SOLUTION TO HW 4

1) a) Let $f(x)=x^3$ By the definition of derivative,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^3 - x^3}{h} =$$

$\boxed{\text{Hint}}$

$$= \lim_{h\to 0} \frac{(\cancel{x}+h-\cancel{x})\left((x+h)^2 + (x+h)x + x^2\right)}{h} =$$

$$= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}} \left(\underbrace{(x+h)^2}_{\downarrow\, x^2} + \underbrace{(x+h)x}_{\downarrow\, x^2} + \underbrace{x^2}_{\downarrow\, x^2}\right) =$$

$$= x^2 + x^2 + x^2 =$$

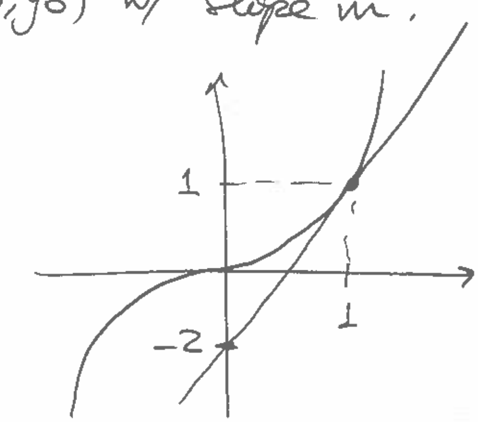
$$= 3x^2$$

b) • the slope of the tangent line to the graph of $f(x)=x^3$ at the point $(1,1)$ is: $f'(1) = 3 \cdot 1^2 = 3$ ("m")

• Equation of line through point (x_0, y_0) w/ slope m:
$y - y_0 = m(x - x_0)$

• Plug the above m to get:

$y - 1 = 3(x-1) \Rightarrow \boxed{y = 3x - 2}$



2) a) $\frac{d}{dx}(x^4 - 3x^3 + 2x^2 + x - 1) = 4x^3 - 9x^2 + 4x + 1$

b) $\frac{d}{dx}\left(\sqrt{x} + \frac{\sqrt{3}}{\sqrt{x}} + \sqrt{3}\right) = \frac{1}{2}x^{-\frac{1}{2}} + \sqrt{3}\cdot\left(-\frac{1}{2}x^{-3/2}\right)$

$= \frac{1}{2\sqrt{x}} - \frac{\sqrt{3}}{2x\sqrt{x}}$

c) $\frac{d}{d\theta}(\sin\theta + \cos\theta) = \cos\theta - \sin\theta$

d) $\frac{d}{dt}(e^t + 4t^2 + 3) = e^t + 8t$

e) $\frac{d}{dt}\left(\frac{1}{t^2} - 4\sin t + t^{4/5}\right) = -2t^{-3} - 4\cos t + \frac{4}{5}t^{-1/5}$

$= -\frac{2}{t^3} - 4\cos t + \frac{4}{5t^{1/5}}$.