MAT175, Fall 2018

Practice Problems for Midterm 2

1. Find all the critical points of the following functions:
   a) $f(x) = x^4 - 4x + 1$
   b) $f(x) = x^3 - 12x + 2$
   c) $f(x) = x^2 + x - 5$
   d) $f(x) = |x - 2|$
   e) $f(x) = \pi e^{x^2}$
   f) $f(x) = 2^x$

2. Determine where the functions in Problem 1 are increasing and decreasing.

3. Determine where the functions in Problem 1 are concave up and concave down.

4. Determine whether each critical point found in Problem 1 is a local minimum, a local maximum, or neither.

5. Find the global minimum and global maximum on the interval $I = [-3, 3]$ of each of the functions in Problem 1.

6. What are the global minimum and global maximum of $f(x) = e^x \cos x$ on $I = [0, 2\pi]$?

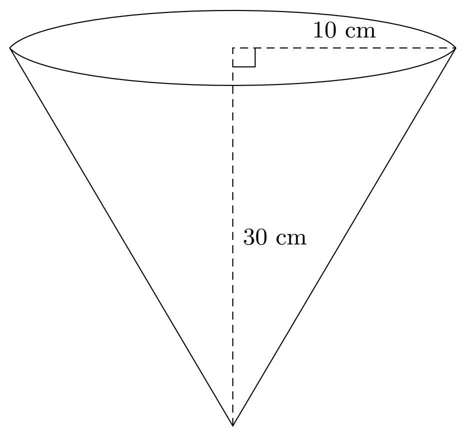
7. Find all the local minima and local maxima of the function $f(x) = 5x^{4/5} - x^2$.

8. The height of a rectangle is increasing at a rate of 8 cm/s and its width is increasing at a rate of 3 cm/s.
   a) When the height is 20 cm and the width is 10cm, how fast is the area of the rectangle increasing?
   b) When the height is 20 cm and the width is 10cm, how fast is the perimeter of the rectangle increasing?

9. A page of a book is to have a total area of 150 cm^2, with 1 cm margins at the top and sides and a 2 cm margin at the bottom. Find the dimensions in cm of the page that has largest possible printed area.

10. A boat is pulled in to a dock by a rope with one end attached to the front of the boat and the other end passing through a ring attached to the dock at a point 5 ft higher than the front of the boat. The rope is being pulled through the ring a‘ate of 0.6 ft/sec. How fast is the boat approaching the dock when 13 ft of rope are out?

11. Water is poured into a conical container at the rate of 10 cm^3/sec. The cone points directly down, and it has a height of 30 cm and a base radius of 10 cm. How fast is the water level rising when the water is 3 cm deep (at its deepest point)?



12. A square swimming pool will be built so that, when completely full, it will hold 32,000 m^3 of water. What are the dimensions (depth and length of the sides) of the swimming pool that will cost the least to cover with tiles?

13. Compute the following indefinite integrals:

a) $\displaystyle\int x^3 - 3x^2 + x + 1 \;\, \mathrm{d}x$

b) $\displaystyle\int 2\sin x - 3\cos x + 5 \;\, \mathrm{d}x$

c) $\displaystyle\int \frac{t^4 + t^2 + 1}{\sqrt{t}} \;\, \mathrm{d}t$

d) $\displaystyle\int e^x - x^e \;\, \mathrm{d}x$

e) $\displaystyle\int \frac{3}{y} - \frac{y}{3} \;\, \mathrm{d}y$

14. Compute the following definite integrals (you can use your answers to Problem 13):

a) $\int_0^1 x^3 - 3x^2 + x + 1 \ \mathrm{d}x$

b) $\int_0^\pi 2\sin x - 3\cos x + 5 \ \mathrm{d}x$

c) $\int_1^4 \frac{t^4 + t^2 + 1}{\sqrt{t}} \ \mathrm{d}t$

d) $\int_0^1 e^x - x^e \ \mathrm{d}x$

e) $\int_1^e \frac{3}{y} - \frac{y}{3} \ \mathrm{d}y$

15. Suppose that $\int_0^7 f(x)\,\mathrm{d}x = 8$, $\int_5^8 f(x)\,\mathrm{d}x = 1$, and $\int_7^8 f(x)\,\mathrm{d}x = -2$. Compute $\int_0^5 f(x)\,\mathrm{d}x$.

16. Find the area under the curve $y = 6x^2 + 2$ from $x = 0$ to $x = 2$.

17. Use Riemann sums to show that $\int_0^1 x^4 \ \mathrm{d}x = \frac{1}{5}$, using the fact that

$$\sum_{i=0}^{n} i^4 = 1^4 + 2^4 + 3^4 + \cdots + n^4 = \frac{n^5}{5} + \frac{n^4}{2} + \frac{n^3}{3} - \frac{n}{30}$$

18. Write, but do not evaluate, a (left) Riemann sum with 5 terms for the integral $\int_1^2 \frac{\sin x}{x} \ \mathrm{d}x$

19. Solve the Initial Value Problem

$$\begin{cases} \dfrac{\mathrm{d}y}{\mathrm{d}x} = x^2 - 6\cos x \\ \\ y(0) = 0 \end{cases}$$

20. Solve the Initial Value Problem

$$\begin{cases} \dfrac{\mathrm{d}s}{\mathrm{d}t} = t^{1/3} + e^t \\ \\ s(0) = \sqrt{3} \end{cases}$$

21. Compute the following derivatives:

a) $\dfrac{\mathrm{d}}{\mathrm{d}x}\displaystyle\int_0^x \sqrt{1+t^4}\ \mathrm{d}t$

b) $\dfrac{\mathrm{d}}{\mathrm{d}x}\displaystyle\int_0^{x^2} \sqrt{1+t^4}\ \mathrm{d}t$

c) $\dfrac{\mathrm{d}}{\mathrm{d}x}\displaystyle\int_{-\pi}^{\cos x} 4e^s\ \mathrm{d}s$

d) $\dfrac{\mathrm{d}}{\mathrm{d}z}\displaystyle\int_1^{1/z^2} \ln(y)\ \mathrm{d}y$

22. Use substitutions to compute the following indefinite integrals:

a) $\displaystyle\int 4x\cos(x^2)\ \mathrm{d}x$

b) $\displaystyle\int xe^{-x^2}\ \mathrm{d}x$

c) $\displaystyle\int z\sqrt{4-z^2}\ \mathrm{d}z$

d) $\displaystyle\int e^x\sqrt{1-e^x}\ \mathrm{d}x$