Gauge / gravity duality
in
everyday life

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Outline

1. About the title...*
2. What is it?
3. What is it good for?
4. My own interest: gauge => gravity
5. Cosmology?

*credit to Hirosi Ooguri

Gauge / gravity duality, or AdS / CFT duality, has been around for a while.

J. Maldacena,
27 November '97

(building on many previous hints and insights)

Since its inception gauge / gravity duality has become part of everyday life, at least for many in the theory community.

> 13 000 citations to the '97 paper

Sociology, not science.

What is it?

The basic idea is that there is an equivalence between

conformal quantum field theory, e.g. $\mathcal{N} = 4$ susy SU(N) Yang-Mills in 3+1 dimensions

and

quantum gravity with AdS boundary conditions, e.g. IIB supergravity on $\mathrm{AdS}_5 \times S^5$

This is conjectured to be an exact equivalence.

Conformal QFT?

A quantum theory with no dimensionfull parameters, so that it's invariant under scale transformations.

$$(t, \vec{x}) \to (\lambda t, \lambda \vec{x})$$

In 3+1 dimensions the Coulomb potential

$$V(r) = \frac{e^2}{r}$$

is (classically) scale invariant. One can rig up quantum theories that have scale invariance.

One can do this in any number of dimensions. For instance the action in 0+1 dimensions

$$S = \frac{1}{2}\int dt\left(\dot{x}^2 - \frac{\gamma}{x^2}\right)$$

de Alfaro, Fubini, Furlan

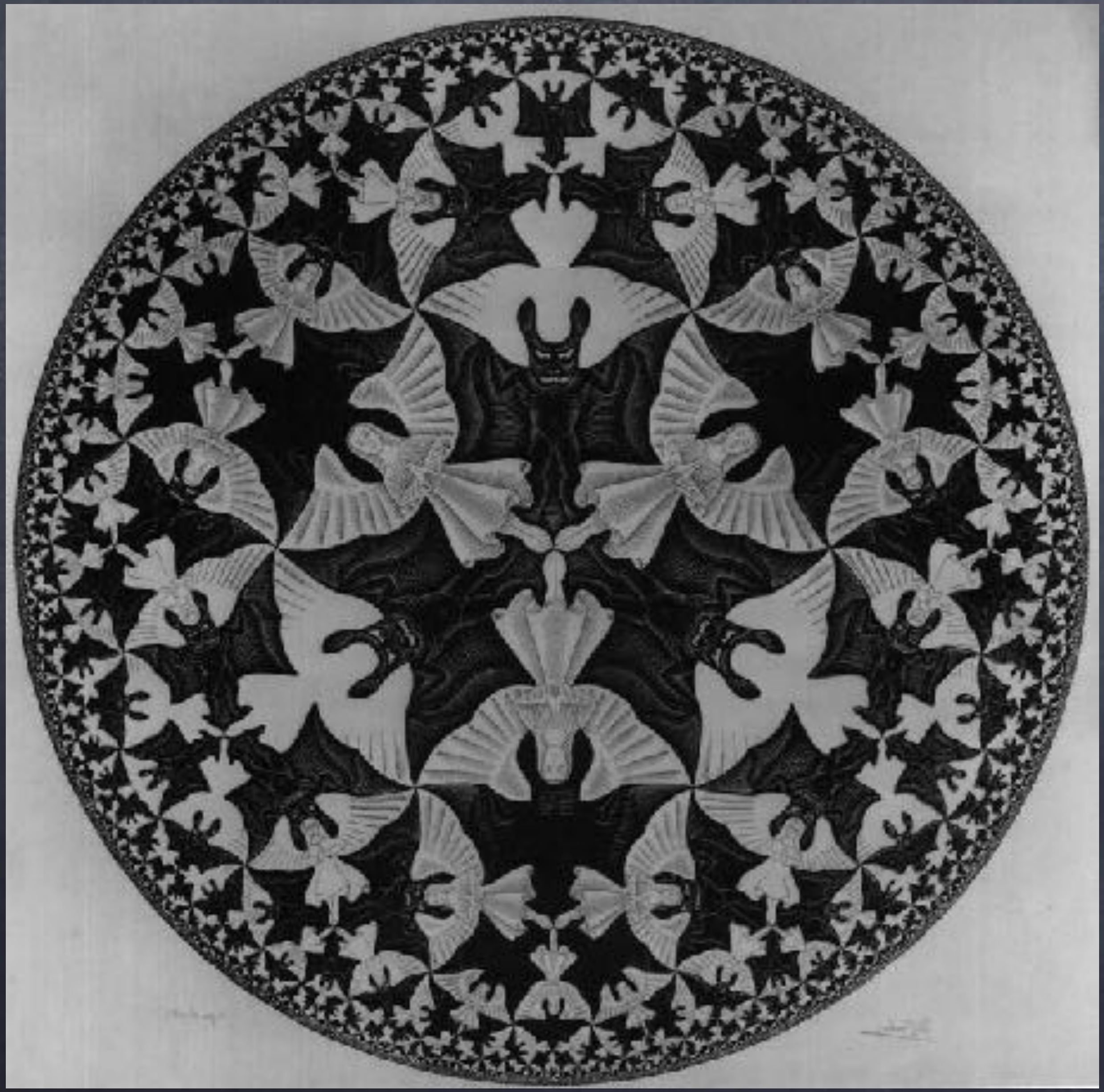
$[x] \sim t^{1/2}$, γ dimensionless

is scale invariant. In fact it has a larger conformal symmetry

$$SL(2,\mathbb{R}): \qquad t \to \frac{at+b}{ct+d}$$

AdS boundary conditions?

Near spatial infinity, the geometry has constant negative curvature.

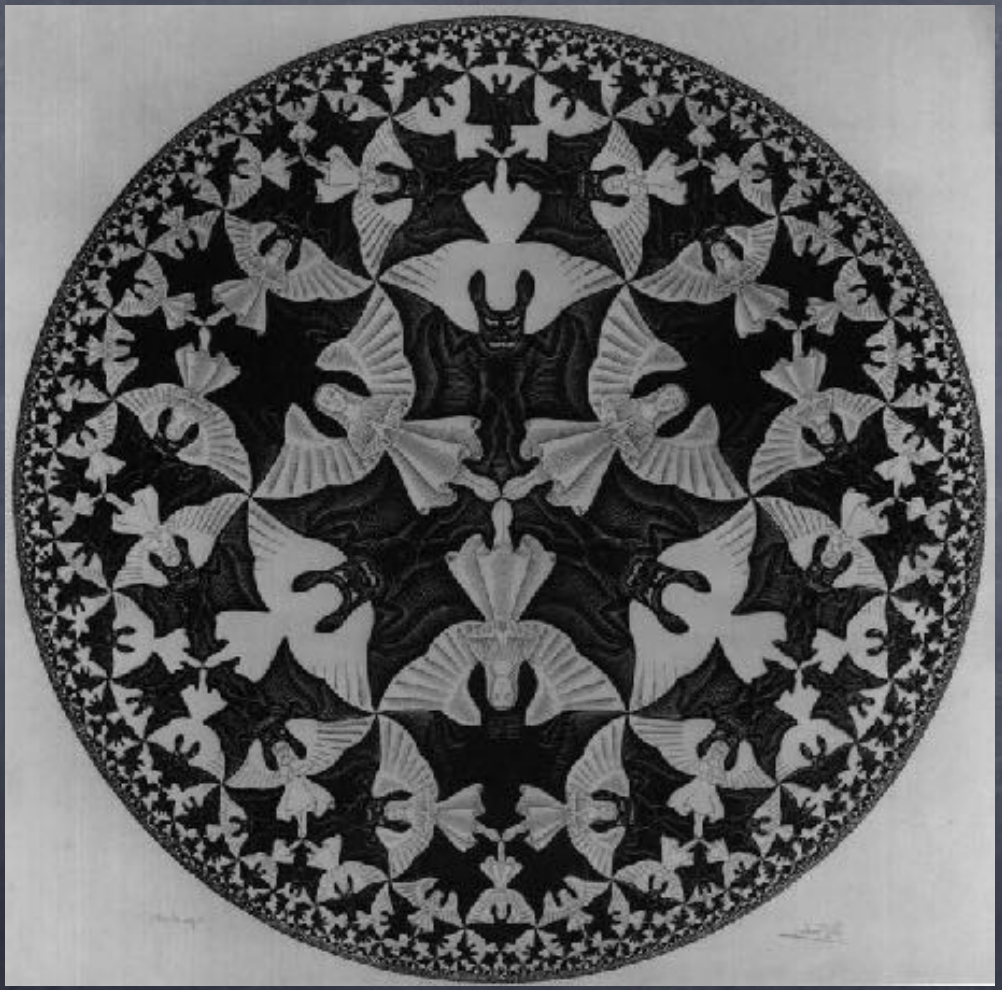


Euclidean
2 dimensional

$$ds^2 = \frac{dr^2 + r^2 d\theta^2}{(1 - r^2)^2}$$

$SL(2, \mathbb{R})$ isometry

To each conformally invariant field theory on the boundary, there should be an equivalent quantum gravity theory in the bulk.



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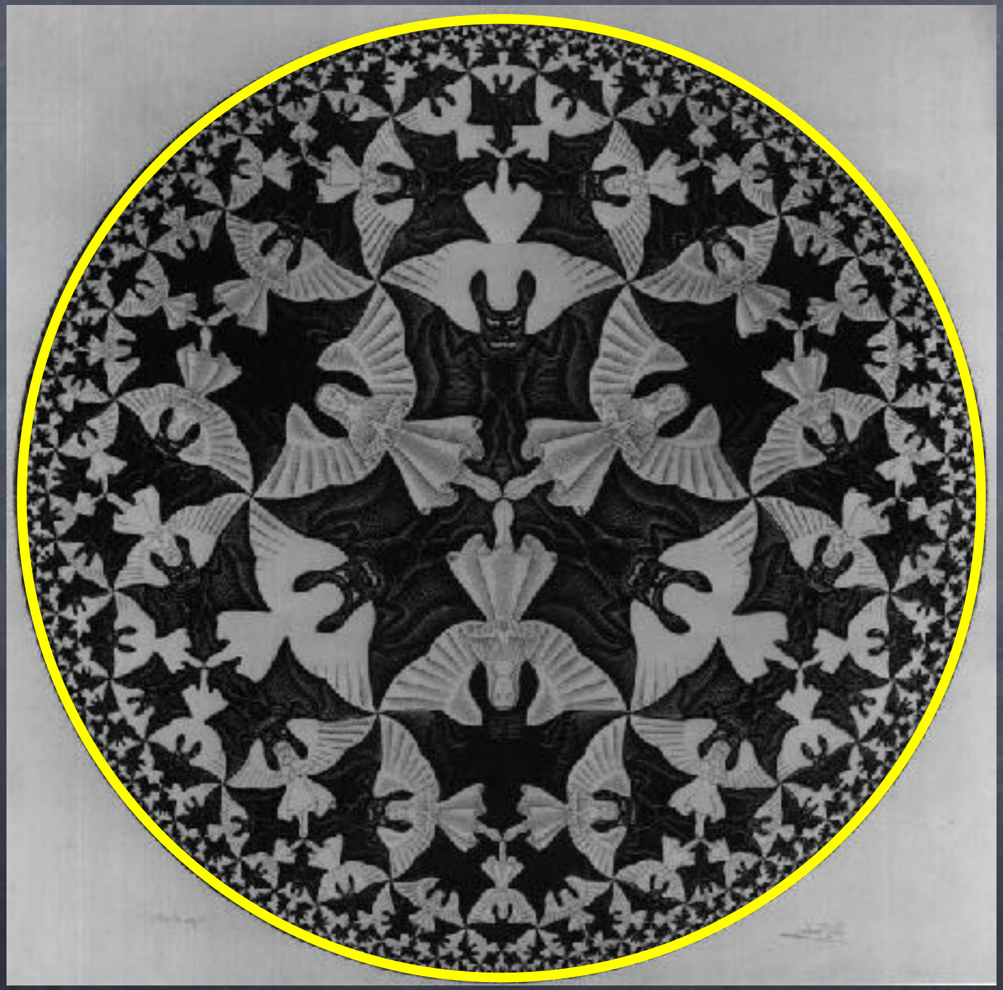

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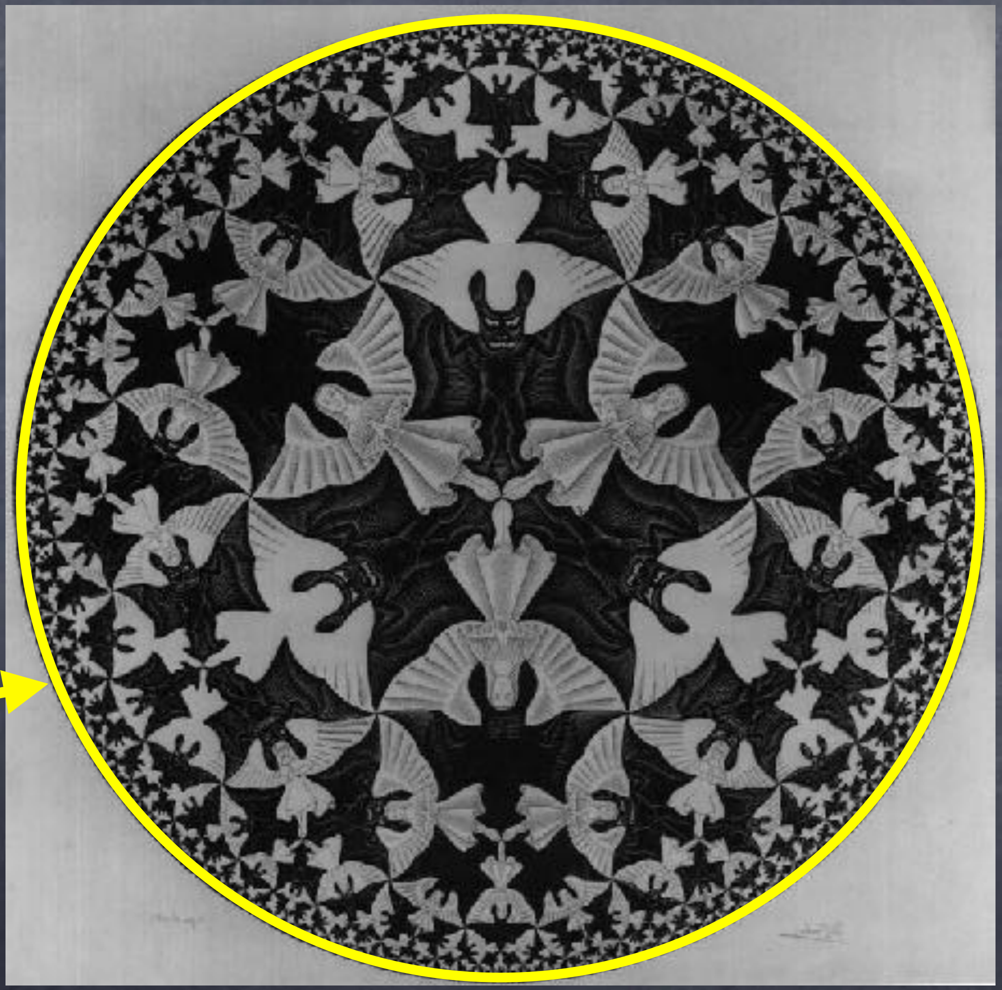
CFT on boundary



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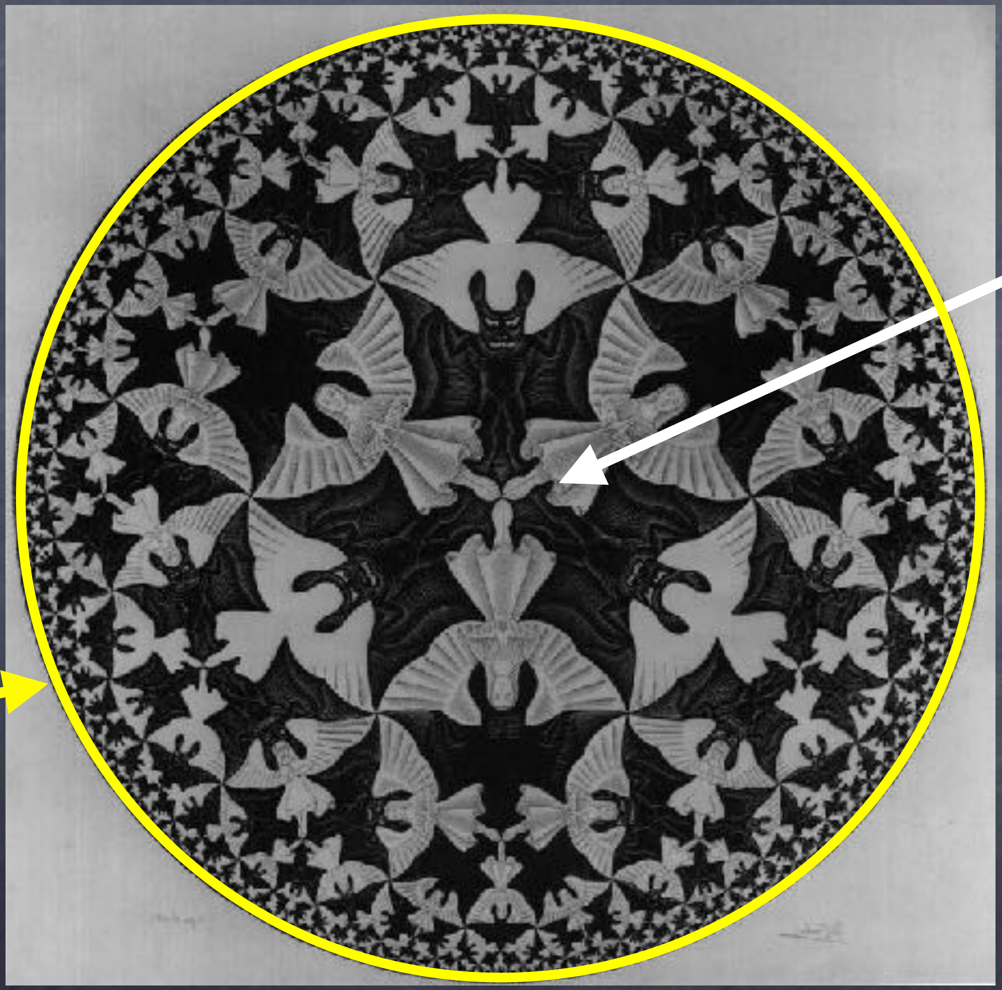

gravity in the bulk

CFT on boundary

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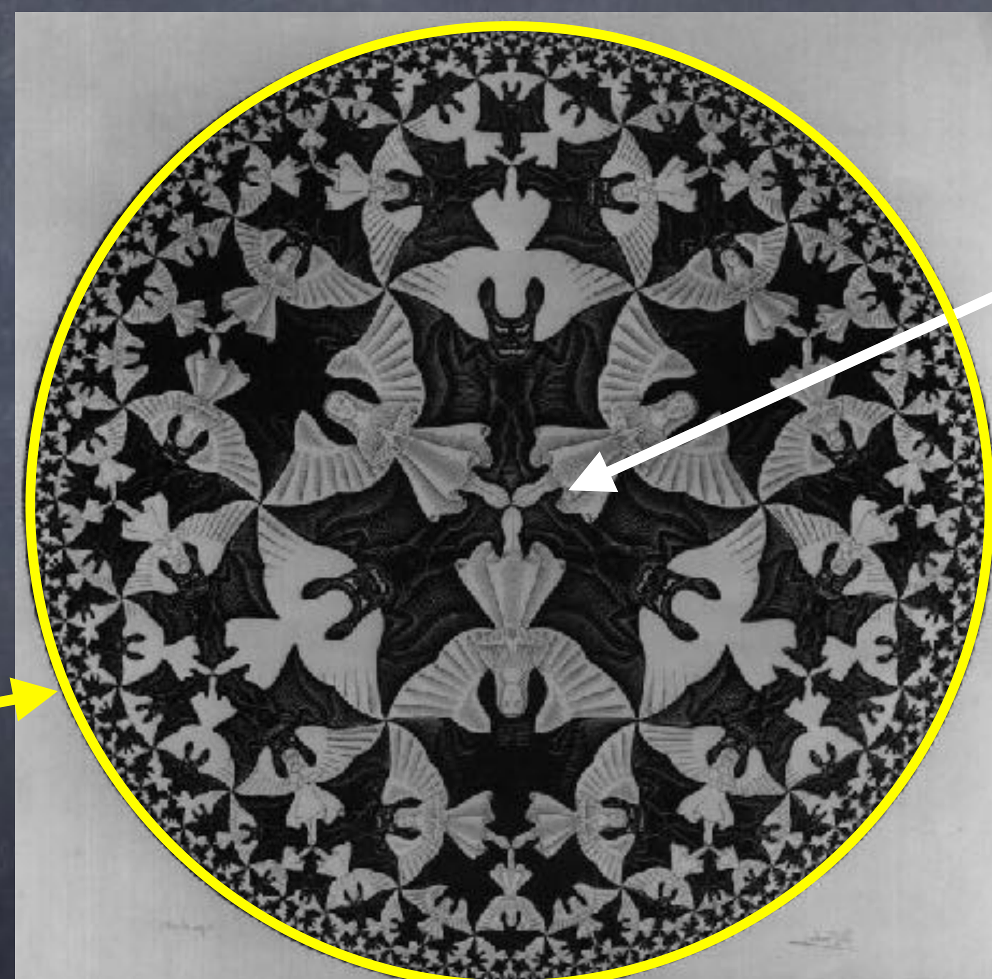

gravity in the bulk

CFT on boundary

This works both ways: (bulk gravity) <=> (boundary)

The power (and the challenge) comes from the fact that when the gravity theory is weakly coupled the CFT is strongly coupled, and visa versa.

For instance for $\mathcal{N}=4$ SU(N) Yang-Mills

gauge coupling $\lambda = g_{\mathrm{YM}}^2 N$

AdS radius of curvature $R_{\mathrm{AdS}} = \lambda^{1/4}\,\ell_{\mathrm{string}}$

What is it good for?

Can use bulk gravity as a tool to study strongly-coupled CFT's. Let's heat up the CFT.

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$$= \frac{\pi^2}{2} N^2 T^3 V$$

for $\mathcal{N} = 4$ SYM

Gubser, Klebanov, Peet 1996

What about viscosity at finite temperature?

CFT

$$\eta = \frac{i}{\omega} \int dt d\mathbf{x}\, e^{i\omega t} \theta(t) \left\langle [T(t,\mathbf{x}), T(0,0)] \right\rangle \Big|_{\omega \to 0}$$

perturb the CFT and see how the stress tensor responds

gravity

perturb the metric with a low-energy graviton and ask if it's absorbed by the black hole

$$\eta = \frac{\text{area}}{16\pi G}$$

Kovtun, Son, Starinets
2004

Comparing the two expressions

$$S = \frac{\text{area}}{4G} \qquad \eta = \frac{\text{area}}{16\pi G}$$

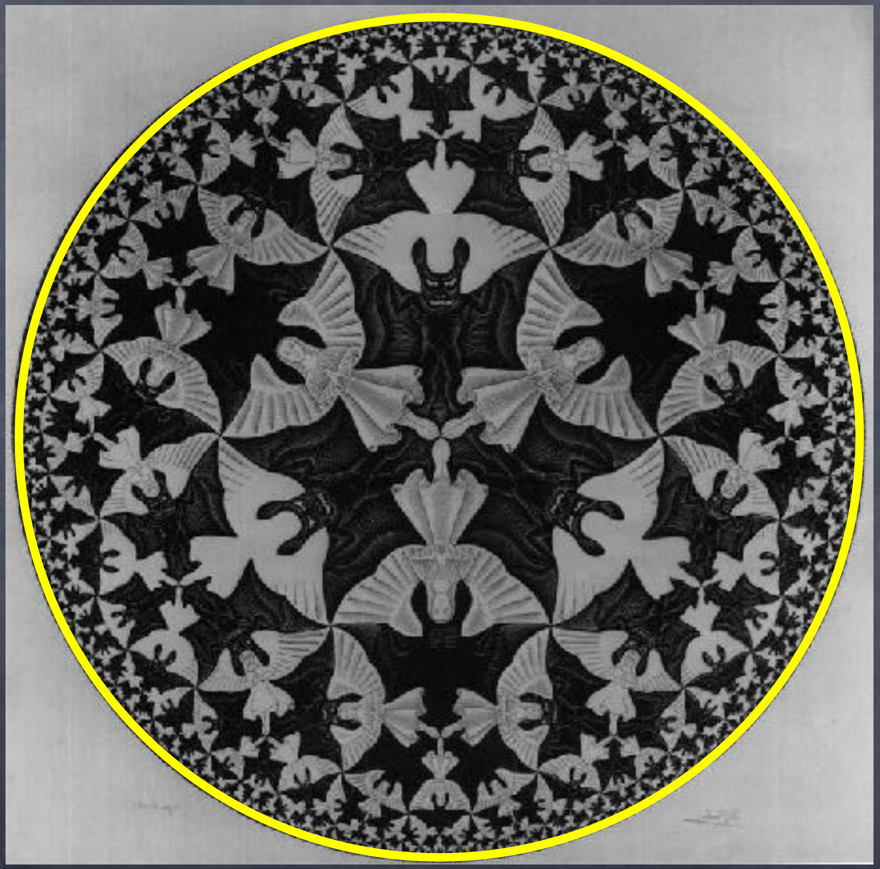
gravity predicts that at strong coupling

$$\frac{\eta}{S} = \frac{1}{4\pi} = \frac{\hbar}{4\pi k_B}$$

Conjectured to be a lower bound.

Gravity from conformal field theory

My main interest: the CFT is well-defined and (relatively) well-understood. Use it as a definition of quantum gravity.



At first sight the CFT has nothing to do with gravity. Can we describe bulk excitations (fields) in terms of the CFT?

Take a free massless scalar field ϕ in AdS_2.

Lorentzian AdS

$$ds^2 = \frac{R^2}{z^2}\left(-dt^2 + dz^2\right)$$

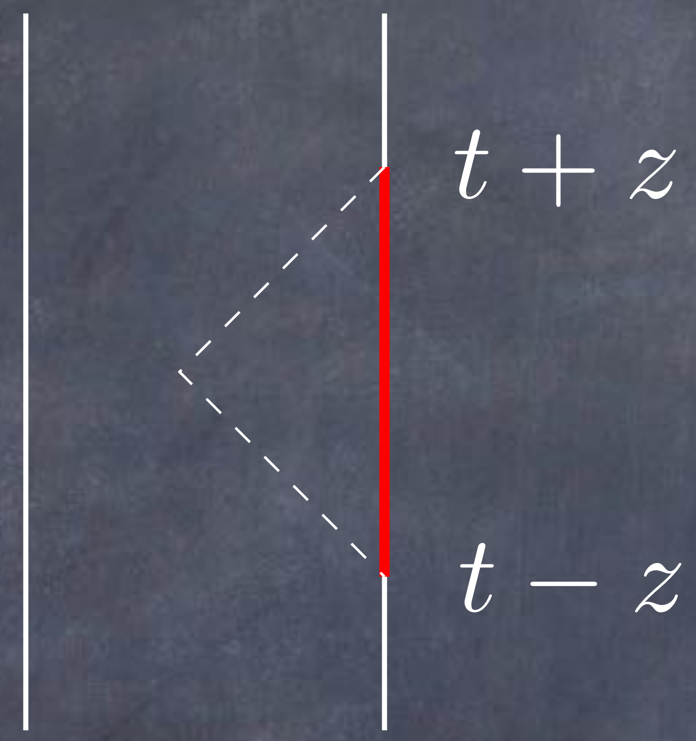
wave eqn $\qquad \Box\phi = 0$

bdy cond $\qquad \phi(t,z) \sim z\,\mathcal{O}(t) \qquad$ as $z \to 0$

identify $\mathcal{O}$ with a dimension-1 operator in the CFT

Can we express ϕ in terms of $\mathcal{O}$?

Solution: $$\phi(t,z) = \frac{1}{2}\int_{t-z}^{t+z} dt'\,\mathcal{O}(t')$$



support at spacelike separation

Works for bulk gauge fields and gravity as well.

$$A_\mu(x,z) = \int K\, j_\mu(x)$$

$$h_{\mu\nu}(x,z) = \int K\, T_{\mu\nu}(x)$$

What about interacting fields?

In higher dimensions, when there's a weakly-coupled bulk description (a CFT with a 1/N expansion), we know how to proceed.

In perturbation theory

$$\phi = \int K\mathcal{O} + \sum_i a_i \int K_{\Delta_i}\mathcal{O}_i$$

The coefficients a_i can be fixed, order-by-order in perturbation theory, by requiring bulk locality.

Kabat, Lifschytz, Lowe

What happens beyond perturbation theory?

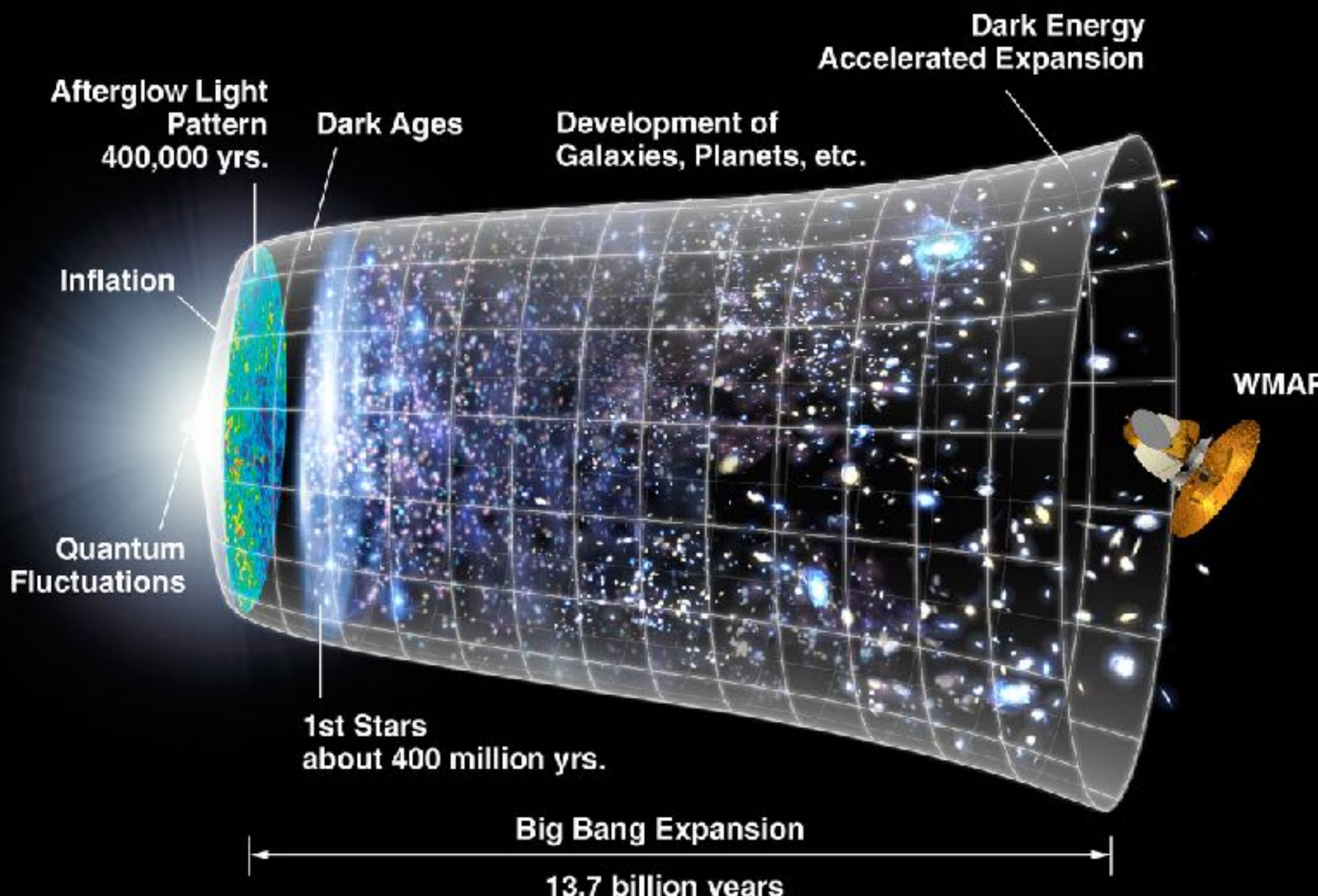
Cosmology?

Maybe AdS / CFT is just an example of something bigger at work.

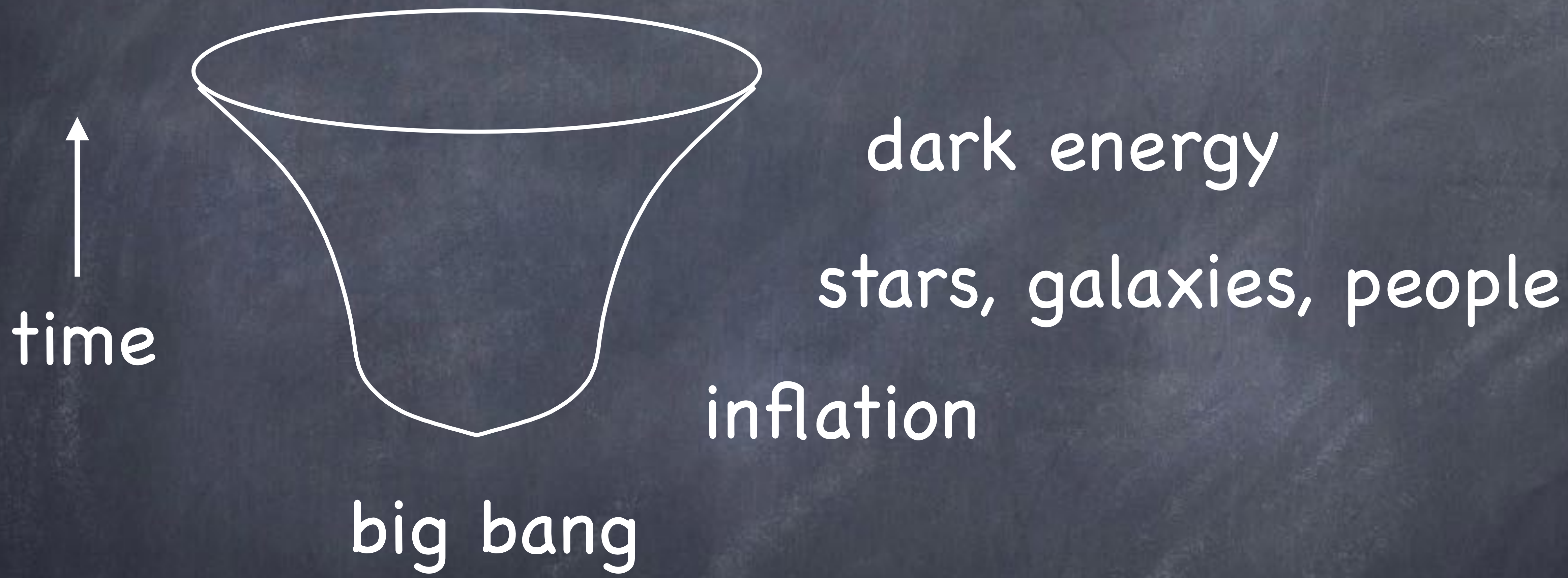
> "Holographic principle" - a gravitational system can be completely described by theory on the boundary.
>
> 't Hooft, Susskind

Does this apply to our universe?

Maybe. Where's the boundary?



If this accelerated expansion continues forever the history of the universe looks like



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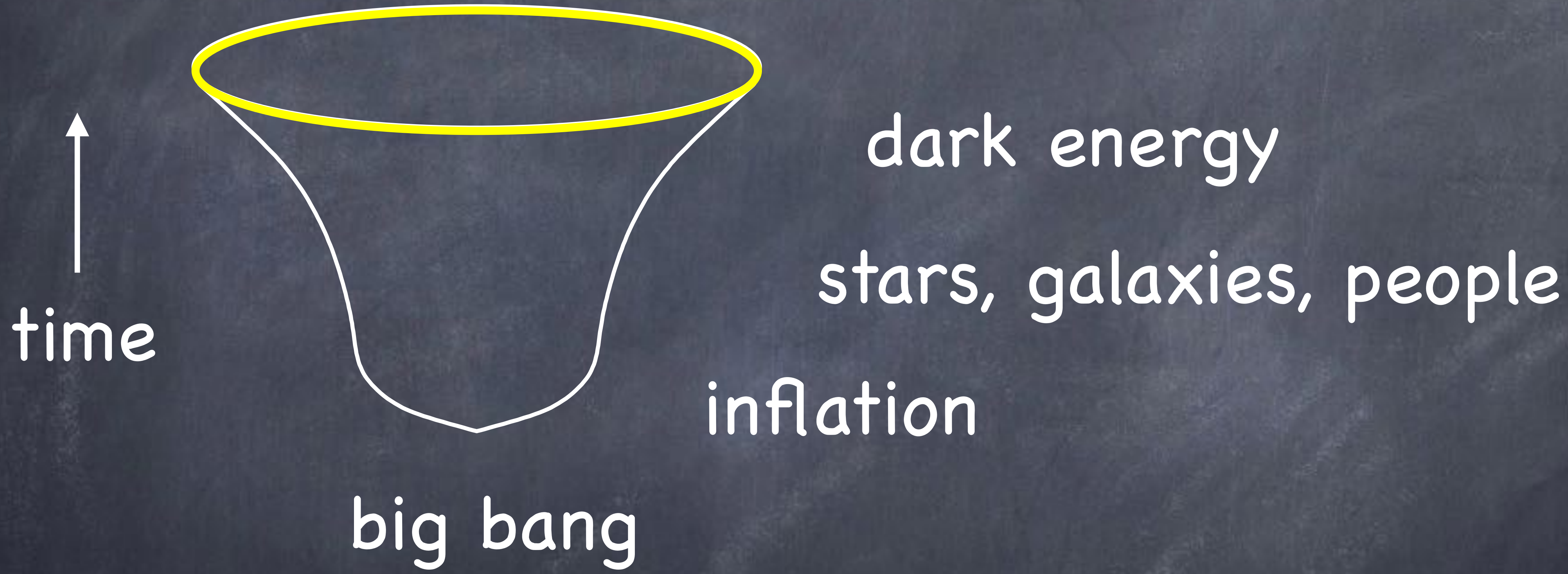

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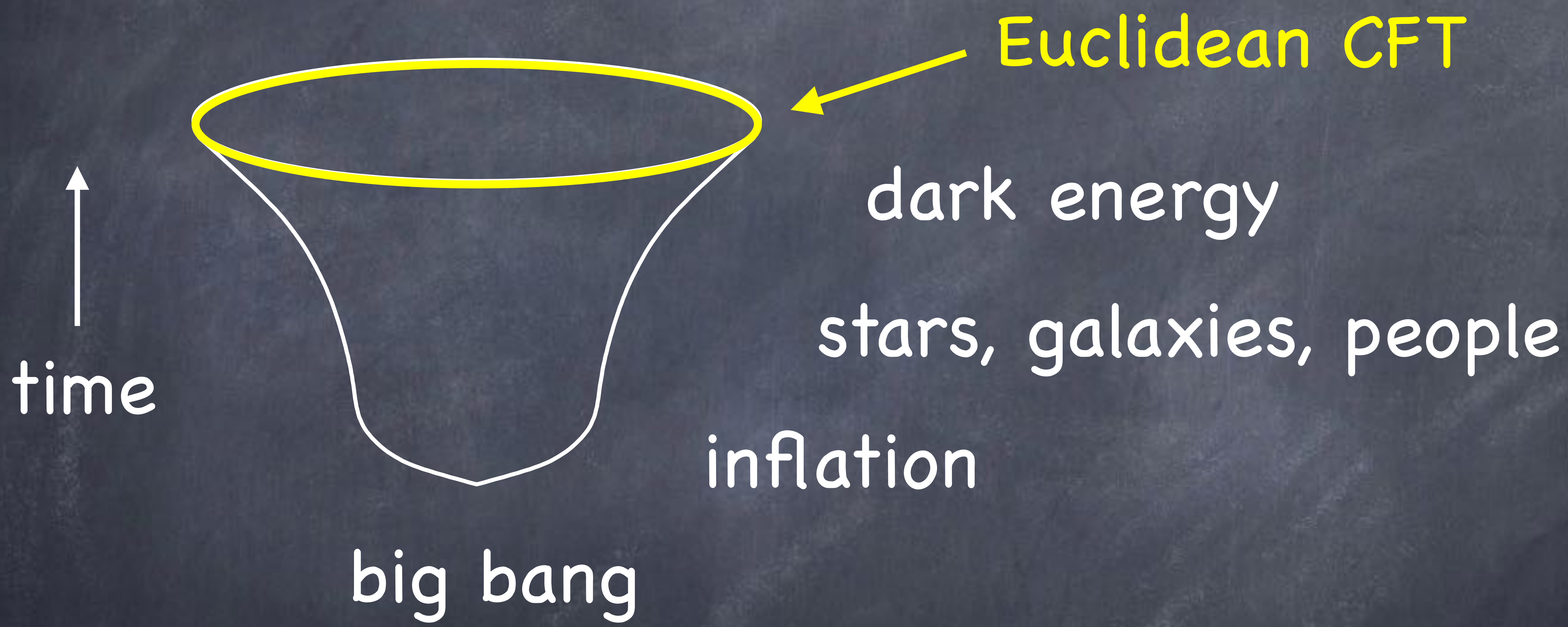

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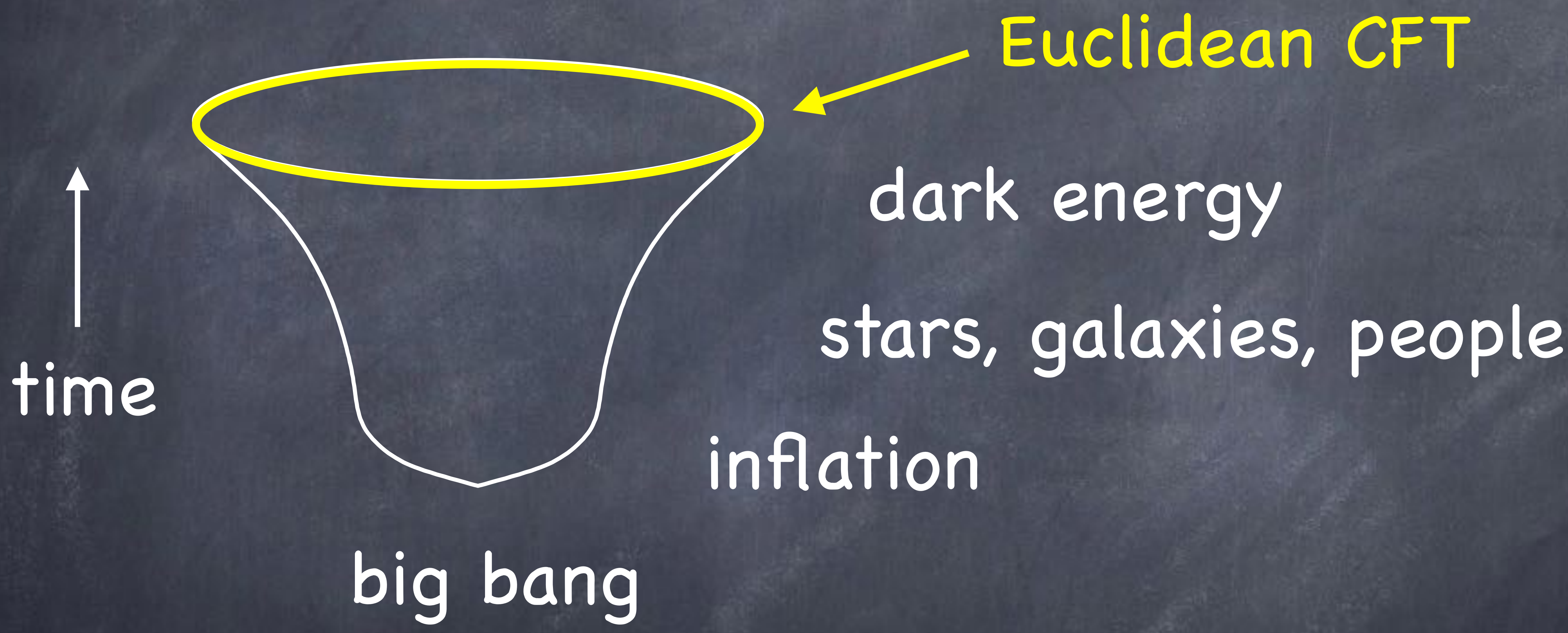

Our universe is approaching the symmetries of a Euclidean CFT.

We'd like to know

- ⋆ What's the CFT?
- ⋆ Do formulas like $\phi = \int K\,\mathcal{O}$ describe everyday life?

Thank you!