistance and capacitance RC is called the time constant. It can be calculating using equation 14.3.

$$\text{Time Constant, RC circuit} \quad \tau = RC \quad (14.3)$$

Capacitance multiplied by resistance is verified to be the same units as time (seconds) by dimensional analysis as follows:

$$1 F (\text{Farad}) = 1 \text{ s}^4 \cdot \text{A}^2 \cdot \text{m}^{-2} \cdot \text{kg}^{-1}$$

$$1 \Omega = 1 \text{ kg m}^2 \text{ s}^{-3} \cdot \text{A}^{-2}$$

$$\therefore F \cdot \Omega = \frac{\text{s}^4 \cdot \text{A}^2}{\text{m}^2 \cdot \text{kg}} \cdot \frac{\text{kg} \cdot \text{m}^2}{\text{s}^3 \cdot \text{A}^2} = \text{s}$$

The time constant is the characteristic time for the charging and discharging behavior of an RC circuit. For instance it represents the time it takes for the voltage across a discharging capacitor to decrease by a factor of $e^{-1} \approx 0.368$. It also represents the time it takes for the voltage across a charging capacitor to reach a fraction $1 - e^{-1} \approx 0.632$ of its final value. These relationships are shown graphically in figure 14.3.

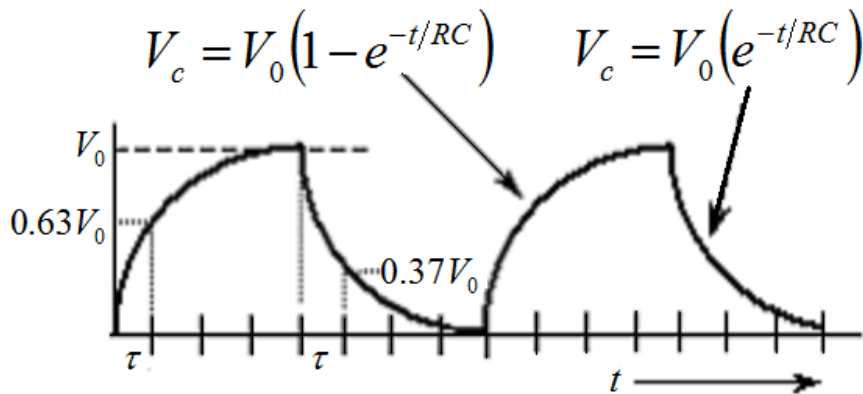


Figure 14.3 The capacitor voltage functions and the approximations at intervals of 1τ

ACCEPTED VALUES

The accepted value for the time constant τ is calculated as $\tau = RC$. The capacitor in this lab has a rating of $0.5\mu F$, so $\tau = R \cdot 0.5\mu F$, which calculates the following accepted values for τ :

- @ 1000Ω , $\tau = 5.0 \times 10^{-4}$ seconds
- @ 400Ω , $\tau = 2.0 \times 10^{-4}$ seconds
- @ 100Ω , $\tau = 5.0 \times 10^{-5}$ seconds

- @40Ω, $\tau = 2.0 \times 10^{-5}$ seconds

APPARATUS

- oscilloscope
- BK Toolkit 2704B multimeter
- resistor box
- capacitor
- wires
- BNC cables with probes
- function generator
- DC power supply

The oscilloscope is versatile and powerful research tool. The essential component of an oscilloscope is the cathode ray tube (CRT). The CRT is a basic component of the television, computer monitor, electrocardiograph, etc. The CRT creates an image on a screen by generating a controlled electron beam that strikes a fluorescent screen so that it glows at that spot. The electrons in the beam are collimated (focused) into a fine beam by magnets and accelerated by a high voltage (the acceleration potential). The beam can be deflected both horizontally and vertically by means of plates to which and electric potential is applied. By the time-dependent modulation of these plate voltages, a trace is obtained.

The oscilloscope is especially useful for displaying alternating currents and voltages which vary with time. When an alternating signal is applied to the input of the oscilloscope a “trace” is seen which graphically depicts the change in voltage (vertical trace) with respect to time (horizontal trace). Any signal that can be represented by a voltage that changes with time can be viewed with an oscilloscope.

CONTROLS OF THE OSCILLOSCOPE

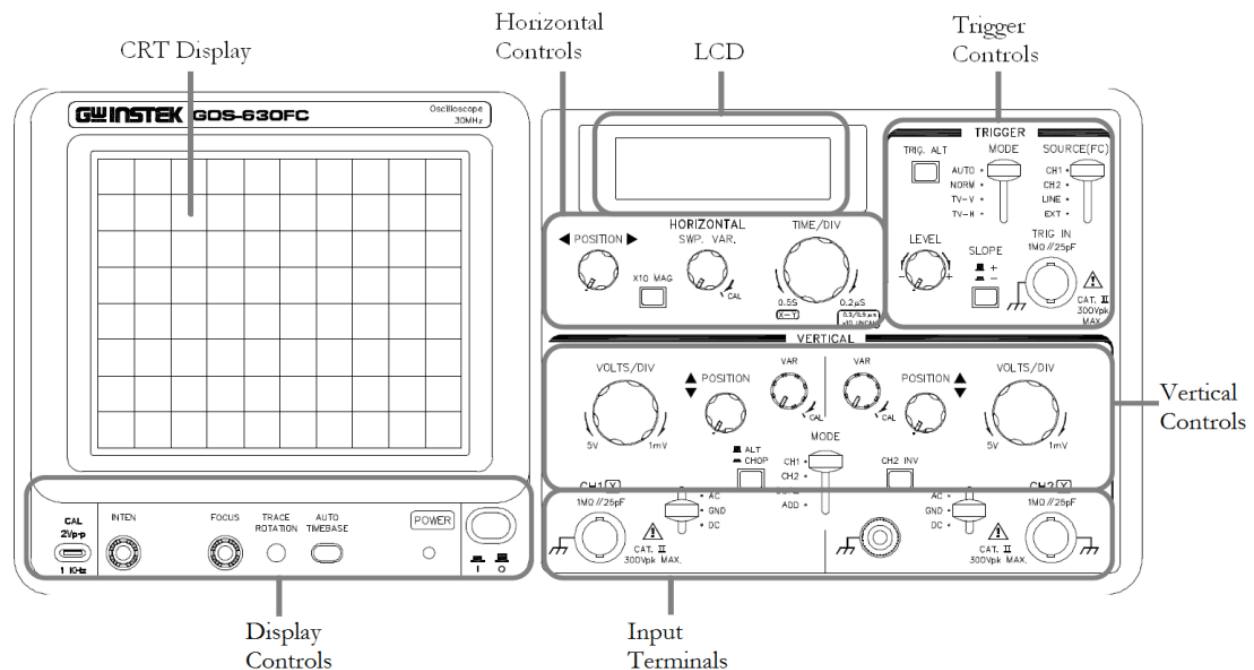


Figure 14.4 The control groups and displays of the Instek GDS-630FC analog oscilloscope

The horizontal trace generally represents time. The “Time/Div” control in horizontal controls determines the rate at which the beam sweeps across the screen for both channels. For example, if the Time/Div is set at 1 ms (millisecond) it takes 10 ms to sweep across the ten divisions on the

display screen. Once it reaches the end of the screen, the beam is deflected almost instantaneously back to where the sweep started, and the next sweep begins. The sweep can be set relatively slow (about 2 seconds to cross the screen) or very fast (2 micro seconds to cross the screen).

The vertical trace generally represents the signal strength (voltage). The “Volts/Div” control for each channel determines how great a deflection results from an applied voltage. If the knob is set to 1 volt/div, and +3 V is applied, the beam deflects “up” 3 divisions. Conversely if -3 V were applied, the beam would deflect “down” 3 divisions.

The LCD displays the settings for Time/Div and Volts/Div for both channels in the LCD, as shown in figure 14.5.

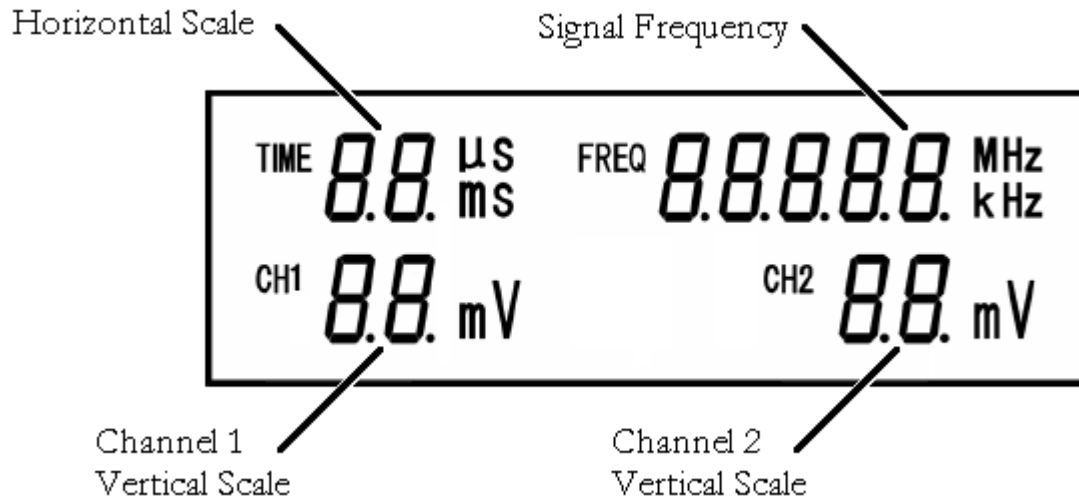
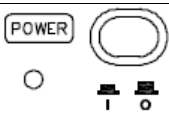




Figure 14.5 The LCD displays the settings adjusted by the Time/Div (horizontal) and Volts/Div (vertical) controls

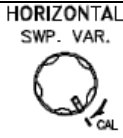


The signal frequency is the same for both channels and can be compared to the signal frequency generated by signal generator. If there is a large mismatch between the frequency displayed on the signal generator and the oscilloscope, there is likely a problem with the circuit.

DEFAULT SETTINGS

Before powering up the oscilloscope, set up the front panel as follows.

POWER switch		Off
INTEN Knob		Center
FOCUS Knob		Center

Vertical MODE Switch		CH1
ALT/CHOP Switch		Released (ALT)
CH 2 INV Switch		Released (Inversion disabled)
CH1/CH2 Vertical POSITION Knob		Center
CH1/CH2 VOLTS/DIV		Center
CH1/CH2 VARIABLE Knob		CAL (calibrate)
CH1/CH2 Coupling Switch		DC
Trigger SOURCE Switch		CH1
Trigger SLOPE Switch		Released (+ Positive slope)
TRIG ALT Switch		Released (alternating trigger disabled)
Trigger MODE Switch		AUTO
TIME/DIV Knob		Center

Horizontal SWP.VAR Knob		CAL (calibrate)
Horizontal POSITION Knob		Center
x10 MAG Switch		Released (x10 magnification disabled)

CONTROLS OF THE FUNCTION GENERATOR

A function generator is a source of AC voltage with adjustable frequency set by the buttons and knobs on its face, as shown in figure 14.6.

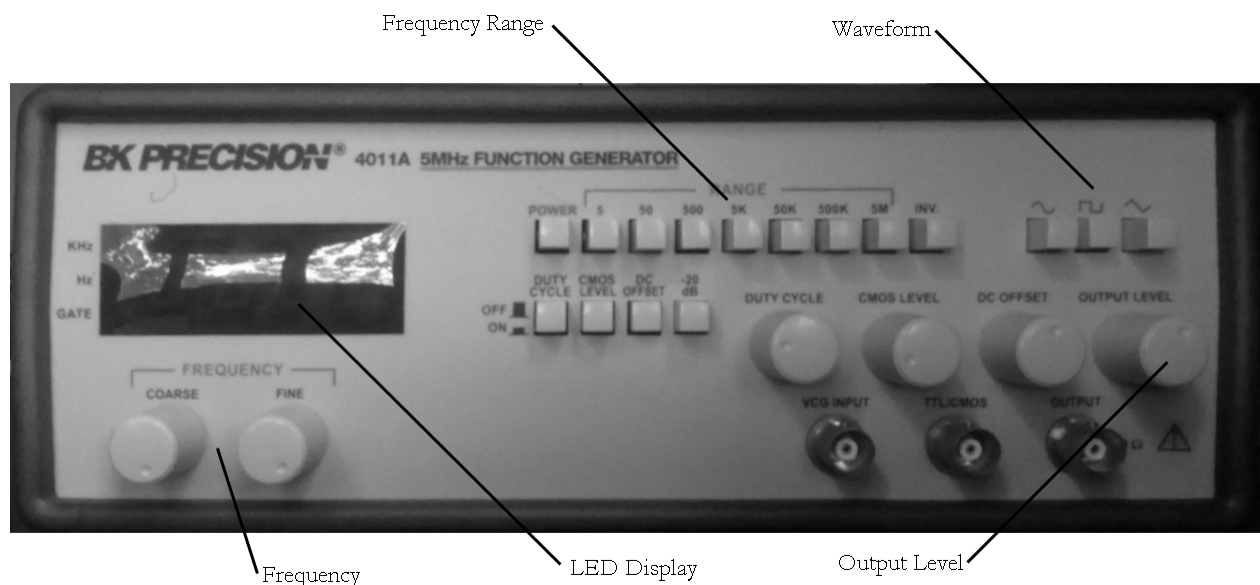


Figure 14.6 the front panel of the BK Precision 4011A Function Generator used in this experiment

The voltage from the generator is obtained from the OUTPUT terminal, to which a BNC cable can be connected. The approximate frequency of the output is displayed in the digital frequency display. To adjust the output frequency, first set the desired first two significant figures of the frequency using the FREQUENCY-COARSE knob. Then set the multiplier by pushing the appropriate button in the RANGE section of the front panel, which changes the frequency threshold to the number of Hz shown above each RANGE button.

The amplitude of the output signal can be adjusted from the OUTPUT LEVEL knob. For this lab, it is convenient to have it in the 2 – 10 volt range, so the ‘-20 dB’ push button should be left pushed out (OFF). If the button is in, a -20 dB attenuation is applied to the output, which reduces its amplitude to only a few millivolts.

When using the function generator, make sure the DUTY CYCLE, CMOS LEVEL, and DC OFFSET are all OFF. For AC calibration, set the waveform to the sine wave(∞). For RC circuit analysis, set the waveform to the square wave(□).

PROCEDURE

1. Calibrate the oscilloscope using AC input
 - A. Connect the output terminal of the function generator to the COM and V jacks of the multimeter.
 - B. Switch the MODE on the Oscilloscope to CH1.
 - C. Connect the BNC cable and probe to the channel 1 input of the oscilloscope. Attach the probe to the COM and V jacks of the multimeter, as shown in figure 14.8.

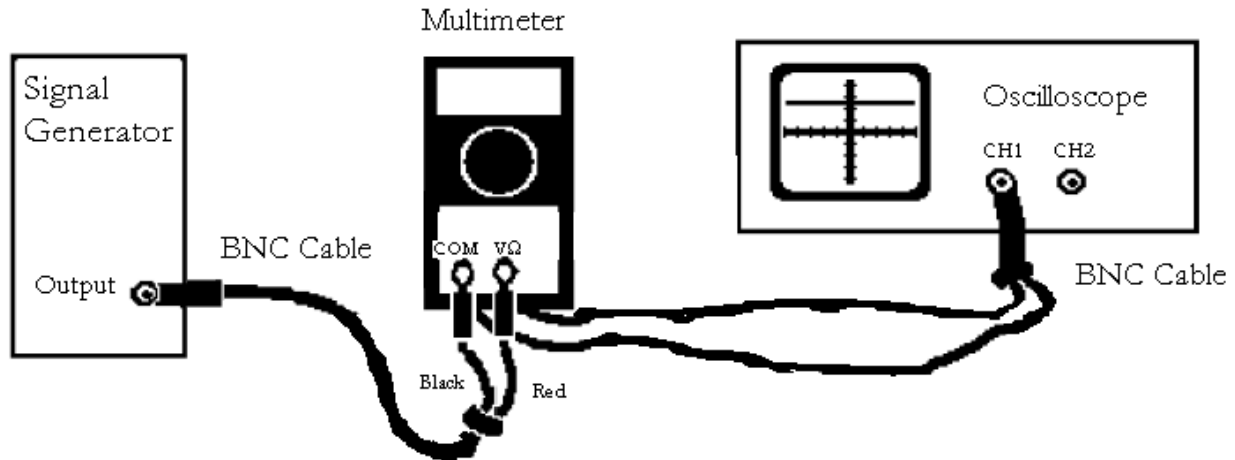


Figure 14.8 Wire connections for the AC calibration of the oscilloscope

- D. Switch the multimeter to the 20 volt AC scale.
- E. On the function generator, push the button for the sine waveform (\sim) and the button for the 500 Hz scale.
- F. Power on the function generator, and adjust the OUTPUT LEVEL knob until the voltmeter reads 1.0 V.
- G. Change the CH1 coupling switch on the oscilloscope to the GND position and adjust the CH1 Vertical POSITION Knob until the trace rests on the horizontal median line of the CRT display. If necessary, adjust the intensity and focus controls so the trace is easily visible, sharp but not overly bright.
- H. Change the CH1 coupling switch on the oscilloscope to the AC position. The frequency displayed in the LCD of the oscilloscope should be very close to the output frequency displayed in the digital display of the function generator. If they are very different, check all wire connections, and notify your lab instructor if the problem persists.
- I. Adjust the CH1 VOLTS/DIV knob on the oscilloscope until the entire wave form is visible in the CRT display. Record the vertical scale (voltage) for CH1 from the LCD of the oscilloscope in your data table for AC Calibration.
- J. Adjust the TIME/DIV knob on the oscilloscope until a clear display of 3 – 5 complete periods of the signal are visible in the CRT display. Record the horizontal scale (time) displayed in the LCD of the oscilloscope, which indicates the amount of time each cm represents on the CRT display.
- K. Measure and record to the nearest millimeter, the peak-to-peak amplitude and the period of the sinusoidal trace as it appears on the CRT display as shown in figure 14.9.

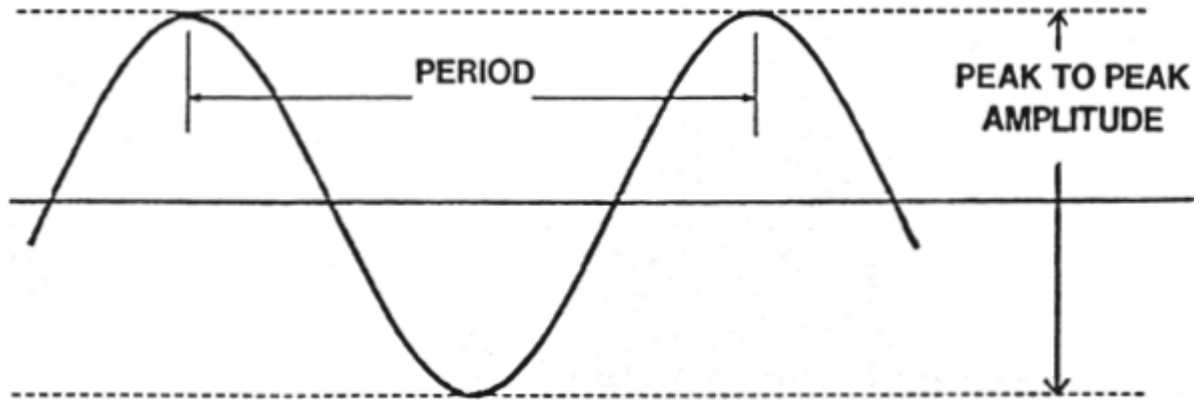


Figure 14.9 Measuring the period and peak-to-peak amplitude of a sinusoidal wave form

- L. Switch the TRIGGER SOURCE (FC) to CH2 and the Vertical MODE switch to CH2 on the Oscilloscope. Repeat steps G through K for the channel 2 input.
2. With calibrations complete, set the following settings on the oscilloscope in preparation for testing an RC circuit:
 - Switch the TRIGGER SOURCE (FC) to CH1
 - Switch the MODE to DUAL
3. Change the waveform on the function generator to square wave and the frequency range to 5000 Hz. Adjust the OUTPUT LEVEL knob until the voltmeter reads about 1.0 V.
4. Power off the oscilloscope. Connect the oscilloscope to an RC circuit with the oscilloscope's Channel 1 input connected to the function generator and Channel 2 input connected in parallel with the capacitor as shown in figure 14.6. You do not need a multimeter in this circuit.

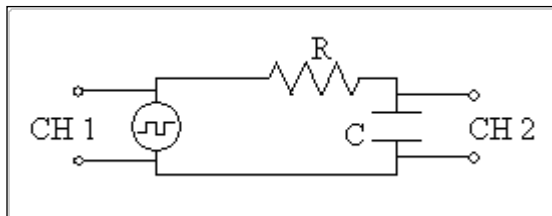


Figure 14.10 Circuit diagram of channel 1 and channel 2 inputs connections to the RC circuit

5. Measure the time constant 1τ for the charging capacitor by performing the following steps:
 - A. Set the variable resistor to 1000Ω .
 - B. Power on the oscilloscope. Adjust the TIME/DIV knob so that 3-5 complete periods of the cycle are visible. Record the horizontal scale from the LCD (ms/cm) in your data table on the 1000Ω row.
 - C. Adjust the VOLTS/DIV for CH2 so that the entire waveform across the capacitor is visible in the CRT display.
 - D. Adjust the vertical POSITION knob until the beginning of the ascending trace starts on the horizontal median line of the CRT display.
 - E. The voltage across the capacitor is displayed on the channel 2 trace. Adjust the horizontal POSITION knob until the channel 2 trace crosses the y-axis at 63% of its maximum value. The x-intercept of the trace is then 1τ as shown in figure 14.11.

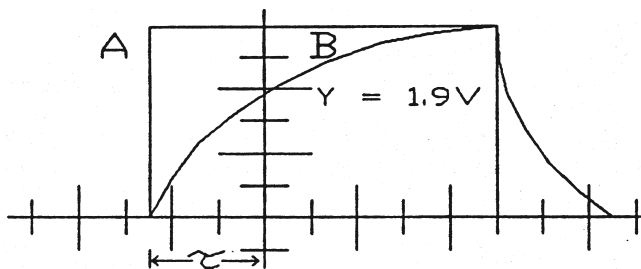


Figure 14.11 measuring the time constant 1τ for the charging capacitor wave

- F. Measure and record the time constant 1τ from the CRT display to the nearest millimeter and record it on your data table.
 - G. Multiply your result for τ by the horizontal scale to get τ in seconds and record it on the data table.
 - H. Repeat steps A through G for $R = 400\Omega$, $R = 100\Omega$ and for $R = 40\Omega$.
6. Measure the time constant 1τ for the discharging capacitor by performing the following steps:
- A. Set the variable resistor to 1000Ω .
 - B. Adjust the TIME/DIV knob so that 3-5 complete periods of the cycle are visible. Record the horizontal scale from the LCD (ms/cm) in your data table on the 1000Ω row.
 - C. Adjust the VOLTS/DIV for CH2 so that the entire waveform across the capacitor is visible in the CRT display.
 - D. Adjust the vertical POSITION knob until the end of the descending trace rests on the horizontal median line of the CRT display.
 - E. Adjust the horizontal POSITION knob until the channel 2 trace crosses the y-axis at approximately 37% of its maximum value. This fall in voltage occurs at 1τ as shown in figure 14.12.

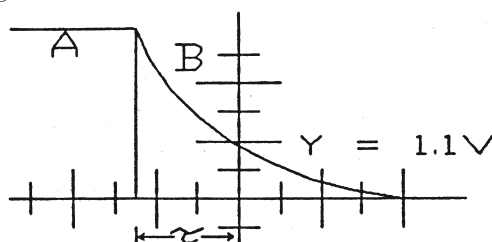


Figure 14.12 measuring the time constant 1τ for a discharging capacitor

- F. Measure and record the time constant 1τ from the CRT display to the nearest millimeter and record it on your data table.
- G. Multiply your result for τ by the horizontal scale to get τ in seconds and record it on the data table.
- H. Repeat steps A through G for $R = 400\Omega$, $R = 100\Omega$ and for $R = 40\Omega$.

DATA

AC Calibration

CH1 Peak-to-Peak Amplitude (cm) = _____

CH2 Peak-to-Peak Amplitude (cm) = _____

Period (cm) = _____

Horizontal Scale (ms/cm) = _____

CH1 Vertical Scale (V/cm) = _____

CH2 Vertical Scale (V/cm) = _____

Charging Capacitor

Resistance, R (Ω)	Horizontal Scale (ms/cm)	1τ (cm)	1τ (sec)
1000			
400			
100			
40			

Discharging Capacitor

Resistance, R (Ω)	Horizontal Scale (ms/cm)	1τ (cm)	1τ (sec)
1000			
400			
100			
40			

CALCULATION AND ANALYSIS

1. During the AC calibration, you set the power source to a root mean square (rms) voltage of 1.0 volts. This corresponds to a peak voltage V_p of $1.0\sqrt{2}$ or 1.41 volts, and a peak-to-peak voltage of twice V_p or 2.82 volts. Multiply your measured peak-to-peak amplitude (in cm) by the vertical scale to find your measured peak-to-peak voltage. Compare your measured value with the expected value, 2.82 volts. Does your measurement agree with the expected value within the uncertainty due to the oscilloscope scale reading accuracy alone?
2. For each of your time constant measurements, compare the experimental value with the expected value given by the product RC.
3. Can the difference between the experimental and expected results be explained by the combined effects of reading accuracy and the $\pm 5\%$ uncertainty of the time scale calibration?
4. For each value of RC, do you find that the time constant for the charging and discharging capacitor is the same?

Experiment 15: Force on a Current-Carrying Conductor in a Uniform Magnetic Field

OBJECTIVES

The force on a current carrying conductor in a uniform magnetic field depends on the length of the conductor and on the current flowing through it. To test this mathematical relationship, a good experiment must isolate each contributing component and vary it independently of the others. In this experiment, you measure the force on a conductor while varying only the length or only the current. You also measure the current required to produce a constant force on the conductor while varying its length. The objectives of this experiment are as follows:

1. To build a circuit in which you can vary the current and length of the wire
2. To measure the force on a current carrying wire in a magnetic field
3. To calculate the strength of the magnetic field for a permanent magnet

THEORY

If a current is sent through a conductor in a magnetic field there is a force on the conductor which is perpendicular to both the conductor and the direction of the field, shown in figure 15.1.

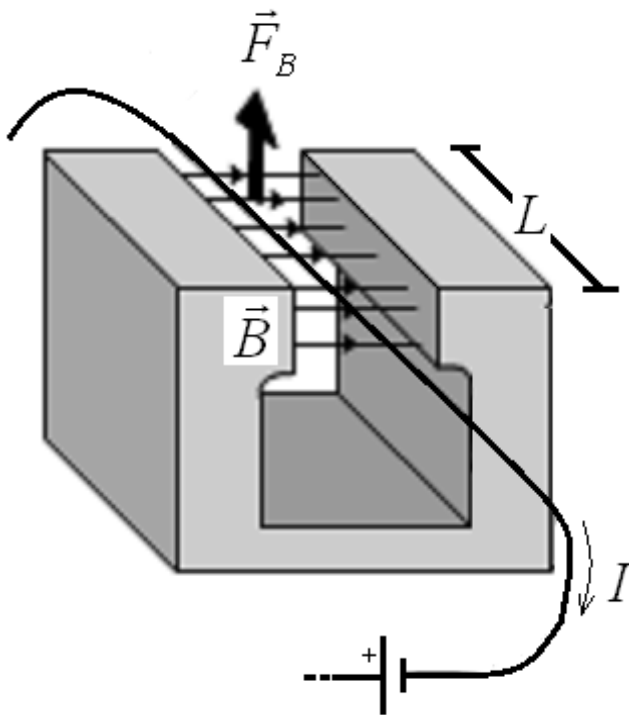


Figure 15.1 Magnetic force on a current-carrying wire

If the magnetic field is uniform and perpendicular to the conductor, the magnitude of this force is the product of the current on the wire, the length of the wire in the magnetic field, and the magnetic field, as shown in equation 15.1.

$$\text{Magnetic Force, Magnitude} \quad F_B = BIL \quad (15.1)$$

Here,

- B is the strength of the magnetic field in teslas. This is constant for a permanent magnet.
- L is the length of the conductor in meters
- I is the current moving through the conductor in amperes
- F_B is the magnitude of the magnetic force in newtons

In this experiment, the magnetic field is supplied by an Alnico permanent magnet. In permanent magnets, the magnetic field is constant and does not change throughout the experiment.

Opposing the magnetic force F_B on the conductor is the gravitational force F_G . By Newton's second law, the sum of these forces is zero when the conductor is stationary. By Newton's third law, there is a force on the magnet equal but opposite to the force on the conductor. The magnet is placed on the pan of a balance and you measure the force on the magnet rather than the equal and opposite force on the conductor.

You test the validity of equation 15.1 in three ways. The magnetic field of the permanent magnet B cannot be varied in this experiment.

- Part A: You determine the magnetic force F_B as a function of I when L and B are constant.
- Part B: You determine the magnetic force F_B as a function of L when I and B are constant.
- Part C: You determine the current I as a function of L when F_B and B are constant.

ACCEPTED VALUES

There is no accepted value for the strength of the magnetic field B generated by the magnet because all the magnets are different for each laboratory setup. You may compare the different values of B you calculate for each part and calculate variance and uncertainty, but you cannot calculate percent error.

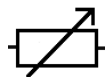
APPARATUS

- multimeter
- linear rheostats
- switch
- DC power supply
- magnetic force apparatus

In the diagram for the circuit you construct for this experiment, there are several specific components denoted by the following symbols:



Switches interrupt the current flowing in a conductor by physically interrupting the current.



Rheostats are resistors for which the resistance can be varied on a sliding scale.

\vec{B} Magnets through which a circuit passes have no standard symbol in circuit diagrams. Parallel lines above and below the conductor indicate where the wire passes through the magnetic field.

The circuit diagram for the magnetic force apparatus is shown in figure 15.2.

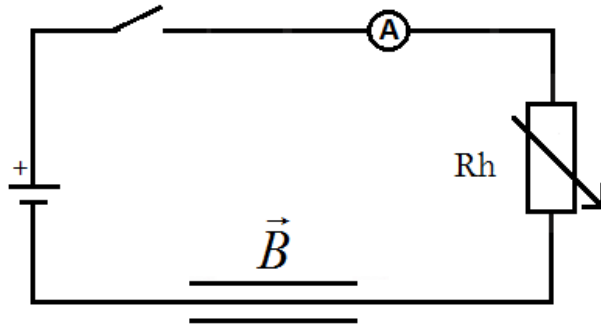


Figure 15.2 The circuit diagram for a circuit in which you can vary current through a magnetic field

The multimeter in this experiment is used as an ammeter to measure I . In part A, you vary I by changing the resistance on the circuit using two rheostats. Moving the sliders up and down the rheostats changes the resistance they apply to the circuit and by Ohm's law, this changes the current.

The magnetic force apparatus is equipped with a three-beam balance, which measures the gravitational force on the permanent magnet. You use the three-beam balance in a similar way to the platform balance by varying the position of the sliders on the beams until the pointer to the right of the beams aligns with fixed scale mark, as shown in figure 15.3.

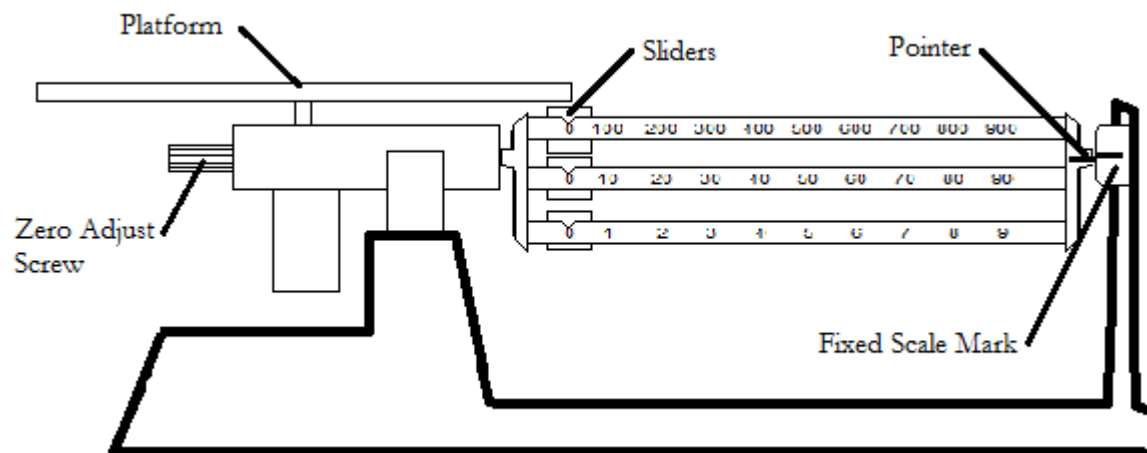


Figure 15.3 Measuring weight with the three-beam balance

PROCEDURE

1. Connect the circuit with the switch, multimeter, rheostats, and wire through the magnetic force apparatus in series. **DO NOT CONNECT TO THE POWER SUPPLY UNTIL THE CIRCUIT HAS BEEN CHECKED BY YOUR INSTRUCTOR.**
2. While the switch is open, adjust the balance for equilibrium using only the beam slider weights. When this balance is reached, be sure that the wire is parallel to the long side of the magnet and slightly below the top of the air gap. Record the balance measurement as the initial mass m_0 on the data sheet.

PART A: FORCE GENERATED BY VARYING CURRENT

3. Adjust the clips on the wire through the magnetic force apparatus so that the current is supplied to the entire length L of the wire. The entire length of the wire L is 0.152 meter. Record this length as the initial length L_0 on the data sheet for Part A.
4. Move the slider to the middle of each rheostat and close the switch. If this causes a repulsive force between the magnet and the wire, and the magnet moves downward, proceed to step 5. If the magnet and wire attract each other and the magnet lifts, open the switch, switch the clips connected the wire passing through the magnetic force apparatus, close the switch, and proceed to step 5.

CAUTION: To increase the current in the circuit, first decrease the resistance of the rheostat with the larger resistance. Do not reduce the resistance of the other rheostat until the first rheostat has been reduced to zero resistance.

5. Adjust the rheostats until the multimeter displays a current of 1.00 ampere and adjust the balance for equilibrium. Record the balance measurement in the mass (g) column in the 1.00 amperes row in the data table for Part A.
6. Repeat step 5 for currents of 2.00, 3.00, 4.00, 5.00, 6.00, and 7.00 amperes.

PART B: FORCE GENERATED BY VARYING LENGTH

7. Change the clips so that the current is supplied to $5/6L$ and measure the balance equilibrium when the current is 5.00 amperes. Record the balance measurement in the mass (g) column in the 0.1267 meter row in the data table for Part B. Record 5.00 amperes as the value for the initial current I_0 .
8. Repeat step 7 for lengths $4/6L$, $3/6L$, $2/6L$, and $1/6L$ with a constant current of 5.00 amperes in each case. When changing the length, use the connections such that the length of wire through which the current is flowing is in the center of the magnetic field.

PART C: EQUILIBRIUM CURRENT VERSUS LENGTH

9. Set the balance reading at a value 2.00 grams greater than the equilibrium value you measured in step 2. The goal is to balance the resulting 2 grams of additional gravitational force against the magnetic force. Multiply $2g$ by $g=9.80 \text{ m/s}^2$ and record this as the constant gravitational force F_i on the data sheet for Part C.
10. Adjust the clips so that current is supplied to the entire length of the wire and adjust the current until the balance reaches equilibrium. Record the current measurement from the multimeter in the Current (A) column in the 0.1520 meter row in the data table for Part C.
11. Repeat step 10 for lengths $5/6L$, $4/6L$, $3/6L$, $2/6L$, and $1/6L$.

DATA

$$m_0 \text{ (g)} = \underline{\hspace{2cm}}$$

Part A: Force generated by varying current

$$L_0 \text{ (m)} = \underline{\hspace{2cm}}$$

Current (A)	Mass (g)
1.00	
2.00	
3.00	
4.00	
5.00	
6.00	
7.00	

Part B: Force generated by varying length

$$I_0 \text{ (A)} = \underline{\hspace{2cm}}$$

length (m)	Mass (g)
0.1267	
0.1013	
0.0760	
0.0507	
0.0253	

Part C: Equilibrium current versus length F_i (Newtons) = _____

length (m)	Current(A)
0.1520	
0.1267	
0.1013	
0.0760	
0.0507	
0.0253	

CALCULATION AND ANALYSIS

1. Calculate the difference in force ΔF between the initial force and the measured force for each row in the data table for Part A. You should express ΔF in Newtons. It is given by the formula $\Delta F = (m - m_0)g$ where $g=9.80 \text{ m/s}^2$.
2. Plot a graph of the change in force ΔF (in Newtons) against current I (in Amperes) using your data from Part A. From the slope of the graph and the initial length L_0 , calculate the strength of the magnetic field B .
3. Calculate the difference in force ΔF in grams between the initial force and the measured force for each row in the data table for Part B. Again it is given by the formula $\Delta F = (m - m_0)g$ where $g=9.80 \text{ m/s}^2$.
4. Plot a graph of the change in force ΔF (in Newtons) against length L (in meters) using your data from Part B. From the slope of this graph and the initial current I_0 , calculate the strength of the magnetic field B .
5. Calculate the average value of $I \times L$ (ampere meters) using the data from the data table for Part C. From this average value and the constant gravitational force F_i , calculate the strength of the magnetic field B .
6. Calculate the average value (mean) of your three measurements of B .
7. Calculate the uncertainty in your average value for B using equation 0.6.

Experiment 16: The Specific Charge of the Electron

OBJECTIVES

The purpose of this experiment is to observe the trajectory of moving charges (electrons) in a uniform magnetic field and to calculate the charge-to-mass ratio of an electron. In this experiment a beam of electrons travels through a uniform magnetic field produced by a pair of Helmholtz coils. The magnetic field bends the beam into circular path, and by measuring the radius of the circle you can calculate the charge-to-mass ratio of a single electron. The objectives of this experiment are as follows:

1. To measure the radius of a circular electron beam in a magnetic field
2. To calculate the apparatus constant for a pair of Helmholtz coils
3. To calculate the ratio of charge to mass for an individual electron

THEORY

In this experiment two devices are used. The electrons are emitted by a heated cathode, also known as an electron gun, into a glass bulb filled with low pressure Hydrogen gas. Two identical coils of wire separated by a distance equal to their radius, called Helmholtz coils, produce a uniform magnetic field through the gas-filled bulb as shown in figure 16.1.

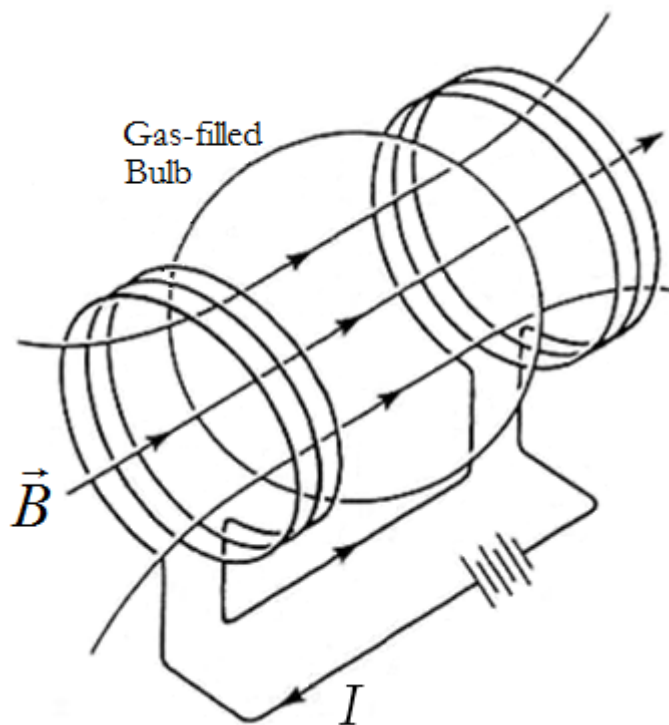


Figure 16.1 A magnetic field produced by Helmholtz coils through a gas-filled bulb

Electrons emitted by a heated cathode are accelerated in an electric field. They acquire kinetic energy in accord with equation 16.1.

$$\text{Kinetic energy (electrons)} \quad \frac{1}{2}mv^2 = eV \quad (16.1)$$

Here, m is the mass of the electron, v is magnitude of the velocity of the electron, e is the charge of the electron, and V is the anode potential/Voltage. The electrons are assumed to have an initial velocity of zero as they emerge through an opening in the anode. The tube can be turned so that the electron beam is directed at right angles to the magnetic field, which bends the beam into a circular path. The electrons gain no energy from the magnetic field because the magnetic force acts at right angles to the path of the electrons. Therefore, they move with constant speed v . The magnetic deflecting force is shown in equation 16.2.

$$\text{Magnetic Deflecting Force} \quad F_B = Bev \quad (16.2)$$

Here B is the strength of the magnetic field. Because this force acts radially, it is also a centripetal force, which is equivalent to the product of the mass and the velocity squared divided by the radius of the circular path, as shown in equation 16.3.

$$\text{Centripetal Force} \quad F_C = Bev = \frac{mv^2}{r} \quad (16.3)$$

Here r is the radius of the circular path the electrons follow. Solving equation 16.3 for r yields equation 16.4.

$$\text{radius} \quad r = \frac{mv}{Be} \quad (16.4)$$

To eliminate v from equation 16.4, square equation 16.4 and substitute the quantity of v^2 from equation 16.1. After some algebra, the final value for the ratio of charge to mass of the electron e/m is shown in equation 16.5.

$$\text{Ratio of charge to mass} \quad \frac{e}{m} = \frac{2V}{r^2 B^2} \quad (16.5)$$

The magnetic field produced by a pair of Helmholtz coils is a function of the current I passing through them. A single Helmholtz coil of radius R produces the magnetic field along the axis of symmetry at distance x from the center, as shown in equation 16.6.

$$\text{Magnetic field (one coil)} \quad B = \frac{\mu_0}{2} \frac{NR^2}{(R^2 + x^2)^{3/2}} I \quad (16.6)$$

Here, N is the number of turns in the coil and μ_0 is the magnetic permeability of a vacuum ($4\pi \times 10^{-7}$ Tm/A). The pair of Helmholtz coils in the apparatus are separated by a distance R from each other, so halfway between them $x=R/2$. At this distance, the magnetic field in the region between the coils is nearly uniform. Substituting $x=R/2$ into equation 16.6 and doubling the result for the pair of coils, the apparatus constant is calculated using equation 16.7.

$$\text{Apparatus Constant} \quad B = CI \rightarrow C = \frac{\mu_0}{(5/4)^{3/2}} \frac{N}{R} \quad (16.7)$$

From equations 16.5 and 16.7, e/m is the calculated quantity in equation 16.8.

$$\text{Ratio of charge to mass} \quad \frac{e}{m} = \frac{2V}{r^2 I^2 C^2} \quad (16.8)$$

ACCEPTED VALUES

The accepted value for the ratio of charge to mass of an electron to three significant figures is 1.756×10^{11} Coulombs/kilogram. The manufacturer claims the voltmeter and ammeter are accurate to within $\pm 2.5\%$.

APPARATUS

- specific charge apparatus
- transparent half-meter stick

The bulb contains hydrogen gas at a pressure of about 10 mm Hg. As the electron beam passes through the gas it leaves a glowing track. The bulb is placed between two coils of wire which are separated by a distance equal to their radius. The apparatus is self-contained, as shown in figure 16.2.

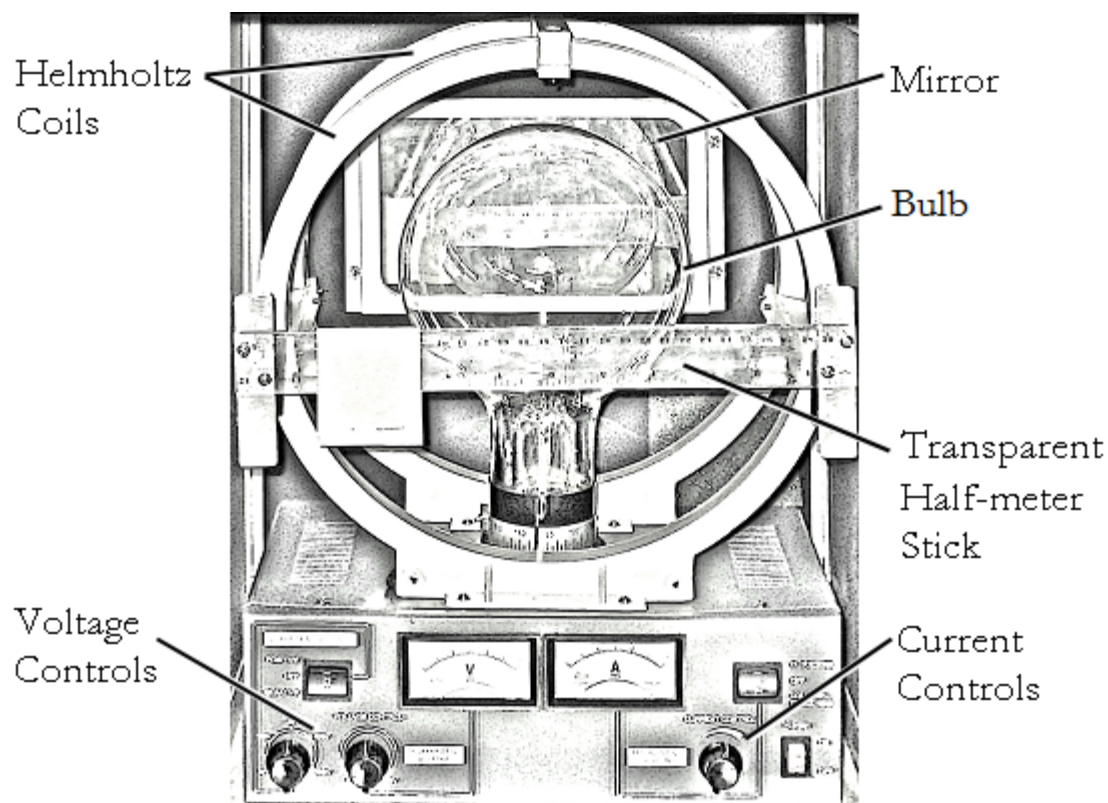


Figure 16.2 The specific charge apparatus

This particular arrangement of coils, known as Helmholtz coils, produces a very uniform magnetic field in the space between the coils. To measure the diameter of the electron beam, measure each edge of the beam individually using the transparent half-meter stick on the front of the apparatus. By positioning your eyes level with the half-meter stick, align the edge of the electron beam with the edge of the reflection of the electron beam in the mirror behind the CRT, as shown in figure 16.3.

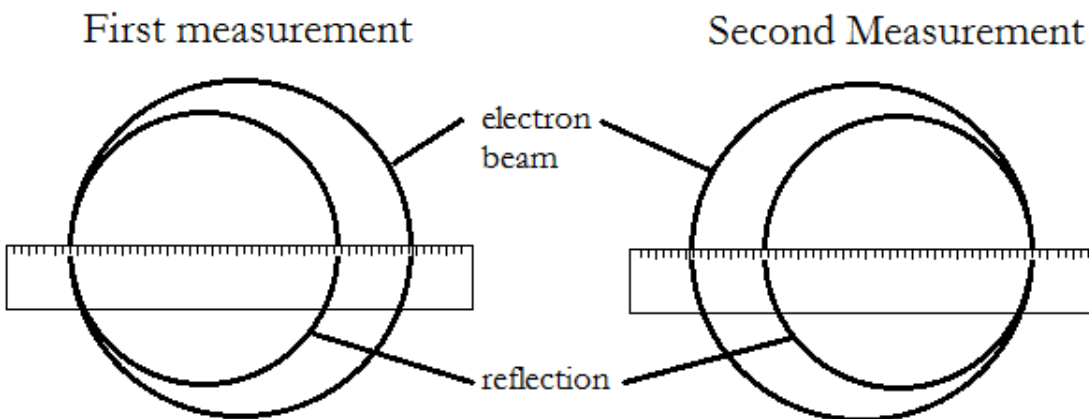


Figure 16.3 Aligning the electron beam and reflection to measure the beam diameter.

The difference between the readings on the left and right is the diameter of the circular path of the electron beam $2r$.

PROCEDURE

1. Set the anode voltage to 150 V and the Helmholtz coil current to 1 A.
2. Increase the filament current slowly until the electron beam appears. If the beam is not clearly visible, increase the filament current slightly. If the electron beam is helical, rotate the tube until it is circular. If the beam does not appear, reverse the connections to the Helmholtz coils.
3. Measure the diameter of the path of the electrons using the procedure described in Figure 16.3. Record your result in the data table.
4. Repeat step 3 with:
 - Helmholtz coil current = 1.5 amperes, anode voltage $V = 150$ volts
 - Helmholtz coil current = 1.0 amperes, anode voltage $V = 200$ volts
 - Helmholtz coil current = 1.5 amperes, anode voltage $V = 200$ volts

DATA

anode voltage, V (V)	Helmholtz coil current, I (A)	beam diameter, $2r$ (m)
150	1.0	
150	1.5	
200	1.0	
200	1.5	

CALCULATION AND ANALYSIS

1. Calculate the value of C in equation 16.7 for the apparatus in this experiment. For this apparatus $R=0.14$ m and $N = 140$.
2. Calculate e/m from each row in the data table using equation 16.8.
3. Calculate the average value of e/m , the uncertainty in the average value, and the % uncertainty in the average value using equations 0.1, 0.6, 0.7.
4. Calculate the % error in your average value of e/m . Do your measurements support the accepted value to within the uncertainty?
5. What effect would you expect Coulomb electrical forces between electrons to have on the beam? Explain.

Experiment 17: Refraction

OBJECTIVES

The transmission of light across a boundary between two media is accompanied by a change in both the speed and wavelength of the wave. This can result in a change of direction at the boundary, a phenomenon known as refraction. In this experiment you measure the change in direction of light beams as they refract or reflect at a boundary to determine the index of refraction of a transparent object. The objectives of this experiment are as follows:

1. To measure the angles of incidence and refraction at a boundary between media
2. To observe total internal reflection at a boundary between media
3. To calculate the critical angle of a boundary between media

THEORY

The index of refraction is a property of transparent substances that has been independently discovered several times, but is attributed to Willebrord Snellius whose name is associated with the law (you can't make this stuff up). Mathematically, Snell's law describes the relationship between the angle of incidence of a beam of light as it intersects a new transparent medium and the angle of refraction as enters that transparent medium.

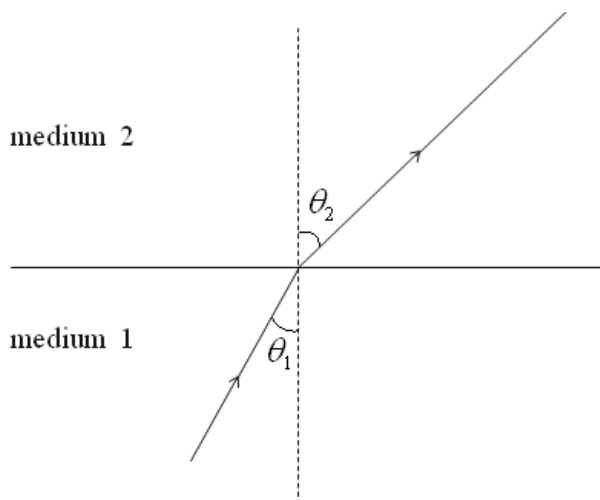


Figure 17.1 angles of incidence for a light as it intersects a new medium

Snell's law quantifies the relationship as shown in equation 17.1.

$$\text{Snell's Law} \quad n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad (17.1)$$

Here,

- n_1 is the index of refraction of the initial medium
- n_2 is the index of refraction of the new medium
- θ_1 is the angle of incidence
- θ_2 is the angle of refraction

The index of refraction of a medium is the ratio of the speed of light in a vacuum to the speed of light in the medium, as shown in equation 17.2.

$$\text{Index of Refraction} \quad n = \frac{c}{v} \quad (17.2)$$

Here $c = 3 \times 10^8$ m/s, the speed of light in a vacuum, and v is the speed of light in the medium. For air to a very good approximation $n = 1$.

The critical angle is the angle of incidence at which light begins to reflect, rather than refract, when it enters a new medium that has a lower index of refraction. This happens when the angle of refraction reaches 90° . Substituting $\theta_2 = 90^\circ$ in equation 17.1 and solving for θ_1 calculates the critical angle as shown in equation 17.3.

$$\text{Critical Angle} \quad \theta_1 = \sin^{-1}\left(\frac{n_2}{n_1}\right) \quad (17.3)$$

A prism is a transparent medium composed of two non-parallel polished surfaces. When light travels through a prism such that it enters one side and exits another, the total deviation δ , in the ray leaving the second side from the original angle of incidence is dependent on the index of refraction, n , and the angle between the two sides of the prism α , as shown in figure 17.2.

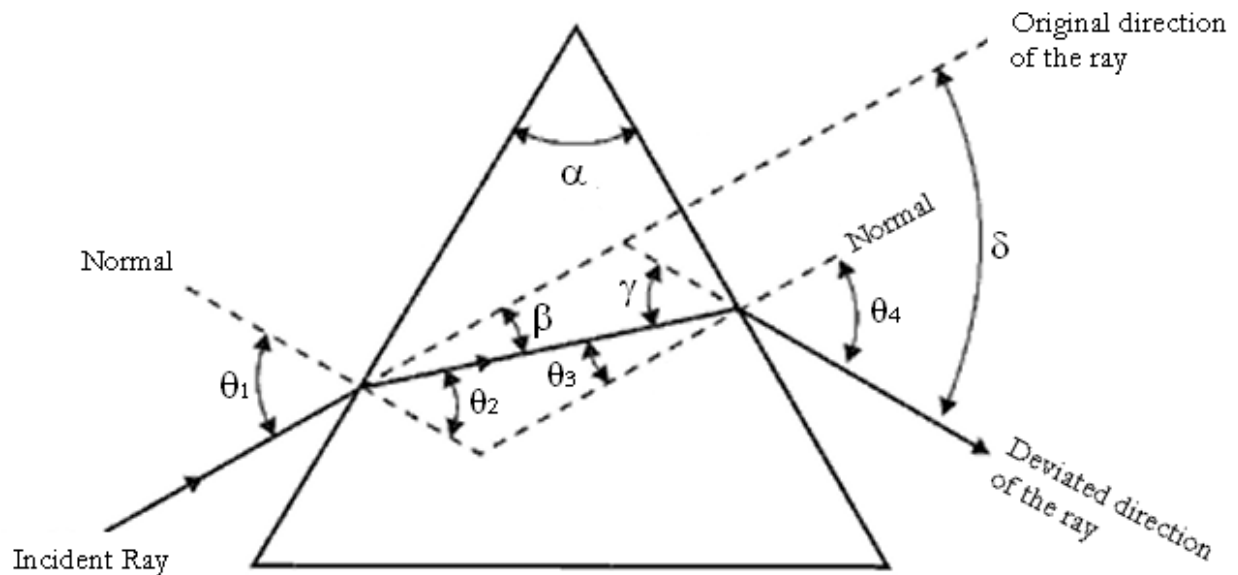


Figure 17.2 Total deviation of a light ray through a prism

The value of δ depends on the angle of incidence of the light, θ_i , at the first face of the prism. When the light passes symmetrically through the prism so that the angle of refraction at the second face is equal to angle of incidence at the first face, $\theta_i = \theta_r$, then δ has a minimum value δ_m . If the light enters the prism from a near vacuum medium, for example air with an index of refraction of 1.00, then the index of refraction is calculable in terms of the angle between the two sides of the prism α and the total deviation δ , as shown in equation 17.4.

$$\text{Index of Refraction} \quad n = \frac{\sin \frac{\delta m + \alpha}{2}}{\sin \frac{\alpha}{2}} \quad (17.4)$$

ACCEPTED VALUES

The accepted value for the index of refraction for Lucite is 1.50.

APPARATUS

- Straight filament lamp
- ruler
- protractor
- Lucite prism

PROCEDURE

1. Place a sheet of blank paper on your desk. Arrange the light source and prism so that a narrow beam of light is incident on the prism as shown in figure 17.3.

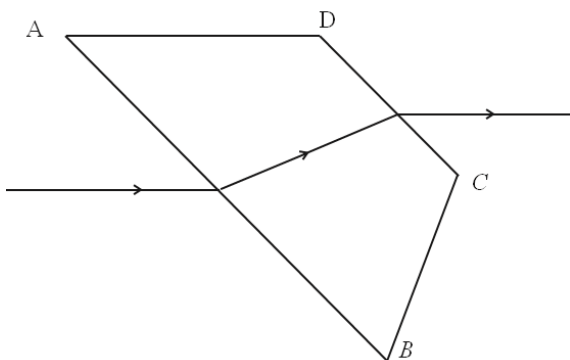


Figure 17.3 Trace of a light ray refracting at two surfaces.

2. Adjust the prism so that the angle of incidence on the first surface is between 45° and 60° . Using two dots made with a sharp pencil to determine the direction of a straight line, trace the outline of the prism and the ray of light onto the paper.
3. Repeat steps 1 and 2 with a second surface with an angle of incidence greater than 60° .
4. Arrange the prism and the beam of light as in figure 17.4 so that the angle of refraction at the second surface is 90° . Trace, as in step 2, the outline of the prism and the ray of light.

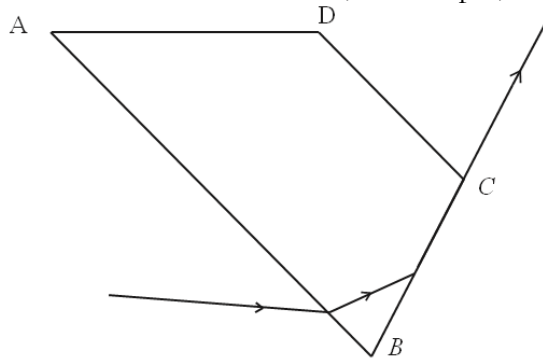


Figure 17.4 Trace of a light ray refracting and then reflecting internally

5. Arrange the prism and the beam of light as in figure 17.5. Rotate the prism until the angle of deviation δ is a minimum and then trace the outline of the prism and the beam of light. Repeat, using the other angle of the prism which is less than 90° as the refracting angle.

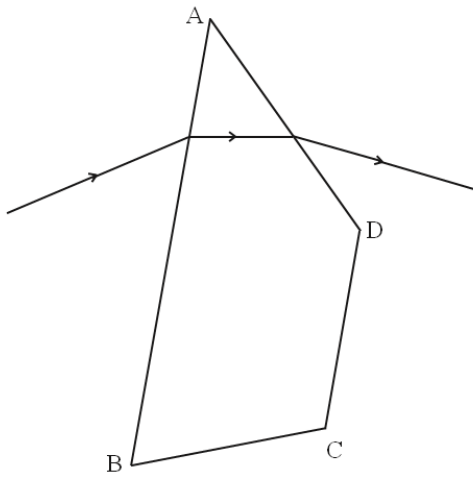


Figure 17.5 Trace of a light ray through the angle of minimum deviation

DATA

The data sheet for this lab is the blank sheet of paper with tracings of the light rays and prism. From this sheet, you measure angles using a protractor, arranged in a data table like the following.

Measurement	Value (in degrees)
Angle of incidence (first surface)	
Angle of refraction (first surface)	
Angle of incidence (second surface)	
Angle of refraction (second surface)	
Critical Angle	
Angle of minimum deviation (narrow end)	
Angle of prism (narrow end)	
Angle of minimum deviation (wide end)	
Angle of prism (wide end)	

CALCULATION AND ANALYSIS

1. Measure the angles of incidence and refraction at both surfaces of the prism in the tracings of procedures step 2 and 3. Calculate the index of refraction for the Lucite prism from these measurements.
2. Measure the critical angle from the tracing of procedure step 4. Calculate the index of refraction for the Lucite prism from the critical angle.
3. Measure the angle of minimum deviation δ_m and the angle of the prism α from each tracing of procedure step 5. Calculate the index of refraction for the Lucite prism from these angles.
4. Find the average (mean) value for the index of refraction of the prism.
5. Calculate the velocity of light in the prism.

Experiment 18: Mirrors and Lenses

OBJECTIVES

Lenses and mirrors bend light in a predictable fashion. In this experiment you'll study concave mirrors and converging and diverging lenses and see what types of images they form. The objectives of this experiment are as follows:

1. To study the imaging properties of mirrors and lenses.
2. To measure the image distances and heights produced by various arrangements of mirrors and lenses.
3. To calculate focal lengths and magnifications.

THEORY

When you measure all distances from the mirror or lens, the object (source) distance is d_o , the image distance is d_i , and the focal length is f . The focal point is the position of the image for incident rays parallel to the axis. For mirrors and lenses the equation relating these distances is given by equation 18.1.

$$\text{Mirror and Lens Equation} \quad \frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o} \quad (18.1)$$

The sign of f is specific to each type of mirror or lens. For concave mirrors and converging lenses, f is positive. For convex mirrors and diverging lenses, f is negative.

An image is said to be inverted if its orientation is opposite to that of the object. Otherwise it is said to be upright. The magnification of the image is given by equation 18.2.

$$\text{Magnification} \quad m = \frac{h_i}{h_o} = -\frac{d_i}{d_o} \quad (18.2)$$

Real images actually have light passing through them focused by the mirror or lens from the source; otherwise the image is virtual. Review these topics from your textbook.

ACCEPTED VALUES

The accepted value for the focal length of a converging lens is the image distance when the object is very far away. This is the only accepted value needed in this lab.

APPARATUS

- optical bench
- illuminated object
- screens
- converging and diverging lenses
- concave mirror
- plane mirror
- half meter stick

The optical bench aligns mirrors, lenses, and screens along a fixed one-dimensional axis. The illuminated object must be aligned along this axis to produce measurable images.

CONCAVE MIRROR

For a concave spherical mirror the focal distance f , measured from the mirror, is half the radius of curvature of the sphere from which the mirror is made. A real inverted image forms at a distance d_i from the mirror when the object is at a distance d_o . Light rays reflected by a concave mirror are shown in figure 18.1, first for the case $d_o = d_i = 2f$, then for the case $d_o > 2f$.

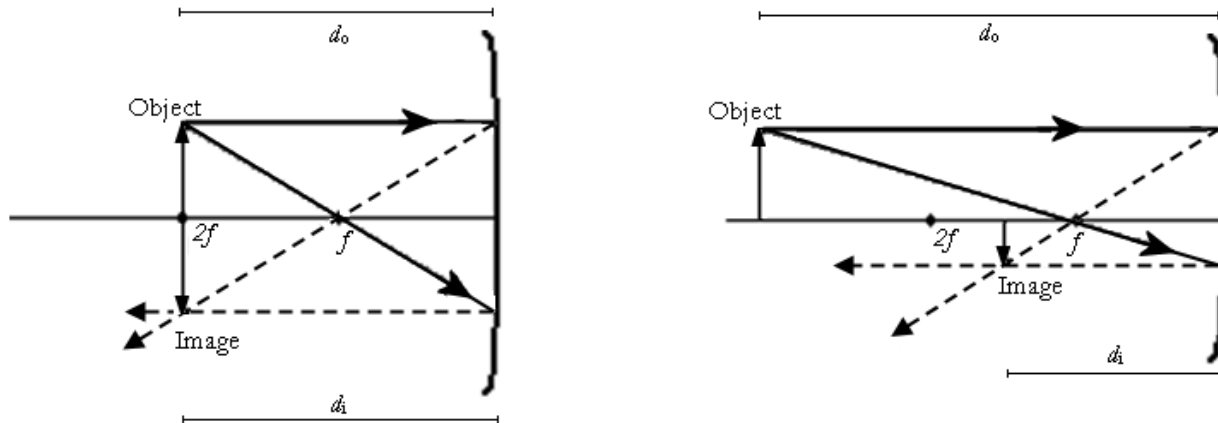


Figure 18.1 light rays reflected by concave mirrors

CONVERGING LENS

Converging lenses, which are thicker at the center than at the edges, focus incident light rays to form a real inverted image on the opposite side of the lens from the object when the object distance d_o is greater than f . When d_o is less than f , a virtual upright image forms on the same side of the lens as the object, as shown in figure 18.2, and the image distance d_i is negative

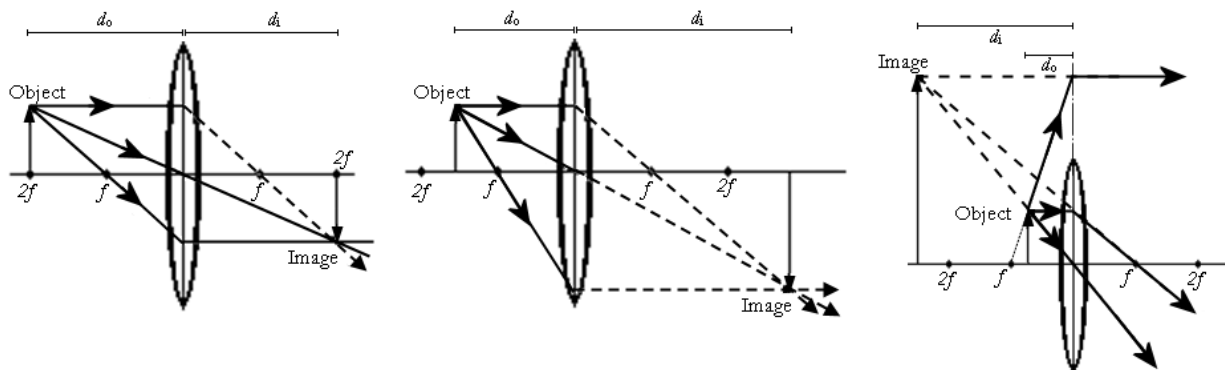


Figure 18.2 light rays refracted through converging lenses

DIVERGING LENS

Diverging lenses, which are thinner in the center than at the edges, spread incident light rays as they pass through the lens. These rays appear to diverge from a point on the same side of the lens as the object. For these lenses, the image is virtual and the focal length and the image distance are negative. These virtual images are upright and on the same side of the lens as the object.

Because diverging lenses make virtual images it's not so easy to measure their focal length. One way to do it is with a converging lens. Suppose a converging lens is used to make an image of an object on a screen. Now suppose a diverging lens is placed between the converging lens and the screen. If the distance from the diverging lens to the screen is exactly the focal length of the diverging lens, then after going through the diverging lens the light rays are parallel to the axis. If a mirror is placed in front of the screen it will reflect these parallel rays back and form an image at the location of the original object. This is shown in figure 18.3, where we've moved the mirror away from the focal point for clarity.

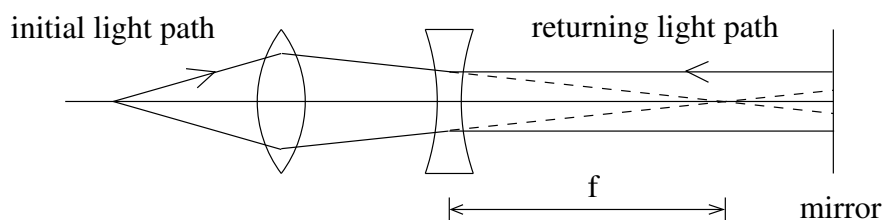


Figure 18.3 light rays refracted through diverging and converging lenses

With such a system we can determine the focal length of a diverging lens: it is the distance from the diverging lens to the screen, when everything is arranged to form an image at the position of the original object.

PROCEDURE

For each part of the experiment, draw a graph with a horizontal line representing the optical axis perpendicular to the lens or mirror. Record the position of each component of the system in the data tables. The distance for any object, image, or focal point is always measured from the lens or mirror.

PART A: CONCAVE MIRROR

1. Arrange the illuminated object and the concave mirror so that a clear image forms at the same position as the object. For this arrangement the image and object distances are the same.
2. Measure the object distance, image distance, object height, image height, and image orientation and record them in the data table for Part A.
3. Arrange the illuminated object, half-screen and concave mirror so that a real image is formed with the object distance approximately twice the image distance.
4. Measure the object distance, image distance, object height, image height, and image orientation and record them in the data table for Part A.

PART B: CONVERGING LENS

5. Arrange the illuminated object, converging lens and screen so that a real image is formed with the object distance approximately equal to the image distance.
6. Measure the object distance, image distance, object height, image height, and image orientation and record them in the data table for Part B.
7. Arrange the object, lens, and screens so that the real image is formed with the object distance approximately twice the image distance ($d_o \approx 2d_i$).
8. Measure the object distance, image distance, object height, image height, and image orientation and record them in the data table for Part B.

9. Keeping the object and screen positions the same as in part 7, move the lens toward the object until an image is again formed on the screen.
10. Measure the object distance, image distance, object height, image height, and image orientation and record them in the data table for Part B.
11. Choose a distant object, such as a building far from the laboratory or a window in a separate wing of Gillet Hall, and form an image of a distant object with the converging lens and the screen. The image distance is equal to the focal length of the lens, because a very distant object is produced by rays that are essentially parallel.
12. Measure the image distance and image orientation, and record them in the data table for Part B.

PART C: DIVERGING LENS

13. Adjust the illuminated object, screen, and converging lens so that an image is formed on the screen approximately 30 cm from the lens. Place the diverging lens between the converging lens and the screen, and place a mirror in front of the screen. Move the diverging lens until an image forms at the position of the object.
14. Measure the distance from the diverging lens to the screen as the focal length in the data table for Part C.

DATA

Part A: Concave Mirror

Arrangement	Object Distance (cm)	Image Distance (cm)	Object Height (cm)	Image Height (cm)	Image Orientation
Projected back to object distance					
Projected back to 2x object distance					

Part B: Converging Lens

Arrangement	Object Distance (cm)	Image Distance (cm)	Object Height (cm)	Image Height (cm)	Image Orientation
image and object distance ~ the same					
object at ~ 2x image distance					
Reformed image after moving lens toward the object					
Distant object	∞				

Part C: Diverging Lens

Arrangement	Focal Length
diverging lens between converging lens and screen, image reflected off a mirror onto the plane of the object	

CALCULATION AND ANALYSIS

1. Calculate the focal length of the concave mirror twice, using the data from the two rows of the data table for part A.
2. Calculate the magnification of the concave mirror for each object distance analyzed.
3. Calculate the focal length of the converging lens three times, using the data from the first three rows of the data table for part B.
4. Use the image distance for a distant object as the accepted value for the focal length of a converging lens. Calculate the % error for the focal lengths you obtained in step 4.
5. Report the focal length of the diverging lens.

Experiment 19: The Grating Spectrometer

OBJECTIVES

A diffraction grating is a way to measure the wave properties of light. The grating produces a diffraction pattern that depends on the wavelength of the light that passes through it. In this experiment you observe the diffraction pattern produced by light from excited Mercury atoms. This allows you to determine the emission spectrum of Mercury. The objectives of this experiment are as follows:

1. To observe the emission spectrum of Mercury using a diffraction grating
2. To measure the angles to various lines in the diffraction pattern
3. To calculate some of the wavelengths of light emitted by Mercury

THEORY

A diffraction grating consists of a surface with regularly-spaced parallel slits through which light can pass. Light passing through different slits travels different distances from the grating to the screen (or spectrometer telescope), as shown in figure 19.1.

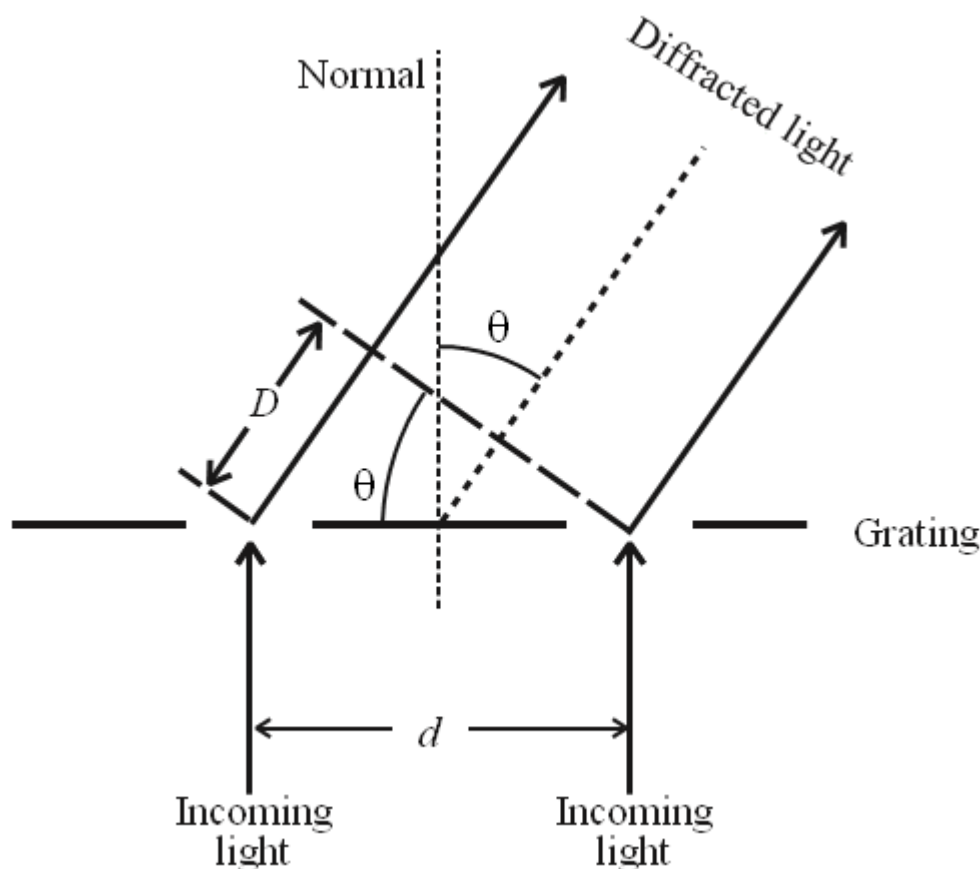


Figure 19.1 Diffraction angles for light passing through a diffraction grating

When the light reaches the screen there can be constructive interference, which produces a bright line, or destructive interference, in which case no light is observed.

Consider two slits with a separation distance d . The light passing through the slits is observed at an angle θ with respect to the normal to the diffraction grating. As shown in figure 19.1, the ray to the left travels an extra distance D . D is calculable using trigonometry, as shown in equation 19.1.

$$\text{Extra distance traveled (light)} \quad D = d \sin \theta \quad (19.1)$$

Constructive interference occurs when the path difference for the two rays is exactly an integer number of wavelengths, as shown in equation 19.2.

$$\text{Constructive Interference} \quad D = n\lambda = d \sin \theta_n \quad (19.2)$$

Here

- n is an integer (0, 1, 2, ...)
- θ_n is the corresponding angle at which constructive interference occurs
- $n\lambda$ is the path difference
- $d = 1.69 \times 10^{-6} \text{m} = 1690 \text{ nm}$

The angle for constructive interference depends on the wavelength λ , so by measuring the angle you can determine the wavelength of the light. You observe light from Mercury atoms, which is a mixture of different wavelengths (colors). You measure the angle for constructive interference from a grating for which we know the line spacing d and use equation 19.2 to calculate the wavelength, λ , of the light. The telescope is initially set perpendicular to the grating ($\theta = 0$) where all wavelengths (colors) of the beam constructively interfere, which appears as a bright white line.

To observe the separated colors the spectrum, swing the telescope away from $\theta = 0$ until you observe separate color lines from violet to red, which comprise the $n = 1$ angular position for the lines of the spectrum. The angular position of the telescope for the first appearance of a given color line to the left of the initial position is a poor estimate of θ_n for that spectrum line. To get a more accurate estimate, also measure the angular position of the telescope for the first appearance of same color line to the right of the initial position. Half of the difference between the angular positions of these two observations is a good estimate of θ_n for that color. Measuring the angular position to the left and then to the right of the perpendicular direction corrects for any error in positioning the grating perpendicular to the collimator beam. This process must be repeated for each color line in each spectrum, $n=1$ and $n=2$, which correspond to the first order and second order constructive interference patterns of the light. The $n=2$ spectrum is more spaced out than the $n=1$ spectrum.

ACCEPTED VALUES

In one fringe, you can typically make out four distinct color bands, similar to the spectrum shown in figure 19.2.



Figure 19.2 A typical diffraction pattern for one fringe from the light emitted by Mercury

A Mercury lamp emits light at specific wavelengths. The colors and wavelengths of the strongest visible lines in the Mercury spectrum are:

Color	Wavelength (nm)	Intensity
Violet	404.6	moderate
	407.8	weak
blue-violet	435.8	strong
green	546.1	strong
yellow	577.0	strong
	579.0	strong

APPARATUS

- Spectrometer
- Diffraction grating
- mercury arc lamp

The grating is clamped in a rotating table so that it is perpendicular to the collimator which focuses the light from the lamps. The telescope is initially set perpendicular to the grating ($\theta = 0$) where all wavelengths (colors) of the beam constructively interfere. The goniometer angular scales display different initial angles through the magnifying glasses, neither of which may be equal to zero, as shown in figure 19.3. Make sure you choose one of the goniometer angular scales to use for the entire experiment. We will call the angle measured on the goniometer scale φ . For each line you will measure the angle twice, once to the right (φ) and once to the left (φ'). The deflection angle θ is then given by half the difference,

$$\text{Deflection angle} \quad \theta = \frac{\varphi - \varphi'}{2} \quad (19.3)$$

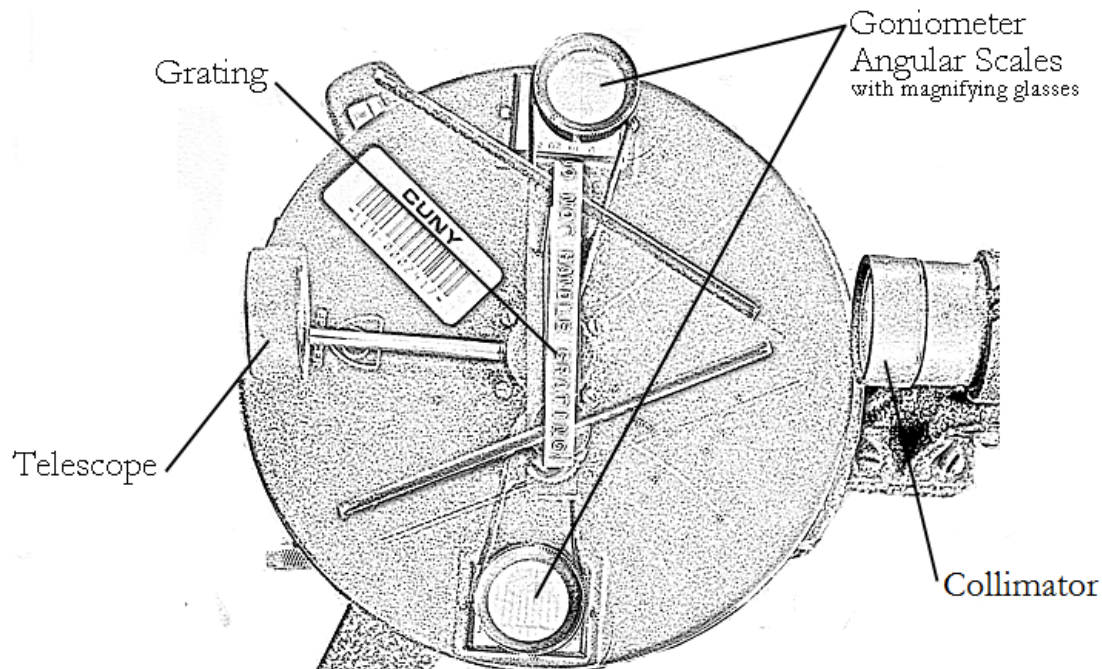


Figure 19.3 An overhead view of the parts of the diffraction grating spectrometer

PROCEDURE

1. Position the telescope so that it is perpendicular to the grating. Move it until you see a bright white band in the cross hairs. Important: always read the angle from the same goniometer angular scale of the apparatus!
2. Swing the telescope to the right until you observe the separated colors of the spectrum.
3. Set the telescope cross hairs to the first mercury line to the right. Record its color. Then read the angle from your chosen goniometer angular scale and record the angle in your data table as φ_1 .
4. Swing the telescope to find the same color line to the left. Read the angle from your chosen goniometer angular scale and record the angle in your data table as φ'_1 .
5. Repeat procedures 3 through 5 for the other colors. Take care that you are observing the first line of each color on either side.
6. Now measure the angles (left and right) for the second appearance of just the green and yellow lines. Record the angles as φ_2 and φ'_2 .

DATA

Spectrum Measurements

color	φ_1	φ'_1	φ_2	φ'_2

CALCULATION AND ANALYSIS

1. Calculate the wavelength for each of the lines you measured. Recall that for each line you measured the angle twice. To get the deflection angle θ use equation 19.3.
2. Try to match your calculated wavelength values with the values in the table of accepted values. Calculate the percentage error for each wavelength calculation.

Appendix: Algebra and Trigonometry Review Topics

REFRESHING

High-School Mathematics

and beyond..

Numbers and symbols

Symbolic and numeric calculations

Physics students have to be able to operate with symbols (such as a , b , x , y , Q , etc.) that stand for numbers. Calculations should be done, as a rule, in a symbolic form, and the analytical (that is, symbolic) result should be obtained. Only after that concrete numbers should be plugged into the analytical result, to obtain the numerical result. Doing this is not a big problem since all operations on numbers can be done on symbols as well. Symbolic operations have important advantages, however, such as better overview of the manipulations, possibility of backtracking and checking, possibility of using multiple sets of numerical values without a necessity of doing similar calculations many times, possibility of operating with quantities, numerical values of which are unknown but irrelevant.

Basic identities

$$a + b = b + a$$

$$a + (b + c) = (a + b) + c = (a + c) + b = a + b + c$$

$$ab = ba$$

$$a(bc) = (ab)c = (ac)b = abc = acb = bca = \dots$$

$$a(b + c) = ab + bc$$

$$(a + b)^2 = a^2 + 2ab + b^2 \text{ (binomial formula)}$$

(operations in brackets are performed first)

Fractions

$$a/b = \frac{a}{b} = a \frac{1}{b} = a \times b^{-1}$$

$$\frac{b}{a} \frac{c}{c} = \frac{bc}{ac} = ab \frac{1}{c} = ab/c$$

$$\frac{ac}{bd} = \frac{ac}{bd}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad + bc}{bd}$$

Inserted for convenience

Compound Fractions

Fractions containing fractions are sometimes confusing, such as

$$\frac{a}{\frac{b}{\frac{c}{d}}} \text{ or } \frac{\frac{a}{b}}{\frac{c}{d}}$$

To avoid confusion, we can distinguish between external and internal fractions and make external fractions longer and/or bolder. If we divide

$$a \text{ or } \frac{a}{b} \text{ over } \frac{c}{d}$$

we write

$$\frac{a}{\frac{b}{\frac{c}{d}}} \quad \begin{array}{l} \text{internal fractions} \\ \text{external fraction} \end{array}$$

and simplify the fractions as follows

$$\frac{a}{\frac{b}{\frac{c}{d}}} = \frac{ac}{b} = \frac{a}{\frac{b}{c}} = \frac{ad}{bc}$$

Manipulations above can be justified using powers instead of reciprocals:

$$\frac{a}{\frac{b}{\frac{c}{d}}} = a \left(\frac{b}{\frac{c}{d}} \right)^{-1} = a \frac{c}{b} = \frac{ac}{b}$$

$$\frac{a}{\frac{b}{\frac{c}{d}}} = \frac{a}{b} \left(\frac{c}{d} \right)^{-1} = \frac{a}{b} \frac{d}{c} = \frac{ad}{bc}$$

Fractions can also be written as

$$\frac{a}{b} = a/b$$

Expression a/bc can be confusing. If the writer means $a/(bc)$, it should be written explicitly so. Otherwise it means

$$a/bc = (a/b)c = \frac{a}{b}c = \frac{ac}{b}$$

according to all programming languages.

Exponents

Products of several equal numbers can be represented by powers of these numbers, such as

$$\underbrace{a \times a \times \dots \times a \times a}_{b \text{ times}} = a^b$$

Here b is the exponent and a is the base. Bases and exponents can be, in fact, any numbers, not necessary natural. In particular, negative exponents are used for reciprocals, such as

$$a^{-b} = \frac{1}{a^b}$$

and fractional exponents represent roots

$$a^{1/2} = \sqrt{a}$$

Properties of powers:

$$a^m a^n = a^{m+n}$$

$$(a^m)^n = a^{mn}$$

$$a^1 = a, \quad a^0 = 1$$

Examples:

$$\frac{3 \times 5 \times 3 \times 5 \times 3}{5 \times 3 \times 5 \times 5} = 3^{3-1} 5^{2-3} = 3^2 5^{-1}$$

$$(\sqrt{2})^2 = (2^{1/2})^2 = 2^{1/2 \times 2} = 2^1 = 2$$

$$\sqrt{2^2} = (2^2)^{1/2} = 2^{2 \times 1/2} = 2^1 = 2$$

Scientific notation for numbers

In physics one has frequently to deal with very large and very small numbers. The best way to write these numbers is using the scientific notation. For instance, the scientific notation for 12345.678 is 1.2345678×10^4 . The exponent 4 shows that we have moved the decimal point by 4 positions to the left. Here one sees more clearly how large the number is, its order of magnitude is 10^4 . For this reason, the factor in front (the so-called simple part) should be kept maximally close to 1. The number 9123.456 is better to write as 0.9123456×10^4 than as 9.123456×10^3 because the order of magnitude of this number is 4 and not 3. Similarly the number 0.000001234 is written in the scientific notation as 1.234×10^{-6} , where the exponent -6 shows that we have moved the decimal point by six positions to the right. Remember that negative powers describe reciprocals, so that in this case we divide 1.234 by 10 six times.

Example: The mass of electron m_e is approximately 0.911×10^{-30} kg, difficult to write in the usual notation!

When several numbers are multiplied or divided, one can operate the simple parts and powers of 10 independently:

$$\frac{1.2 \times 10^5 \times 3.4 \times 10^7 \times 0.68 \times 10^{-21}}{0.56 \times 10^{12} \times 4.4 \times 10^{-30}} = \frac{1.2 \times 3.4 \times 0.68}{0.56 \times 4.4} 10^{5+7-21-12+30} = 1.13 \times 10^9$$

For an order-of-magnitude estimation, you can simplify the task and drop all simple parts, that yields 10^9 in the example above.

Algebraic equations

Solving physical problems usually involves solving algebraic equations and systems of equations. In most cases these equations are linear.

An example of a linear equation (usually we use a, b, c, \dots for knowns and x, y, z, \dots for unknowns):

$$ax + b = c$$

Equations remain valid if the same quantity is added or subtracted to their right-hand side (rhs) and left-hand side (lhs) and if both rhs and lhs are multiplied or divided by the same quantity. This can be used to isolate unknowns in one of the sides of the equation, that is to solve the equation. For the equation above it is done in the following way:

$$ax + b = c \Rightarrow ax + b - b = c - b \Rightarrow ax = c - b \Rightarrow x = \frac{c - b}{a}$$

(Frame your final result!)

Solving physical problems, we use standard notations adopted in physics rather than just a, b, c and x, y, z . One should understand which quantities are known and which are unknown. If, for instance, acceleration a has to be found from Newton's second law $F = ma$, then we consider a as the unknown and solve the algebraic equation as follows:

$$F = ma \Rightarrow a = \frac{F}{m}$$

The framed expression is our final symbolic, or so-called analytical, result. We now plug numerical values for F and m into it and obtain our final numerical result for a .

Systems of algebraic equations

In many cases one has to solve systems of linear algebraic equations such as

$$a_1x + b_1y = c_1$$

$$a_2x + b_2y = c_2$$

where x and y are unknowns. One of different ways to do it is, say, to

- (i) find y from the first equation;
- (ii) plug the result into the second equation;
- (iii) solve the second equation for x ;
- (iv) plug the result for x into the expression for y obtained in (i)

that is

$$a_1x + b_1y = c_1 \Rightarrow y = \frac{c_1 - a_1x}{b_1}$$

$$a_2x + b_2y = c_2 \Rightarrow a_2x + b_2 \frac{c_1 - a_1x}{b_1} = c_2 \Rightarrow \left(a_2 - a_1 \frac{b_2}{b_1} \right) x + c_1 \frac{b_2}{b_1} = c_2$$

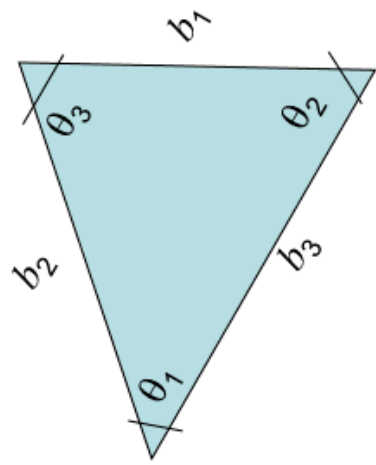
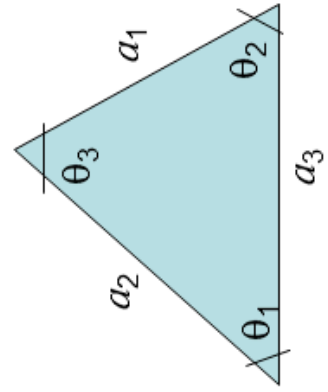
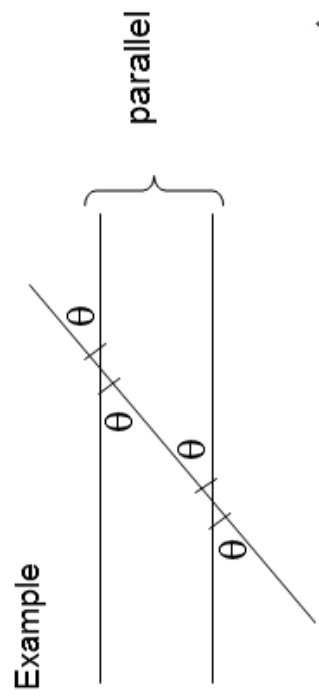
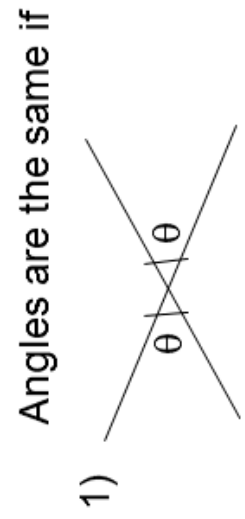
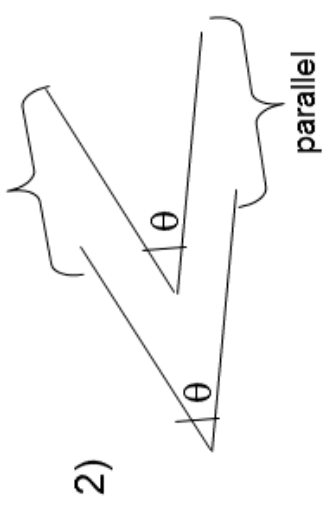
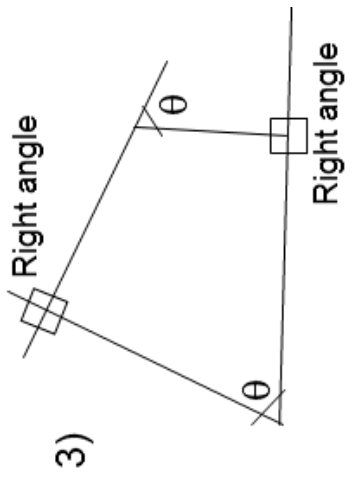
$$\Rightarrow x = \frac{\frac{b_2}{b_1} \frac{c_2 - c_1}{a_2 - a_1}}{\frac{c_2 b_1 - c_1 b_2}{a_2 b_1 - a_1 b_2}} = x \quad \text{(do not forget to simplify your results!)} \quad \boxed{\frac{c_2 b_1 - c_1 b_2}{a_2 b_1 - a_1 b_2} = x}$$

and then we perform (iv) that after simplification yields

$$y = \frac{c_2 a_1 - c_1 a_2}{b_2 a_1 - b_1 a_2}$$

Remember that in a well-behaved system of equations the number of unknowns is equal to the number of equations. You should always perform the count of unknowns and equations before you start with solving a system of equations. 8

Lines, Angles, and Triangles



Similar triangles

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$$

Angles

Angles are usually denoted by Greek letters such as $\alpha, \beta, \theta, \varphi$, etc.

Angles can be measured in

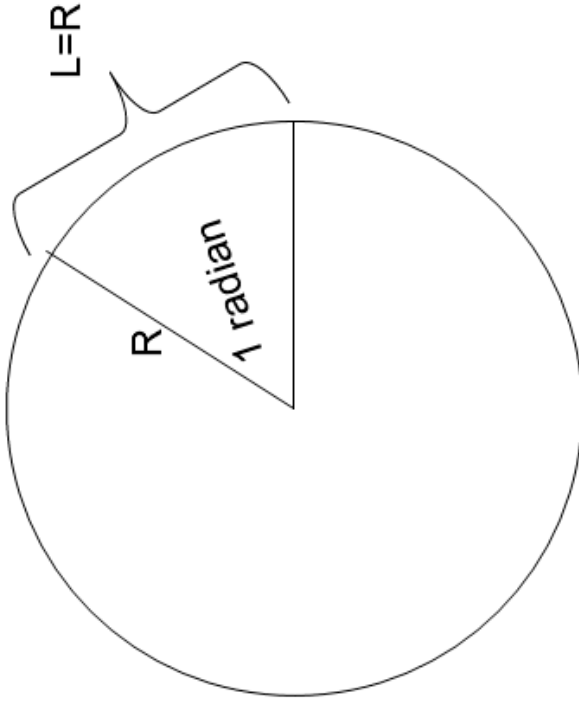
- degrees (seldom used in physics)
- revolutions (360°) (used in engineering)
- radians (mostly used in physics)

Radian is such an angle, for which the length of the arc is equal to the radius. In other words, the angle in radians is given by L/R and it is dimensionless.

Revolution corresponds to $L = 2\pi R$, thus $360^\circ = 2\pi$ radians and

$$1 \text{ radian} = 360^\circ / (2\pi) = 57.3^\circ$$

$$1 \text{ revolution} = 360^\circ = 2\pi \text{ radian}$$

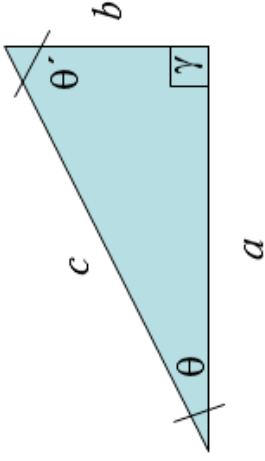


Trigonometric functions

Properties of triangles with right angle ($\gamma = \pi/2$):

$$\theta + \theta' = \pi/2 = 90^\circ$$

$$c^2 = a^2 + b^2 \quad (\text{Pythagoras theorem})$$



For these triangles one defines trigonometric functions as follows:

$$\sin \theta = \frac{b}{c} = \frac{b}{\sqrt{a^2 + b^2}}, \quad \cos \theta = \frac{a}{c} = \frac{a}{\sqrt{a^2 + b^2}}, \quad \tan \theta = \frac{b}{a} = \frac{\sin \theta}{\cos \theta}, \quad \cot \theta = \frac{a}{b} = \frac{\cos \theta}{\sin \theta} = \frac{1}{\tan \theta}$$

sin and cos are called, as a group, sinusoidal functions. They satisfy

$$\sin^2 \theta + \cos^2 \theta = 1$$

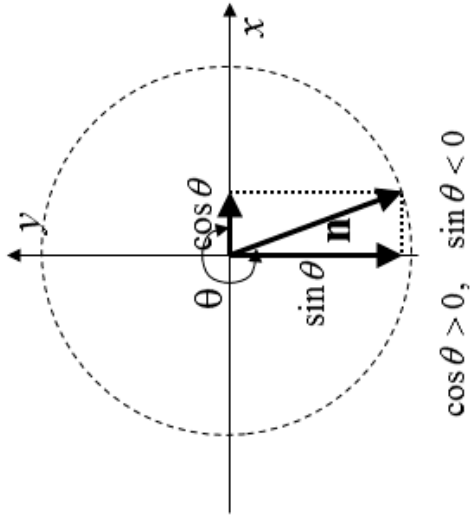
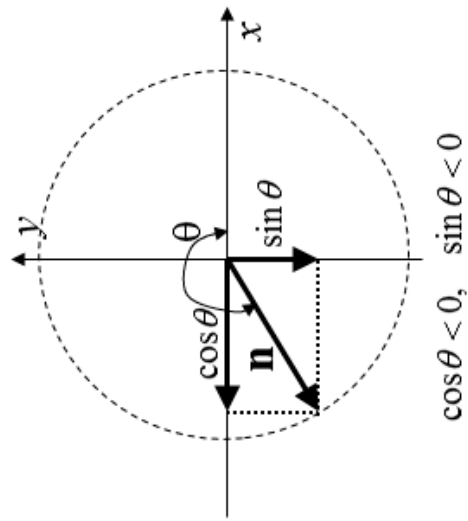
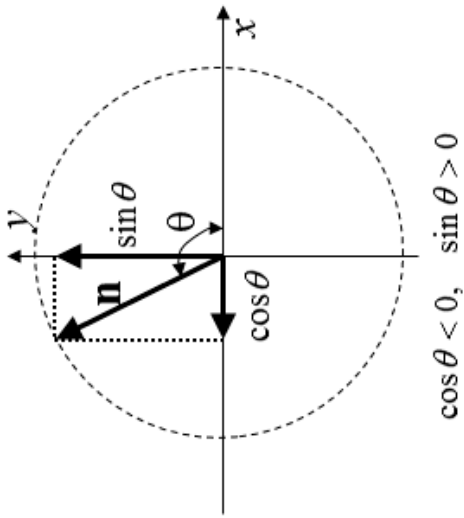
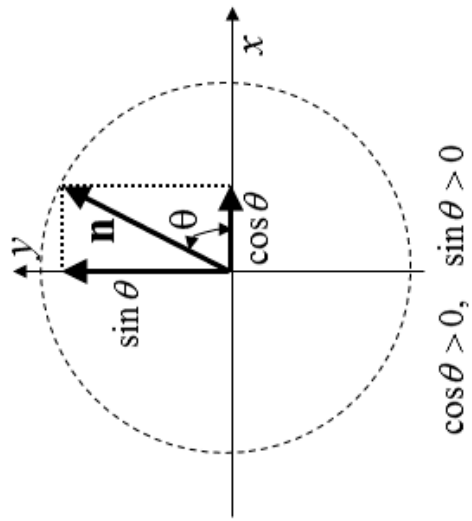
On the other hand, one can define trigonometric functions for the angle θ' of the triangle in the same way:

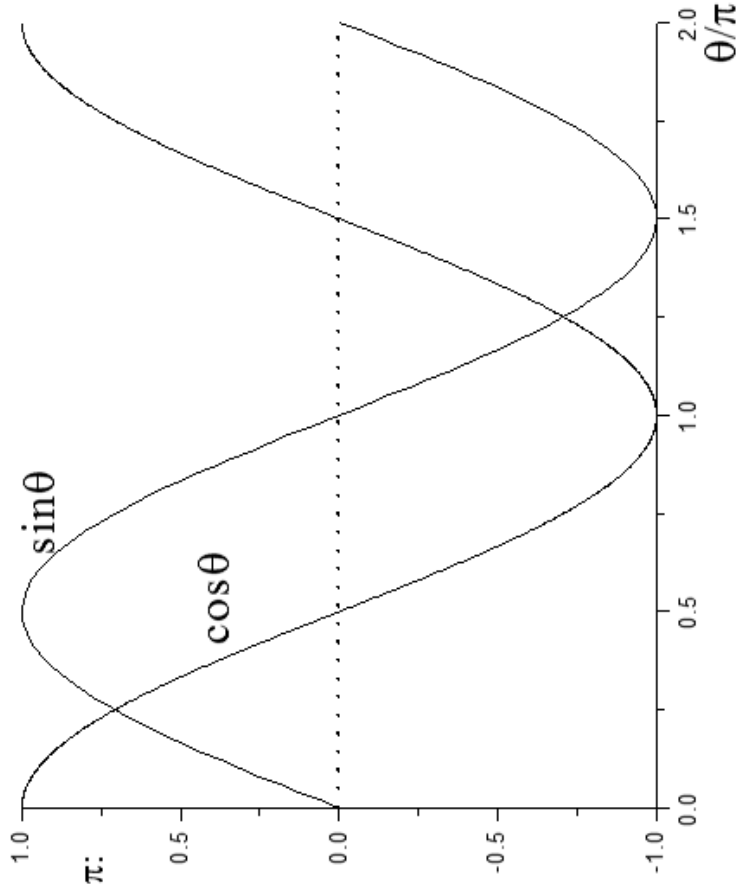
$$\sin \theta' = \frac{a}{c} = \frac{a}{\sqrt{a^2 + b^2}}, \quad \cos \theta' = \frac{b}{c} = \frac{b}{\sqrt{a^2 + b^2}}, \quad \tan \theta' = \frac{a}{b}, \quad \cot \theta' = \frac{b}{a}$$

Comparison with the above yields

$$\sin \theta' = \sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta, \quad \cos \theta' = \cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta, \quad \tan \theta' = \tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta, \quad \dots \quad 11$$

Above we have defined trigonometric functions with the help of triangles, so that θ is limited to the interval $0 \leq \theta \leq \pi/2$. One can, however, define trigonometric functions for arbitrary arguments with the help of components of a two-dimensional unit vector \mathbf{n} ($|\mathbf{n}| = 1$) as follows





Trigonometric functions are periodic with period 2π :

\sin and \cos differ by an argument shift and there are a lot of corresponding relations

Symmetry:

$$\cos(-\theta) = \cos \theta$$

$$\sin(-\theta) = -\sin \theta$$

$$\tan(-\theta) = -\tan \theta$$

$$\cot(-\theta) = -\cot \theta$$

Inverse trigonometric functions

$$\sin \theta = x \Rightarrow \theta = \arcsin x$$

$$\cos \theta = x \Rightarrow \theta = \arccos x$$

$$\tan \theta = x \Rightarrow \theta = \arctan x$$

$$\cot \theta = x \Rightarrow \theta = \text{arccot } x$$

Scalars and Vectors


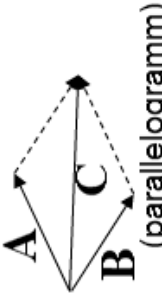
Most of the physical quantities are scalars or vectors.

Scalars are objects that are represented by numbers, such as time t , mass m , electric charge q or Q , temperature T or T . Some scalar quantities are always positive or nonnegative, such as mass m , volume V , kinetic energy E_k , absolute temperature T , etc. Most of scalar quantities can be either positive or negative, such as electric charge q or Q , time t , etc.

Vectors are mathematical and/or physical objects that are characterized by (i) their magnitude or absolute value or length and (ii) their direction in space. Many physical quantities are vectors, such as position, velocity, force, electric and magnetic fields, etc. Vectors can be added, subtracted, and multiplied. Vectors can be divided by a scalar but one cannot divide by vector. Vectors are denoted by symbols with overhead arrows (\vec{A}) in handwritten texts and by boldface symbols (\mathbf{A}) in printed texts.

Magnitude (length) of a vector \mathbf{A} is denoted as $|\mathbf{A}|$ or simply as A . Vectors of unit length, $|\mathbf{A}| = 1$, describe directions. Each vector can be represented in the form $\mathbf{A} = A\mathbf{n}$, where $A > 0$ and \mathbf{n} is a unit vector directed along \mathbf{A} .

Addition of vectors can be done graphically with the help of either the tail-to-tip rule or the parallelogram rule

$$\mathbf{A} + \mathbf{B} = \mathbf{C}$$



Subtraction of vectors:

$$\mathbf{A} - \mathbf{B} = \mathbf{C}$$



because $\mathbf{A} = \mathbf{B} + \mathbf{C}$

Vectors and Coordinate Systems

To perform operations on vectors numerically, it is convenient to introduce a coordinate system. The latter is defined by the origin O and three mutually perpendicular axes x , y , and z . The tail of the vector \mathbf{A} is in the origin of the coordinate system. We project the vector \mathbf{A} onto the axes of the coordinate system by drawing the three lines from its tip towards all three axes perpendicularly to the latter. As the result, \mathbf{A} is represented as the sum of three vectors:

$$\mathbf{A} = A_x \mathbf{e}_x + A_y \mathbf{e}_y + A_z \mathbf{e}_z$$

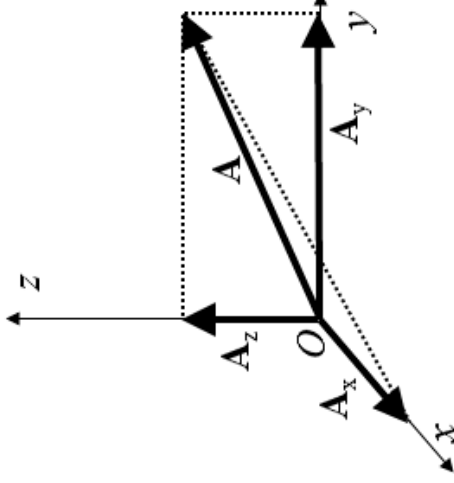
Here we have introduced the unit vectors \mathbf{e}_x , \mathbf{e}_y , and \mathbf{e}_z , ($|\mathbf{e}_x| = 1$ etc.) that are directed along different axes. The scalar quantities A_x , A_y and A_z are components of the vector \mathbf{A} in this coordinate system or its projections on the axes of this coordinate system. Note that components of a vector can be both positive and negative.

With the above definitions, one obtains many useful formulas. Addition of vectors can be done as

$$\mathbf{A} + \mathbf{B} = A_x \mathbf{e}_x + A_y \mathbf{e}_y + A_z \mathbf{e}_z + B_x \mathbf{e}_x + B_y \mathbf{e}_y + B_z \mathbf{e}_z = (A_x + B_x) \mathbf{e}_x + (A_y + B_y) \mathbf{e}_y + (A_z + B_z) \mathbf{e}_z$$

$$\mathbf{A} + \mathbf{B} = \mathbf{C} = C_x \mathbf{e}_x + C_y \mathbf{e}_y + C_z \mathbf{e}_z \Rightarrow C_x = A_x + B_x, \quad C_y = A_y + B_y, \quad C_z = A_z + B_z$$

That is, to add vectors, one has just to add their components, and similar for subtraction.



Multiplication or division of a vector by a positive scalar changes its length but does not change its direction. If \mathbf{A} is a vector and $\varphi > 0$ is a scalar, then $\mathbf{B} = \varphi \mathbf{A} = \varphi A \mathbf{n} = B \mathbf{n}$, that is, $B = \varphi A$. Multiplication of a vector by a negative scalar additionally inverts its direction. In components one obtains

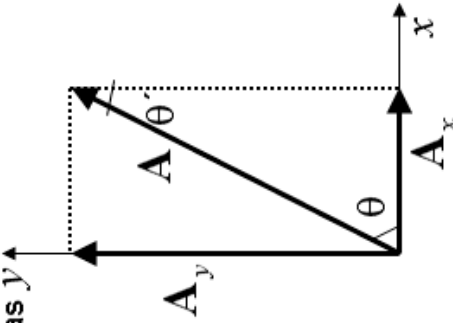
$$\mathbf{B} = \varphi A \mathbf{e}_x + \varphi A \mathbf{e}_y + \varphi A \mathbf{e}_z = B_x \mathbf{e}_x + B_y \mathbf{e}_y + B_z \mathbf{e}_z$$

thus one has simply to multiply components of the vector by the scalar: $B_x = \varphi A_x$, etc., for both signs of φ .

The length of a vector can be obtained in components from the Pythagoras theorem:

$$A = |\mathbf{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Using trigonometric functions, one can express components (projections) of a vector as y



where α , β , and γ are the angles between vector \mathbf{A} and the axes x , y , and z , respectively. In particular, for a vector that is confined to a plane (that is, has only two components) one has

$$A_x = A \cos \theta, \quad A_y = A \sin \theta$$