Strings, gauge theory and gravity

Storrs, Feb. '07

Outline

- Motivation – why study quantum gravity?

- Intro to strings

- Gravity / gauge theory duality

- Gravity => gauge: Wilson lines, confinement

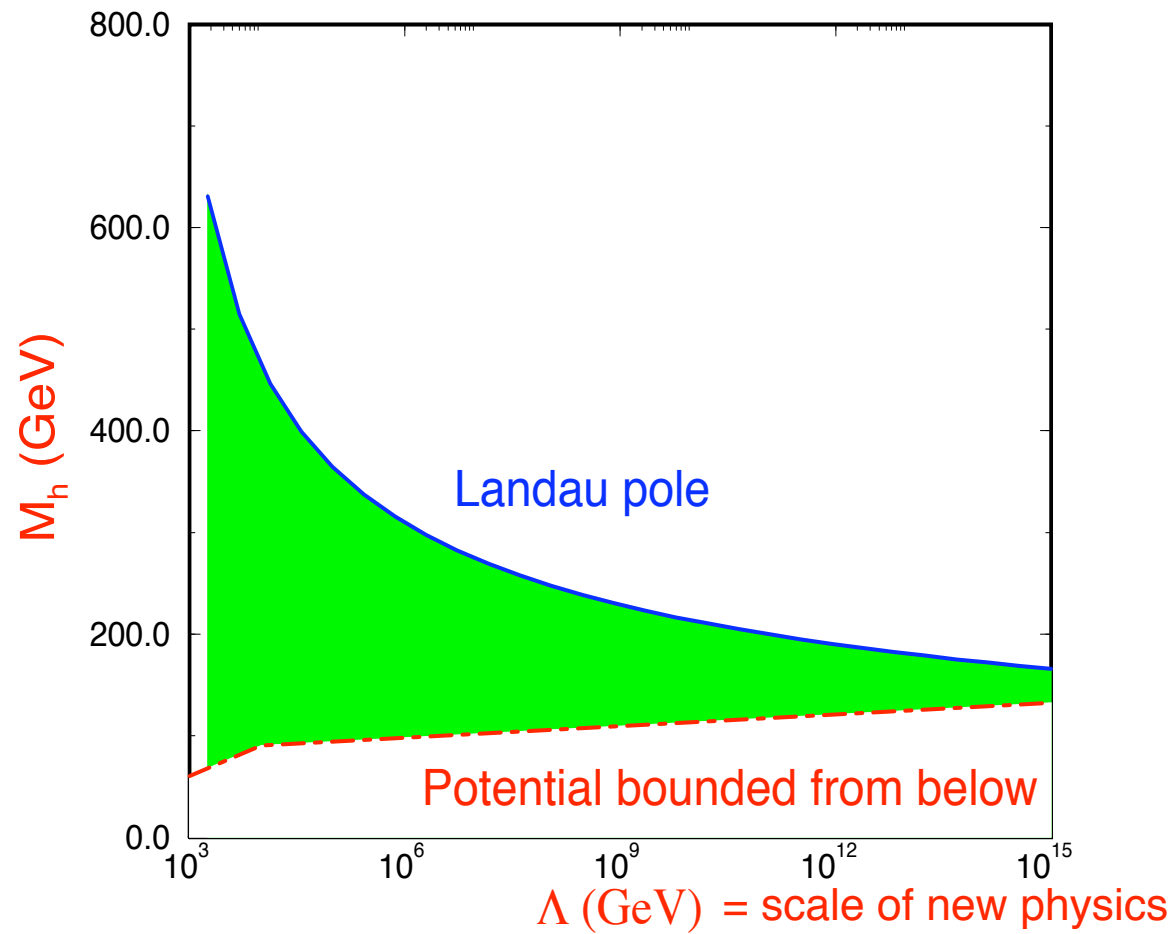- Gauge => gravity: black holes from gauge theory

Motivation

Good theories often carry the seeds of their own destruction. For example blackbody radiation can't be understood using classical E&M.
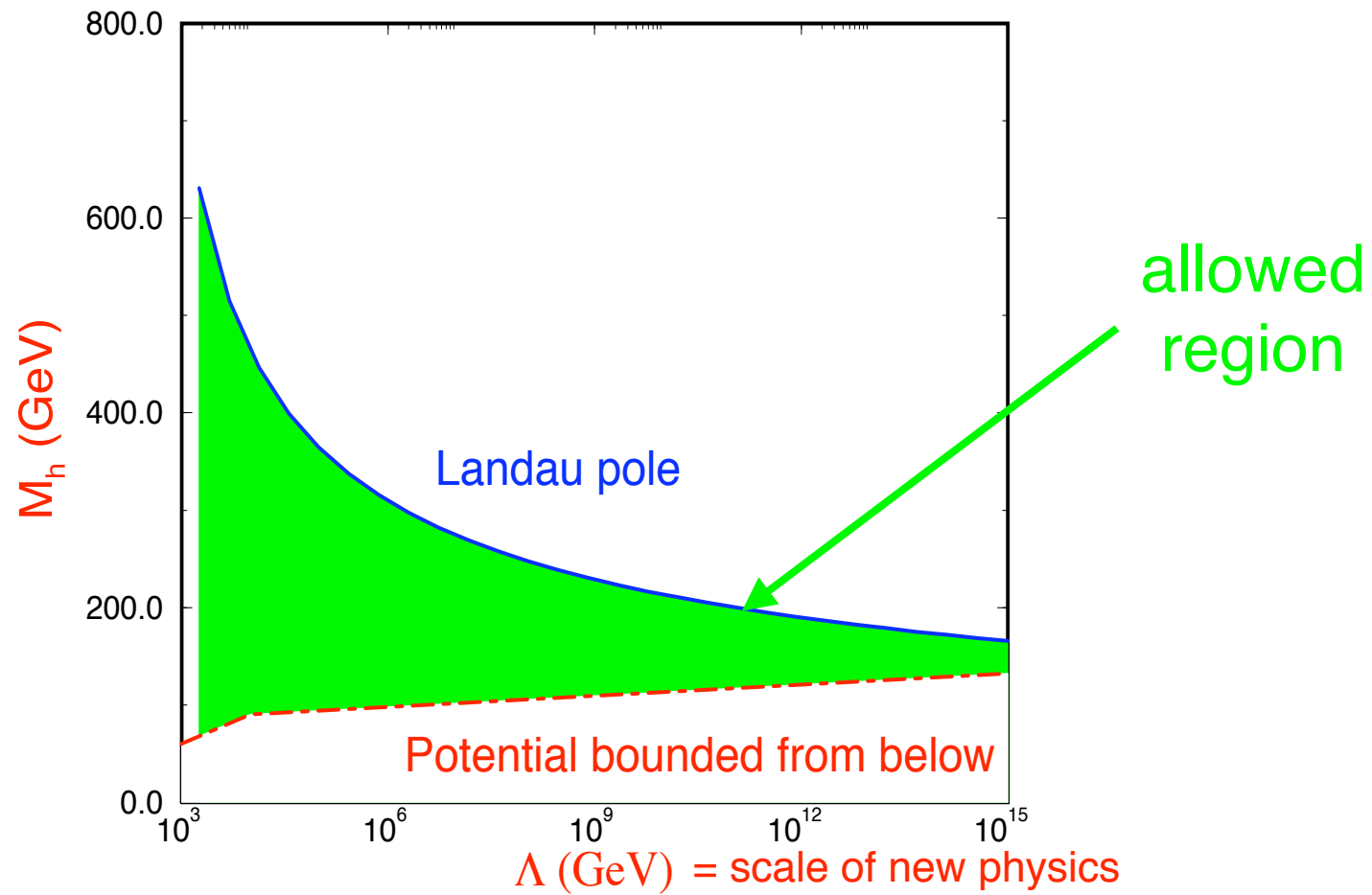
But the standard model of particle physics is a remarkably robust theory. It could well be valid to extremely high energies.

For example, consider dialing the one parameter in the standard model that hasn't been directly observed: the Higgs mass M_h. As a function of M_h you can plot the energy scale Λ where the standard model breaks down.
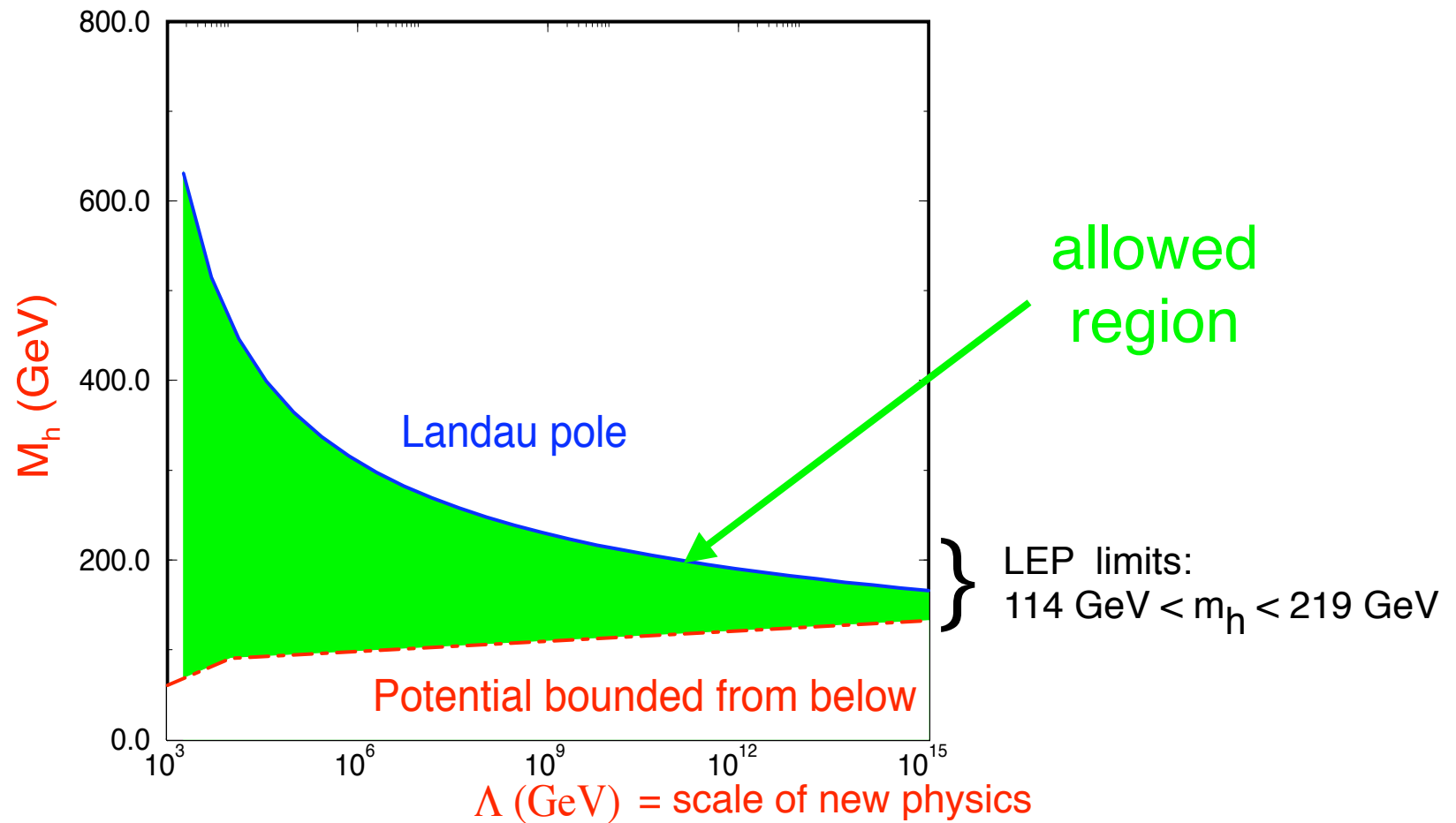
From Dawson, hep-ph/9901280:

M_h (GeV)

800.0

600.0

400.0

Landau pole

200.0

Potential bounded from below

0.0

10^3 10^6 10^9 10^{12} 10^{15}

Λ (GeV) = scale of new physics

From Dawson, hep-ph/9901280:

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allowed
region

M_h (GeV)

Landau pole

LEP limits:
114 GeV $< m_h <$ 219 GeV

Potential bounded from below

Λ (GeV) = scale of new physics

So we don't get much guidance from staring at the standard model. What's a poor theorist to do?

- Experimental input: neutrino masses, dark matter, accelerated expansion,...

- Theoretical puzzles: hierarchy problems (explain electroweak scale, vacuum energy,...)

but the one I'd like to focus on is

- Theoretical problem: the one force that's left out of the standard model - gravity - doesn't play by the usual rules of the game.

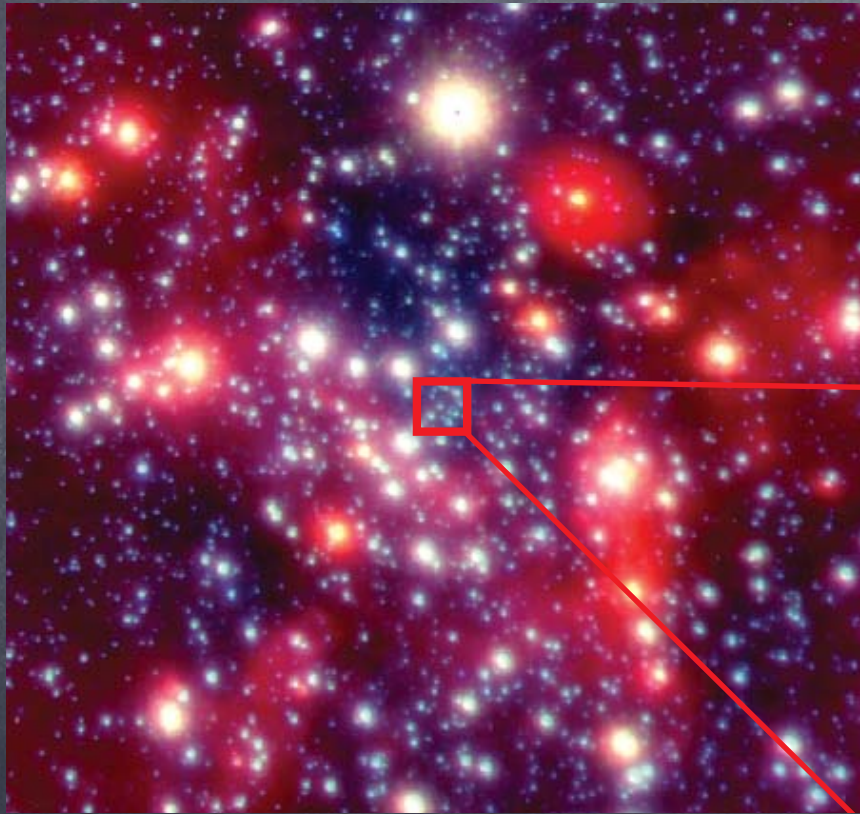The problem is that gravity becomes strong at high energies.

$$F = G\frac{m_1 m_2}{r^2} \quad \Rightarrow \quad F = G\frac{E_1 E_2}{r^2 c^4}$$

This makes it hard to construct a theory of quantum gravity (virtual particles can have arbitrarily large energies).

Why worry, since classical GR describes the world so well?

Singularity theorems => short distance / high energy phenomena are unavoidable (big bang, black holes).

The universe had to begin in a state where the short distance behavior of gravity was important. Besides, right in our neighborhood
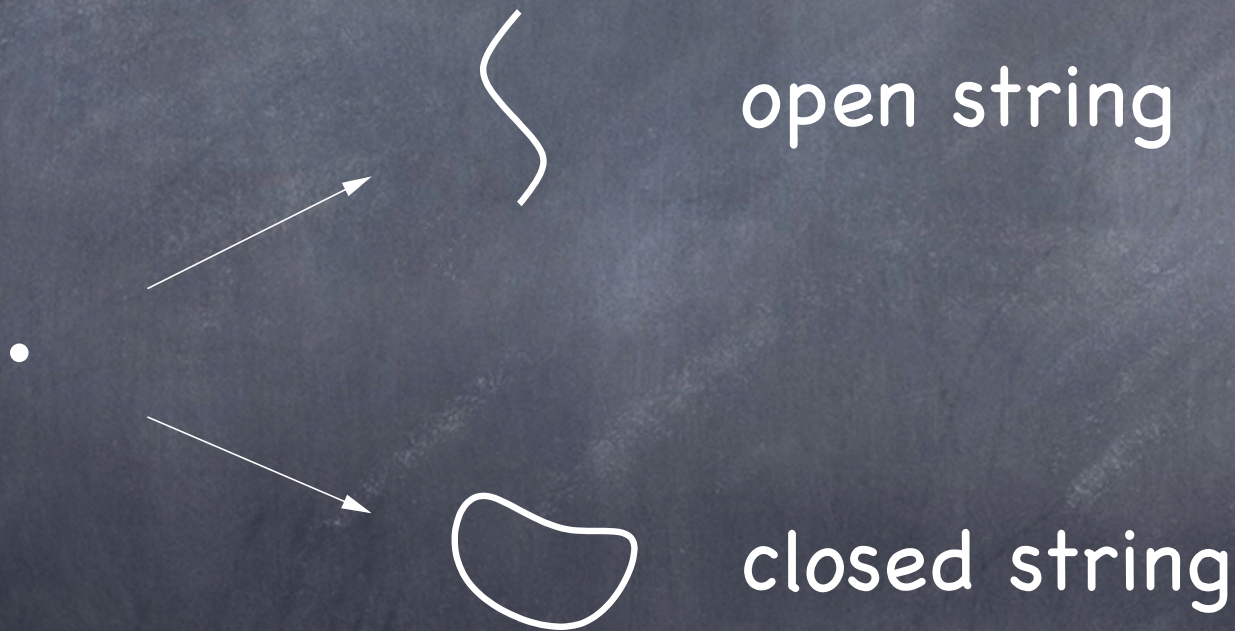

galactic center

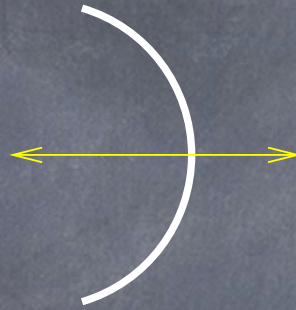$4 \times 10^6 \ M_\odot \ \text{in} \ 1''$

We need a well-behaved quantum theory of gravity!

Intro to strings

The best-developed approach to quantum gravity is string theory. Based on a very simple idea: replace point particles with strings.

open string

closed string

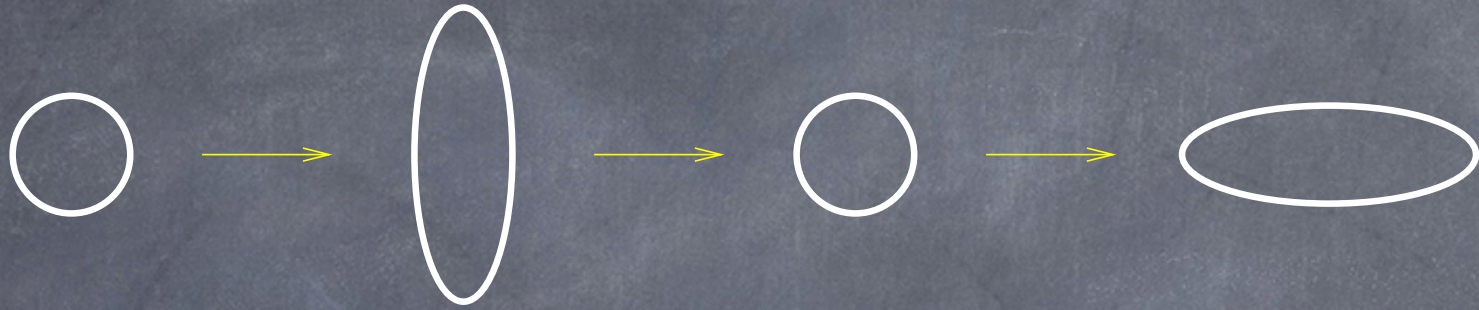Compare the first excited state of an open string
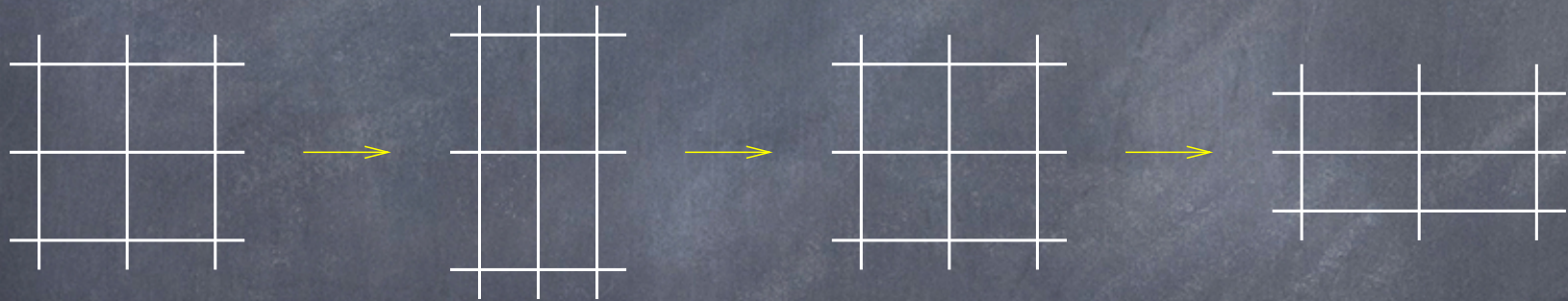

to an electromagnetic wave

$\vec{E}$

Both carry spin 1, and both are massless (in 26 dimensions).

=> open strings incorporate gauge fields!

Compare the first excited state of a closed string

to a gravity wave

Both carry spin 2, and both are massless (in 26 dimensions).

=> closed strings incorporate gravity!

You get the right types of interactions, from strings joining and splitting.

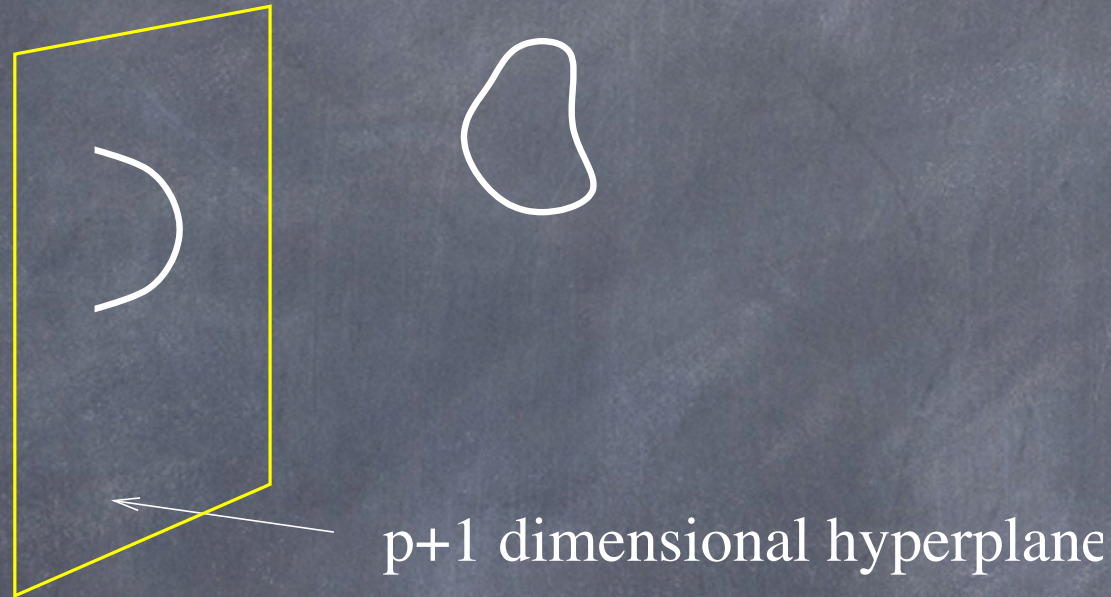This could describe a photon emitting a graviton.

So string theory is automatically a theory of quantum gravity, and it has all the ingredients one needs to build models of particle physics (gauge fields, fermions,...).

Main topic:

It leads to surprising connections between gravity and gauge theory.

Gravity / gauge duality

Open strings can be restricted to end on "D-branes."

p+1 dimensional hyperplane

This gives rise to

- a gauge theory in p+1 dimensions

- gravity in all dimensions

Usually both open and closed strings are important. But in a special "decoupling limit" one can have a complete description either in terms of open strings, or in terms of closed strings.
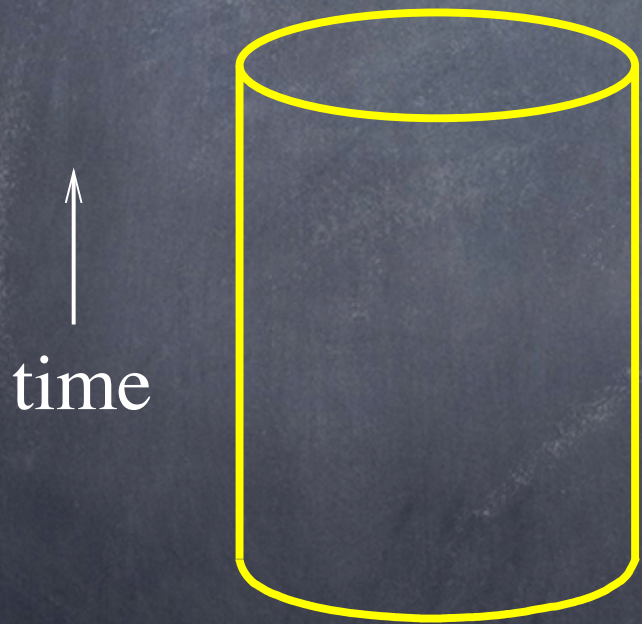
Decoupling?

Go to low energies => only massless open strings important, but gravitational redshift => closed strings survive.

The two descriptions are equivalent. This leads to gravity / gauge duality.

Prototype example: (Maldacena)

Gravity (closed string theory) on $\mathrm{AdS}_5 \times S^5$ is equivalent to a gauge theory ($\mathcal{N} = 4$ susy Yang-Mills) on the boundary of AdS
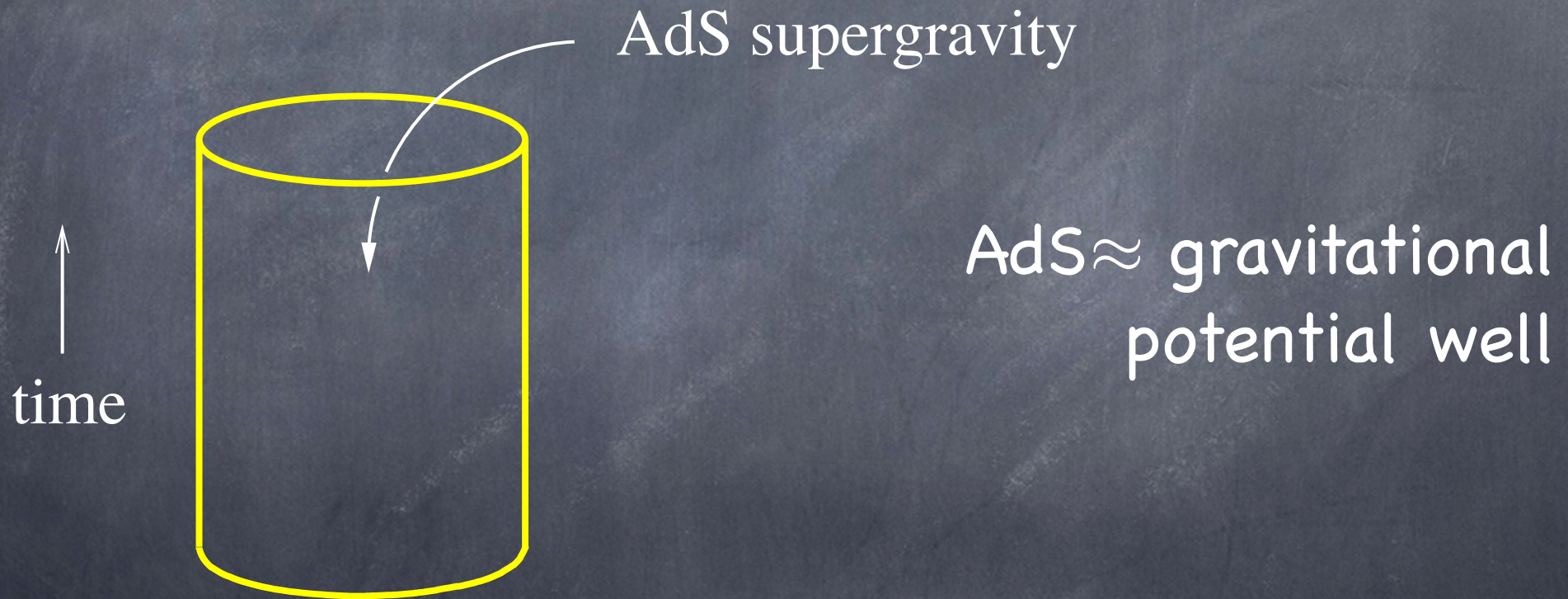

time

AdS $\approx$ gravitational potential well
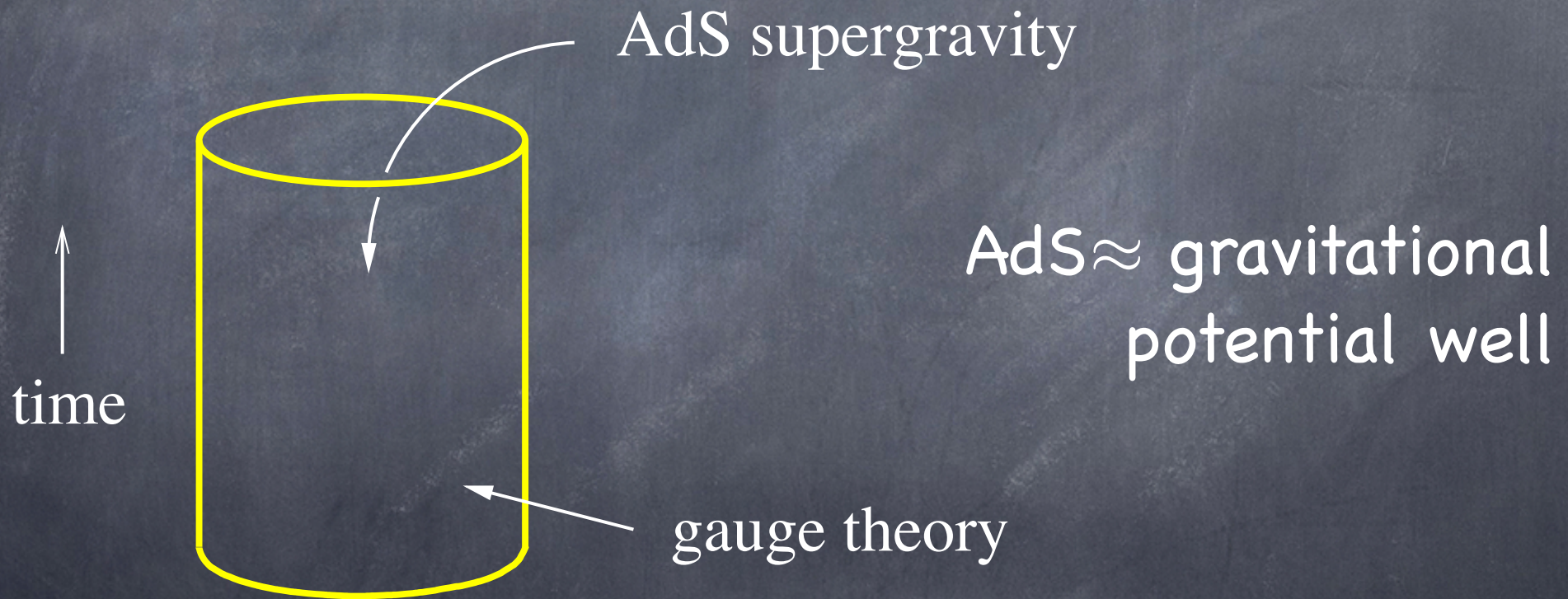
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AdS supergravity

AdS $\approx$ gravitational potential well

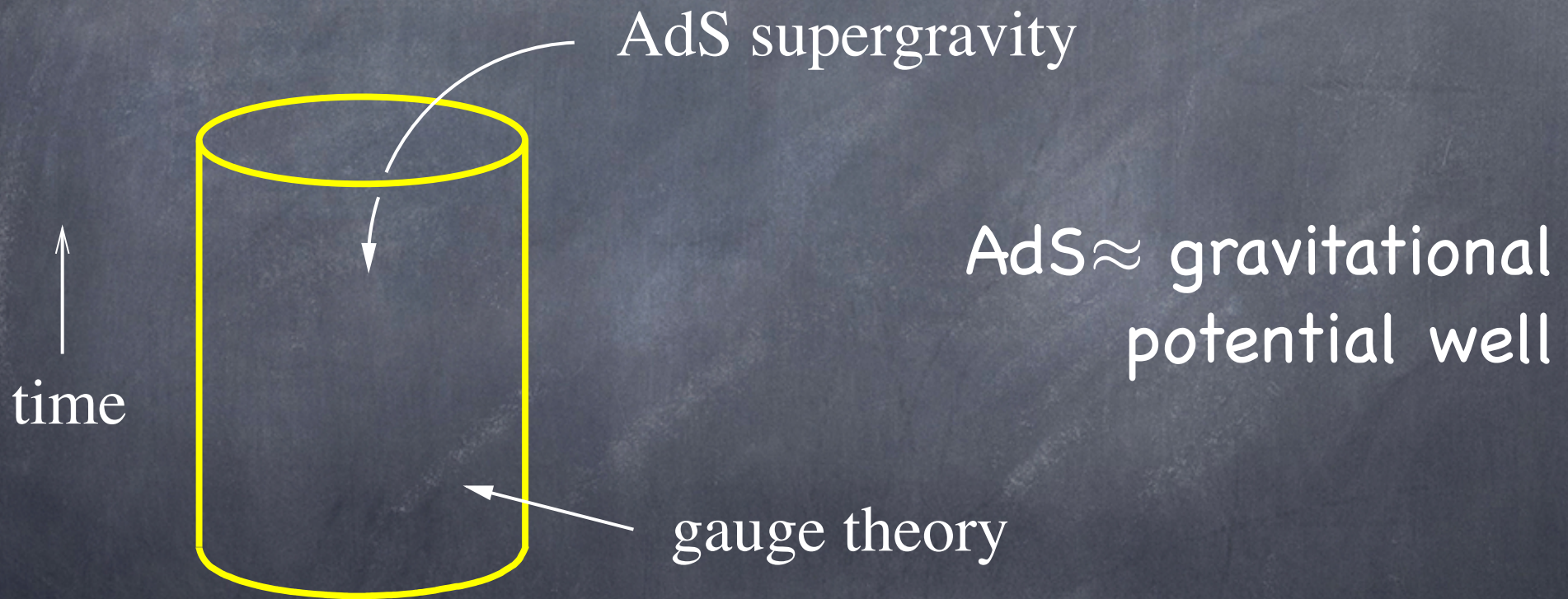time

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gauge theory

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gauge theory

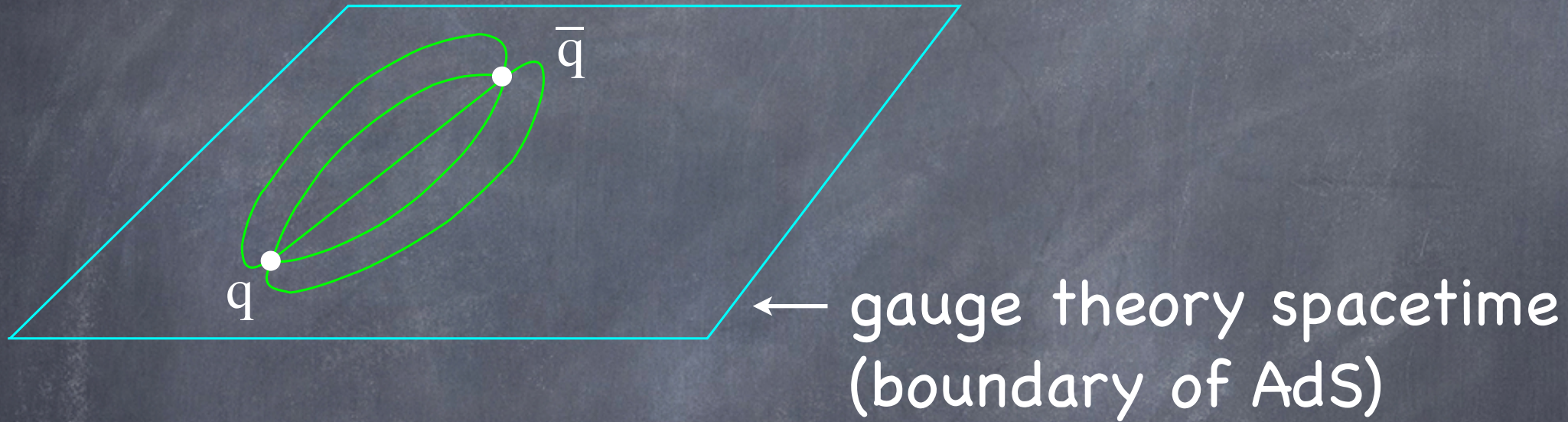Can take this as the definition of AdS supergravity.

Gravity => gauge theory

An interesting quantity in the gauge theory is the interquark potential, or equivalently the Wilson loop expectation value.

$$\bullet \overset{\longleftarrow}{} \quad r \quad \overset{\longrightarrow}{} \bullet \qquad\qquad V(r) = ?$$

q $\qquad\qquad\qquad\qquad$ $\overline{q}$

Need to compute

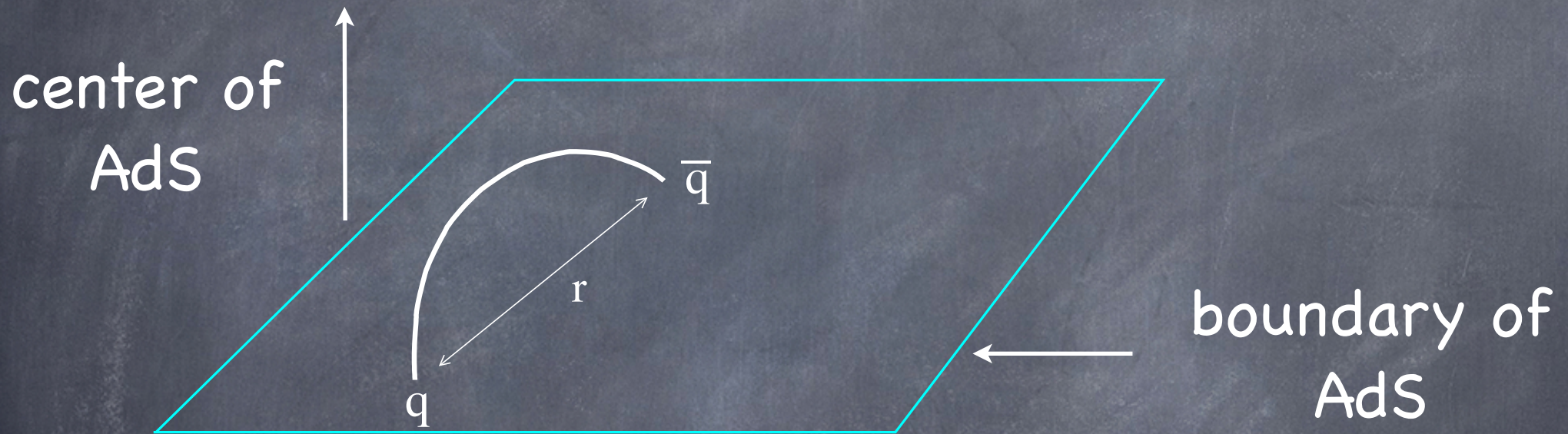$$\langle 0 | \operatorname{Tr} P \, e^{i \oint A_\mu dx^\mu} \, | 0 \rangle$$

In the gauge theory this is a hard calculation. One has to study the electric field

$\bar{q}$

q

← gauge theory spacetime (boundary of AdS)

Non-abelian theory so the flux lines self-interact.

How does it look in supergravity? The quark and antiquark are connected by a string which sags towards the center of AdS.
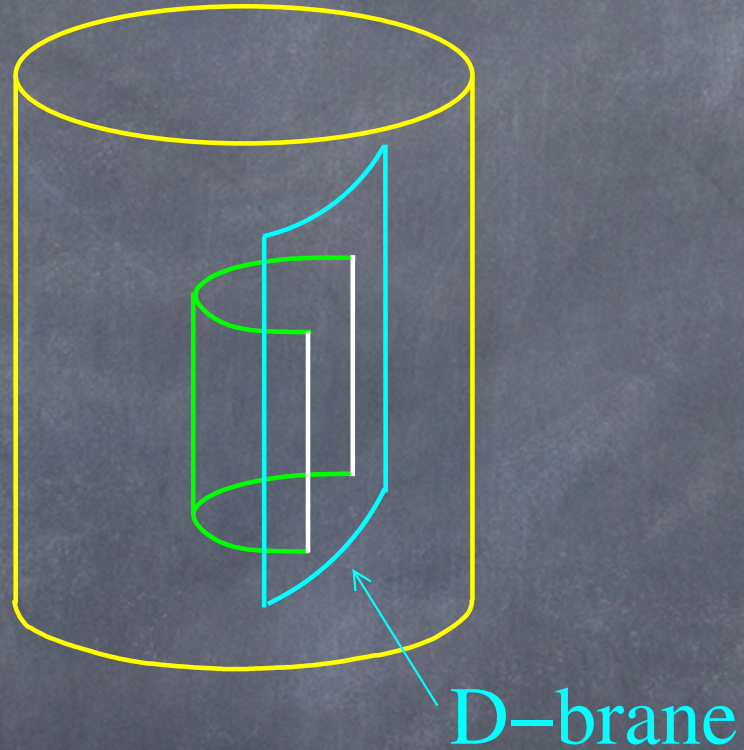
center of
AdS

$\overline{q}$

r

q

boundary of
AdS

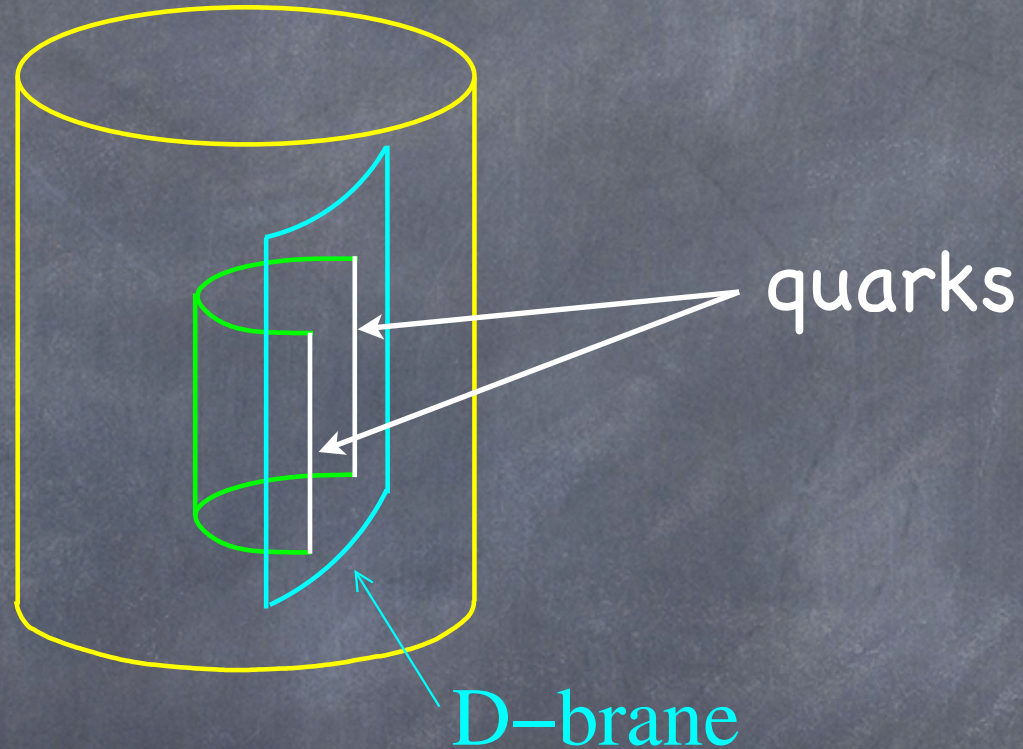You can easily compute the energy of the string.

$$\Rightarrow \quad V(r) = -\frac{4\pi^2 \sqrt{2g_{YM}^2 N}}{\Gamma(1/4)^4 \, r}$$
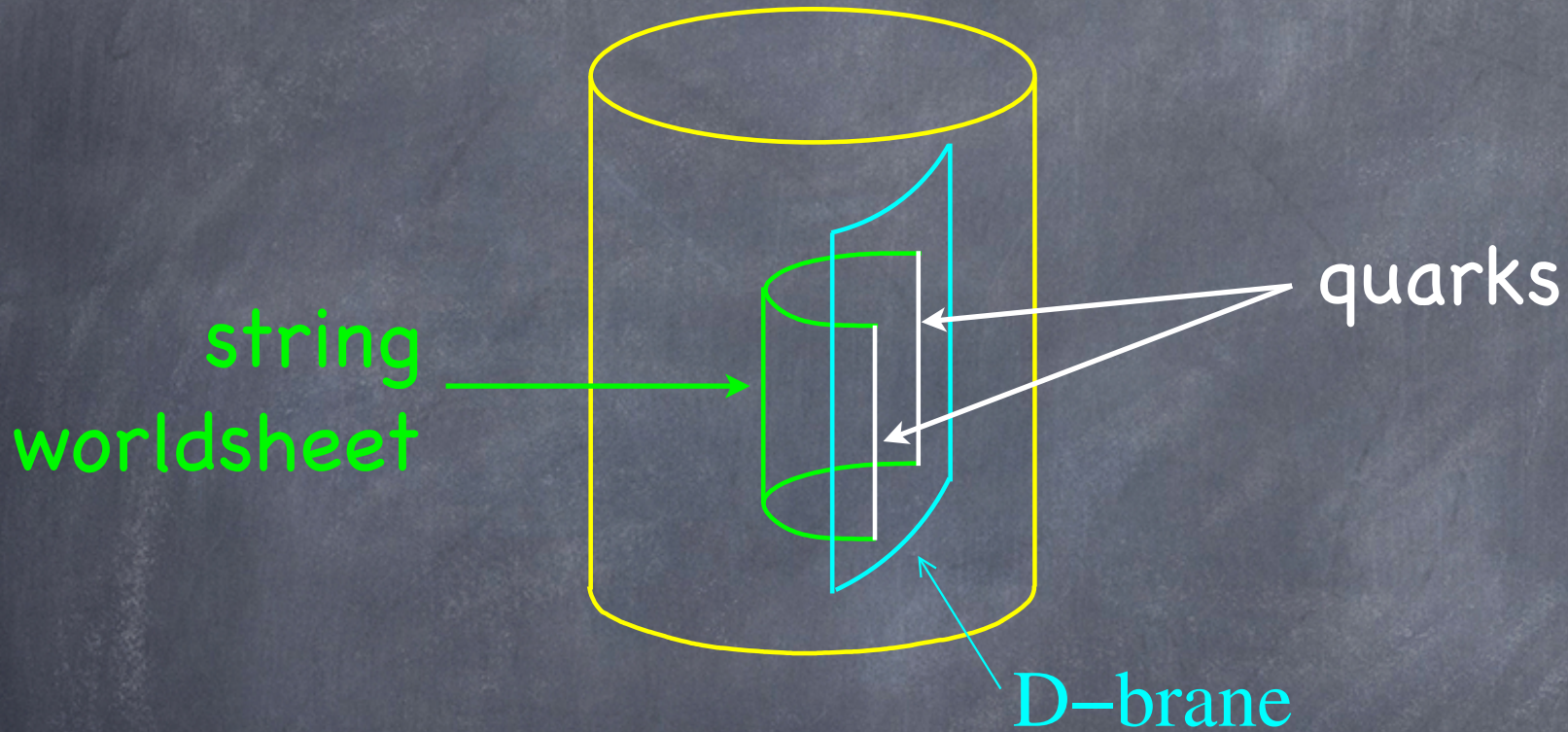
(Maldacena, Rey & Yee)

Cute extension: you can put a D-brane in AdS, so the strings don't have to end on the boundary.
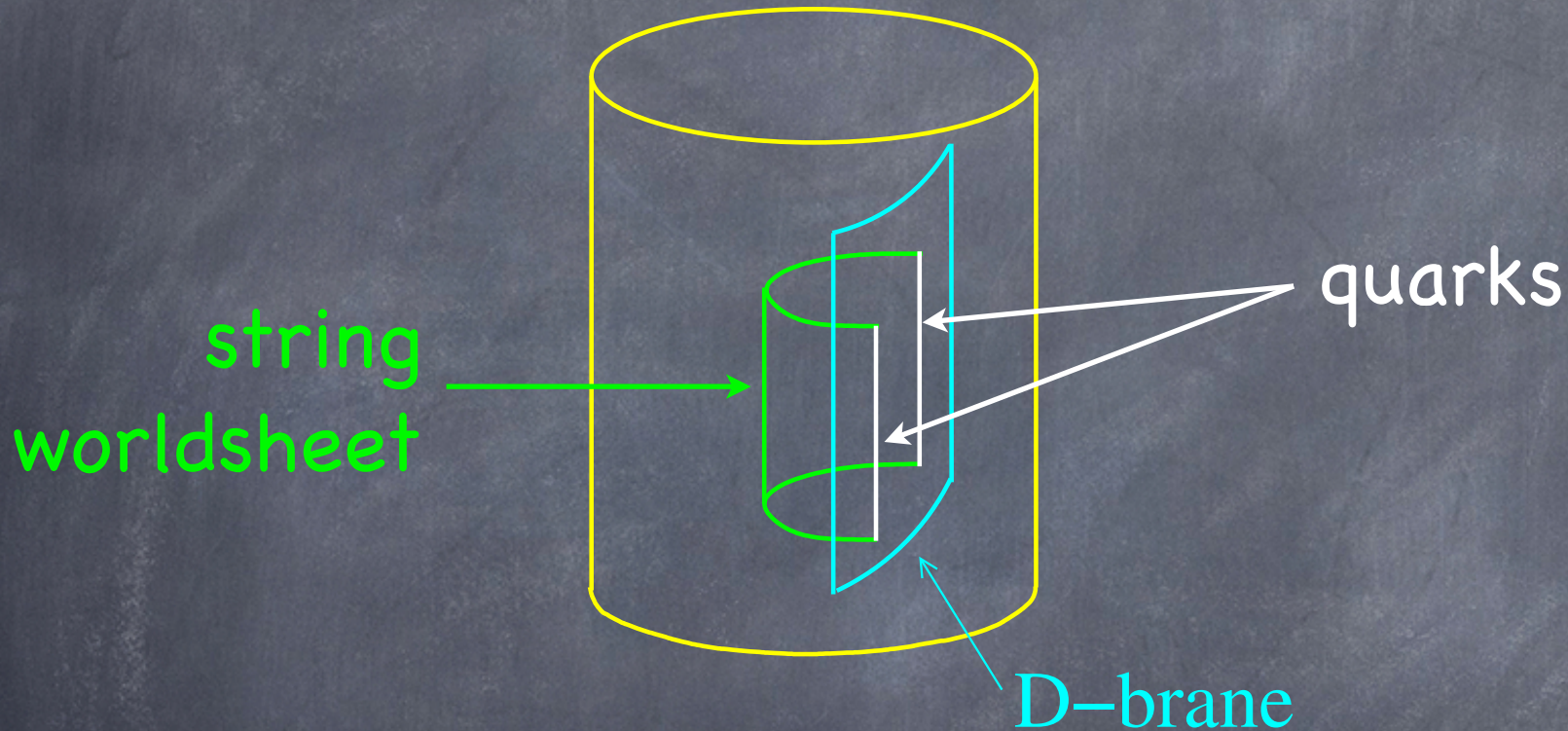
D-brane

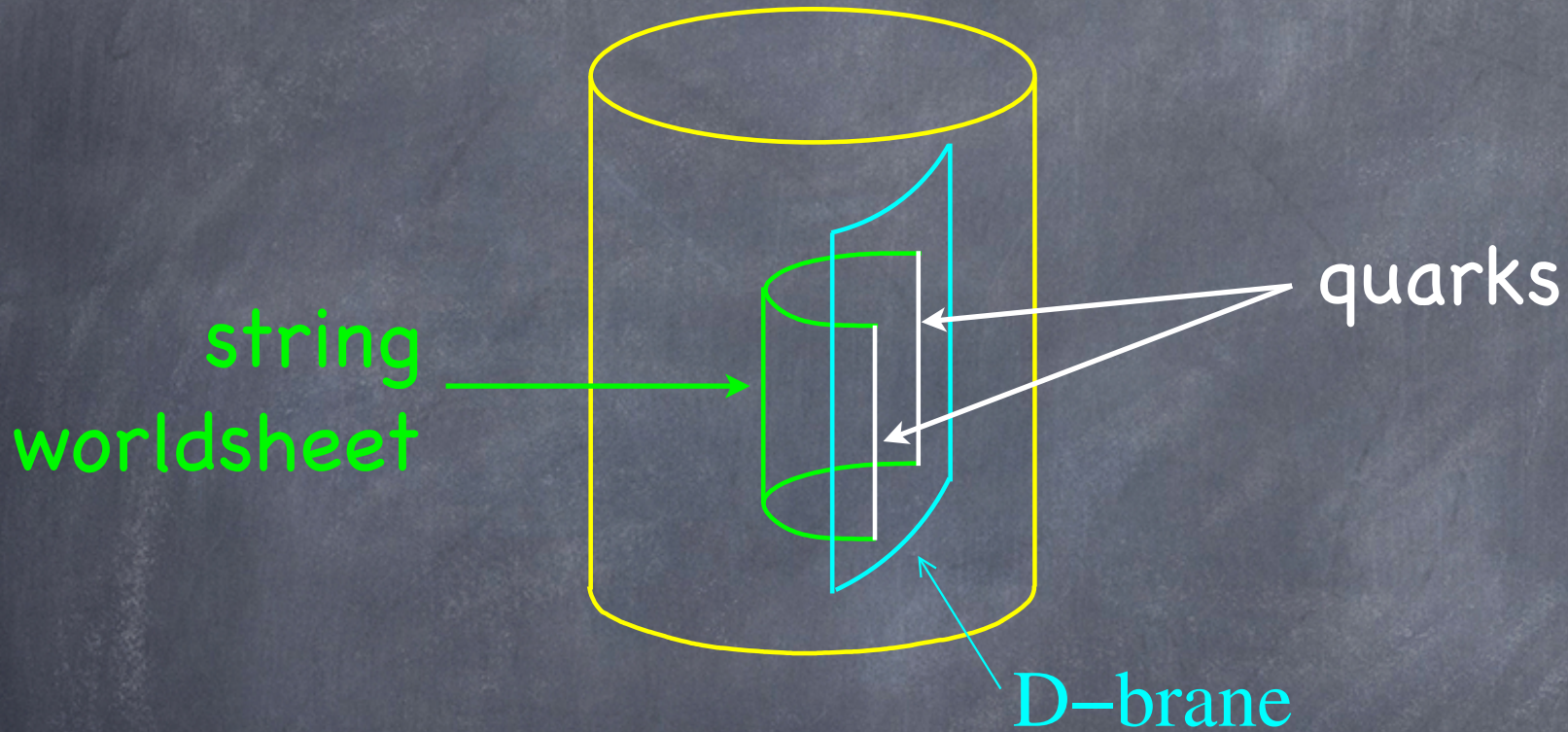Cute extension: you can put a D-brane in AdS, so the strings don't have to end on the boundary.

quarks

D–brane

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quarks

string worldsheet

D–brane

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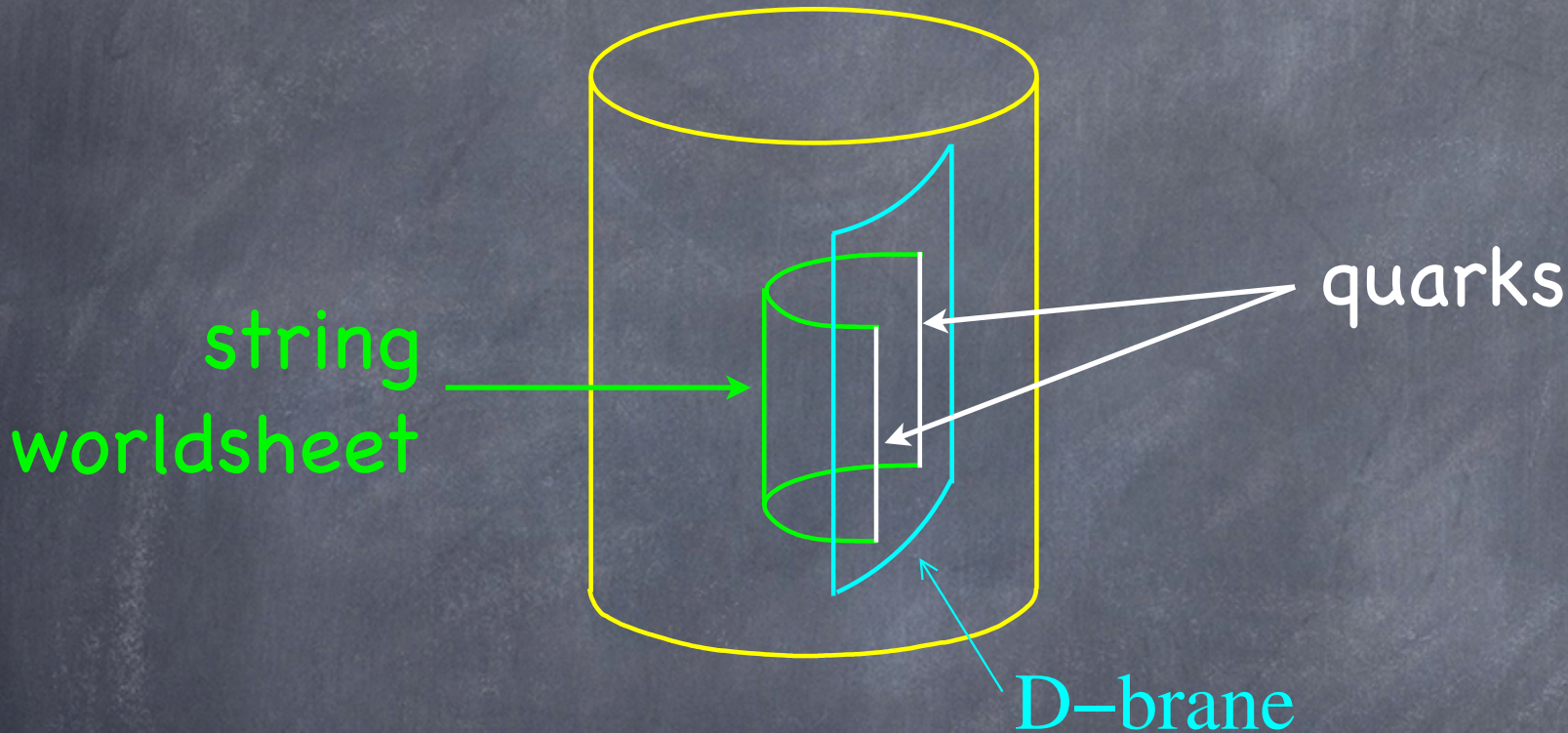

string worldsheet

quarks

D-brane

This gives the quarks a finite mass. You can compute the potential $V(m, r)$.   (DK & Lifschytz)

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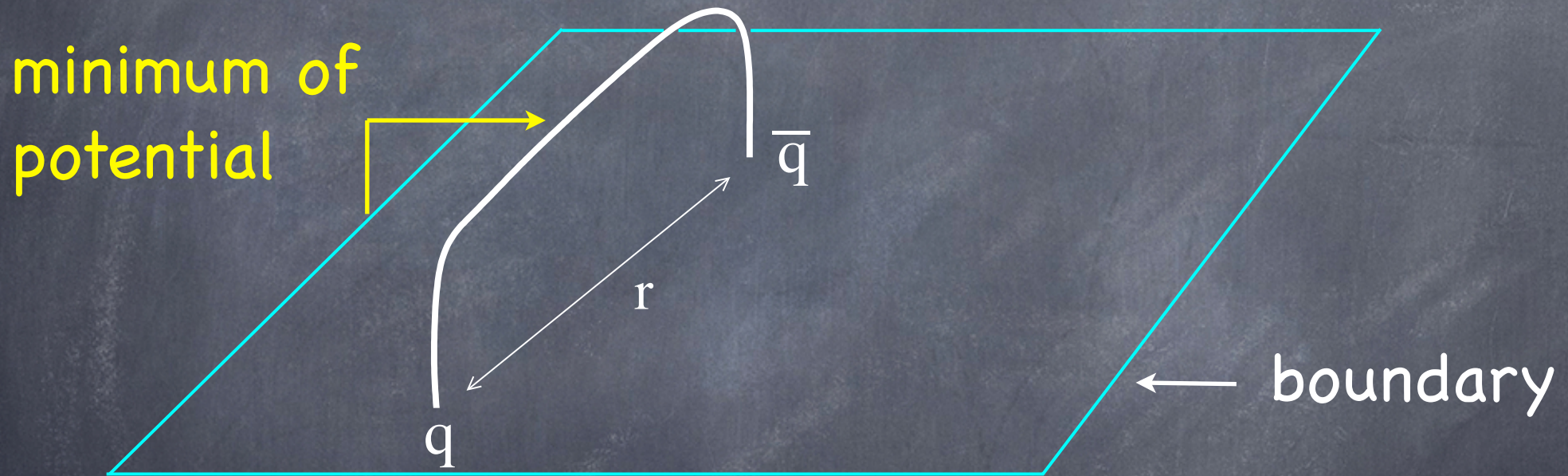
string worldsheet

quarks

D—brane

Cute extension: you can put a D-brane in AdS, so the strings don't have to end on the boundary.

string worldsheet

quarks

D-brane

$$V(m, z) = \frac{m z^{1/2}}{(z+1)^{1/4}} F(\tfrac{1}{4}, \tfrac{1}{2}, \tfrac{3}{2}, -z)$$

$$r(m, z) = \frac{1}{\pi m} \left(\frac{g_{\mathrm{YM}}^2 N}{2} \right)^{1/2} z^{1/2} (z+1)^{1/4} F(\tfrac{5}{4}, \tfrac{1}{2}, \tfrac{3}{2}, -z)$$

How would things look in a confining theory like QCD? The potential well would cut off at a finite depth. Finite redshift => finite energy / unit length for a string parallel to the boundary.

minimum of potential

$\overline{q}$

r

q

← boundary

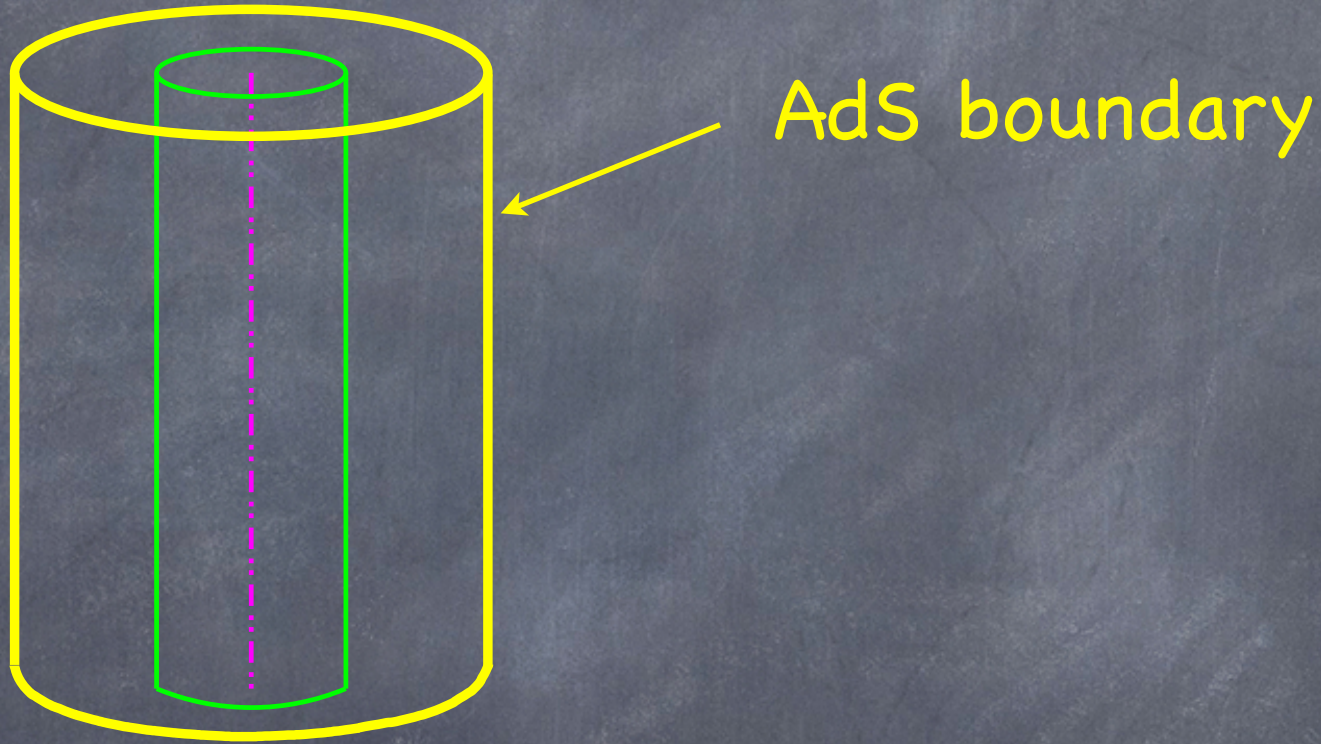The hard part is understanding the geometry near the boundary - it isn't AdS any more.   (Witten,...)

Gauge theory => gravity

Gravity / gauge duality gives a new perspective on long-standing problems of quantum gravity.
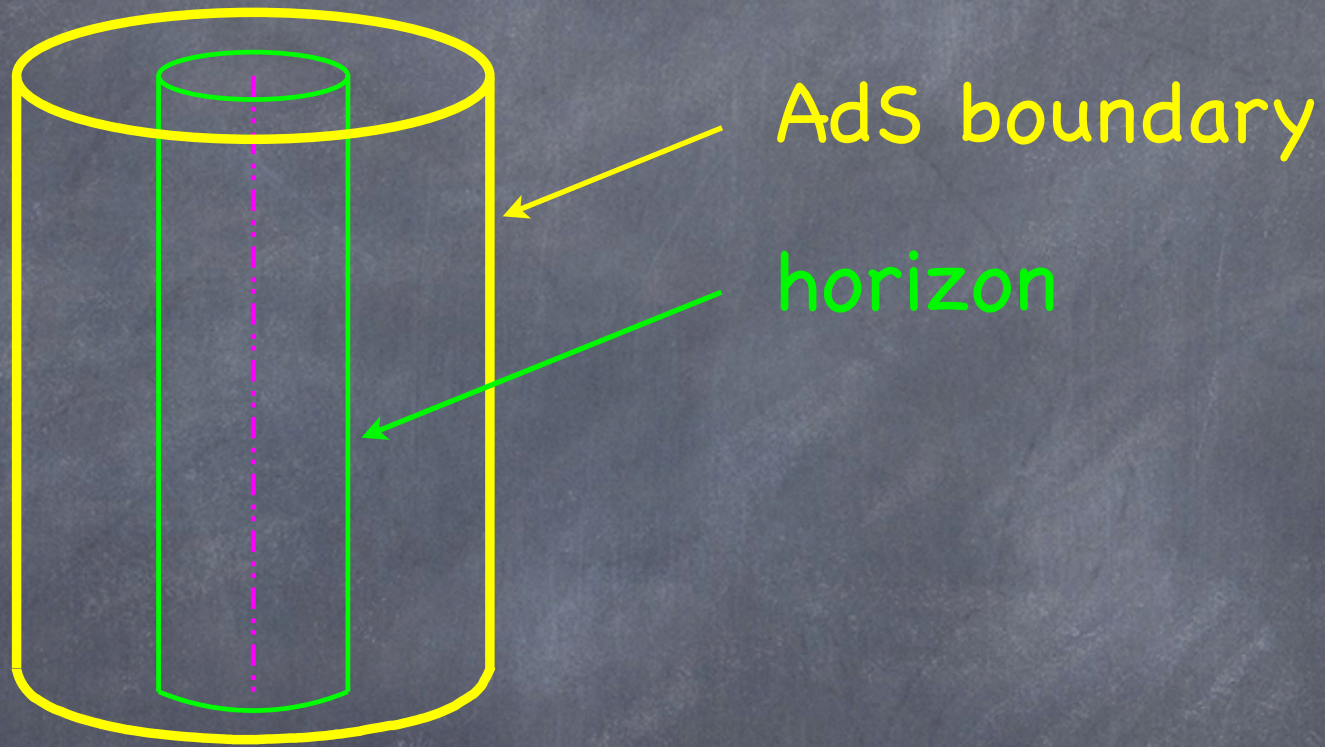
For example, can we understand black holes from the gauge theory perspective?

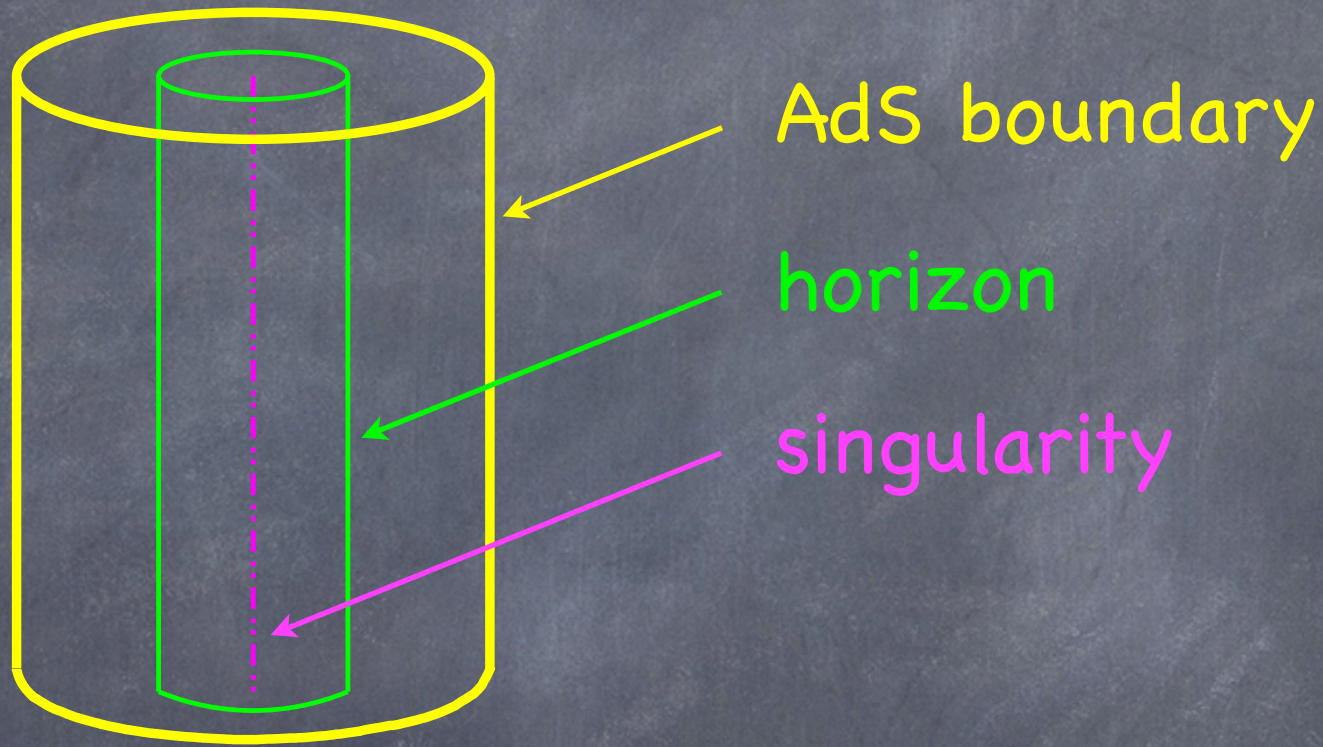Study the duality at finite temperature.

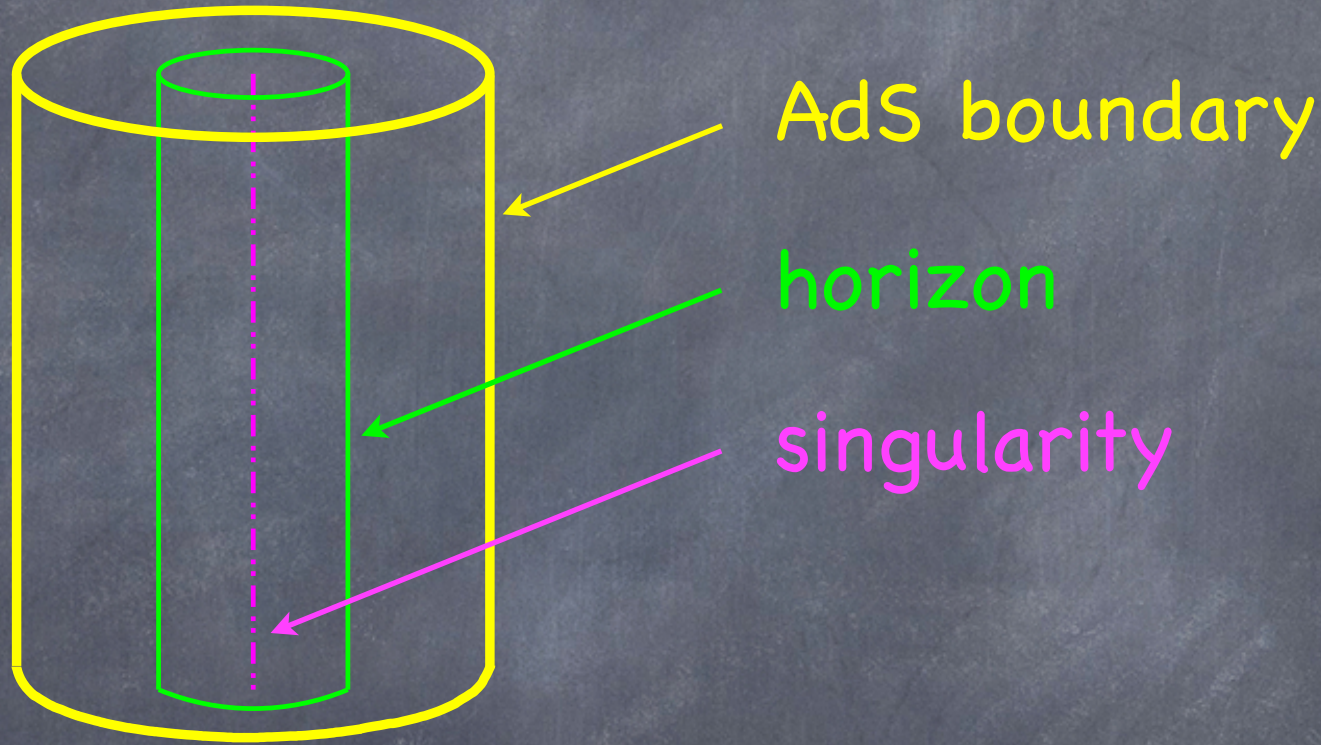gravity description: get a black hole in AdS

AdS boundary

gravity description: get a black hole in AdS

AdS boundary

horizon

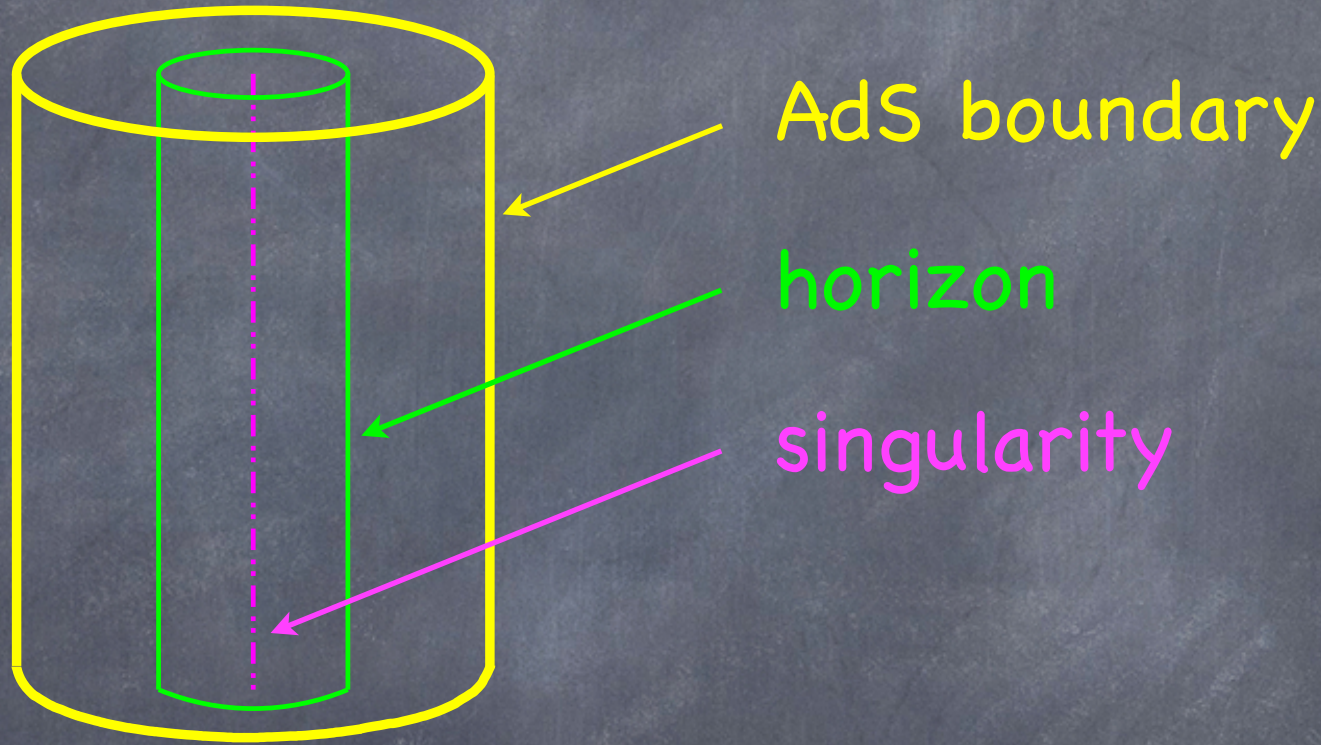# gravity description: get a black hole in AdS

AdS boundary

horizon

singularity

gravity description: get a black hole in AdS

AdS boundary

horizon

singularity

gauge theory description: thermal plasma

gravity description: get a black hole in AdS

AdS boundary

horizon

singularity

gauge theory description: thermal plasma

black hole $\approx$ thermal gauge theory

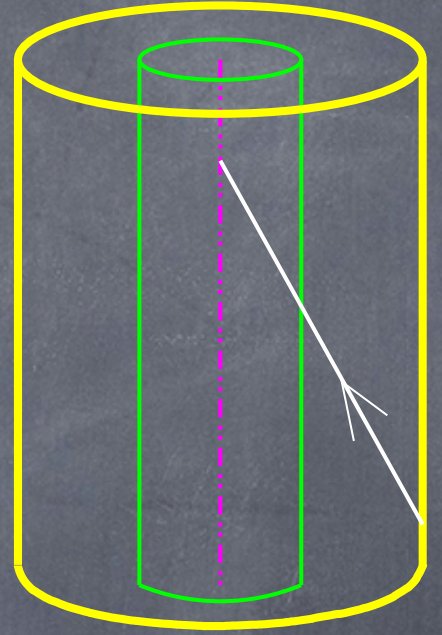# What does absorption by a black hole look like?

gravity description:

object falls towards center of AdS,
trapped once it crosses the horizon
absorption time τ_{abs}

gauge theory description:

approach to thermal equilibrium
equilibration time τ_{equil}

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There's a simple model of the gauge theory in which
the timescales agree.   **Iizuka, DK, Lifschytz, Lowe**

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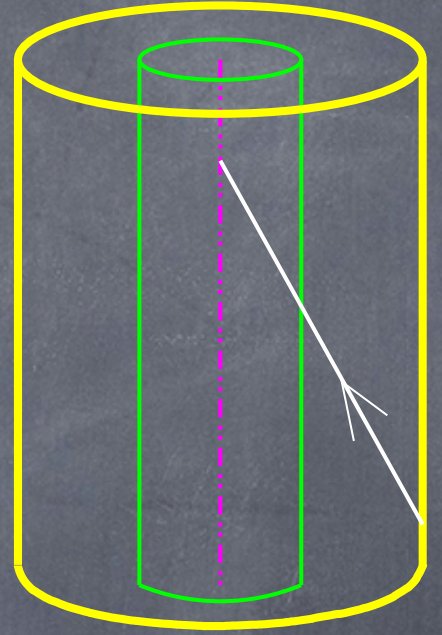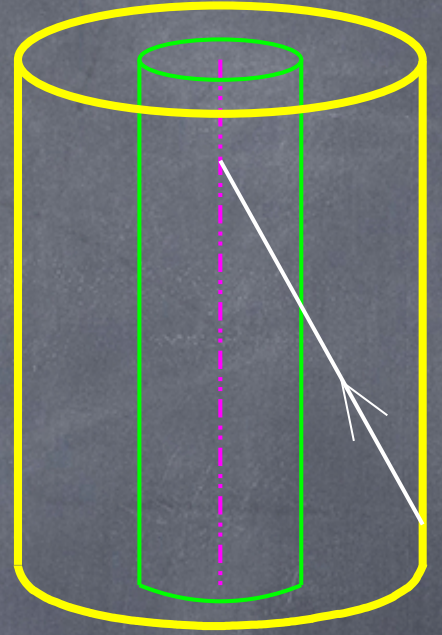
gravity description:
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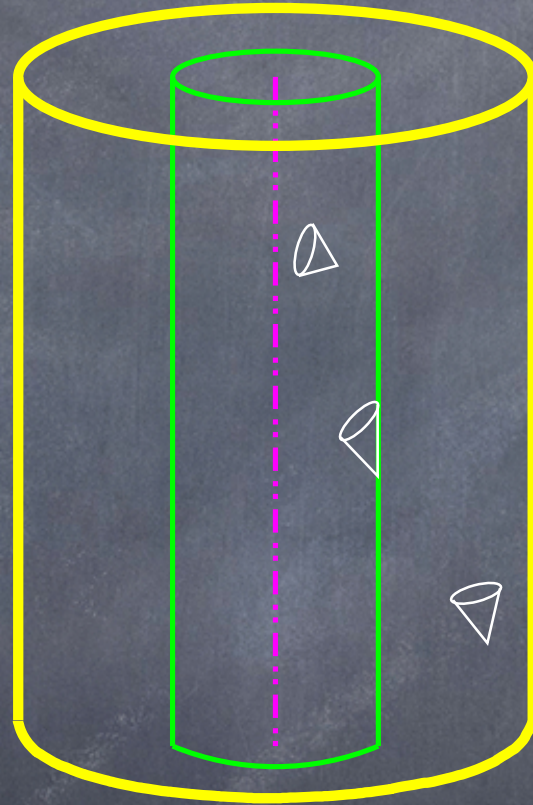gauge theory description:
  approach to thermal equilibrium
  equilibration time τ_{equil}

There's a simple model of the gauge theory in which
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What happens inside the horizon?

The region inside the horizon can't be seen from the boundary - the light cones tip over.

So how can it be described in the gauge theory?

Local operators in the bulk of AdS correspond to non-local operators on the <u>complexified</u> boundary.

For example in AdS_3

radial coordinate in AdS

$\phi(T, X, Z)$ local bulk operator (massless scalar)

$\mathcal{O}(T, X)$ local boundary operator (dimension 2)

$$\phi(T, X, Z) \Leftrightarrow \frac{1}{\pi} \int_R dT' dY' \, \mathcal{O}(T + T', X + iY')$$

$$R = \{T'^2 + Y'^2 < Z^2\}$$
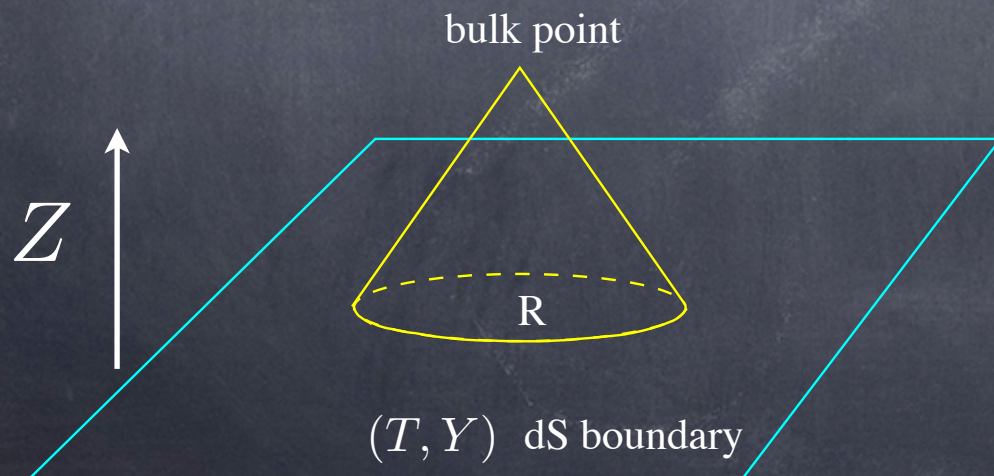
complex spatial coordinates on bdy

Why?

$$ds^2_{AdS} = \frac{1}{Z^2}\left(-dT^2 + dX^2 + dZ^2\right)$$

Wick rotate $X = iY$

$$\Rightarrow \quad ds^2 = \frac{1}{Z^2}\left(-dT^2 - dY^2 + dZ^2\right)$$

This is de Sitter space (Z = time coordinate).

bulk point

Z

R

(T, Y) dS boundary

Conventional initial value problem in de Sitter space.

This construction applies directly to $\mathrm{BTZ} = \mathrm{AdS}_3/\mathbb{Z}$.

You can use it to represent local operators inside the BTZ horizon, or even at the BTZ singularity.

An operator at the singularity corresponds to a non-normalizeable operator in the boundary theory, with divergent correlation functions.

Conclusions

Gravity / gauge duality is a powerful tool for studying gauge theory, and provides a new perspective on quantum gravity.

So far, it has only been precisely formulated in anti-de Sitter space.

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Q. Is there a dual gauge-theory-like description of a realistic cosmology?