



Laboratory Manual

Physics 166, 167, 168, 169

Lab manual, part 1
For PHY 166 and 168 students

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Writing a laboratory report

OBJECTIVES

The main way to communicate scientific information today is through articles and reports in scientific journals. Traditionally these were distributed in print, but can now be read in digital format as well, as shown in table 1.

Table 1. Online resources for digitally distributed scientific publications

Resource	Topics	URL
arXiv	Physics, Mathematics, Computer Science	http://arxiv.org/
NASA Scientific and Technical Information (STI)	Astronomy, Aerospace Engineering	www.sti.nasa.gov/STI-public-homepage.html
SOA/NASA Astrophysics Data System	Astronomy, physics	adswww.harvard.edu/

In college physics, you write a laboratory report for each experiment that contains the essential information about the experiment. For scientific information, a consistent format is helpful to the reader (and your lab instructor). Each laboratory report you turn in contains a subset of the sections found in a professional scientific publication for experimental topics, as shown in table 2.

Table 2. A comparison of the sections of a laboratory report and a professional scientific publication

Laboratory Report	Professional Publication
1. Name, date and title of the experiment	1. Cover page: name, date, and title
2. Abstract	2. Abstract
	3. Introduction
	4. Methods and procedure
3. Data	5. Raw data and graphs
4. Calculations and analysis	6. Calculations and analysis
	7. Results
5. Conclusion	8. Discussion and conclusion

The content to include in each section is detailed below. Your lab instructor requires all five sections to evaluate your work, so be sure to include every section in every report.

A laboratory report must be typed. Photocopies of the manual are not accepted. Your laboratory instructor can tell you whether your laboratory report must be printed or can be delivered in a digital format such as email.

ABSTRACT

Describe in your own words what you did in the experiment and why. Your abstract should include one or two sentences each for Purpose, Methods and Conclusions.

- **Purpose:** What physical principle or law does this experiment test?
- **Methods:** What apparatus did you use? How did you analyze the data?
- **Conclusions:** Do your results support the physical law or principle? You should describe any significant experimental errors or uncertainties.

Note that the abstract should be no more than 5 or 6 sentences long, and should not include too much detail. The goal of the abstract is to sum up the experiment quickly and succinctly.

DATA

The data section includes all the raw data you collected in the laboratory without any calculation or interpretation. At a minimum, include the following information:

- A copy of the data table with all fields and rows filled with measurements.
- Any drawings or sketches you were required to make in the laboratory. You must deliver drawings with a printed lab report. You can take a digital photograph of your drawings and import it to a document as needed.

CALCULATIONS AND ANALYSIS

In the calculations and analysis section, you write out all of your calculations and results as explained in the instructions for the experiment. Be sure to answer all of the questions in the lab manual. Include the following information as instructed:

- The equations you used to make all calculations
- Tables of calculated values
- Graphs of the raw data or calculated values
- Average values, uncertainty, and % uncertainty calculations
- Error and % error calculations

CONCLUSION

In the conclusion section, interpret the results you obtained by analyzing the data. Include the following information:

- Do your data and calculations support the physical principle or law being tested?
- What are the important sources of experimental error and uncertainty?
- Are there ways you could have improved your experimental results?
- Also answer any specific questions posed by your lab instructor.

Introduction: Measurement and uncertainty

No physical measurement is ever completely precise. All measurements are subject to some uncertainty, and the determination of this uncertainty is an essential part of the analysis of the experiment.

Experimental data include three components: 1) the value measured, 2) the uncertainty, and 3) the units. For example a possible result for measuring a length is $3.6 \pm 0.2 \text{ m}$. Here 3.6 is the measured value, ± 0.2 specifies the uncertainty, and *m* gives the units (meters).

ERRORS AND UNCERTAINTIES

The accuracy of any measurement is limited. An *uncertainty* is our best estimate of how accurate a measurement is, while an *error* is the discrepancy between the measured value of some quantity and its true value. Errors in measurements arise from different sources:

- a) A common type of error is a blunder due to carelessness in making a measurement, for example an incorrect reading of an instrument. Of course these kinds of mistakes should be avoided.
- b) Errors also arise from defective or improperly calibrated instruments. These are known as systematic errors. For example, if a balance does not read zero when there is no mass on it, then all of its readings will be in error, and we must either recalibrate it, or be careful to subtract the empty reading from all subsequent measurements.
- c) Even after we have made every effort to eliminate these kinds of error, the accuracy of our measurements is still limited due to so-called statistical uncertainties. These uncertainties reflect unpredictable random variations in the measurement process: variations in the experimental system, in the measuring apparatus, and in our own perception! Since these variations are random, they will tend to cancel out if we average over a set of repeated measurements. To measure a quantity in the laboratory, one should repeat the measurement many times. The average of all the results is the best estimate of the value of the quantity.
- d) Besides the uncertainty introduced in a measurement due to random fluctuations, vibrations, etc., there are also so-called instrumental uncertainties which are due to the limited accuracy of the measuring instruments we use. For example, if we use a meter stick to measure a length, we can, at best, estimate the length to within about half of the smallest division on the stick or 0.5 millimeters. Beyond that we have no knowledge. It is important to realize that this kind of uncertainty persists, even if we obtain identical readings on repeated trials.

CALCULATING AVERAGES

There are several important steps we will follow to help us quantify and control the errors and uncertainties in our laboratory measurements.

Most importantly, in order to minimize the effect of random errors, one should always perform several independent measurements of the same quantity and take an average of all these readings. In taking the average the random fluctuations tend to cancel out. In fact, the larger the number of measurements taken, the more likely it is that random errors will cancel out.

When we have a set of n measurements x_1, x_2, \dots, x_n of a quantity x , our *best estimate* for the value of x is the average value \bar{x} , is defined as follows.

$$\text{Average value:} \quad \bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n} \quad (0.1)$$

The average value is also known as the mean value. Note that when making repeated measurements of a quantity, one should pay attention to the consistency of the results. If one of the numbers is substantially different from the others, it is likely that a blunder has been made, and this number should be excluded when analyzing the results.

REPORTING ERRORS

Quite often in these labs one has to compare a value obtained by measurement with a standard or generally accepted value. To quantify this one can compute the *percent error*, defined as follows.

$$\text{Percent error:} \quad \% \text{ Error} = \left| \frac{\text{measured value} - \text{accepted value}}{\text{accepted value}} \right| \times 100 \quad (0.2)$$

Sometimes one has to report an error when the accepted value is zero. You'll encounter this situation in experiment 3. The procedure to follow is described at the end of that experiment.

CALCULATING UNCERTAINTIES

To estimate the uncertainty associated with our best estimate of x , we begin by examining scatter of the measurements about the mean \bar{x} . Specifically, we start by determining the absolute value of the deviation of each measurement from the mean:

$$\text{Deviation:} \quad \Delta x_i = |x_i - \bar{x}| \quad (0.3)$$

Next we have to compare the deviation to the systematic or reading uncertainty due to limited accuracy of the instrument used. If this systematic uncertainty R is bigger than the deviation Δx_i , then the result of our measurements can be written as

$$\bar{x} \pm R \quad (0.4)$$

If, however, the deviation is larger than the systematic or reading error, then we must determine how big the random uncertainty in our measurements is. This is given by the standard deviation, defined as follows.

$$\text{Standard deviation:} \quad \sigma = \sqrt{\frac{(\Delta x_1)^2 + (\Delta x_2)^2 + \dots + (\Delta x_n)^2}{n - 1}} \quad (0.5)$$

The standard deviation has the following meaning: if we were to make one single additional measurement of the quantity x , there is 68% probability of obtaining a value which lies between $\bar{x} - \sigma$ and $\bar{x} + \sigma$. The uncertainty in the average value \bar{x} is smaller (that's the whole point of

taking an average!). In fact the uncertainty in \bar{x} is the standard deviation divided by the square root of the number of measurements:

$$\text{Uncertainty in } \bar{x}: \quad U = \frac{\sigma}{\sqrt{n}} \quad (0.6)$$

Sometimes it is useful to express this as a percent uncertainty, defined as one hundred times the uncertainty divided by the average value.

$$\% \text{ uncertainty in } \bar{x}: \quad \% \text{ uncertainty} = \frac{U}{\bar{x}} \times 100 \quad (0.7)$$

EXAMPLE

To illustrate the calculation of \bar{x} and the associated uncertainty U , suppose we are measuring the length of a stick and have obtained, in five separate measurements, the results tabulated below.

Length x (cm)	Deviation Δx (cm)	$(\Delta x)^2$
54.84	0.43	0.1849
53.92	0.49	0.2401
54.46	0.05	0.0025
54.55	0.14	0.0196
54.30	0.11	0.0121
sum: 272.07		sum: 0.4592

From this information we can calculate

$$\text{Average:} \quad \bar{x} = \frac{272.07 \text{ cm}}{5} = 54.41 \text{ cm}$$

$$\text{Standard deviation:} \quad \sigma = \sqrt{\frac{0.4592}{4}} = 0.3388 \text{ cm}$$

$$\text{Uncertainty:} \quad U = \frac{\sigma}{\sqrt{n}} = \frac{0.3388}{\sqrt{5}} = 0.1515 \text{ cm}$$

Thus our final result for the length is $54.41 \pm 0.15 \text{ cm}$.

SIGNIFICANT FIGURES

A number expressing the result of a measurement, or of computations based on measurements, should be written with the proper number of *significant figures*, which just means the number of reliably known digits in a number. The number of significant figures is independent of the position of the decimal point, for example 2.163 cm, 21.63 mm and 0.02163 m all have the same number of significant figures (four).

In doing calculations, all digits which are not significant can be dropped. (It is better to round off rather than truncate). A result obtained by multiplying or dividing two numbers has the same number of significant figures as the input number with the fewest significant figures.

EXAMPLE

Suppose that we want to calculate the area of a rectangular plate whose measured length is 11.3 cm and measured width is 6.8 cm. The area is found to be

$$\text{Area} = 11.3 \text{ cm} \times 6.8 \text{ cm} = 76.84 \text{ cm}^2$$

But since the width only has two significant figures we can round to two figures and report that the area is 77 cm^2 .

GRAPHING, SLOPE AND INTERCEPTS

In almost every laboratory exercise, you plot a graph based on the data measured or calculated. A graph lets you visualize the relation between two physical quantities. In plotting a graph, use the following steps:

1. Arrange the data into a table with two columns listing the values for the two measured or calculated quantities. For example, the first column could list the values for time and the second column could list the values for the average velocity.
2. Decide which of the two quantities to plot along each axis. Graphs have two perpendicular axes, the x-axis and the y-axis and by convention you plot the independent quantity along the x-axis and the dependent quantity along the y-axis.
3. Choose the scale for each axis to cover the range of variation of each quantity. You should choose the scale so that the final curve spans the largest area possible on the graph paper.
4. Label each axis with the quantity plotted on that axis and the units used.
5. Mark the main divisions along each axis.
6. Mark each data point on the graph using the values in each row of the data table. Data points must align with the value of each quantity on their respective axes. Make each data point clearly visible on the graph.
7. Fit and draw a smooth curve through the data points so that the curve comes as close as possible to most of data points. Do not force the curve to go exactly through all the points or through the origin of the coordinate system. The fact that not all points lie along the fitted curve just indicates that measurements are subject to some uncertainty.

In many cases the fitted curve is a straight line. The best straight line fit has nearly the same number of data points above and below the line. The equation for a straight line is given by

$$\text{Straight Line} \quad y = mx + b \quad (0.8)$$

The quantity b is the intercept: it is the value of y when $x = 0$. The quantity m is the slope of the curve. Given two points on the straight line, $\{y_1 = mx_1 + b, y_2 = mx_2 + b\}$, called basis points, the slope is defined as the ratio of the change in y to the change in x between these points, as shown in equation 0.9.

Slope $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$ (0.9)

Basis points are NOT experimental points. They should be chosen as far from each other as possible to increase the precision of m , as shown in figure 0.1.

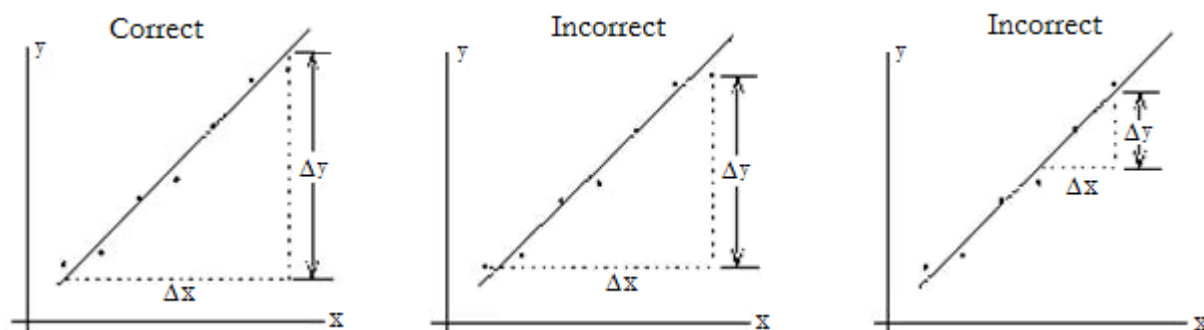


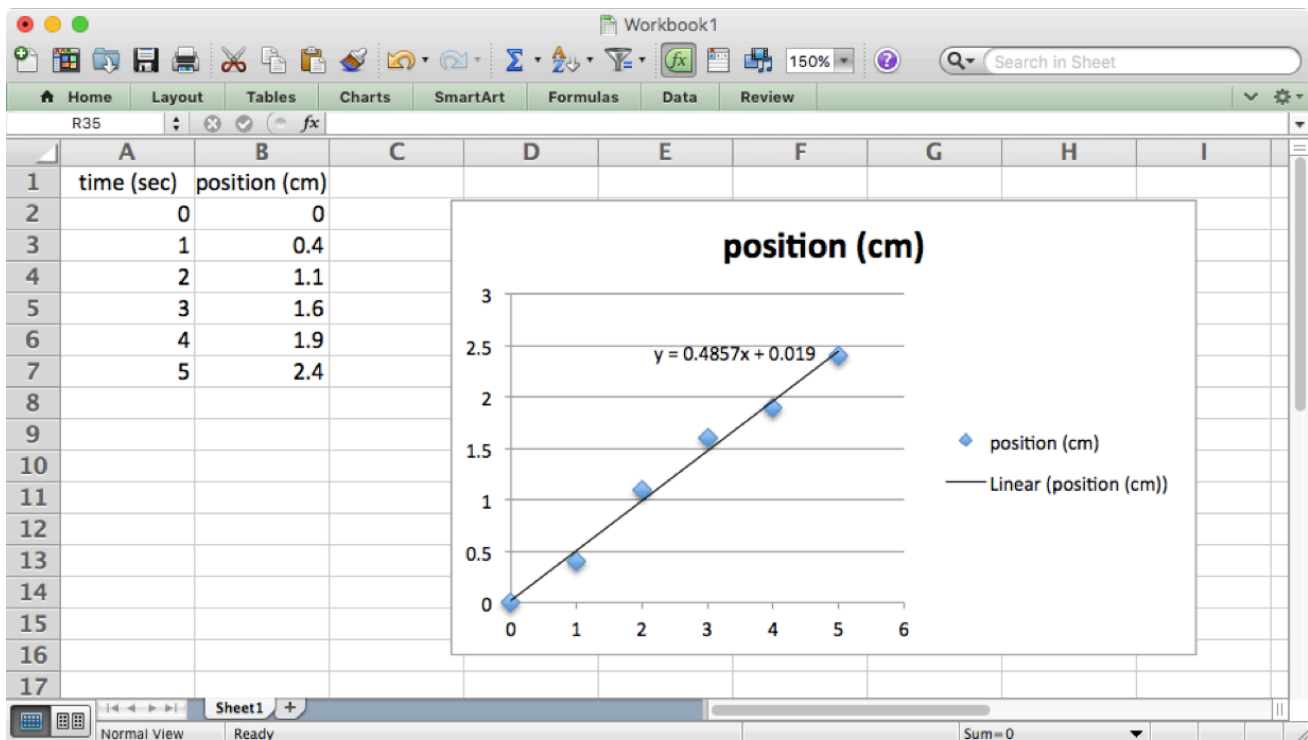
Figure 0.1 Choosing the correct basis points to calculate the slope

PLOTTING USING A COMPUTER

After you have some experience making plots by hand, it's easier and more accurate to let a computer do the work. Any standard spreadsheet or plotting software should work. The steps might depend on the type of software you're using. But in Excel, for example, you would start by entering your data in two columns. The first column gives the x values and the second column gives the corresponding y values. Select the data you want to plot, then

- Charts → Scatter → Marked Scatter will produce a plot of your data
- Chart Layout → Trendline → Linear Trendline will add a best-fit line to your plot
- To see the equation of the best-fit line go to Chart Layout → Trendline → Trendline Options... then in the dialog box that appears go to Options and check "Display equation on chart".

Here's a screenshot of a typical Excel plot.



PRACTICE CALCULATIONS

1. The accepted value of the acceleration due to gravity on Earth is $g = 980 \text{ cm/s}^2$. When trying to measure this quantity, we performed an experiment and got the following five values for g .

Trial	$g \text{ (cm/s}^2\text{)}$
1	1004
2	992
3	978
4	985
5	982

a) Find the average value and standard deviation of our measurements of g .

b) Find the uncertainty in our average value for g .

c) What is the percent error in our measurement of g ?

2. A box is moving along a frictionless inclined plane. Experimental measurements of velocity at various times are given below.

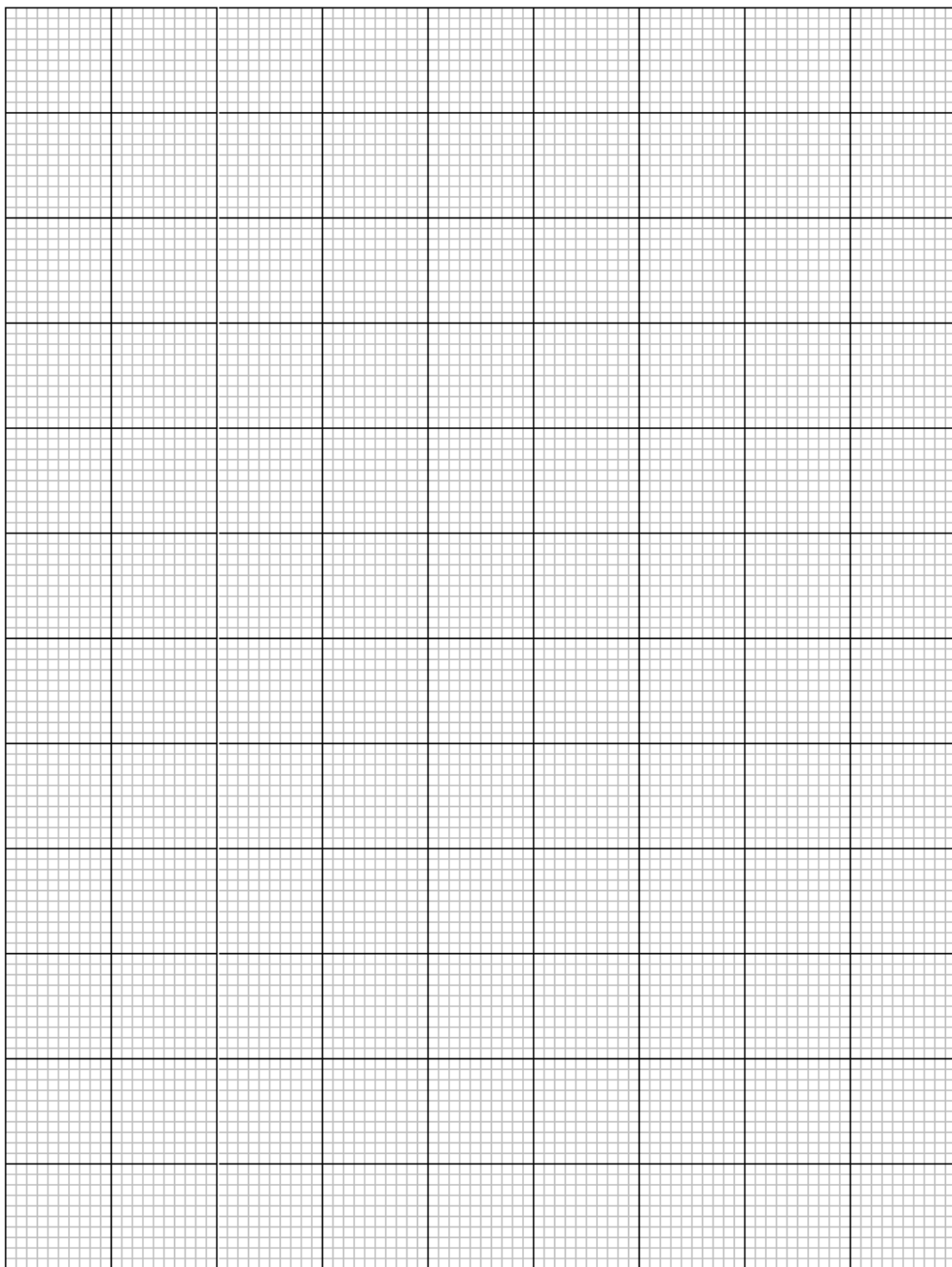
time $t \text{ (s)}$	velocity $v \text{ (m/s)}$
0	0
1	5.2
2	10.1
3	14.8
4	19.9
5	25

a) Plot a graph of v versus t . Can the data be represented by a straight line? (You can use the graph paper on the next page.)

b) Calculate the slope.

c) What physical quantity does this slope represent?

d) From your estimate of the slope, what would the velocity be at $t = 10 \text{ s}$?



Introduction: Units and conversions

Every measurement requires a choice of units. It's important to keep track of the units you're using, and to know how to convert between different common units. Here are a few examples.

The SI unit of length is the meter (m). But we sometimes measure lengths in centimeters (cm) or millimeters (mm). The conversions are

$$1\text{ m} = 100\text{ cm} = 1000\text{ mm}$$

For example 70 cm is the same as 0.7 m , because

$$70\text{ cm} = 70\text{ cm} \times \frac{1\text{ m}}{100\text{ cm}} = 0.7\text{ m}$$

The SI unit of mass is the kilogram (kg), but we sometimes measure mass in grams (g). The conversion is

$$1\text{ kg} = 1000\text{ g}$$

For example 150 g is the same as 0.15 kg , because

$$150\text{ g} = 150\text{ g} \times \frac{1\text{ kg}}{1000\text{ g}} = 0.15\text{ kg}$$

It's also important to recognize the difference between mass (measured in kilograms) and weight (measured in Newtons). Weight is another name for the force of gravity. It's given by the formula $F = mg$ where m is the mass and $g = 9.8\text{ m/s}^2$ is the acceleration due to gravity. For example a mass of 150 g has a weight of 1.47 Newtons, because 150 g is the same as 0.15 kg and

$$F = 0.15\text{ kg} \times 9.8\text{ m/s}^2 = 1.47\text{ kg m/s}^2 = 1.47\text{ N}$$

Note that we had to convert the mass to kilograms in order to get the right answer in Newtons!

Experiment 1: Density

OBJECTIVES

Density describes how much matter is distributed within any given region of space. Quantitatively, it is the amount of mass contained in a unit of volume. In this experiment, you measure the mass and spatial dimensions of a specimen of an unknown metal, calculate its density, and use the result to identify the metal. The objectives of this experiment are as follows:

1. To identify the limits of precision for different measuring devices and calculate the uncertainty for measurements made with them
2. To measure the mass and spatial dimensions of a specimen
3. To calculate the volume and density of a metal specimen

THEORY

The density of a substance is constant for any pure sample at a constant temperature and pressure. For example, the density of a sample of pure Iron has the same density as any other sample of pure Iron at the same temperature and pressure. The density of any substance is its mass per unit volume, calculated using equation 1.1.

$$\text{Density} \quad \rho = \frac{m}{V} \quad (1.1)$$

Where

- m = mass in grams (g), which you measure with a balance.
- V = volume in cubic centimeters (cm^3). The specimens in this lab are rectangular solids, so you calculate volume as the product of the specimen's length, width, and height, each of which you measure using a different instrument.
- ρ = density in grams per cubic centimeter (g/cm^3), which you calculate using equation 1.1.

ACCEPTED VALUES

Metals in their solid state do not change significantly in density with minor changes in temperature and pressure. The accepted values for the density at 20°C of a pure sample of the metals analyzed in this lab are as follows:

- Aluminum: $2.6989 \text{ g}/\text{cm}^3$
- Copper: $8.96 \text{ g}/\text{cm}^3$
- Iron: $7.874 \text{ g}/\text{cm}^3$

APPARATUS

- | | | |
|--------------------|-----------------------|--------------------|
| • meter stick | • micrometer calipers | • platform balance |
| • Vernier calipers | • metal specimens | |

THE PLATFORM BALANCE

The platform balance can measure the mass of a specimen to the nearest 0.05 grams. The balance measures the mass of specimens on the left platform against the known masses of weights placed on the right platform and the sliders suspended from the two beams, as shown in Figure 1.1.

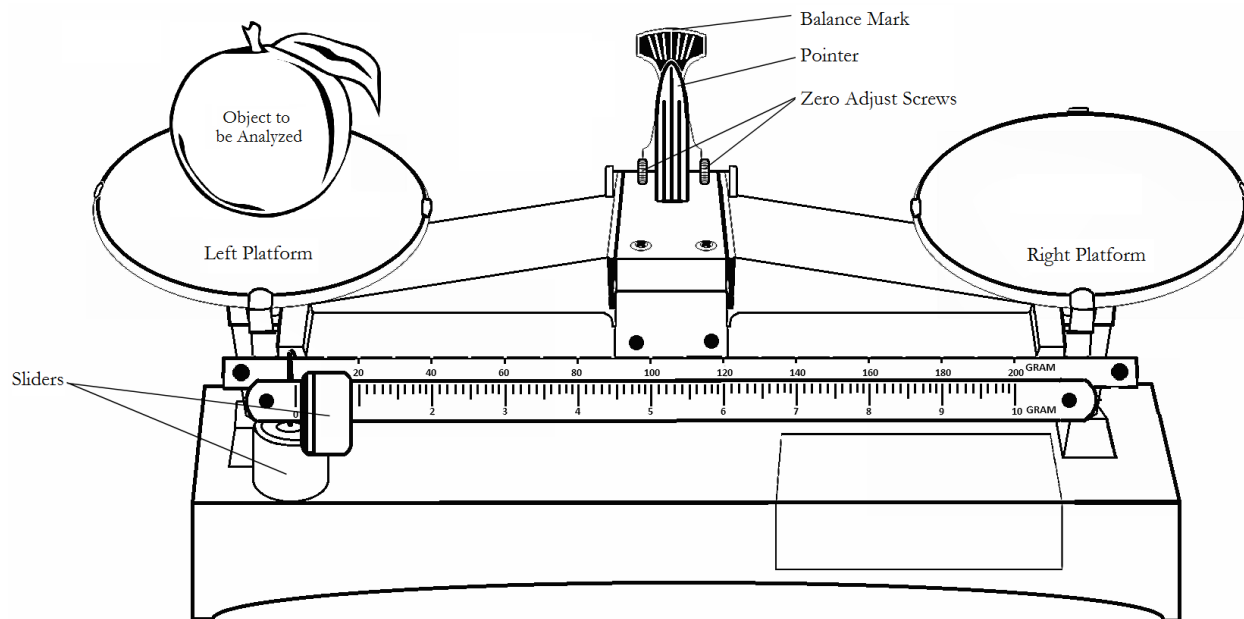


Figure 1.1 The platform balance

To zero the balance, move both sliders to their leftmost positions so that the balance reads zero. If the pointer does not align with the balance mark, then calibrate the balance by turning the zero adjust screws behind the pointer until the pointer aligns with the balance mark.

To measure the mass of a specimen, place the specimen on the left platform. Move the slider along the rear beam, letting it stop at each notch, until the pointer moves to the right of the balance mark. Then, move the slider on the rear beam to the notch immediately to the left of its current position, which measures the mass of the specimen to within 10 grams. Move the slider on the front beam to the right until the pointer aligns with the balance mark. Measure the grams from the front beam using the boldface numbers and tenths of a gram using hash marks between the boldface numbers. If the slider rests between hash marks, add an additional 0.05 grams. If the slider rests on a hash mark, add an additional trailing zero to the right of the decimal point. For example, if you measure 5.6 grams on the front beam and the slider rests exactly on the 5.6 gram hash mark, the precise measurement is 5.60 grams. Add the mass measured on the front beam to the mass measured on the rear beam to measure the precise mass of the specimen.

THE METER STICK

The meter stick can measure distance to the nearest half a scale division, 0.5 millimeters (0.05 cm). The ends of the specimen being measured seldom line up exactly with a mark on the scale, so estimate any fractional part of the smallest division at both ends.

THE VERNIER CALIPERS

The Vernier calipers can measure distance to the nearest tenth of a millimeter (0.01 cm). This instrument is shown in Figure 1.2. The Vernier scale on the slide measures a fractional part of the

main scale. To take a measurement, place the specimen between the large jaws of the instrument. The hash mark on the main scale that aligns with the zero mark on the Vernier scale indicates the length of the specimen to within 1 millimeter (0.1 cm). The zero mark of the Vernier scale is the long line at the left end.

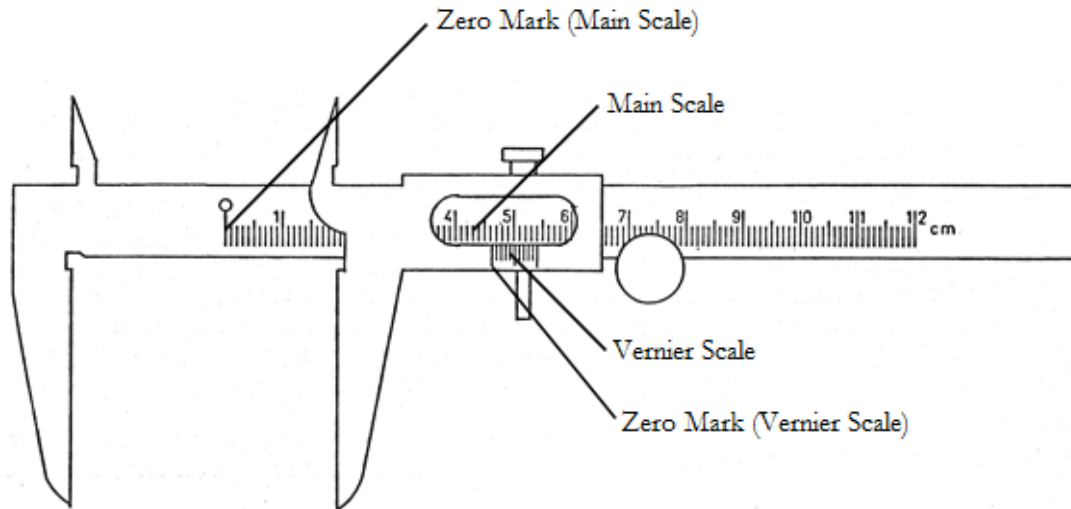


Figure 1.2 The Vernier calipers

On the Vernier scale, the 10 divisions have the length of only 9 divisions on the main scale. Figure 1.3 shows the relation between the two scales when the calipers are closed.

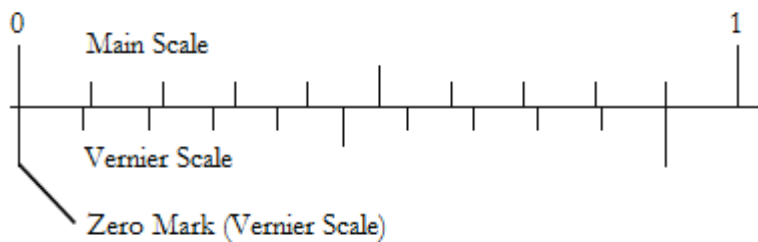


Figure 1.3 The initial positions of the main scale and Vernier scale

The main scale is divided into 0.1 cm intervals and the Vernier scale is 0.09 cm long, which provides an additional level of precision to one hundredth of a centimeter. In Figure 1.4, the zero line of the Vernier scale lies between the 2.3 cm and 2.4 cm marks of the main scale and the fourth mark on the Vernier scale aligns with a main scale line.

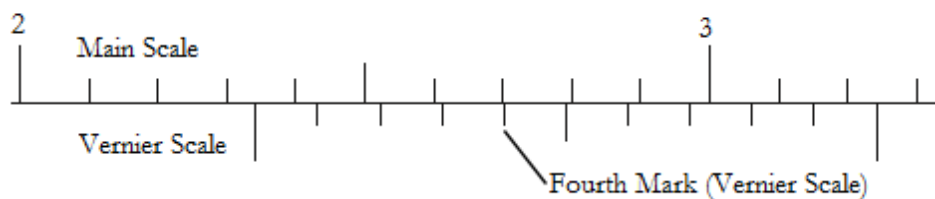


Figure 1.4 An example of the Vernier scale, which reads 0.04 cm past 2.3 cm or 2.34 cm

The difference between the size of a main scale division and a Vernier division is 0.01 cm. Therefore, the difference between the 2.3 main scale mark and the Vernier zero mark is 0.04 cm and the precise measurement is 2.34 cm. In general, if the n th line of the Vernier scale coincides with a main scale division, the Vernier zero mark is at a distance $(n \times 0.01 \text{ cm})$ beyond the main scale division immediately to the left of the Vernier zero mark. To find the length of the specimen, add this distance to the main scale measurement. For example, if line 1 on the Vernier scale coincides with a main scale line, the Vernier zero mark is 0.01 cm beyond the mark on the main scale. If line 2 on the Vernier scale coincides with a main scale line, the Vernier zero mark is 0.02 cm beyond the mark on the main scale, and so on.

THE MICROMETER CALIPERS

The micrometer calipers can measure distance to the nearest hundredth of a millimeter (0.001 cm). The construction of this instrument is shown in figure 1.5. To measure a specimen, place it between the jaws (A and B). The spindle (B) moves by turning a precision screw connected to the thimble (D). Turning the thimble opens or closes the jaws. The distance between the jaws is given by the scale on the sleeve (C), which is ruled in millimeters. There are 50 divisions on the circular scale of the thimble. It takes two turns of the thimble to advance the spindle 1 mm, so each division on the circular scale of the thimble corresponds to an advancement of one hundredth of a millimeter.

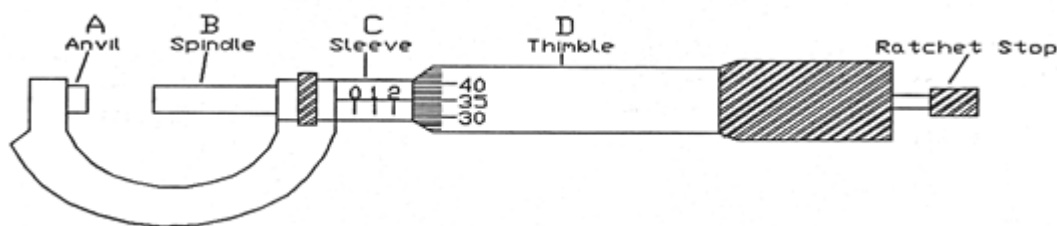


Figure 1.5 The micrometer calipers

To measure the length of a specimen, add the highest number of millimeters visible on the sleeve (C) to the hash mark on the thimble (D) that aligns with the horizontal axis on the sleeve. For example, if the edge of thimble (D) lies between 2.0 and 2.5 mm on the sleeve, as shown in figure 1.5, and the thirty-fifth mark of the circular scale is aligned with the horizontal mark of the sleeve scale, the distance between the jaws is 2.350 mm (0.2350 cm). If the thimble were rotated until the measurement on the circular scale was again 0.350 mm, then the sleeve scale measurement is between 2.5 mm 3.0 mm, and the total distance between the jaws is 2.850 mm (0.2850 cm).

Before measuring lengths with either the Vernier calipers or the micrometer calipers, check the measurement when the jaws are completely closed to observe the systemic uncertainty caused by a consistent error in the measuring instruments. Record the measurement when the jaws are closed as the **offset**. If the **offset** is not zero, correct all other measurements by that amount. For example, if the micrometer measurement is 0.02 mm when the jaws are closed, then correct all measurements made by the micrometer by subtracting 0.02 mm from the micrometer measurement. This avoids a systematic error due to irregularities in the measuring instrument.

PROCEDURE

1. Zero the balance so that it reads zero when the platform is empty. Do not wait for the pointer to come to rest. Adjust the zero adjust screws behind the pointer until the pointer swings equally to the left and to the right in successive swings.
2. Measure the mass of the specimen, recording the value to the nearest 0.05 grams.
3. Measure the length of the specimen using the meter stick. Estimate to the nearest 0.05 cm. Avoid using the ends of a wooden meter stick, as they may be worn.
4. Follow your teacher's instructions for correct use and reading of the Vernier calipers. Measure the width of the specimen. Estimate to the nearest 0.01 cm.
5. Follow your teacher's instructions for correct use and reading of the micrometer calipers. Measure the height of the specimen. Estimate to the nearest 0.001 cm.

DATA

1.1 Mass, Length, Width, and Height

mass (g)	length (cm)	width (cm)	height (cm)

CALCULATION AND ANALYSIS

1. Calculate the volume of the specimen.
2. Calculate the density of the specimen.
3. How many significant figures do your measurements of the mass, length, width and height have?
4. How many significant figures does your result for the volume have? Recall that multiplying two numbers gives a result with the same number of significant figures as the input number with the fewest significant figures.
5. How many significant figures does your result for the density have? Recall that dividing two numbers gives a result with the same number of significant figures as the input number with the fewest significant figures.
6. Identify the metal from which your specimen is made using the standard density values from the theory section of this lab.
7. Calculate the % error in your result for the density of the specimen, using the standard density as the accepted value.
8. Suggest a hypothesis that could explain the % error and how you could change the experiment to reduce that error.

Experiment 2: Acceleration of a Freely Falling Object

OBJECTIVES

Acceleration is the rate at which the velocity of an object changes over time. An object's acceleration is the result of the sum of all the forces acting on the object, as described by Newton's second law. Under ideal circumstances, gravity is the only force acting on a freely falling object. In this lab, you measure the displacement of a freely falling object, calculate the average velocity of a falling object at set time intervals, and calculate the object's acceleration due to gravity. The objectives of this experiment are as follows:

1. To measure the displacement of a freely falling object
2. To test the hypothesis that the acceleration of a freely falling object is uniform
3. To calculate the uniform acceleration of a falling object due to gravity, g

THEORY

The average acceleration of an object measures how quickly its velocity is changing. If the velocity changes by an amount Δv during a time Δt , the average acceleration is calculated as shown in equation 2.1.

$$\text{Average Acceleration} \quad \bar{a} = \frac{\Delta v}{\Delta t} \quad (2.1)$$

In this experiment we study the motion of an object falling vertically down, that is, one-dimensional motion. We take the y axis to point down. Because the acceleration is constant, the average acceleration is equal to g . If the velocity of the object at $t = 0$ is v_0 , the velocity v at time t is

$$\text{Velocity} \quad v = v_0 + gt \quad (2.2)$$

If the position of an object at $t = 0$ is y_0 , then the position y at time t is

$$\text{Position} \quad y = y_0 + v_0 t + \frac{1}{2}gt^2 \quad (2.3)$$

Because the velocity of an accelerating object constantly changes, it is not possible to calculate the velocity at an exact time algebraically from measuring positions over time. However, you can approximate the velocity at the midpoint of any time interval by calculating the average velocity over that time interval. The average velocity \bar{v} of an object in one dimension as it moves along the y -axis is defined as the change in its position, Δy , over time, Δt , as shown in equation 2.4.

$$\text{Average Velocity} \quad \bar{v} = \frac{\Delta y}{\Delta t} \quad (2.4)$$

The change in position, or displacement, of an object in one dimension as it moves along the y -axis is defined as the change in its position, Δy , from an initial position y_i to a new position y_{i+1} , as shown in equation 2.5.

Displacement $\Delta y = y_{i+1} - y_i$ (2.5)

ACCEPTED VALUES

The acceleration due to gravity varies slightly, depending on the latitude and the height above the earth's surface. In this experiment the change in height of the falling object is negligible and can be approximated as 0 km for its entire descent. The acceleration due to gravity at $40^\circ 52' 21''$ N latitude (the latitude of Lehman College) and 0 km altitude is

$$g = 980.2 \text{ cm/s}^2$$

APPARATUS

- synchronized sparking device
- meter stick
- earth's gravity

THE SYNCHRONIZED SPARKING DEVICE

The synchronized sparking device records the position of a falling object at regular time intervals. The sparking device consists of a pair of parallel vertical wires with a high electrical potential between them, a strip of heat sensitive recording paper along the rear wire, and an object made of conducting material that falls between the wires, as showing in figure 2.1.

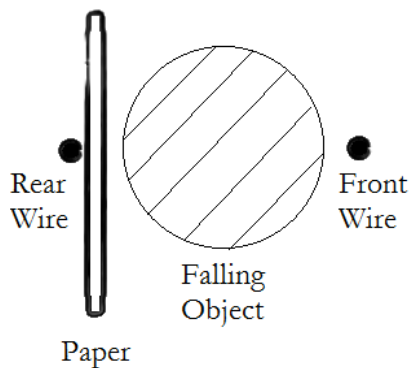


Figure 2.1 A horizontal cross-section of the sparking device and falling object

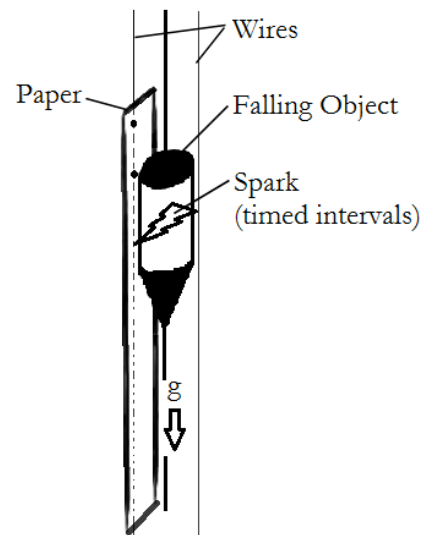


Figure 2.2 Sparks mark the position of the falling object every $1/60$ of a second

The conductive object, held initially by an electromagnet, falls between the wires along the recording paper so that sparks pass through the object and the paper from one wire to the other wire every $1/60$ of a second, as shown in figure 2.2. Sparks leave a mark on the recording paper to mark the object's position at the time each spark is generated without changing the motion of the object as it falls.

To quantify the data from the recording paper, remove it from the device and measure the distance between the marks using a meter stick to determine the displacement of the object between the marks. To minimize errors due to the initial release of the object and the fact that the sparks do not always travel exactly horizontally, ignore the first two data points. To measure intervals for acceleration calculations, record only every third mark, such that the interval between two successive recorded marks corresponds to a time interval, Δt , of $3/60$ of a second or 0.05 second.

PROCEDURE

1. Recording paper tapes are prepared before the lab begins. The instructor can explain and demonstrate the method used in marking the recording paper.
2. Lay your group's recording paper on your table. Have one person hold each end flat and be sure not to stretch it.
3. Skip the first two spark marks and circle the third spark mark on the recording paper. Then, circle every third spark mark after that.
4. Place a meter stick on its edge over the recording paper so that the millimeter and centimeter hash marks lie next to the spark marks on the recording paper, as shown in Figure 2.3.
5. Record the distance of each of the circled spark marks from the starting position in the distance y (cm) column of your data table. Don't move your ruler between measurements!
6. Record the difference in distance between each pair of successive circled spark marks in the displacement Δy (cm) column of your data table.
7. Have the instructor check your recording paper and measurements and rerecord any values that are incorrect.

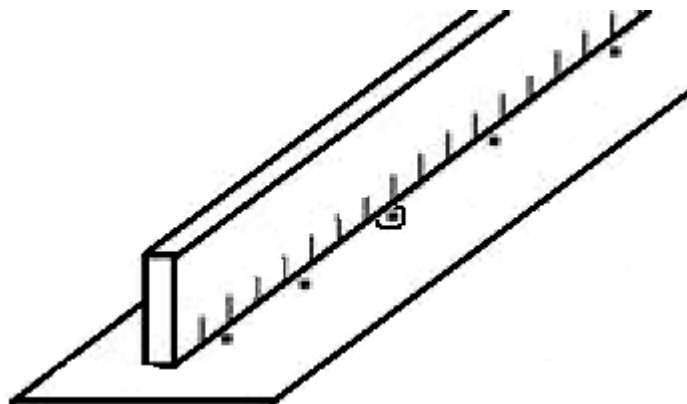


Figure 2.3 Placing a meter stick over the recording paper

DATA

2.1 Position of a falling object

time t (s)	distance y (cm)	displacement Δy (cm)
0.050		
0.100		
0.150		
0.200		
0.250		
0.300		
0.350		
0.400		
0.450		
0.500		

2.2 Average Velocity

time t at the middle of an interval (s)	average velocity \bar{v} (cm/s)
0.075	
0.125	
0.175	
0.225	
0.275	
0.325	
0.375	
0.425	
0.475	

CALCULATION AND ANALYSIS

- Fill in table 2.2 by calculating the average velocity \bar{v} for each time interval. Record your results in the second column of the table. Remember that Δt for each interval is the same, $3/60$ of a second or 0.05 seconds, so the formula is $\bar{v} = \frac{\Delta y}{0.05 \text{ s}}$.
- Using plotting software or good plotting paper, plot a graph of the average velocity \bar{v} versus time t at the middle of an interval. Be sure to observe the following rules:
 - Time t at the middle of an interval is the independent variable and goes on the horizontal x -axis.

- b. Start both axes at zero and choose scales large enough so that your data points fill most of the plot
 - c. Label each axis with the variable and units of the axis
5. Fit the plotted data points to a straight line using a straight edge or graphing software. An ideal straight line fit has nearly the same number of points above and below the line. The experimental points should lie around a straight line that is nearly identical to the line generated by graphing equation 2.2, which is $v = gt + v_0$ when arranged in y-intercept form with g as the slope and v_0 as the y-intercept.
 6. Find the slope of your graph g by choosing two basis points that lie on the line and plugging into equation (0.9). Make sure you follow the correct procedure for choosing basis points illustrated in Figure 0.1.
 7. Calculate the % error for your value for g , as defined in equation (0.2).
 7. What value do you get for the y-intercept and x-intercept of your best straight line? Give units for these intercepts. What is the meaning of these numbers?

Experiment 3: Static Equilibrium

OBJECTIVES

When all the external forces acting on object do not accelerate the object, the object is in a state of mechanical equilibrium. If the object is also at rest, the object is in a state of static equilibrium. In this experiment, you arrange sets of forces to put an object into static equilibrium, measure the vector quantities of these forces, and calculate the net force acting upon an object in equilibrium. The objectives of this experiment are as follows:

1. To measure vector quantities for forces using the force table
2. To calculate the net force on an object using vector addition
3. To test the hypothesis that an object in equilibrium has no net force acting upon it

THEORY

According to Newton's second law of motion, an object accelerates in direct proportion to the net force acting on it. An object in static equilibrium is not moving, so has an acceleration of zero, and the net force on the object is also zero. Therefore, the necessary condition for equilibrium is that the vector sum of all external forces acting on the object is zero, as shown in equation 3.1.

$$\text{Equilibrium} \quad \vec{F}_{total} = \sum_i \vec{F}_i = 0 \quad (3.1)$$

In this experiment, you apply forces to an object in two dimensions until it is in static equilibrium, measure the vector forces, and calculate the vector sum. All forces are applied in the plane (two dimensions), so one can project the forces on the x - and y -axes as shown in equations 3.2.

$$\begin{aligned} \text{2-D Equilibrium} \quad F_{total,x} &= \sum_i F_{i,x} = 0 \\ F_{total,y} &= \sum_i F_{i,y} = 0 \end{aligned} \quad (3.2)$$

To decompose a force vector into its x and y components, it is convenient to choose the x -axis along the direction $\phi = 0^\circ$ and the y -axis along the direction $\phi = 90^\circ$. Then the components of the forces are shown in the pair of equations 3.3.

$$\text{Component Forces} \quad F_{i,x} = F_i \cos \phi_i, \quad F_{i,y} = F_i \sin \phi_i \quad (3.3)$$

ACCEPTED VALUES

The accepted value for the sum of the forces on an object in equilibrium is 0.

APPARATUS

- Horizontal force table
- four pulleys
- one metal ring
- four cords
- four weight hangers
- a degree scale
- Assortment of known weights.

THE FORCE TABLE

A force table consists of a circular platform supported by a heavy tripod base. The circular platform has a graduated degree scale around its rim and a small peg located directly in the center. Four cords are attached to a metal ring placed over a peg in the center of the platform and the cords are connected over pulleys to weight hangers, as shown in Figure 3.1.

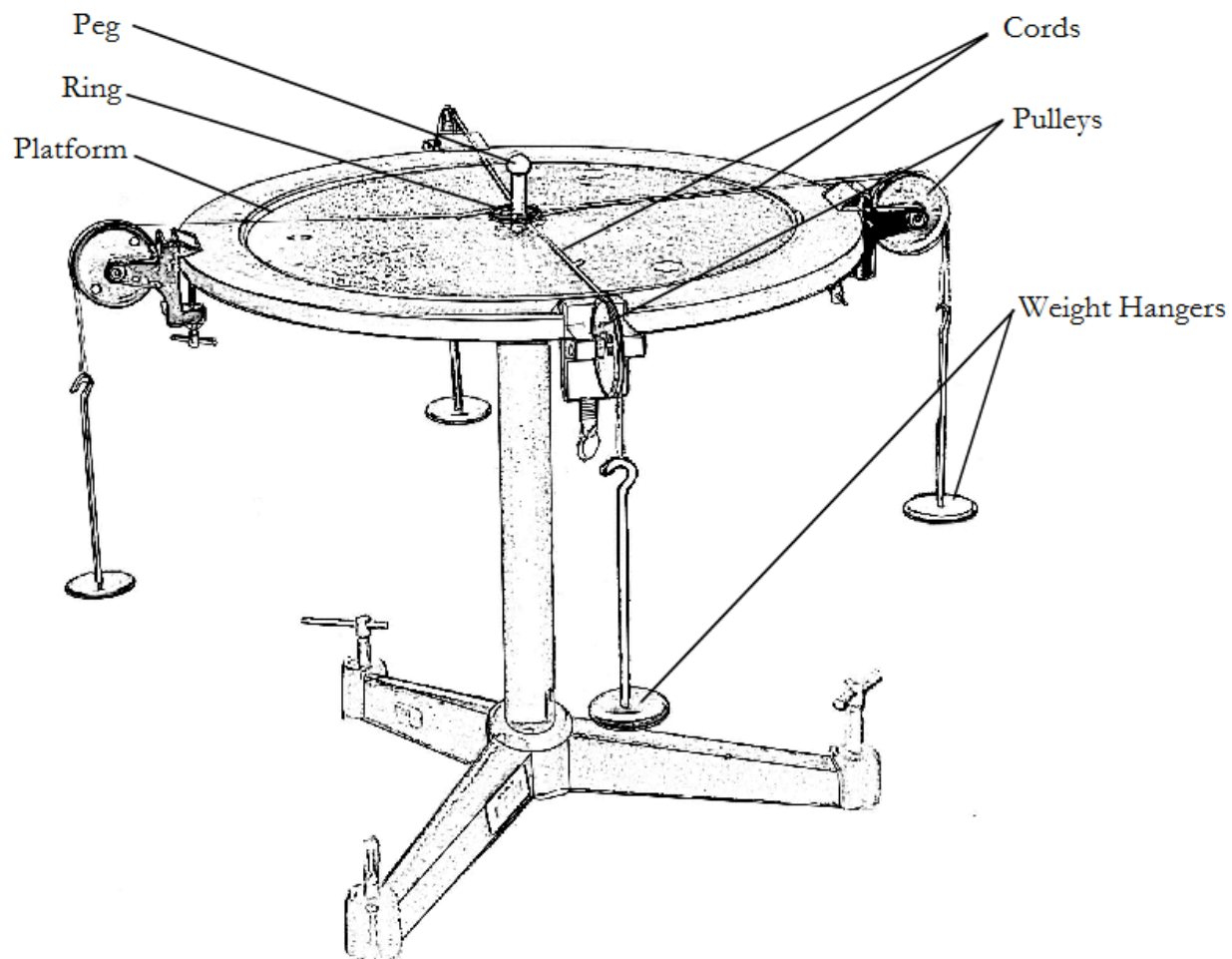


Figure 3.1 An assembled force table

Tension forces are applied to the ring by varying the total mass on each weight hanger and moving the pulleys to change the direction in which each force acts. The ring is in a state of static equilibrium when it is over the peg but not touching the peg, as shown in figure 3.2.

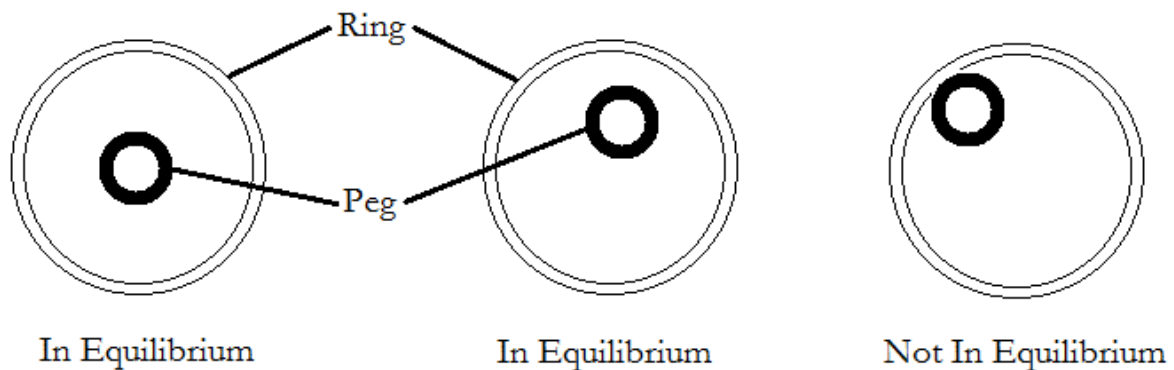


Figure 3.2 Overhead view of ring and peg positions for system in and not in equilibrium

PROCEDURE

1. Mount a pulley at 0° and attach 250 g to the cord running over it. Remember that the holder is part of the mass.
2. Mount a second pulley at 60° with a load of 350 g.
3. Holding a third cord in your hand, find the direction in which a third force should act in order to balance the system. Set the cord on a pulley in the proper position and add weights to the holder until the system is in static equilibrium, as shown in figure 3.2. It may be necessary to adjust the position of the weight holder to achieve equilibrium.
4. Record masses, angles and forces in the data sheet labeled Trial 1.
5. Repeat step 1 through 4 using a 45° angle between the two loads. Record your results in the data sheet labeled Trial 2.
6. Set up four pulleys and suspend unequal loads on the cords running over them. Arrange the system so that it is in equilibrium, and record the masses and angles. Do not have any two cords form an angle of 180° . Record your results in the data sheet labeled Trial 3.
7. Suppose you place a mass, $m = 300$ g, at $\phi = 210^\circ$ mark. Compute the masses m_a and m_b you would place at 0° and 90° to balance mass m . Try it, and see if your solution is correct. Report what masses you had to place at 0° and 90° to balance the mass at 210° .

DATA

Trial 1

i	m_i (grams)	$F_i = m_i g$ (Newtons)	ϕ_i (°)
1			0.0
2			60.0
3			

Trial 2

i	m_i (grams)	$F_i = m_i g$ (Newtons)	ϕ_i (°)
1			0.0
2			45.0
3			

Trial 3

i	m_i (grams)	$F_i = m_i g$ (Newtons)	ϕ_i (°)
1			
2			
3			
4			

CALCULATION AND ANALYSIS

1. Draw a picture showing the three forces for Trial 1. (This is sometimes called a free body diagram.) Be sure to label each vector's direction and magnitude.
2. Calculate the component forces for each force vector, F_{ix} and F_{iy} , using equation 3.3.
3. Calculate $|F_{total}|$ and $\sum_i |F_i|$ using equations 3.5 and 3.6, given below.

4. Calculate the % discrepancy of the force calculations using equation 3.4.
5. Repeat steps 2 through 4 using data from trials 2 and 3.
6. In procedure 7, was the system in equilibrium? If not, suggest some possible reasons.
7. What are the sources of experimental error in this experiment? Do any of these factors help the ring achieve static equilibrium?

REPORTING % ERROR WHEN THE ACCEPTED VALUE IS ZERO

In this experiment, the accepted value of the total force is zero. If you could measure all the forces on the ring with perfect precision, you would find that the net force vanishes. If zero is inserted into our initial % Error equation (0.2), the result is undefined because the denominator is zero. Due to experimental errors and measurement uncertainties, the calculated net force isn't zero. A useful way to characterize the accuracy of our measurements is to divide the magnitude of net force, $|F_{total}|$, by the sum of the magnitudes of all the individual forces, $\sum_i |F_i|$, multiplied by 100% as shown in equation 3.4.

$$\% \text{ Discrepancy} = \frac{|F_{total}|}{\sum_i |F_i|} \times 100\% \quad (3.4)$$

The quantities appearing in this formula are

$$\text{Magnitude of Net Force} \quad |F_{total}| = \sqrt{F_{x,total}^2 + F_{y,total}^2} \approx 0 \quad (3.5)$$

$$\text{Individual Magnitude Sum} \quad \sum_i |F_i| = \sum_i m_i g \quad (3.6)$$

Experiment 4: Newton's Second Law

OBJECTIVES

Newton's second law predicts that acceleration is a function of force and mass. To test this mathematical relationship, a good experiment must isolate each contributing component and vary it independently of the others. In this experiment, you measure the acceleration of an object by varying the force acting upon the object without changing its mass and by varying the object's mass without changing the force. The objectives of this experiment are as follows:

1. To measure the linear acceleration of objects acted on by external forces.
2. To predict the acceleration of an object by applying Newton's Second law.
3. To test the predictions using calculations and graphical methods.

THEORY

Newton's second law in vector form is shown in equation 4.1.

$$\text{Newton's second law} \quad \vec{F} = m\vec{a} \quad (4.1)$$

Here \vec{F} is the net force acting on an object, m is the mass of the object, and \vec{a} is its acceleration. If the force is constant, as, for instance, the force of gravity, the object moves with constant acceleration. Newton's second law also applies to systems of bodies considered as a whole, like two masses connected by a cord. Each of the objects in this experiment moves along a straight line. Thus it is sufficient to consider projections of the vectors on the direction of motion and we can remove the vector notation: $F = ma$.

In this experiment we measure the acceleration of a system consisting of a glider moving along a nearly frictionless air track, and a falling/hanging mass tied to the glider via a cord. The net force on the system is exerted by the gravitational force acting on the hanging mass over a low-friction pulley. If you ignore friction, the acceleration of the system according to Newton's second law is shown in equation 4.2.

$$\text{Acceleration of the glider} \quad a = g \frac{m}{M} \quad (4.2)$$

Here m is the hanging mass, M is the total moving mass, and g is the free-fall acceleration due to gravity.

The acceleration of an object is the rate of change in its velocity. If the velocity changes by an amount $\Delta \mathbf{v}$ during a time Δt , the average acceleration is shown in equation 4.3.

$$\text{Average Acceleration} \quad \vec{a} = \frac{\Delta \mathbf{v}}{\Delta t} \quad (4.3)$$

Here $\Delta t = t_2 - t_1$ and $\Delta \mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1 \equiv \mathbf{v}(t_2) - \mathbf{v}(t_1)$. If Δt becomes very small, equation 4.3 gives the instantaneous acceleration at $t_2 \cong t_1$. For the motion with constant acceleration that we study in this experiment, the average and instantaneous accelerations are the same.

In the experiment, the glider goes through two photo gates, G_1 and G_2 , its flag interrupting the light path in the gates. The computerized system measures three time intervals, T_1 , T' , and T_2 , as depicted in figure 4.1.

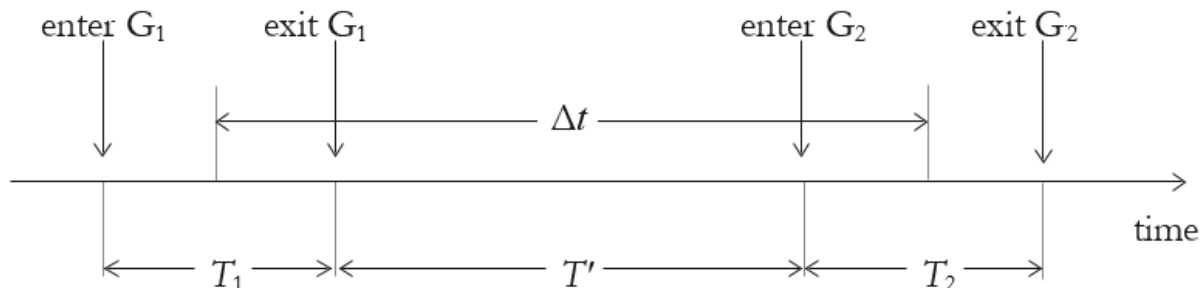


Figure 4.1 The motion of the glider flag through photo gates G_1 and G_2

Time interval T_1 begins when the flag enters the first gate, G_1 . As the flag clears this gate, T_1 ends and the second interval, T' , begins. Then, as the flag enters the second gate, G_2 , T' ends and the third interval, T_2 , begins. It ends when the flag clears gate G_2 .

From the length of the flag L measured in this experiment, you can calculate the average velocities of the glider crossing gates 1 and 2, as shown in equation 4.4.

$$\text{Average velocities} \quad \bar{v}_1 = \frac{L}{T_1}, \quad \bar{v}_2 = \frac{L}{T_2} \quad (4.4)$$

For motion with constant acceleration the velocity changes linearly with time, so these average velocities coincide with instantaneous velocities in the middle of the time intervals T_1 and T_2 . Thus the time interval Δt corresponding to the velocity change $\Delta v = v_2 - v_1$ is shown in equation 4.5.

$$\text{Elapsed time} \quad \Delta t = T' + \frac{1}{2}T_1 + \frac{1}{2}T_2 \quad (4.5)$$

Therefore, the formula for experimentally determining the acceleration is the difference of the average velocities divided by the elapsed time, as shown in equation 4.6.

$$\text{Acceleration (experimental)} \quad a = \frac{\Delta v}{\Delta t} = \frac{\left(\frac{L}{T_2} - \frac{L}{T_1} \right)}{\left(T' + \frac{1}{2}T_1 + \frac{1}{2}T_2 \right)} \quad (4.6)$$

ACCEPTED VALUES

The expected value of the acceleration for all of the mass distributions in this experiment is the result of equation 4.2. For Part A, you change the masses of the falling/hanging object and the total moving mass, so obtain a different expected value for each setup.

For Part B, the same force is acting in each case, and therefore Newton's second law predicts that the product of the total moving mass M_1 and its acceleration a_1 is equal to the product of any other mass and its acceleration under the same conditions, as shown in equation 4.7.

Newton's second law

$$M_1 a_1 = M_2 a_2 = F = mg$$

(4.7)

APPARATUS

- glider
- air track
- photo gates
- computer with PASCO interface.
- cord
- pulley
- flags
- earth's gravity

THE AIR TRACK AND GLIDERS

In this experiment you measure the acceleration of a system consisting of a glider moving along a nearly frictionless air track, and a falling/hanging mass tied to the glider via a string, as shown in figure 4.2.

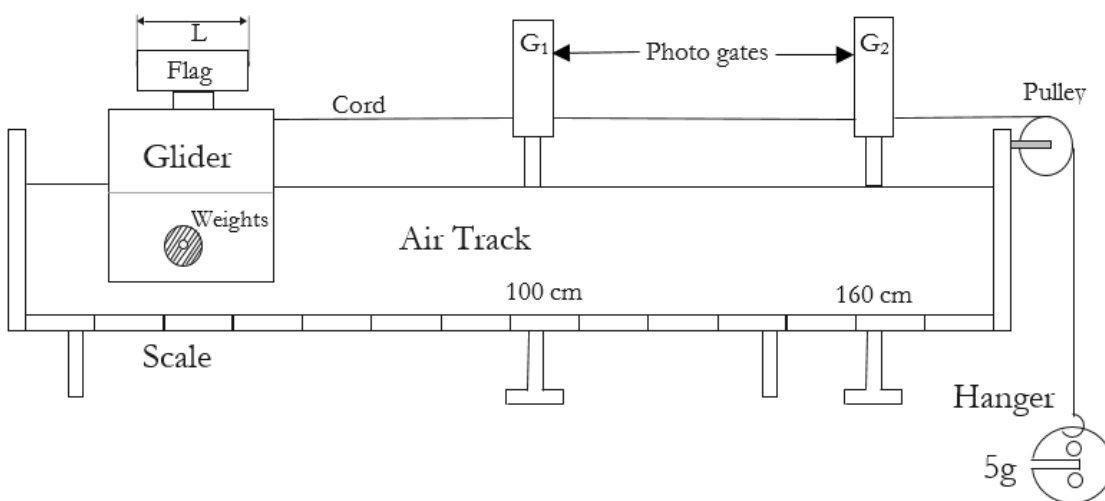


Figure 4.2 System consisting of a glider on a nearly-frictionless air track connected to a hanging weight by a cord over a low-friction pulley

The net force on the system is exerted by the gravitational force acting on the hanging mass. The glider is supported by a cushion of air coming out of the holes in the horizontal frame, so that friction is almost eliminated and can be neglected.

The glider has two rods on which weights may be set. You should have four fifty gram weights (shiny cylinders) and five 5 gram weights (flat slotted disks). The slotted disks stay firmly on the glider if the fifty-gram weights are placed on top of them.

Your experimental station also includes a pair of photo gates, connected to a computer, which function as an electric stopwatch.

PROCEDURE

PART A: ACCELERATION AS A FUNCTION OF FORCE WITH A CONSTANT MASS

1. Measure the length of the flag, using the scale on the air track and record this as length of the flag, L .

2. Measure the mass of the weight hanger and record this value as the mass of the weight hanger, m_{hanger} .
3. Put two 50 gram weights, five 5 gram weights, the flag, the cord, and the weight hanger on the glider, and weigh everything together. Record this as the total moving mass, M , in the data table for Part A.
4. Place Photo gate G_1 at 110 cm, and photo gate G_2 at 160 cm. Make sure that G_1 is connected to "DIGITAL CHANNELS" input 1 of the interface, and G_2 to input 2, respectively.
5. Place the glider on the air track. Turn on the blower and run the glider slowly through the gates. Make sure that the flag blocks the light but does not hit the gates. Then turn off the blower.
6. Turn on the PC with Interface turned ON and log in as "student" using the password provided by the lab instructor.
7. From desktop click the icon "Newton Second Law".
8. Move the glider to 100 cm on the air track.
9. Place two 5g weights and one 50 g weight onto the rods on each side of the glider.
10. Let the cord fall over the pulley at the end of the track and attach a 5 gram weight to the hanger, as shown in figure 4.2.
11. Measure the motion of the glider with a 5 gram hanging/falling weight by performing the following steps:
 - A. Click "Start" on the PC monitor: The system is ready to collect the data.
 - B. Hold the glider at 100 cm and turn on the air blower, wait for the pitch of the blower to reach a stationary level.
 - C. Let the glider go. The times T_1 , T' , and T_2 , respectively, appear in the first row of the table with columns labeled "Timer 1", "Timer2", and "Timer 3". Be careful not to let the glider bounce back into the second gate!
 - D. Remove the glider from the air track and replace it at 100 cm without disturbing the photo gates. Do not stop the PC data acquisition after the glider crosses the two gates and do not switch off the air blower.
 - E. Repeat runs described by steps C and D four times. Each time a new row in the table is added. If one or more of the runs yields times substantially different from the results of other runs, an error occurred. In this case clear the data sheets and repeat the experiment.
 - F. Record the average values for T_1 , T' , and T_2 on your data sheet.
 - G. To clear all entries, click "Stop", and from "Experiment" menu click "Clear ALL Data Runs".
12. Repeat step 11 by increasing the hanging mass by transferring 5 gram weights from the glider to the hanger. For each value of the mass on the hanger $m = 5, 10, 15, 20$, and 25 grams (plus the mass of the hanger m_{hanger} itself) make five runs. Record the average times T_1 , T' , T_2 in the data sheet for Part A. Always transfer weights from the glider to the hanger so the total mass of the system remains constant. When removing weights from the glider, ensure that the weights on each side of the glider are approximately equal, otherwise the glider can slip off the air cushion and add friction to your measurements.

PART B: ACCELERATION AS A FUNCTION OF MASS WITH CONSTANT FORCE

1. Put two 50 gram weights, one 5 gram weight, the flag, the cord, and the weight hanger on the glider, and weigh everything together. Record this as the total moving mass, M_1 , in the data Section for Part B.

- Place the glider on the track. Place the 50 g weights on the rods, one on each side of the glider. Let the cord fall over the pulley at the end of the track and attach a 5 gram weight to the hanger, as shown in figure 4.2.
- Measure the motion of the glider attached to a 5 gram hanging/falling weight using the same procedure as Part A. Do five runs, and record the average value of T_1 , T' , and T_2 in the first row of the data table for Part B.
- Remove the two 50 gram weights from the glider. Calculate the resulting mass M_2 using the value of M_1 and enter M_2 in the data table for Part B.
- Measure the motion of the glider attached to a 5 gram hanging/falling weight using the same procedure as Part A. Record the average values of T_1 , T' , T_2 in the data table for Part B.

DATA

m (g)	T_1 (s)	T' (s)	T_2 (s)
$5 + m_{\text{hanger}}$			
$10 + m_{\text{hanger}}$			
$15 + m_{\text{hanger}}$			
$20 + m_{\text{hanger}}$			
$25 + m_{\text{hanger}}$			

M (g)	T_1 (s)	T' (s)	T_2 (s)
$M_1 = \underline{\hspace{2cm}}$			
$M_2 = \underline{\hspace{2cm}}$			

CALCULATION AND ANALYSIS

- Calculate the acceleration for each of your five m values using your data from Part A. To do this use equation 4.6 and the average values for T_1 , T' , and T_2 recorded by the computer.
- Calculate and record the accelerating force, $F = mg$, for each of your five m values. Use $g = 9.80 \text{ m/s}^2$.
- Draw a graph of the accelerating force F versus the acceleration a , with a on the x -axis. Draw a best fit straight line through the five points on your graph.
- Find the slope of the straight line you fit to the points and compare it with the total mass of the system M . By what percent does the slope differ from M ?
- Calculate the acceleration for your two M values using your data from Part B. To do this use equation 4.6 and the average values for T_1 , T' , and T_2 recorded by the computer.

6. Using the average acceleration a_1 of the system with mass M_1 , and the average acceleration a_2 for mass M_2 from Part B, calculate and compare M_1a_1 and M_2a_2 . By what percent do the two values differ?

Experiment 5: Conservation Laws in Collisions

OBJECTIVES

The conservation laws for linear momentum and energy state that the total momentum and energy of an isolated system remain constant. This is true at all times in the system, even if some momentum or energy is transferred from one component of the system to another. In this experiment, you measure the motion and mass of a system comprised of colliding objects and calculate the energy and momentum of the system before and after the collision. The objectives of this experiment are as follows:

1. To measure the motion of objects that undergo elastic and inelastic collisions
2. To calculate changes in energy and momentum in elastic and inelastic collisions
3. To test the conservation laws for linear momentum and energy

THEORY

A conservation law states that a measurable property of an isolated physical system does not change with time. Two conservation laws are particularly important: conservation of linear momentum and conservation of energy

CONSERVATION OF LINEAR MOMENTUM

The law of conservation of linear momentum states that in a system where the sum of external forces is zero, the total momentum of a system does not change. In a system composed of n objects, the total momentum is given by the vector sum shown in equation 5.1.

$$\text{Total Momentum} \quad p = \sum_{i=1}^n m_i \vec{v}_i = m_1 \vec{v}_1 + m_2 \vec{v}_2 + \dots + m_n \vec{v}_n \quad (5.1)$$

Here, m_i and \vec{v}_i are the mass and velocity of object number i , respectively. As the objects interact with one another, the individual velocities may change, but the total momentum \mathbf{p} remains constant. In this experiment, you study collisions between two objects. Before the collision suppose one object has mass m_1 and is moving at velocity \vec{v}_{1i} and the other object has mass m_2 and is moving at velocity \vec{v}_{2i} . After the collision their velocities are \vec{v}_{1f} and \vec{v}_{2f} . The law of conservation of momentum predicts that the total momentum is the same before and after the collision, as shown in equation 5.2.

$$\text{Conservation of Momentum} \quad m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f} \quad (5.2)$$

Velocities are vector quantities, with direction as well as magnitude. In this experiment they act along a straight line so they have components only along one axis. However velocity in one direction (e.g. to the right) must be taken as positive while a velocity in the opposite direction must be taken as negative.

In an inelastic collision, two objects stick together and become one combined object. Momentum is still conserved, but the calculation changes to meet the new condition as shown in equation 5.3.

$$\text{p for Inelastic Collisions} \quad m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = (m_1 + m_2) \vec{v}_f \quad (5.3)$$

Here, before the collision one object has mass m_1 and is moving with velocity \vec{v}_{1i} and the other object has mass m_2 and is moving with velocity \vec{v}_{2i} . After the collision the combined mass is $(m_1 + m_2)$ and the velocity is \vec{v}_f .

CONSERVATION OF ENERGY

The kinetic energy of an object of mass m moving at speed $|\vec{v}|$ is defined as $\frac{1}{2} m |\vec{v}|^2$. Speed is the magnitude of the velocity vector; both speed and kinetic energy are scalar quantities. The law of conservation of energy states that the total energy of an isolated system is constant, but this does not imply that the kinetic energy is constant. In a collision, the objects may be deformed or set into vibration. Then some or all of the kinetic energy is converted into heat or other forms of energy.

You study two kinds of collisions in this experiment: collisions which are nearly elastic (only a small fraction of the kinetic energy is lost), and inelastic collisions, in which a large fraction of the kinetic energy is lost. By using a rubber band so that the gliders bounce away from each other with little loss of kinetic energy, we obtain (nearly) elastic collisions. In an elastic collision the total kinetic energy does not change, as shown in equation 5.4.

$$\text{Energy in Elastic Collisions} \quad KE_i = KE_f \quad (5.4)$$

Here KE_i and KE_f are the initial and final kinetic energies, as shown in equations 5.5 and 5.6.

$$\text{Initial Kinetic Energy} \quad KE_i = \frac{1}{2} m_1 |\vec{v}_{1i}|^2 + \frac{1}{2} m_2 |\vec{v}_{2i}|^2 \quad (5.5)$$

$$\text{Final Kinetic Energy} \quad KE_f = \frac{1}{2} m_1 |\vec{v}_{1f}|^2 + \frac{1}{2} m_2 |\vec{v}_{2f}|^2 \quad (5.6)$$

If we fix a system so that the objects stick together after colliding, we obtain the maximum possible loss in kinetic energy. The initial and final kinetic energies for an inelastic collision when one object has no initial velocity ($|\vec{v}_{2i}| = 0$) are shown in equations 5.7 and 5.8.

$$\text{Initial Kinetic Energy} \quad KE_i = \frac{1}{2} m_1 |\vec{v}_{1i}|^2 \quad (5.7)$$

$$\text{Final Kinetic Energy} \quad KE_f = \frac{1}{2} (m_1 + m_2) |\vec{v}_f|^2 \quad (5.8)$$

To calculate the percent kinetic energy lost in any collision, use equation 5.9.

$$\text{Percent Kinetic Energy Lost} \quad \%KE_{lost} = \frac{KE_i - KE_f}{KE_i} \times 100 \quad (5.9)$$

Here, KE_i and KE_f are the energies calculated in equations 5.5 through 5.8.

ACCEPTED VALUES

In this experiment the mass and velocity values depend on the experimental conditions, so there is no single accepted value. The accepted value of the total energy in a collision is the kinetic energy of the objects before colliding KE_p , given in equation 5.5 (elastic) or equation 5.7 (inelastic).

For momentum, you measure the masses of the objects and test to what degree the velocities support the law of conservation of momentum. For an inelastic collision when one object has no initial velocity ($|\vec{v}_{2i}| = 0$), we can rearrange equation 5.3 algebraically to compare the ratio of the initial and final masses to the initial and final velocities, as shown in equation 5.10.

$$\text{Ratio for Inelastic Collisions} \quad \frac{|\vec{v}_f|}{|\vec{v}_{1i}|} = \frac{m_1}{(m_1 + m_2)} \quad (5.10)$$

Here, the ratio of the masses before and after the collision $\frac{m_1}{(m_1 + m_2)}$ is the accepted value to test the law of conservation of momentum.

For an elastic collision, we can rearrange equation 5.2 algebraically to compare the ratio of the initial and final masses to the initial and final velocities, as shown in equation 5.11.

$$\text{Ratio for Elastic Collisions} \quad \frac{v_{2f} - v_{2i}}{v_{1i} - v_{1f}} = \frac{m_1}{m_2} \quad (5.11)$$

Since v_{1i} and v_{2f} are negative we can multiply numerator and denominator by (-1) and write this as

$$\text{Ratio for Elastic Collisions} \quad \frac{|v_{2f}| + |v_{2i}|}{|v_{1i}| + |v_{1f}|} = \frac{m_1}{m_2} \quad (5.12)$$

Hence the ratio of the masses $\frac{m_1}{m_2}$ is the accepted value to test the law of conservation of momentum.

APPARATUS

- | | | |
|-------------|---------------------------------|---------------------------------|
| • Air track | • photo-gates | • 2 metal flags |
| • 2 gliders | • elastic and inelastic bumpers | • computer with PASCO interface |
| • weights | | |

COLLIDING GLIDERS ON AN AIR TRACK

The colliding objects in this experiment are gliders, mounted on an air track to minimize friction. The photo gates are used to measure the speed of the gliders, as in experiment 4. You use the inelastic bumpers for inelastic collisions, and the elastic bumpers for nearly elastic collisions.

The inelastic bumpers stick together when two gliders collide. The pin on one glider sticks in the wax-filled hole on the other, as shown in figure 5.1. The flag is used to measure the speed of a glider before and after the collision by timing its passage through the photo gates.

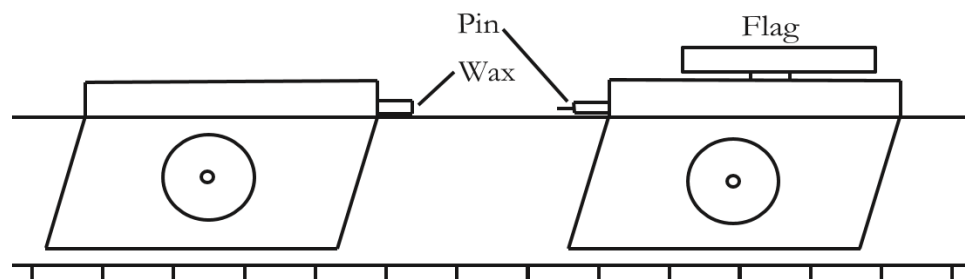


Figure 5.1 Gliders fitted with the inelastic bumpers and one flag

The elastic bumper prevents the loss of energy due to heat and vibration when two gliders collide. The bumper is attached to one glider only, as shown in figure 5.2. The flags are used to measure the speed of both gliders before and after the collision by timing their passage through the photo gates.

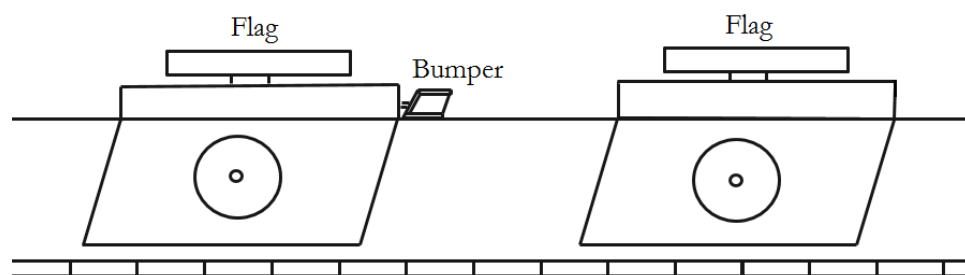


Figure 5.2 Gliders fitted with flags and one elastic bumper

PROCEDURE

PART A: INELASTIC COLLISION

1. Attach the flag to one glider and the inelastic bumpers to both gliders.
2. Weigh the two gliders individually. Record the mass of the glider with the flag as m_1 and the mass of the other glider as m_2 in the data table for Part A.
3. Measure the length of the flag and record it as L in the data table for Part A.
4. Adjust the height of the photo-gates so that the light is blocked by the flag, not by the entire body of the glider.
5. Place the photo-gates at about 70 and 130 cm.
6. Place the glider with the flag on the air track. Turn on the blower and run the glider slowly through the gates. Make sure that the flag blocks the light but does not hit the gates. Then turn off the blower.
7. Turn on your computer.
8. Double-click the icon labeled "Conservation of energy" in your desktop.
9. Set up the two gliders as shown in figure 5.3 with glider m_1 to the right or left of both photo-gates and glider m_2 between the photo-gates.

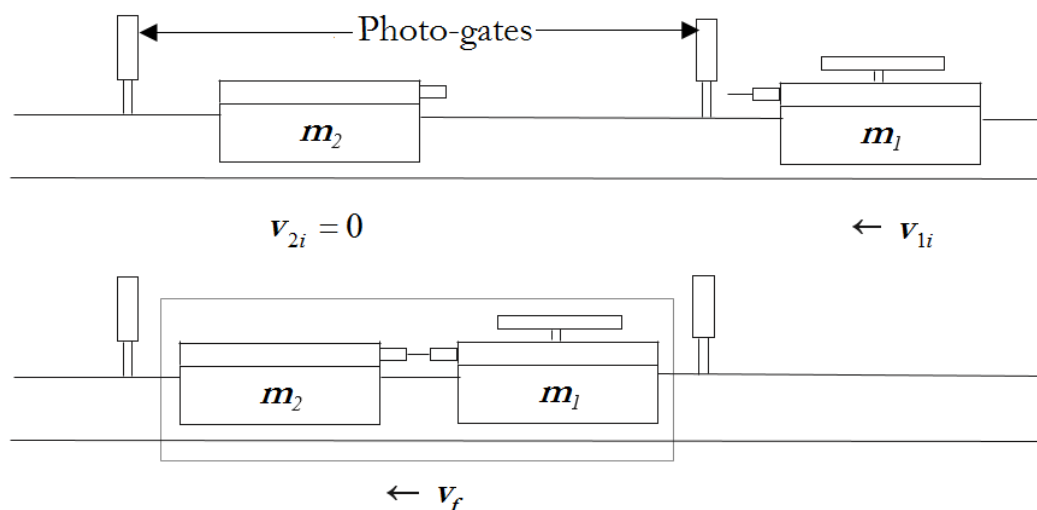


Figure 5.3 The initial and final motion of the gliders in an inelastic collision

10. Choose “Start” on the computer interface.
11. Turn on the air track blower.
12. Release glider m_2 (without moving it) and push glider m_1 towards glider m_2 at a moderate speed. Be sure you have finished pushing before the flag enters the photo-gate. Glider m_1 should strike glider m_2 and stick to it, and the two gliders together should pass through the second photo-gate. Be sure the gliders collide after the flag has passed through the first photo-gate, and that the gliders do not bounce back between the photo-gates.
13. Repeat step 12 four more times.
14. Choose “Stop” on the computer interface.
15. Record the values for T_1 and T_2 for all five trials from the computer in the data table for Part A.

PART B. ELASTIC COLLISION

1. Remove the inelastic bumpers and set them aside.
2. Attach a flag and elastic bumper to the glider without a flag.
3. Weigh each glider and record the mass of the one with the bumper as m_1 and the other as m_2 in the data table for Part B.
4. Measure the length of the flags and record them as L_1 and L_2 in the data table for Part B.
5. Place the photo-gates near 90 and 160 cm.
6. Place the gliders on the air track. Turn on the blower and run the gliders slowly through the gates. Make sure that the flag on glider m_1 blocks photo-gate 1 and the flag on glider m_2 blocks photo-gate 2. Then turn off the blower.
7. Place the gliders as in the upper diagram in figure 5.4, with glider m_1 to the right of both photo-gates and glider m_2 to the left of both photo-gates.

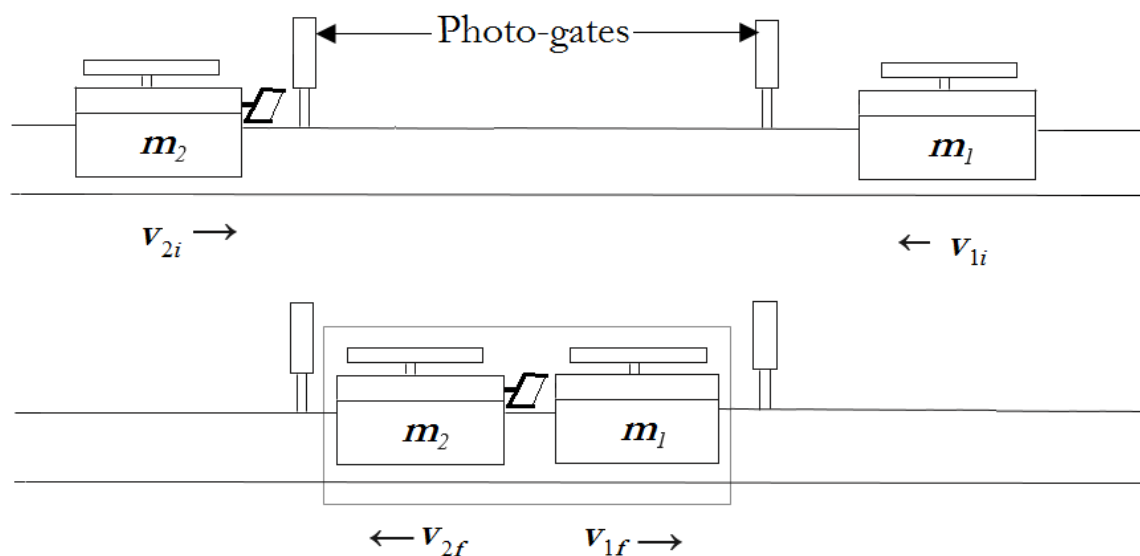


Figure 5.4 The initial and final motion of the gliders in an elastic collision

8. Turn on the blower and choose "Start" on the computer interface.
9. Push the gliders towards each other so that they collide between the gates. Be sure you have stopped pushing them before they enter the gates. The computer records the time it takes for each glider to pass through the photo-gates, both before (*i*) and after (*f*) the collision. The computer records the passage of the gliders before the collision in one row and the passage of the gliders after the collision in the next row. Record the data from the first row as T_{1i} and T_{2i} in the data table for Part B. Record the data from the second row as T_{1f} and T_{2f} .
10. Without choosing "Stop" on the computer, repeat procedure 9 four more times so that the gliders collide a total of five times, which the computer records in a total of 10 rows.
11. Choose "Stop" on the computer interface.
12. Record the data from each pair of rows on the computer into the data table for Part B. Record the data from the first row of each pair as T_{1i} and T_{2i} and the data from the second row of each pair as T_{1f} and T_{2f} .

DATA

Part A: Inelastic Collision

$$m_1(\text{g}) = \underline{\hspace{2cm}}$$

$$m_2(\text{g}) = \underline{\hspace{2cm}}$$

$$\frac{m_1}{m_1+m_2} = \underline{\hspace{2cm}}$$

$$L(\text{m}) = \underline{\hspace{2cm}}$$

i	$T_{1i}(\text{s})$	$T_{2i}(\text{s})$
1		
2		
3		
4		
5		

Part B: Elastic Collision

$$m_1(\text{g}) = \underline{\hspace{2cm}}$$

$$m_2(\text{g}) = \underline{\hspace{2cm}}$$

$$\frac{m_1}{m_2} = \underline{\hspace{2cm}}$$

$$L_1(\text{m}) = \underline{\hspace{2cm}}$$

$$L_2(\text{m}) = \underline{\hspace{2cm}}$$

i	$T_{1i}(\text{s})$	$T_{2i}(\text{s})$	$T_{1f}(\text{s})$	$T_{2f}(\text{s})$
1				
2				
3				
4				
5				

CALCULATION AND ANALYSIS

1. For inelastic collisions, calculate the values of \mathbf{v}_{1i} and \mathbf{v}_f by dividing the length of the flag L by the time it took for the flag to pass through the photo-gate, T_{1i} and T_f and then calculate $\frac{|\mathbf{v}_f|}{|\mathbf{v}_{1i}|}$ for all five of your trials.
2. Calculate the average (mean) of the five values $\frac{|\mathbf{v}_f|}{|\mathbf{v}_{1i}|}$, the deviation of each value, the uncertainty of the average (mean), and the percent uncertainty for the average (mean) using equations 0.1, 0.3, 0.6 and 0.7.
3. Compare your average value of $\frac{|\mathbf{v}_f|}{|\mathbf{v}_{1i}|}$ with the accepted value of $\frac{m_1}{(m_1+m_2)}$. Calculate the percent error of your calculation using equation 0.2. Is the % error less than or greater than the % uncertainty? Does the experimental evidence support the conservation of momentum?
4. For ONE of your collisions, calculate the kinetic energy of \mathbf{m}_1 before the collision and the kinetic energy of $\mathbf{m}_1 + \mathbf{m}_2$ after the collision. Calculate the percentage of the initial kinetic energy lost in the collision, as shown in equation 5.9.
5. For each of the five elastic collisions studied, calculate the values of \mathbf{v}_{1i} , \mathbf{v}_{1f} , \mathbf{v}_{2i} and \mathbf{v}_{2f} by dividing the length of the flags L_1 and L_2 by the time it took for the flag to pass through the photo-gate, T_{1i} , T_{2i} , T_{1f} and T_{2f} . Then calculate the value of $\frac{|\mathbf{v}_{2f}| + |\mathbf{v}_{2i}|}{|\mathbf{v}_{1i}| + |\mathbf{v}_{1f}|}$ for all five of your trials.
6. Calculate the average (mean) of your five values of $\frac{|\mathbf{v}_{2f}| + |\mathbf{v}_{2i}|}{|\mathbf{v}_{1i}| + |\mathbf{v}_{1f}|}$, the deviation of each value, the uncertainty of the average (mean), and the percent uncertainty for the average (mean).
7. Compare your average value of $\frac{|\mathbf{v}_{2f}| + |\mathbf{v}_{2i}|}{|\mathbf{v}_{1i}| + |\mathbf{v}_{1f}|}$ with the accepted value $\frac{m_1}{m_2}$. Calculate the percent error of your calculation using equation 0.2. Is the % error less than or greater than the % uncertainty? Does the experimental evidence support the conservation of momentum?
8. For ONE of your collisions, calculate the kinetic energy of the two masses before and after the collision, using the definition of the kinetic energy. Calculate the percentage of the initial kinetic energy lost in the collision, as shown in equation 5.9.
9. How does this compare with what happened in the inelastic collision?

Experiment 6: The Ballistic Pendulum

OBJECTIVES

An inelastic collision is a collision in which kinetic energy is not conserved. The total energy is always conserved, but in an inelastic collision some of the initial kinetic energy is converted to other forms such as heat and vibration. The ballistic pendulum provides a means to measure the gravitational potential energy converted from kinetic energy in an inelastic collision. The objectives of this experiment are as follows.

1. To observe the motion of a ballistic pendulum.
2. To calculate the initial and final velocities and the change in kinetic energy during the collision.
3. To make a comparison to the initial velocity determined by observing ballistic motion of a projectile.

THEORY

When two objects collide, they exert equal and opposite forces on each other. If there are no external forces, this implies that the total momentum is conserved, namely the total momentum before and after the collision is the same. This is true for any collision whether elastic (kinetic energy conserved) or inelastic (kinetic energy not conserved).

The ballistic pendulum is a device sometimes employed to determine the speed of a bullet. The pendulum consists of a large bob hollowed out to receive the projectile and is suspended by a light rod. When the projectile is fired into the pendulum bob it remains imbedded in it. This is an inelastic collision. Conservation of momentum implies that the total momentum of the bob and the projectile just after the collision is equal to the total momentum of the projectile just before the collision (the momentum of the bob just before the collision is zero). Momentum in such a collision is conserved, as shown in equation 6.1.

$$\mathbf{p} \text{ for Inelastic Collisions} \quad m_1 \vec{v}_{1i} = (m_1 + m_2) \vec{v}_f \quad (6.1)$$

Here, the projectile has mass m_1 and is moving with an initial velocity \vec{v}_{1i} and the bob has mass m_2 . After the collision the combined system (projectile and bob) has mass $(m_1 + m_2)$ and moves with a common velocity \vec{v}_f . The kinetic energies in this situation before and after the collision are shown in equations 6.2 and 6.3.

$$\text{Initial Kinetic Energy} \quad E_i = \frac{m_1 |\vec{v}_{1i}|^2}{2} \quad (6.2)$$

$$\text{Final Kinetic Energy} \quad E_f = \frac{(m_1 + m_2) |\vec{v}_f|^2}{2} \quad (6.3)$$

As a result of the collision, the pendulum with the imbedded projectile swings about its point of support and rises to a height h (see figure 6.1). Nearly all the kinetic energy of the system just after the collision is converted to potential energy at the top of the pendulum swing. The conversion of

kinetic energy to potential energy results in an equation to calculate the final velocity $|\vec{v}_f|$ of the system right after the collision as shown in equation 6.4.

$$\begin{array}{ll} \text{Energy Conservation} & E_f = \frac{(m_1+m_2)|\vec{v}_f|^2}{2} = (m_1 + m_2)gh \\ \text{Final Velocity} & |\vec{v}_f| = \sqrt{2gh} \end{array} \quad (6.4)$$

The initial velocity $|\vec{v}_{li}|$ is calculable by substituting $|\vec{v}_f|$ from equation 6.4 and the values of m_1 and m_2 into equation 6.1, as shown in equation 6.5.

$$\text{Initial Velocity} \quad |\vec{v}_{li}| = \frac{(m_1 + m_2)}{m_1} |\vec{v}_f| = \frac{(m_1 + m_2)}{m_1} \sqrt{2gh} \quad (6.5)$$

LOSS OF ENERGY IN A COLLISION

The energy lost in the collision is the difference between the initial and final kinetic energies. For the ballistic pendulum where only the projectile has an initial velocity, substituting the value of equation 6.1 into the difference in initial and final energies results in a simplified way to calculate the loss of energy shown in equation 6.6.

$$\text{Energy Lost} \quad \Delta E \equiv E_i - E_f = \frac{m_1 m_2 |\vec{v}_{li}|^2}{2(m_1 + m_2)} \quad (6.6)$$

Equation 6.6 predicts the energy lost in an inelastic collision using the initial velocity $|\vec{v}_{li}|$.

ACCEPTED VALUES

To obtain an accepted value for the initial velocity of the bullet we observe ballistic motion of the projectile without the pendulum.

You can calculate the initial velocity of the projectile $|\vec{v}_{li}|$ by firing it horizontally and allowing it to fall on the floor without striking the pendulum (see figure 6.2). In the x-direction the initial velocity is $|\vec{v}_{li}|$ and there is no acceleration (air resistance is neglected). The horizontal displacement of the projectile x is shown in equation 6.7.

$$\text{Horizontal displacement} \quad x = |\vec{v}_{li}|t \quad (6.7)$$

Here t is the time of flight of the projectile. In the y-direction the initial velocity is zero and the motion is that of a freely falling object starting from rest. Therefore the vertical displacement is calculated using equation (6.8).

$$\text{Vertical displacement} \quad y = \frac{1}{2}gt^2 \quad (6.8)$$

Here the positive direction along the y-axis is down. Eliminating t from equations (6.7) and (6.8), the initial velocity is calculated using equation (6.9).

$$\text{Initial Velocity} \quad |\vec{v}_{li}| = x \sqrt{\frac{g}{2y}} \quad (6.9)$$

The value calculated in equation (6.9) is the accepted value for the initial velocity $|\vec{v}_{li}|$. From this initial value, the accepted value for the energy lost in the collision is calculated using equation 6.6.

APPARATUS

- | | | |
|----------------------|---------------|----------------|
| • ballistic pendulum | • level | • plumb line |
| • balance | • meter stick | • carbon paper |

BALLISTIC PENDULUM

The apparatus used in this experiment is a combination of a ballistic pendulum and a spring gun for propelling the projectile as shown in figure 6.1.

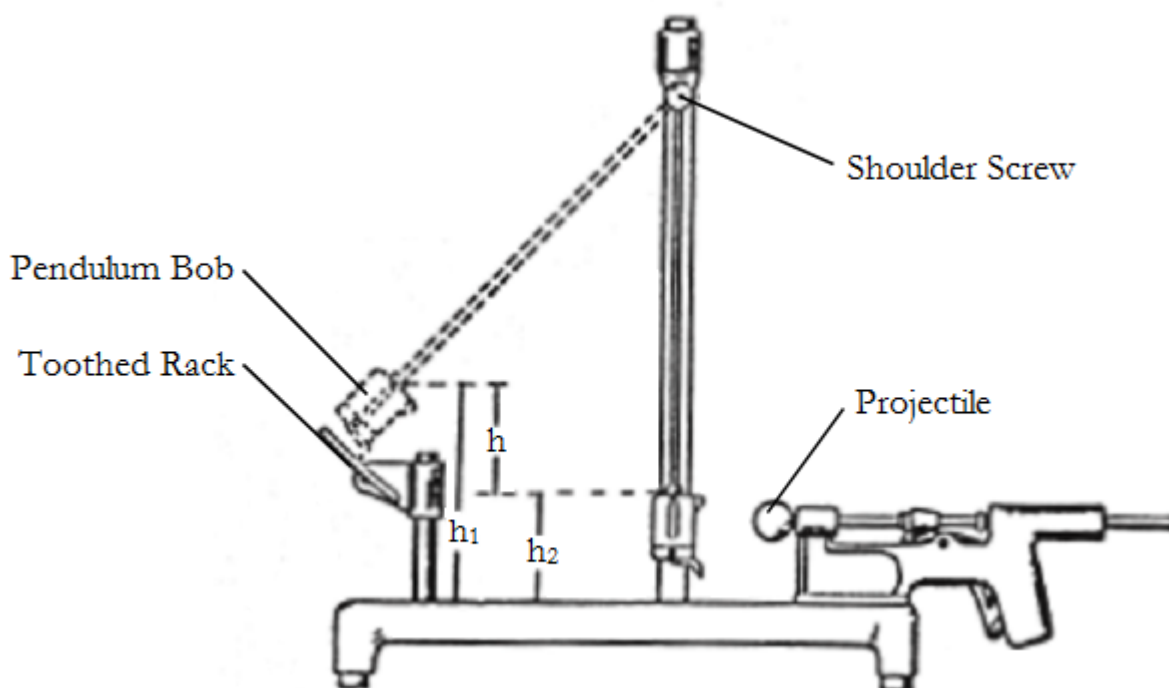


Figure 6.1 The parts of the ballistic pendulum, showing the height of pendulum bob before and after the collision with the projectile

The pendulum consists of a massive cylindrical bob hollowed out to receive the projectile and suspended by a strong, light rod. The pendulum may be removed from its supporting yoke by unscrewing the shoulder screw at the yoke. The projectile is a brass ball which, when propelled into the pendulum bob, is caught and held in such a position that its center of gravity lies on the axis of the suspension rod. A pointer on the side of the bob indicates the height of the pendulum-projectile system. When the projectile is shot into the pendulum, the pendulum swings upward and is caught at its highest point by a pawl which engages a tooth on the curved rack. The height of rise is determined by measuring the heights of the center of gravity above the table before and after the impact. This gives a direct measurement of the height, h , the ball and pendulum rise. The numbers on the toothed rack do not measure height, but the number of teeth if counted from right to left.

The projectile is launched from a spring-loaded gun that can be fired with the pendulum out of the path of the projectile. When the projectile is shot without the pendulum to catch it, the projectile

moves as a projectile with a measurable range along the x-axis when fired from a specific height on the y-axis, as shown in figure 6.2. This alternate setup is used to obtain the accepted value for the initial velocity of the projectile using equations 6.7, 6.8, and 6.9.

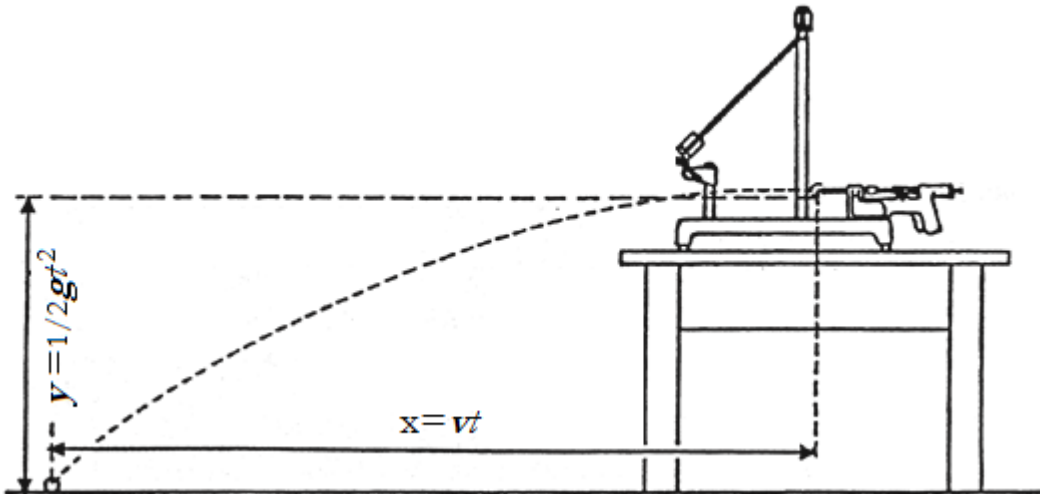


Figure 6.2 Measuring the range and fall of the projectile fired without the pendulum

PROCEDURE

PART A: INELASTIC COLLISION WITH A BALLISTIC PENDULUM

1. Place the pendulum in its central (straight down) position and allow it to swing freely. Get the gun ready for firing by compressing the spring and, when the pendulum is at rest, pull the trigger, thereby firing the projectile into the pendulum bob. This causes the pendulum with the projectile inside it to swing up along the rack where it is caught at its highest point. Record the notch on the curved scale reached by the pawl when it catches the pendulum in the data table for Part A. To remove the projectile from the pendulum, push it out with your finger or a rubber-tipped pencil while holding up the spring catch.
2. Repeat procedure 1 five more times, recording the position of the pendulum on the rack each time.
3. Calculate the average (mean) value of the highest notch reached by the pendulum and record it in the data table for Part A.
4. Raise the pendulum until its pawl is engaged in the notch corresponding most closely to the average (mean) value of the highest notch reached by the pendulum. Measure h_1 , the vertical distance from the base of the apparatus to the index point attached to the pendulum as shown in figure 6.1. This index point is at the vertical position of the center of gravity of the pendulum and projectile.
5. Release the pendulum and allow it to hang in its lowest position. Measure h_2 , the vertical distance from the base of the apparatus to the index point.
6. Calculate the difference between these two values and record it as the average height h , which is the vertical distance through which the center of gravity of the system is raised after shooting the projectile.

7. Loosen the outer shoulder screw and carefully remove the pendulum from its support. It is important that only the outer thumbscrew is loosened-not both-since this ensures proper alignment of the pendulum arm with the gun. Measure the masses of the pendulum and of the projectile and record the results in the data table for Part A. Replace the pendulum and carefully adjust the thumb screw.

PART B: BALLISTIC TRAJECTORY OF A PROJECTILE

1. Set the apparatus near one edge of a level table or stool. Swing the pendulum up onto the rack so that it does not interfere with the free flight of the projectile as shown in figure 6.2. Check that the base of the apparatus is accurately horizontal using a level; if not, place sheets of paper under the feet as required.
2. Fire the projectile out across the floor and make note of the approximate spot where it hits the floor. Place on this spot a sheet of white paper and secure it with masking tape. Cover this sheet with a sheet of carbon paper, carbon side down, so that a record is made of the exact spot where the projectile strikes. Fire the projectile six times. Note: It is important that the apparatus not move on the bench between shots.
3. Measure the range for each spot; this is the horizontal distance from the point of projection to the point of contact with the floor as shown in figure 2. Measure the vertical height from the bottom of the projectile to the floor.

DATA

Part A: Inelastic collision with a ballistic pendulum

Trial	pawl's position (notch number)
1	
2	
3	
4	
5	
6	
Average	

$$m_1(\text{g}) = \underline{\hspace{2cm}} \quad m_2(\text{g}) = \underline{\hspace{2cm}}$$

$$m_1 + m_2(\text{g}) = \underline{\hspace{2cm}}$$

$$h_1(\text{m}) = \underline{\hspace{2cm}} \quad h_2(\text{m}) = \underline{\hspace{2cm}}$$

$$h(\text{m}) = \underline{\hspace{2cm}}$$

Part B: Ballistic trajectory of a projectile

$$y(\text{m}) = \underline{\hspace{2cm}}$$

Trial	x = Range of Projectile (m)
1	
2	
3	
4	
5	
6	
Average	

CALCULATION AND ANALYSIS

1. Calculate the velocity $|\vec{v}_f|$ of the system immediately after collision and the experimental value for the initial velocity $|\vec{v}_{li}|$ of the projectile using data from part A.
2. Calculate the accepted value for the initial velocity $|\vec{v}_{li}|$ of the projectile using the data from Part B.
3. Calculate the percent error of the experimental value for the initial velocity $|\vec{v}_{li}|$ of the projectile.
4. Calculate the total translational kinetic energy before (E_i) and after (E_f) the impact using the calculated velocities from Part A.
5. Calculate the experimental value for the energy lost by subtracting E_f from E_i .
6. Calculate the accepted value for the energy lost by using the accepted value for the initial velocity $|\vec{v}_{li}|$ of the projectile in equation 6.6.
7. Calculate the percent error of the experimental value for the loss of energy.
8. Calculate the percentage of the kinetic energy converted to potential energy in this experiment.

Experiment 7: Rotational Equilibrium

OBJECTIVES

When the forces acting on an object do not make the object rotate, the object is in a state of rotational equilibrium. In this experiment you arrange forces that put an object into rotational equilibrium. You measure the vector quantities of these forces, calculate the torques exerted by these forces, and calculate the net torque acting on the object. The objectives of this experiment are as follows:

1. To measure the forces on an object in rotational equilibrium
2. To calculate the torques exerted by these forces
3. To test the hypothesis that an object in rotational equilibrium has no net torque acting on it

THEORY

An object is rigid if its shape remains unchanged when forces are applied. A rigid object is in translational equilibrium when it has no linear acceleration. This was studied in Experiment 3. A rigid object is in rotational equilibrium when it has no angular acceleration. Therefore, to be in equilibrium, two conditions must be satisfied.

1. As described by Newton's second law, the vector sum of the forces acting on the object and labeled by the index i must be zero, as shown in equation 7.1.

$$\text{Translational Equilibrium} \quad F_{total} = \sum_i F_i = 0 \quad (7.1)$$

If all the forces F_i are applied in a plane, then one can project the forces on the x and z axes as shown in equation 7.2.

$$\begin{aligned} \text{2-D Translational Equilibrium} \quad F_{total,x} &= \sum_i F_{i,x} = 0 \\ F_{total,z} &= \sum_i F_{i,z} = 0 \end{aligned} \quad (7.2)$$

Here the x-axis is horizontal and the z-axis is vertical.

2. The sum of the torques acting on the object and labeled by the index i must be zero, as shown in equation 7.3.

$$\text{Rotational Equilibrium} \quad \tau_{total} = \sum_i \tau_i = 0 \quad (7.3)$$

Here the torque due to the force F_i about a pivot point O is defined by

$$\text{Torque} \quad \tau_{i,O} = F_{i,\perp} r_i \quad (7.4)$$

In this formula r_i is the length of the lever arm, defined as the distance between the pivot point O and the application point A_i of the i th force, and $F_{i,\perp}$ is the component of F_i perpendicular to

the vector \vec{r}_i . This is illustrated in figure 7.1. The torque condition is true for every pivot point O . Since the choice of pivot point is arbitrary, you can use one that is convenient for calculation.

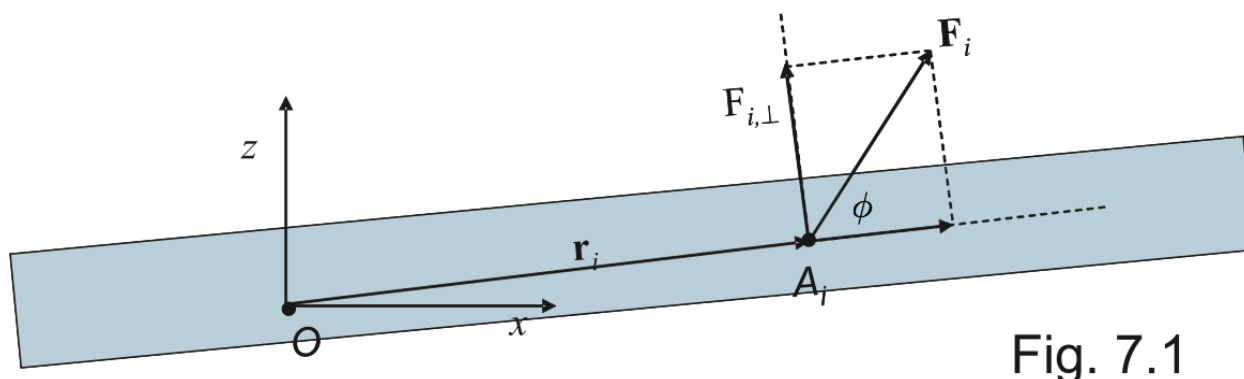


Figure 7.1 The torque due to a force \vec{F}_i about a pivot point O

If the force tends to rotate the object counterclockwise the torque is considered positive. If the force tends to rotate the object clockwise the torque is considered negative. If ϕ is the angle between the force vector \vec{F}_i and lever arm, then the component of the force in the direction perpendicular to \vec{r}_i is given by equation 7.5.

$$\text{Perpendicular force} \quad F_{i,\perp} = |\vec{F}_i| \sin \phi \quad (7.5)$$

The center of gravity of an object is the point, inside or outside the object, where the net force of gravity on all the particles of which the object is composed acts. Since one of the forces always acting on an object is gravity, you must measure the mass of an object and the location of its center of gravity in order to verify the equilibrium conditions above.

ACCEPTED VALUES

As in Experiment 3, the accepted value for the sum of the torques on an object in equilibrium is 0. A useful way to characterize the accuracy of our measurement is to divide the magnitude of the net torque by the sum of the magnitudes of the individual torques multiplied by 100%, as shown in equation 7.6.

$$\% \text{ Discrepancy} = \frac{|\tau_{total}|}{\sum_i |\tau_i|} \times 100\% \quad (7.6)$$

APPARATUS

- A rigid object
- assortment of weights
- knife edge
- 2 spring balances (0-2000 g)
- (hook type)
- balance

The rigid object used in this experiment consists of a metal bar with four pivoted hooks mounted along the bar, two on each side, with protractors to indicate the angles the forces on the hook make

with the major axis of the bar as shown in figure 7.2. We choose the pivot point O to be on the left end of the bar.

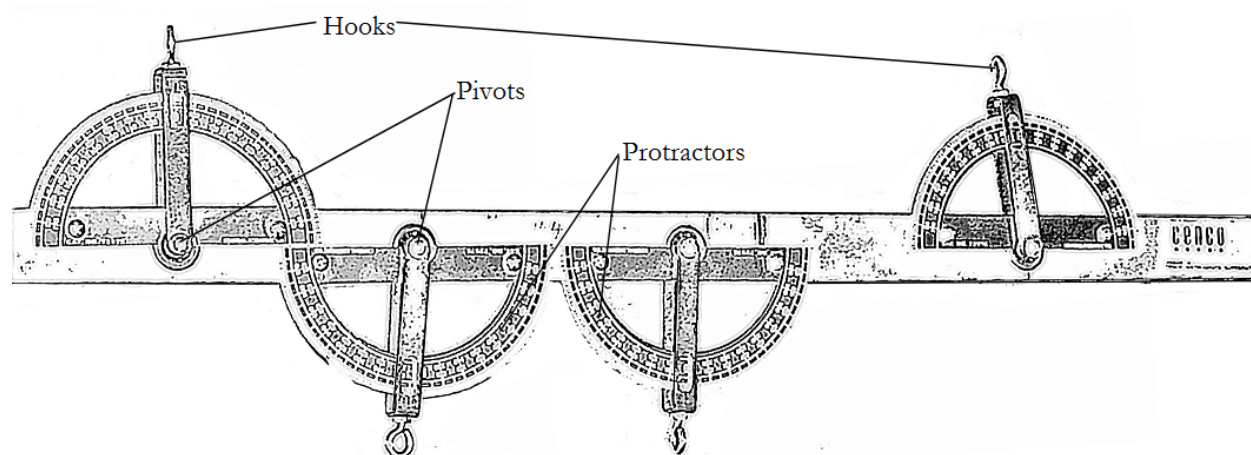


Figure 7.2 The components of the rigid bar with protractors to measure force angles

PROCEDURE

1. Balance the rigid object over a knife edge and measure the distance from the left end of the object to the knife edge. Record the result as the length lever arm r_0 .
2. Weigh the rigid object using your balance and record the result as the mass of the rigid object m_0 in the data table.
3. Check each spring balance with nothing attached. If they do not read zero, turn the adjustment screws until the spring balances all read zero.
4. Support the rigid object by the two spring balances so that each balance pulls vertically on one of the two pivoted hooks nearest the ends of the bar as shown in figure 7.3. Record the forces acting on the rigid object in the data sheet for Part A. Measure the distances from the left end which enables you to calculate the lever arms and thus the torques.

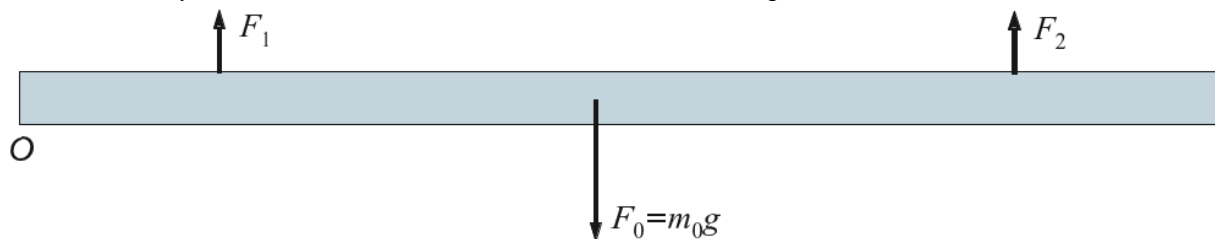


Figure 7.3 The force vectors for Part A which tests the gravitational force only

5. Support the rigid object as in step 4 and add weights to the other two pivoted hooks so that all the external forces acting are vertical and the bar horizontal as shown in figure 7.4. Attach masses that exceed 400 g with one sufficiently different from the other so that the spring balance readings differ by at least 300 g. Record all forces and distances in the data sheet for Part B.

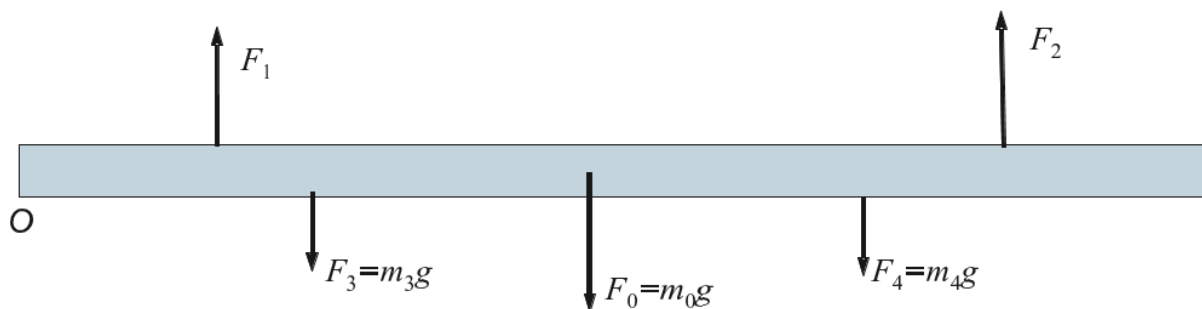


Figure 7.4 The force vectors for Part B which tests two vertically hanging weights

6. Repeat step 5 with the two upward forces pointing outward as shown in figure 7.5. Again, both hanging masses should differ by at least 300 g. Record angles between the upward forces and the bar, all forces, and all distances in the data sheet for Part C.

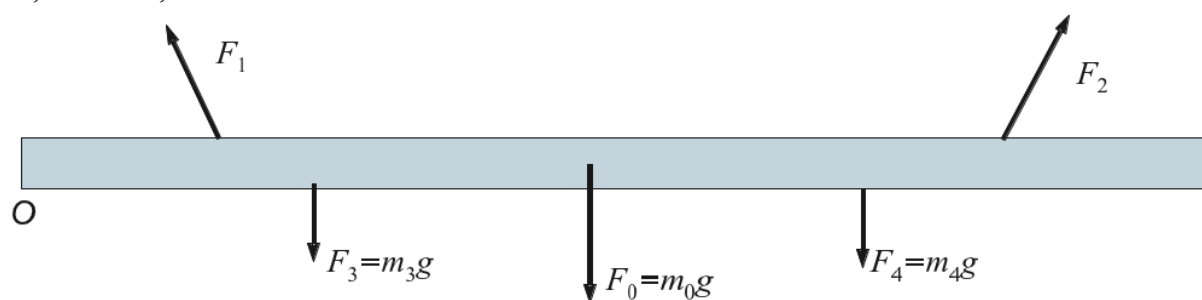


Figure 7.5 The force vectors for Experiment 3 which tests two angled upward forces

DATA

$m_0(\text{g}) =$ _____

Part A

i	$m_i (\text{g})$	$F_i (\text{N})$	$\phi_i (^\circ)$	$r_i (\text{cm})$
0				
1				
2				

Part B

i	m_i (g)	F_i (N)	ϕ_i (°)	r_i (cm)
0				
1				
2				
3				
4				

Part C

i	m_i (g)	F_i (N)	ϕ_i (°)	r_i (cm)
0				
1				
2				
3				
4				

CALCULATION AND ANALYSIS

1. Explain why the center of gravity of the object is located directly above the knife edge position found in procedure 1.
2. Calculate the sum of the torques on the rigid object for Parts A and B.
3. Calculate the % discrepancies of the torques for Part A and B using equation 7.6.
4. Calculate the sum of the horizontal and vertical forces for Part C using equations 7.2.
5. Calculate the sum of the torques on the rigid object for Part C.
6. Calculate the % discrepancies of the torques for Part C using equation 7.6.
7. Would it be possible to achieve equilibrium with only one of the two upward forces vertical and the other at an angle to the vertical? Use a diagram as part of your answer.

Experiment 8: Archimedes' Law

OBJECTIVES

Archimedes discovered that you can measure the volume of a geometrically irregular solid by measuring the displacement of a liquid in which the solid is completely submerged. When scientists quantified the force of buoyancy, they discovered that you can find the density of an irregular object by comparing the weight of the liquid displaced by a submerged object with the apparent loss in the weight of an object. In this experiment, you measure the displacement of liquids by submerged and floating objects and measure the buoyancy force of liquids on submerged objects. The objectives of this experiment are as follows:

1. To measure the liquid displaced by floating and submerged objects
2. To test Archimedes' law
3. To calculate the density of liquids, solid objects that sink, and solid objects that float

THEORY

Archimedes' law states that an object immersed in a liquid is buoyed up by a force equal to the weight of the liquid displaced by the object as shown in equation 8.1.

$$\text{Archimedes' Law} \quad F_b = \rho_L V g \quad (8.1)$$

Here ρ_L is the density of the liquid and V is the volume of the object, so that $\rho_L V$ is the mass of the displaced liquid. This law can measure the density of an irregular solid object ρ_{obj} if the density of the liquid ρ_L is known. Weigh the object in the standard way (without the liquid) determining its weight as a function of density as shown in equation 8.2.

$$\text{Weight (standard)} \quad W = mg = \rho_{obj} V g \quad (8.2)$$

Then weigh the object while immersed in a liquid to determine its apparent weight as shown in equation 8.3.

$$\text{Apparent Weight (in liquid)} \quad W_{app} = W - F_b = (\rho_{obj} - \rho_L) V g \quad (8.3)$$

Dividing equation 8.3 by equation 8.2 eliminates V , which might be difficult to measure directly, and results in the relation shown in equation 8.4.

$$\text{Ratio of Apparent Weight to Standard Weight} \quad \frac{W_{app}}{W} = 1 - \frac{\rho_L}{\rho_{obj}} \quad (8.4)$$

Solving equation 8.4 for ρ_{obj} yields a formula for the density of the object, as shown in equation 8.5.

$$\text{Density of a solid} \quad \rho_{obj} = \rho_L \frac{W}{W - W_{app}} \quad (8.5)$$

Equation 8.5 can also calculate the density of the liquid, such as ethanol, once the density of the submerged object is known, as shown in equation 8.6.

$$\text{Density of a liquid} \quad \rho_L = \rho_{obj} \frac{W - W_{app}}{W} \quad (8.6)$$

In this experiment, you weight the object in two different liquids, water and ethanol, thus it is convenient to label the symbols by the indices W and E. Then equation 8.5 becomes

$$\text{Density of a solid, using water} \quad \rho_{obj} = \rho_W \frac{W}{W - W_{app,W}} \quad (8.7)$$

Substituting equation 8.7 in 8.6 gives a formula for the density of ethanol.

$$\text{Density of Ethanol} \quad \rho_E = \rho_W \frac{W - W_{app,E}}{W - W_{app,W}} \quad (8.8)$$

An object which floats in a liquid displaces a weight of the liquid equal to its own weight. If the object is elongated with length L and constant cross-sectional area S , its volume is $V = LS$ and its mass is $\rho_{obj}V$. If the object is floating in a vertical position and the length of its submerged part is L_{sub} , the volume of the displaced liquid is $V_{sub} = L_{sub}S$ and the mass of displaced liquid is $\rho_L V_{sub}$. So for a floating object $\rho_{obj}V = \rho_L V_{sub}$ which algebraically yields equation 8.10.

$$\text{Density of a solid, floating} \quad \rho_{obj} = \rho_L \frac{L_{sub}}{L} \quad (8.10)$$

In the equations above, it is convenient to measure all the weights in grams instead of Newtons because the acceleration of gravity g cancels in all cases.

ACCEPTED VALUES

The accepted value when testing Archimedes' Law in Part A is the weight of the water displaced by the submerged solid object.

The accepted values for the density of the liquids and solids analyzed in this lab at 20°C and in an atmosphere of 1 bar of pressure are as follows:

- Water: $\rho_L = 998.21 \text{ kg/m}^3$
- Ethanol: $\rho_L = 789.3 \text{ kg/m}^3$
- Aluminum: $\rho_{obj} = 2698.9 \text{ kg/m}^3$

APPARATUS

-
- | | | |
|---------------------|----------------|----------------------|
| • Object | • overflow can | • thread |
| • platform balance | • metal can | • meter stick |
| supported above the | • ethanol | • graduated cylinder |
| table by a stand | • wood block | • beaker |

The object in this lab is suspended by a thread so that it can be immersed in liquids without changing the displaced volume significantly. The overflow can sends displaced water through a spout into the beaker, as shown in Figure 8.1, which can be weighed on a platform balance.

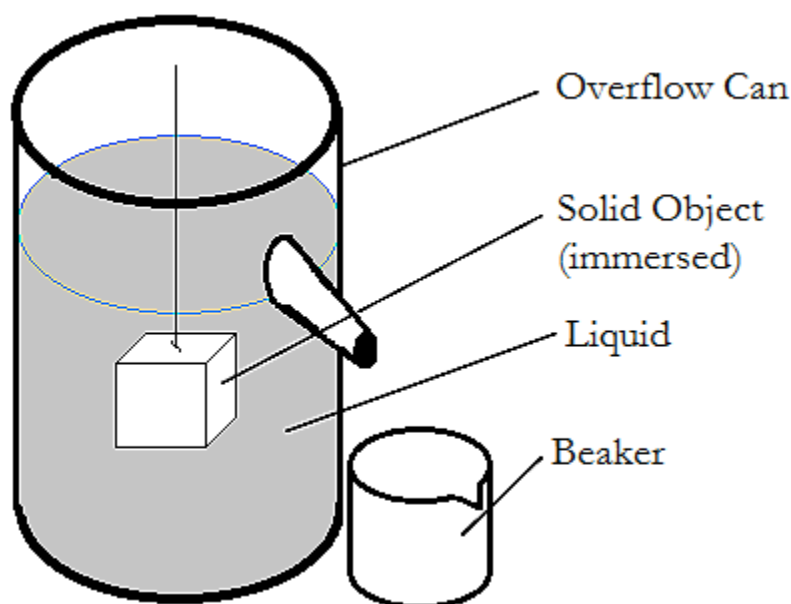


Figure 8.1 An overflow can directs displaced liquid into a beaker

To measure the weight of the object when it is immersed in a liquid, loop the thread over a hook at the end of the platform balance on a stand and make measurements as you would if the object sat upon the balance platform.

PROCEDURE

1. Hang the object from the hook under the left pan of the balance using the thread. Measure the object's weight and record the result as the object weight W in the data table for Part A and again in the data table for Part B.
2. Arrange the overflow can and the beaker so that water can flow from the spout of the overflow can into the beaker. Pour water into the can until it overflows. When the water has stopped dripping from the spout, weigh the beaker with the water in it and record the result as the initial weight of the beaker $W_{b,i}$ in the data table for Part A.
3. Place the beaker and its contents back under the spout. While keeping the object hung from the balance, lower the object by a thread into the water in the overflow can until it is completely immersed. When all the water displaced by the object has flowed into the beaker, weigh the beaker with the water in it and record the result as the final weight of the beaker $W_{b,f}$ in the data table for Part A.
4. Adjust the apparatus so that the object isn't touching the sides or bottom of the overflow can. Measure the apparent weight of the object when immersed in water $W_{app,W}$ and record the result in the data table for Part A and again in the data table for Part B. Set the overflow can aside.
5. Dry the object. Fill the metal can with ethanol. Measure the apparent weight of the object when completely immersed in ethanol, $W_{app,E}$ and record your result in the data table for Part B. Pour the ethanol back into the ethanol bottle and close the bottle.

6. Measure the length of the wood block and record the result as the total length L_{total} in the data table for Part C.
7. Measure and record the width W and the height H of the wood block.
8. Using a balance, measure the mass m of the wood block.
9. Fill the graduated cylinder with approximately 650 cm^3 of water. Lower the wood block into the water in the graduated cylinder until it floats.
10. Measure the length of the wood that is below the level of the water and record the result as the submerged length L_{sub} in the data table for Part C.

DATA

Part A: Testing Archimedes' Law

W (g)	$W_{\text{app},W}$ (g)	$W_{b,i}$ (g)	$W_{b,f}$ (g)

Part B: Calculating the density of a solid object and ethanol

W (g)	$W_{\text{app},W}$ (g)	$W_{\text{app},E}$ (g)

Part C: Calculating the density of a floating object

L_{total} (cm)	W (cm)	H (cm)	m (g)	L_{sub} (cm)

CALCULATION AND ANALYSIS

1. Calculate the weight of the water displaced by the immersed object, $W_{b,f} - W_{b,i}$, using the data from Part A.
2. Calculate the apparent loss of weight of the object when completely immersed in water, $W - W_{\text{app},W}$, using the data from Part A.
3. Archimedes' Law predicts that the weight of the displaced water equals the apparent loss of weight of the object. Do your results support Archimedes' Law?
4. Calculate the density of the object ρ_{obj} using equation 8.7 and the data from Part B.
5. Calculate the % error in your result for ρ_{obj} .
6. Calculate the density of the ethanol ρ_E using equation 8.8 and the data from Part B.
7. Calculate the % error in your result for ρ_E .

8. Calculate the density of the wood block $\rho_{\text{wood, float}}$ using equation 8.10 and the data from Part C.
9. Calculate the volume of the wood block using your data from Part C. (Remember volume = length \times width \times height.)
10. Calculate the density of the wood block $\rho_{\text{wood, actual}} = \frac{m}{\text{volume}}$ using your data from Part C.
11. Using $\rho_{\text{wood, actual}}$ as the accepted value, calculate the % error in your result for $\rho_{\text{wood, float}}$.

Experiment 9: Simple Harmonic Motion

OBJECTIVES

Simple harmonic motion is the motion of an object that is subject to a force that is proportional to the object's displacement. An object attached to a spring undergoes simple harmonic motion. The quantitative relationship between the spring force and the displacement is known as Hooke's Law. In this experiment, you observe the oscillation of an object attached to a spring to test Hooke's law and calculate the spring constant for the spring. The objectives of this experiment are as follows:

1. To measure the period of oscillation of a mass-spring system
2. To test Hooke's law with a spring
3. To calculate the spring constant for a spring

THEORY

When a body is suspended from a spring, its weight causes the spring to elongate. The elongation x is directly proportional to the external force F_{spring} exerted by the spring as shown in equation 9.1.

$$\text{Hooke's Law} \quad F_{spring} = -kx \quad (9.1)$$

Here k is the force constant of the spring. This relationship is known as Hooke's law.

If the spring is oriented to resist the acceleration of gravity and an object is pulled down and then released, the object oscillates about the position of the body when the spring was stationary, known as the equilibrium position. This motion is called simple harmonic motion. The period T of an object in simple harmonic motion is a function of the spring constant k and the moving mass of the system M , as shown in equation 9.2.

$$\text{Period of spring-mass system} \quad T = 2\pi\sqrt{\frac{M}{k}} \quad (9.2)$$

Thus a plot of T^2 vs. M should be a straight line with slope $4\pi^2/k$.

NOTE: In the calculations and analysis, you measure the slope of a plot of T^2 vs. M . In this system the moving mass includes the mass of the weight hanger and some part of the spring mass. Since these weights do not change from run to run, they do not affect the slope calculation. So we can simply take M to be the mass of the suspended body.

ACCEPTED VALUES

The accepted value for the spring constant is calculated using the force of gravity opposing the spring force when the object is stationary. The relationship between the spring constant k and the force of gravity F_G is shown in equation 9.4.

$$\text{Spring constant} \quad F_G = -k\Delta x \rightarrow k = -\frac{F_G}{\Delta x} \quad (9.4)$$

Here, Δx is the displacement of the mass that stretches the spring to a new equilibrium point. Be sure to keep the axis and signs consistent. The force of gravity is a vector pointing down.

APPARATUS

- Hooke's law apparatus
- slotted weights
- stop timer
- half meter stick
- platform balance
- computer with Pasco interface

The spring is equipped with a distance scale, as shown in Figure 9.1, so that you can see the equilibrium point change as you change the weight of the object. However, you do not need the scale to measure the period.

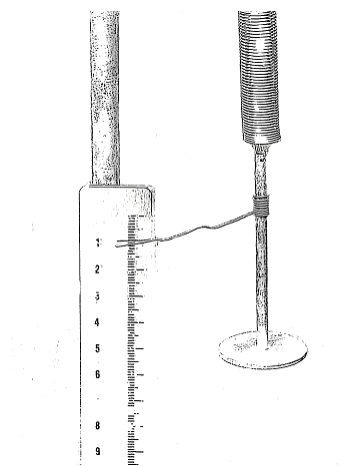


Figure 9.1 Detail of the spring and scale of the Hooke's law apparatus

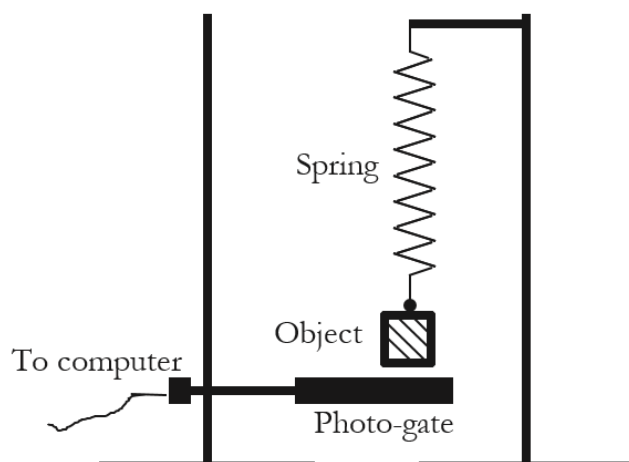


Figure 9.2 Photo-gate positioned to measure the motion of the object

The computer interface measures motion through a photo-gate. The Hooke's law apparatus hangs the object at the end of a spring, which passes through photo-gate, as shown in Figure 9.2.

PROCEDURE

1. Adjust the scale on the Hooke's law apparatus so the pointer aligns with zero.
2. Add a 25 g weight to the hanger and record the new position of the pointer as the equilibrium point x in the data table for Part A.
3. Repeat step 4 seven more times, adding 25 g and recording the equilibrium point next to the total weight added to the apparatus in the data table for Part A until the final load is 200 g.
4. Remove the weight holder from the spring and hook the 100 g mass onto the spring.
5. Turn on the computer and photo-gate interface. *Alternatively, if your lab instructor would like you to use a stopwatch, skip to step 11.*
6. From the desktop choose the "Simple Harmonic Motion" icon.
7. Position the photo-gate so that it is just below the bottom of the mass. Start the system oscillating by gently pulling down on the mass and releasing it. Adjust the position of the photo-gate so that when the system is oscillating, the bottom of the mass interrupts the photo-gate but does not pass completely through the gate. This way the bottom of the mass starts the timer and, when it completes one oscillation, stops the timer.
8. Stop the system oscillating and swing the photo-gate out of the way so that it does not interrupt the oscillating mass.
9. Start the system oscillating by gently pulling down on the mass and releasing it. When the system is oscillating smoothly, with little sideways drift, swing the photo-gate back so that it starts recording data.
10. Choose "Play" on the computer interface to start the computer collecting data. Choose "Stop" when the table is filled. Record the average period displayed at the bottom of the table in the data table for Part B.
11. *If you're using a stopwatch:* start the system oscillating by gently pulling down on the mass and releasing it. When the system is oscillating smoothly use a stopwatch to time 10 periods of oscillation. Divide the time you measure by 10 and record it in the data table for Part B.
12. Repeat steps 8 through 10 if you're using a computer, or step 11 if you're using a stopwatch, with masses of 125 g, 150 g, 175 g and 200 g. Increase the mass by sliding the appropriate slotted masses on top of the 100 g hanging weight. For the 200 g trial you can use the 200 g hanging weight.
13. With a 200 g hanging weight, observe the oscillations for some time as the amplitude slowly decreases. Do you notice any change in the period, or does the period stay roughly constant? Record your observation on the data sheet.

DATA

Part A: Equilibrium point

Added mass (g)	x_0 (cm)
0.0	0.00
25.0	
50.0	
75.0	
100.0	
125.0	
150.0	
175.0	
200.0	

Part B: Oscillation data

Mas (g)	Average T (s)
100.0	
125.0	
150.0	
175.0	
200.0	

Does the period seem to depend on the amplitude? _____

CALCULATION AND ANALYSIS

1. Calculate the gravitational force F_G due to each of the masses in the data table in Part A, using the formula $F_G = mg$.
2. Draw a graph of the force F_G versus the stationary position x using data from the data table for Part A.
3. Calculate k graphically by taking the slope of the graph using equation 0.9.
4. Draw a graph of the period squared T^2 versus the moving mass M using the data from Part B.

5. Find the slope of the graph using equation 0.9. Calculate k using $k = 4\pi^2 / \text{slope}$.
6. Calculate the % error of k calculated from Part B using k calculated from Part A as the accepted value.
7. As the spring oscillates while you measure its period, the amplitude of the oscillation decreases. Do you see any evidence that the period of oscillation changes as the amplitude decreases?

Experiment 10: Boyle's Law

OBJECTIVES

Gases are comprised of small particles in constant random motion separated by relatively large distances. The characteristics of gases can therefore be analyzed in terms of the volume in space they occupy and the pressure their composite particles produce by constant random collisions. The quantitative relationship between an ideal gas's volume and pressure is known as Boyle's law. In this experiment, you test Boyle's law by changing the volume of air in a sealed chamber and observing the corresponding change in pressure using a barometer. The objectives of this experiment are as follows:

1. To measure the changes in pressure that correlate to changes in volume
2. To test Boyle's Law by expanding and contracting volume
3. To calculate an experimental value of a gas constant

THEORY

The volume V of a fixed mass of a gas is a function of temperature T and pressure P . If the temperature is held constant, many gases exhibit the common property that for any given amount of gas, as the pressure P changes, the volume V changes in a manner such that the product of pressure and volume remains constant, as shown in equation 10.1.

$$\text{Boyle's Law} \quad P_1V_1 = P_2V_2 = \text{const.} \quad (10.1)$$

This relation is known as Boyle's law. The value of the constant depends on the temperature and the amount of gas present. Boyle's law accurately describes the pressure-volume behavior of most common gases at moderate temperatures and pressures. Deviations from Boyle's law may become significant if the pressure is too high or the temperature is too low. These limiting values vary drastically with different gases. For example, hydrogen obeys Boyle's law at minus 200°C and 100 atmospheres; sulfur dioxide does not at 20°C and 1 atmosphere.

In this experiment, a column of air of constant cross sectional area S and length L is trapped in a Plexiglas cylinder with a graduated scale along the side. The length L is controlled by the position of the piston which can be read on the graduated scale. Because $V = SL$ and S is constant, then Boyle's law can be written in the form shown in equation 10.2.

$$\text{Boyle's Law (length)} \quad P_1L_1 = P_2L_2 = \text{const.} \quad (10.2)$$

The position of the piston and thus L is controlled by the threaded rod and the pressure is measured by a pressure gauge. By systematically changing the length of the air column and recording the pressure, you can test Boyle's law.

ACCEPTED VALUES

The accepted value for the constant resulting from equation 10.2 depends on the experimental conditions of temperature and amount of gas. However, once an amount of gas is trapped in the cylinder, the accepted value for the constant is known for all subsequent measurements until the

graduated cylinder is ventilated. Therefore the difference between all the values calculated with equation 10.2 has an accepted value of 0.

APPARATUS

- Boyle's law apparatus
- pressure gauge

The Boyle's law apparatus consists of a sealed graduated cylinder that traps gas under a piston. The piston is attached to a threaded screw that when turned changes the volume of the gas in the graduated cylinder. A pressure gauge measures the change in pressure as the volume in the graduated cylinder changes.

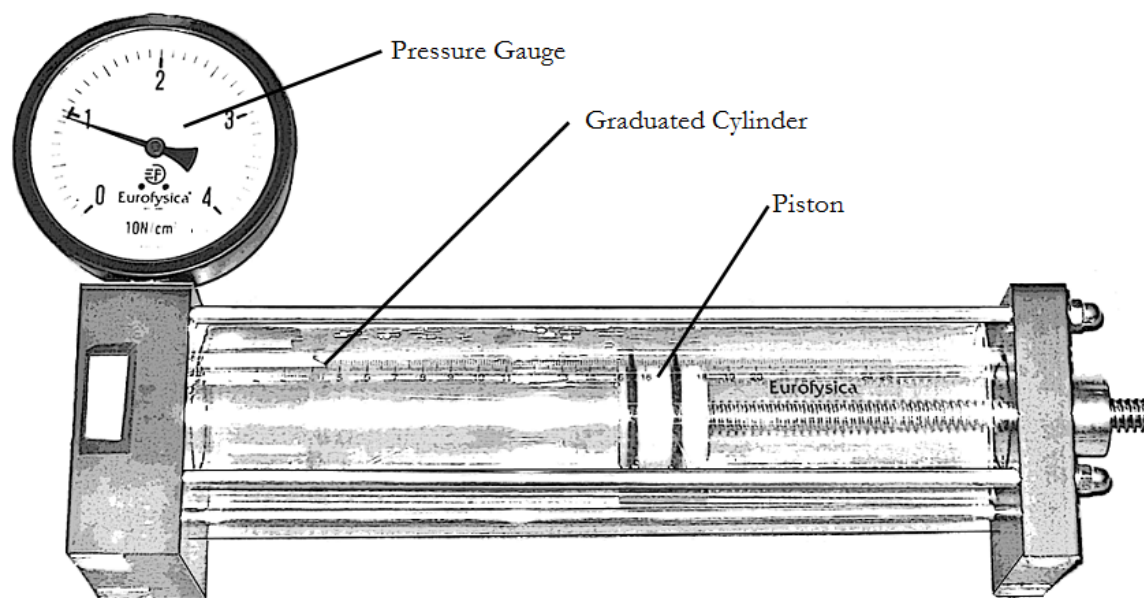


Figure 10.1 the Boyle's law apparatus

PROCEDURE

1. Ventilate the cylinder by turning the hand valve screw on the left end of the cylinder.
2. Set the piston on the 7 cm mark.
3. Close the ventilation valve. You have trapped a column of air in the cylinder at exactly the atmospheric pressure. Check that the pressure gauge reads 1 atm.
4. Before each pressure reading, tap the pressure gauge gently to make sure the needle isn't stuck in a wrong setting.
5. Turn the rotary knob to move the piston to the 8 cm mark. Read the pressure gauge and record the pressure in the data table for Part A. Repeat the procedure in 1 cm steps until the piston reaches the 25 cm mark.
6. Ventilate the cylinder by turning the hand valve screw on the left end of the cylinder.
7. Set the piston on the 25 cm mark.
8. Close the ventilation valve. You have trapped a column of air in the cylinder at exactly the atmospheric pressure. Check that the pressure gauge reads 1 atm.
9. Turn the rotary knob so that the piston moves inward to decrease the volume of the trapped air.

10. Record the pressure at each 1cm decrement in the data table for Part B until the piston reaches 7 cm.

DATA

Part A: Increasing Piston Displacement (L)

L (cm)	P_g (atm)
7.0	
8.0	
9.0	
10.0	
11.0	
12.0	
13.0	
14.0	
15.0	
16.0	
17.0	
18.0	
19.0	
20.0	
21.0	
22.0	
23.0	
24.0	
25.0	

Part B: Decreasing Piston Displacement (L)

L (cm)	P_g (atm)
25.0	
24.0	
23.0	
22.0	
21.0	
20.0	
19.0	
18.0	
17.0	
16.0	
15.0	
14.0	
13.0	
12.0	
11.0	
10.0	
9.0	
8.0	
7.0	

CALCULATION AND ANALYSIS

1. Calculate the product of the pressure P and length L for every row in both data tables.
2. Calculate the deviation for each value of PL .
3. Throwing out any highly deviating values, use plotting software or good plotting paper to plot the graph of the product of the pressure and length PL versus the length L using data from the data table for Part A.
4. Fit the plotted data points to a straight line. Is the best fit line horizontal?
5. Process the data from Part B using steps 3 and 4, plotting PL versus L and drawing the best fit line. Is the best fit line horizontal?
6. If the trapped gas slowly leaked out during your measurement how would the PL values behave? Do you think this occurred?

Appendix: Algebra and Trigonometry Review Topics

REFRESHING

High-School Mathematics

and beyond...

Numbers and symbols

Symbolic and numeric calculations

Physics students have to be able to operate with symbols (such as a , b , x , y , Q , etc.) that stand for numbers. Calculations should be done, as a rule, in a symbolic form, and the analytical (that is, symbolic) result should be obtained. Only after that concrete numbers should be plugged into the analytical result, to obtain the numerical result. Doing this is not a big problem since all operations on numbers can be done on symbols as well. Symbolic operations have important advantages, however, such as better overview of the manipulations, possibility of backtracking and checking, possibility of using multiple sets of numerical values without a necessity of doing similar calculations many times, possibility of operating with quantities, numerical values of which are unknown but irrelevant.

Basic identities

$$a + b = b + a$$

$$a + (b + c) = (a + b) + c = (a + c) + b = a + b + c$$

$$ab = ba$$

$$a(bc) = (ab)c = (ac)b = abc = acb = bca = \dots$$

$$a(b + c) = ab + bc$$

$$(a + b)^2 = a^2 + 2ab + b^2 \quad (\text{binomial formula})$$

(operations in brackets are performed first)

Fractions

$$a/b = \frac{a}{b} = a \frac{1}{b} = a \times b^{-1}$$

$$\frac{b}{a} \frac{c}{c} = \frac{b}{c} \frac{ac}{c} = \frac{ab}{c} \frac{1}{c} = ab/c$$

$$\frac{ac}{bd} = \frac{ac}{bd}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad + bc}{bd}$$

Inserted for convenience

Compound Fractions

Fractions containing fractions are sometimes confusing, such as

$$\frac{a}{\frac{b}{\frac{c}{d}}} \text{ or } \frac{\frac{a}{b}}{\frac{c}{d}}$$

To avoid confusion, we can distinguish between external and internal fractions and make external fractions longer and/or bolder. If we divide

$$a \text{ or } \frac{a}{b} \text{ over } \frac{c}{d}$$

we write

$$\frac{a}{\frac{b}{\frac{c}{d}}} \quad \begin{array}{l} \text{internal fractions} \\ \text{external fraction} \end{array}$$

and simplify the fractions as follows

$$\frac{a}{\frac{b}{\frac{c}{d}}} = \frac{ac}{b} \quad \frac{a}{\frac{b}{\frac{c}{d}}} = \frac{ad}{bc}$$

Manipulations above can be justified using powers instead of reciprocals:

$$\frac{a}{\frac{b}{\frac{c}{d}}} = a \left(\frac{b}{\frac{c}{d}} \right)^{-1} = a \frac{c}{b} = \frac{ac}{b}$$
$$\frac{a}{\frac{b}{\frac{c}{d}}} = \frac{a}{b} \left(\frac{c}{d} \right)^{-1} = \frac{a}{b} \frac{d}{c} = \frac{ad}{bc}$$

Fractions can also be written as

$$\frac{a}{b} = a / b$$

Expression a/bc can be confusing. If the writer means $a/(bc)$, it should be written explicitly so. Otherwise it means

$$a / bc = (a / b)c = \frac{a}{b} c = \frac{ac}{b}$$

according to all programming languages.

Exponents

Products of several equal numbers can be represented by powers of these numbers, such as

$$\underbrace{a \times a \times \dots \times a \times a}_{b \text{ times}} = a^b$$

Here b is the exponent and a is the base. Bases and exponents can be, in fact, any numbers, not necessary natural. In particular, negative exponents are used for reciprocals, such as

$$a^{-b} = \frac{1}{a^b}$$

and fractional exponents represent roots

$$a^{1/2} = \sqrt{a}$$

Properties of powers:

$$a^m a^n = a^{m+n}$$

$$(a^m)^n = a^{mn}$$

$$a^1 = a, \quad a^0 = 1$$

Examples:

$$\frac{3 \times 5 \times 3 \times 5 \times 3}{5 \times 3 \times 5 \times 5} = 3^{3-1} 5^{2-3} = 3^2 5^{-1}$$

$$(\sqrt{2})^2 = (2^{1/2})^2 = 2^{1/2 \times 2} = 2^1 = 2$$

$$\sqrt{2^2} = (2^2)^{1/2} = 2^{2 \times 1/2} = 2^1 = 2$$

Scientific notation for numbers

In physics one has frequently to deal with very large and very small numbers. The best way to write these numbers is using the scientific notation. For instance, the scientific notation for 12345.678 is 1.2345678×10^4 . The exponent 4 shows that we have moved the decimal point by 4 positions to the left. Here one sees more clearly how large the number is, its order of magnitude is 10^4 . For this reason, the factor in front (the so-called simple part) should be kept maximally close to 1. The number 9123.456 is better to write as 0.9123456×10^4 than as 9.123456×10^3 because the order of magnitude of this number is 4 and not 3. Similarly the number 0.000001234 is written in the scientific notation as 1.234×10^{-6} , where the exponent -6 shows that we have moved the decimal point by six positions to the right. Remember that negative powers describe reciprocals, so that in this case we divide 1.234 by 10 six times.

Example: The mass of electron m_e is approximately 0.911×10^{-30} kg, difficult to write in the usual notation!

When several numbers are multiplied or divided, one can operate the simple parts and powers of 10 independently:

$$\frac{1.2 \times 10^5 \times 3.4 \times 10^7 \times 0.68 \times 10^{-21}}{0.56 \times 10^{12} \times 4.4 \times 10^{-30}} = \frac{1.2 \times 3.4 \times 0.68}{0.56 \times 4.4} 10^{5+7-21-12+30} = 1.13 \times 10^9$$

For an order-of-magnitude estimation, you can simplify the task and drop all simple parts, that yields 10^9 in the example above.

Algebraic equations

Solving physical problems usually involves solving algebraic equations and systems of equations. In most cases these equations are linear.

An example of a linear equation (usually we use a, b, c, \dots for knowns and x, y, z, \dots for unknowns):

$$ax + b = c$$

Equations remain valid if the same quantity is added or subtracted to their right-hand side (rhs) and left-hand side (lhs) and if both rhs and lhs are multiplied or divided by the same quantity. This can be used to isolate unknowns in one of the sides of the equation, that is to solve the equation. For the equation above it is done in the following way:

$$ax + b = c \quad \Rightarrow \quad ax + b - b = c - b \quad \Rightarrow \quad ax = c - b \quad \Rightarrow \quad x = \frac{c - b}{a}$$

(Frame your final result!)

Solving physical problems, we use standard notations adopted in physics rather than just a, b, c and x, y, z . One should understand which quantities are known and which are unknown. If, for instance, acceleration a has to be found from Newton's second law $F = ma$, then we consider a as the unknown and solve the algebraic equation as follows:

$$F = ma \quad \Rightarrow \quad a = \frac{F}{m}$$

The framed expression is our final symbolic, or so-called analytical, result. We now plug numerical values for F and m into it and obtain our final numerical result for a .

Systems of algebraic equations

In many cases one has to solve systems of linear algebraic equations such as

$$a_1x + b_1y = c_1$$

$$a_2x + b_2y = c_2$$

where x and y are unknowns. One of different ways to do it is, say, to

- (i) find y from the first equation;
- (ii) plug the result into the second equation;
- (iii) solve the second equation for x ;
- (iv) plug the result for x into the expression for y obtained in (i)

that is

$$a_1x + b_1y = c_1 \Rightarrow y = \frac{c_1 - a_1x}{b_1}$$

$$a_2x + b_2y = c_2 \Rightarrow a_2x + b_2 \frac{c_1 - a_1x}{b_1} = c_2 \Rightarrow \left(a_2 - a_1 \frac{b_2}{b_1} \right) x + c_1 \frac{b_2}{b_1} = c_2$$

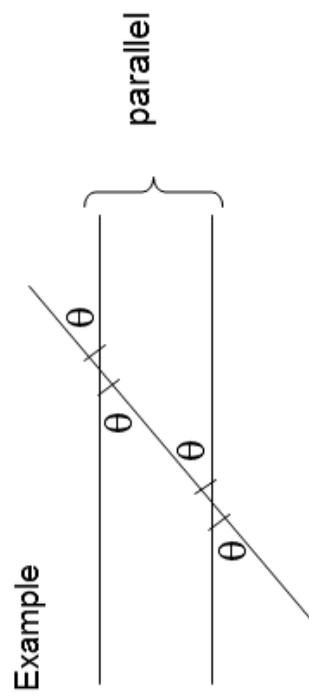
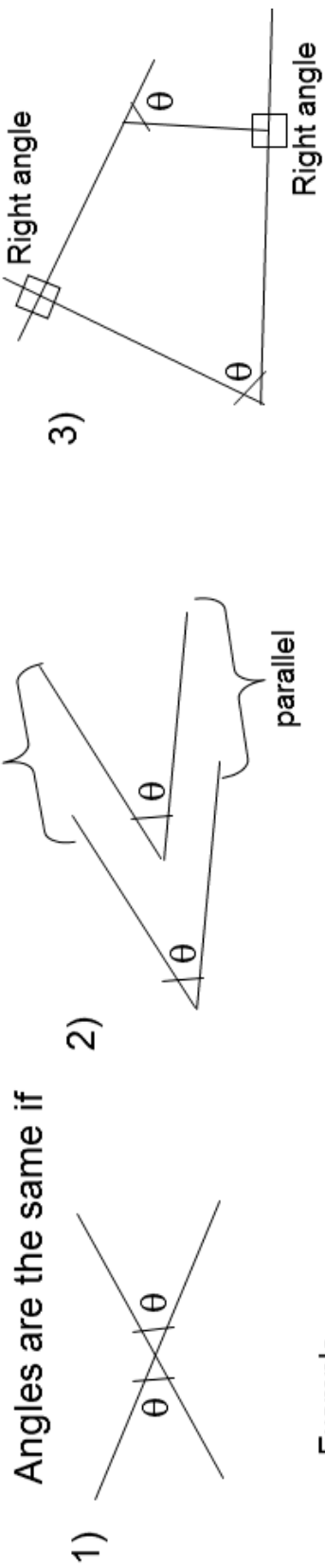
$$\Rightarrow x = \frac{c_2 - c_1 \frac{b_2}{b_1}}{a_2 - a_1 \frac{b_2}{b_1}} = \frac{c_2 b_1 - c_1 b_2}{a_2 b_1 - a_1 b_2} = x \quad \text{(do not forget to simplify your results!)} \quad \boxed{\frac{c_2 b_1 - c_1 b_2}{a_2 b_1 - a_1 b_2} = x}$$

and then we perform (iv) that after simplification yields

$$y = \frac{c_2 a_1 - c_1 a_2}{b_2 a_1 - b_1 a_2}$$

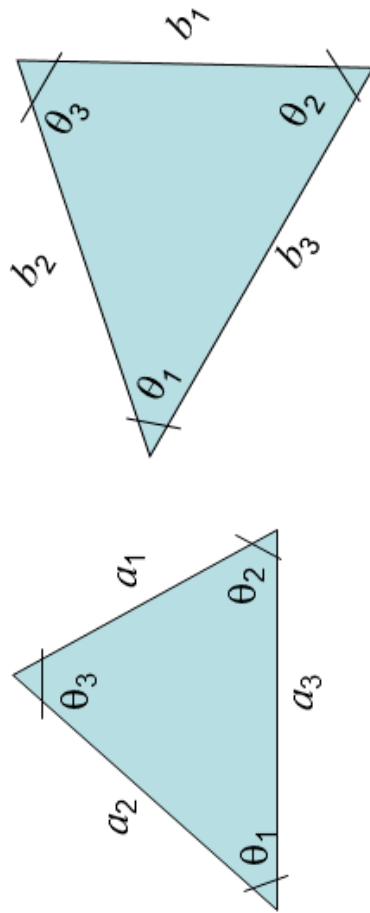
Remember that in a well-behaved system of equations the number of unknowns is equal to the number of equations. You should always perform the count of unknowns and equations before you start with solving a system of equations.

Lines, Angles, and Triangles



Similar triangles

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$$



Angles

Angles are usually denoted by Greek letters such as $\alpha, \beta, \theta, \varphi$, etc.

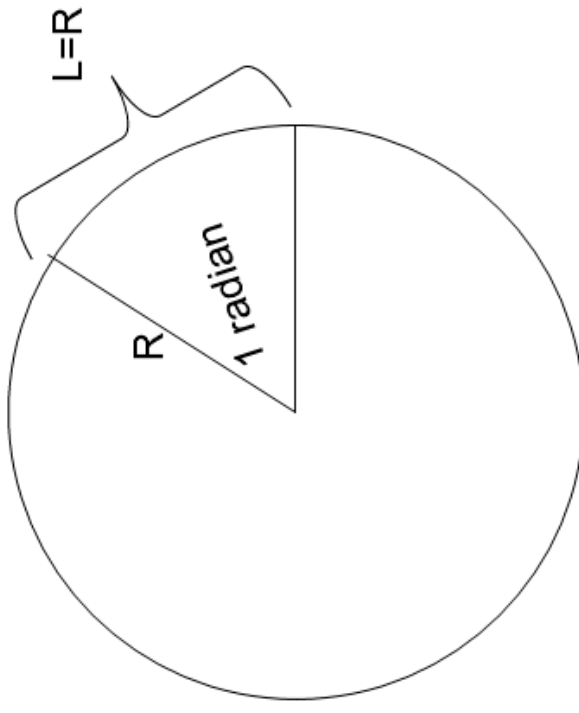
Angles can be measured in

- degrees (seldom used in physics)
- revolutions (360°) (used in engineering)
- radians (mostly used in physics)

Radian is such an angle, for which the length of the arc is equal to the radius. In other words, the angle in radians is given by L/R and it is dimensionless.

Revolution corresponds to $L = 2\pi R$, thus $360^\circ = 2\pi$ radians and

$1 \text{ radian} = 360^\circ / (2\pi) = 57.3^\circ$ $1 \text{ revolution} = 360^\circ = 2\pi \text{ radian}$



Trigonometric functions

Properties of triangles with right angle ($\gamma = \pi/2$):

$$\theta + \theta' = \pi / 2 = 90^\circ$$

$$c^2 = a^2 + b^2 \quad (\text{Pythagor\ae s theorem})$$

For these triangles one defines trigonometric functions as follows:

$$\sin \theta = \frac{b}{c} = \frac{b}{\sqrt{a^2 + b^2}}, \quad \cos \theta = \frac{a}{c} = \frac{a}{\sqrt{a^2 + b^2}}, \quad \tan \theta = \frac{b}{a} = \frac{\sin \theta}{\cos \theta}, \quad \cot \theta = \frac{a}{b} = \frac{\cos \theta}{\sin \theta} = \frac{1}{\tan \theta}$$

sin and cos are called, as a group, sinusoidal functions. They satisfy

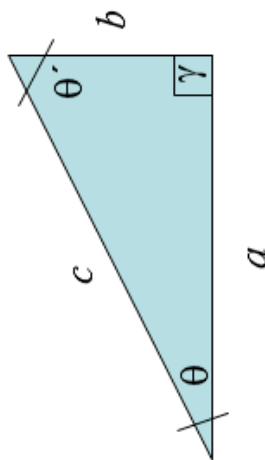
$$\sin^2 \theta + \cos^2 \theta = 1$$

On the other hand, one can define trigonometric functions for the angle θ' of the triangle in the same way:

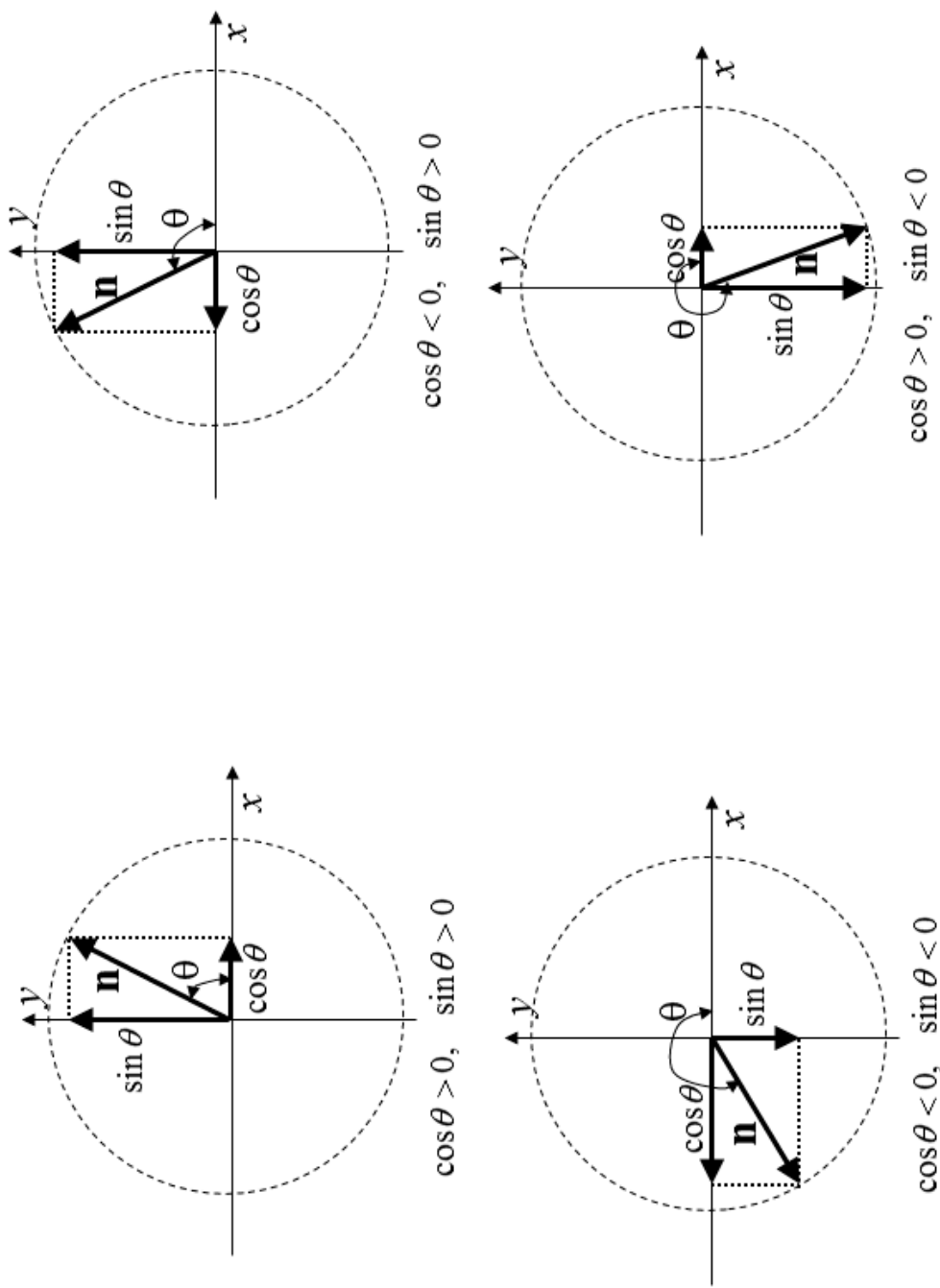
$$\sin \theta' = \frac{a}{c} = \frac{a}{\sqrt{a^2 + b^2}}, \quad \cos \theta' = \frac{b}{c} = \frac{b}{\sqrt{a^2 + b^2}}, \quad \tan \theta' = \frac{a}{b}, \quad \cot \theta' = \frac{b}{a}$$

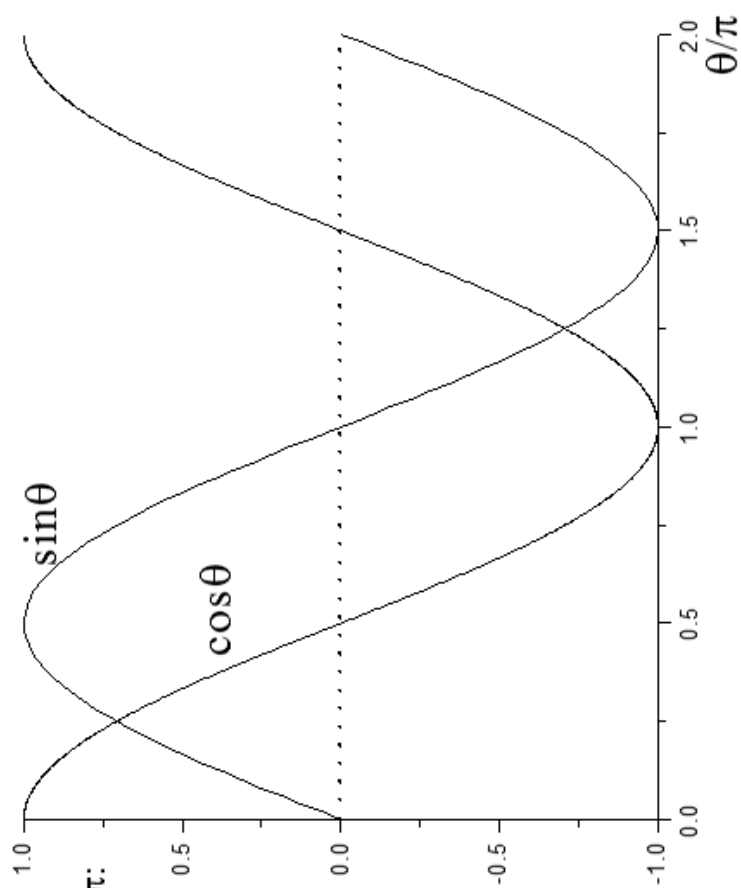
Comparison with the above yields

$$\sin \theta' = \sin \left(\frac{\pi}{2} - \theta \right) = \cos \theta, \quad \cos \theta' = \cos \left(\frac{\pi}{2} - \theta \right) = \sin \theta, \quad \tan \theta' = \tan \left(\frac{\pi}{2} - \theta \right) = \cot \theta, \quad \dots \quad 11$$



Above we have defined trigonometric functions with the help of triangles, so that θ is limited to the interval $0 \leq \theta \leq \pi/2$. One can, however, define trigonometric functions for arbitrary arguments with the help of components of a two-dimensional unit vector \mathbf{n} ($|\mathbf{n}| = 1$) as follows





Trigonometric functions are periodic with period 2π :

\sin and \cos differ by an argument shift and there are a lot of corresponding relations

Symmetry:

$$\cos(-\theta) = \cos \theta$$

$$\sin(-\theta) = -\sin \theta$$

$$\tan(-\theta) = -\tan \theta$$

$$\cot(-\theta) = -\cot \theta$$

Inverse trigonometric functions

$$\sin \theta = x \Rightarrow \theta = \arcsin x$$

$$\cos \theta = x \Rightarrow \theta = \arccos x$$

$$\tan \theta = x \Rightarrow \theta = \arctan x$$

$$\cot \theta = x \Rightarrow \theta = \operatorname{arccot} x$$

Scalars and Vectors

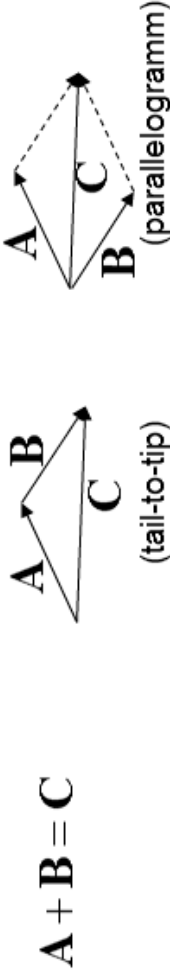
Most of the physical quantities are scalars or vectors.

Scalars are objects that are represented by numbers, such as time t , mass m , electric charge q or Q , temperature t or T . Some scalar quantities are always positive or nonnegative, such as mass m , volume V , kinetic energy E_k , absolute temperature T , etc. Most of scalar quantities can be either positive or negative, such as electric charge q or Q , time t , etc.


Vectors are mathematical and/or physical objects that are characterized by (i) their magnitude or absolute value or length and (ii) their direction in space. Many physical quantities are vectors, such as position, velocity, force, electric and magnetic fields, etc. Vectors can be added, subtracted, and multiplied. Vectors can be divided by a scalar but one cannot divide by vector. Vectors are denoted by symbols with overhead arrows (\vec{A}) in handwritten texts and by boldface symbols (\mathbf{A}) in printed texts.

Magnitude (length) of a vector A is denoted as $|A|$ or simply as A . Vectors of unit length, $|A| = 1$, describe directions. Each vector can be represented in the form $A = A\mathbf{n}$, where $A > 0$ and \mathbf{n} is a unit vector directed along A .

Addition of vectors can be done graphically with the help of either the tail-to-tip rule or the parallelogram rule



Subtraction of vectors:

$$\mathbf{A} - \mathbf{B} = \mathbf{C}$$


because $\mathbf{A} = \mathbf{B} + \mathbf{C}$

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Vectors and Coordinate Systems

To perform operations on vectors numerically, it is convenient to introduce a coordinate system. The latter is defined by the origin O and three mutually perpendicular axes x , y , and z . The tail of the vector \mathbf{A} is in the origin of the coordinate system. We project the vector \mathbf{A} onto the axes of the coordinate system by drawing the three lines from its tip towards all three axes perpendicularly to the latter. As the result, \mathbf{A} is represented as the sum of three vectors:

$$\mathbf{A} = \mathbf{A}_x + \mathbf{A}_y + \mathbf{A}_z = A_x \mathbf{e}_x + A_y \mathbf{e}_y + A_z \mathbf{e}_z$$

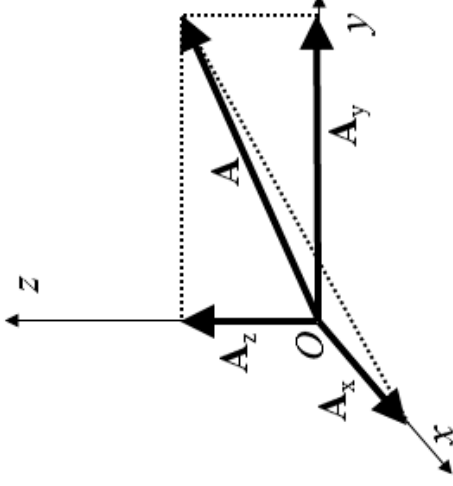
Here we have introduced the unit vectors \mathbf{e}_x , \mathbf{e}_y , and \mathbf{e}_z , ($|\mathbf{e}_x| = 1$ etc.) that are directed along different axes. The scalar quantities A_x , A_y and A_z are components of the vector \mathbf{A} in this coordinate system or its projections on the axes of this coordinate system. Note that components of a vector can be both positive and negative.

With the above definitions, one obtains many useful formulas. Addition of vectors can be done as

$$\mathbf{A} + \mathbf{B} = A_x \mathbf{e}_x + A_y \mathbf{e}_y + A_z \mathbf{e}_z + B_x \mathbf{e}_x + B_y \mathbf{e}_y + B_z \mathbf{e}_z = (A_x + B_x) \mathbf{e}_x + (A_y + B_y) \mathbf{e}_y + (A_z + B_z) \mathbf{e}_z$$

$$\mathbf{A} + \mathbf{B} = \mathbf{C} = C_x \mathbf{e}_x + C_y \mathbf{e}_y + C_z \mathbf{e}_z \Rightarrow \boxed{C_x = A_x + B_x, \quad C_y = A_y + B_y, \quad C_z = A_z + B_z}$$

That is, to add vectors, one has just to add their components, and similar for subtraction.



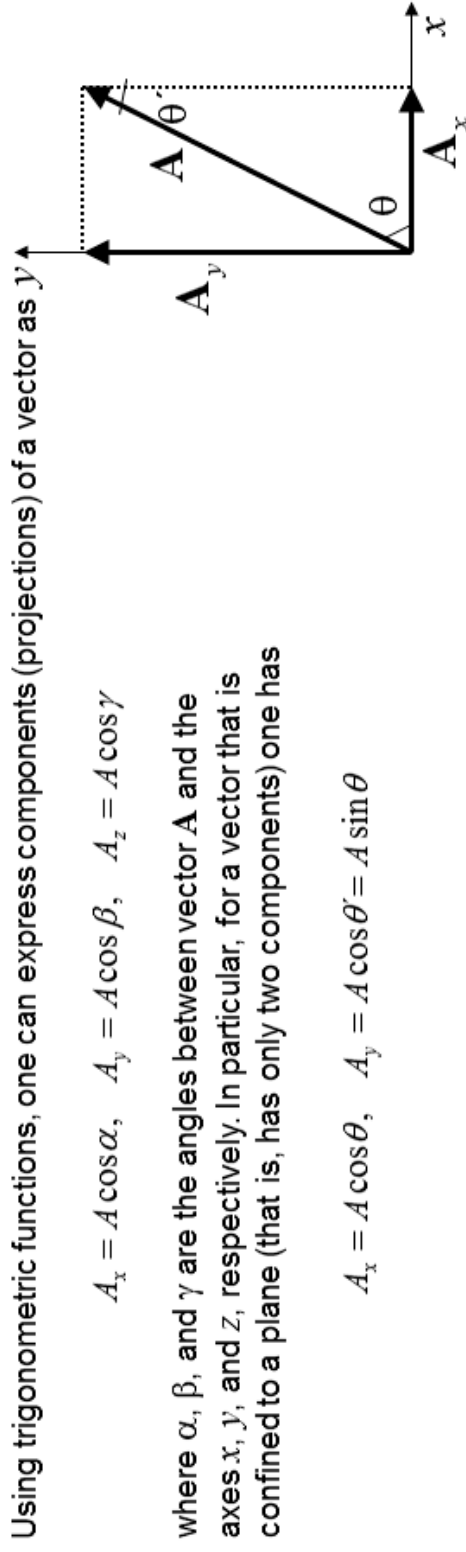
Multiplication or division of a vector by a positive scalar changes its length but does not change its direction. If \mathbf{A} is a vector and $\varphi > 0$ is a scalar, then $\mathbf{B} = \varphi \mathbf{A} = \varphi A \mathbf{n} = B \mathbf{n}$, that is, $\boxed{B = \varphi A}$. Multiplication of a vector by a negative scalar additionally inverts its direction. In components one obtains

$$\mathbf{B} = \varphi \mathbf{A} = \varphi A_x \mathbf{e}_x + \varphi A_y \mathbf{e}_y + \varphi A_z \mathbf{e}_z = B_x \mathbf{e}_x + B_y \mathbf{e}_y + B_z \mathbf{e}_z$$

thus one has simply to multiply components of the vector by the scalar: $\boxed{B_x = \varphi A_x}$, etc., for both signs of φ .

The length of a vector can be obtained in components from the Pythagoras theorem:

$$A = |\mathbf{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$



$$A_x = A \cos \theta, \quad A_y = A \sin \theta$$