MAT175, Spring 2019

Practice Problems for the Midterm Exam

1. Compute the following limits, or explain why they do not exist:

   a) $\lim_{x \to 0} \frac{2\sin(4x)}{3x}$

   b) $\lim_{x \to \pi/2} \frac{2\sin(4x)}{3x}$

   c) $\lim_{x \to 2} \frac{x^2 + 3x - 10}{x - 2}$

   d) $\lim_{x \to 2} \frac{x^2 - 4x + 8}{x - 2}$

   e) $\lim_{a \to 3} \frac{\sqrt{a+1} - 2}{a + 3}$

   f) $\lim_{a \to 3} \frac{\sqrt{a+1} - 2}{a - 3}$

   g) $\lim_{t \to 0} t^2 \cos\left(\frac{\pi}{t}\right)$

2. Compute the following limits at infinity, or explain why they do not exist:

   a) $\lim_{x \to +\infty} \frac{\sin(x)}{x}$

   b) $\lim_{x \to +\infty} \frac{x^3 + 2x + 1}{x^2 - 1}$

   c) $\lim_{x \to -\infty} \frac{x^3 + 2x + 1}{x^2 - 1}$

   d) $\lim_{x \to +\infty} \frac{3x^5 + x^3 + x + 1}{4x^5 - x^2 - 8}$

   e) $\lim_{x \to -\infty} \frac{3x^5 + x^3 + x + 1}{4x^5 - x^2 - 8}$

   f) $\lim_{x \to +\infty} \frac{x^3 + 2x^2 + 1}{-2x^7 + 3x^4 + x^2}$

   g) $\lim_{x \to -\infty} \frac{x^3 + 2x^2 + 1}{-2x^7 + 3x^4 + x^2}$

3. Sketch the graph of the function $f(x)$ defined by

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 2 \\ 2 - x & \text{if } x > 2 \end{cases}$$

   Is $f(x)$ continuous at all points?

4. Sketch the graph of the function $f(x)$ defined by

$$f(x) = \begin{cases} e^x & \text{if } x \leq 0 \\ \cos(x) & \text{if } x > 0 \end{cases}$$

Is $f(x)$ continuous at all points?

5. What is the value of a that makes the following function continuous at all points?

$$f(x) = \begin{cases} \dfrac{2\sin(ax)}{x} & \text{if } x \leq 0 \\ x + a^2 + 1 & \text{if } x > 0 \end{cases}$$

6. Show that the derivative of $f(x) = 2x + 3$ is equal to $f'(x) = 2$ **using the definition as a limit** of a difference quotient.

7. Show that the derivative of $g(x) = 5x^2 - x$ is equal to $f'(x) = 10x - 1$ **using the definition as a limit** of a difference quotient.

8. Compute the first derivative of the following functions:

   a) $f(x) = 1 + 3\pi x^4 - 2x + e^x$

   b) $F(x) = \dfrac{2}{x} - \sqrt{5}x - 5\sqrt{x} + 5\cos(x-1)$

   c) $g(t) = \dfrac{1}{\sqrt{2t+2}} - te^{2t+1} - \tan(4t^2 + \frac{\pi}{4})$

   d) $A(\theta) = 3\sin\theta\cos\theta + e^\theta\cos(5\theta)$

   e) $q(s) = \dfrac{e^{s^2+1} - \sin(\sqrt{s})}{s^2+1}$

9. Find the tangent line to the graph of the functions below at the given point:

   a) $f(x) = 1 + 3\pi x^4 - 2x + e^x$, at the point $(0, 2)$

   b) $F(x) = \dfrac{2}{x} - \sqrt{5}x - 5\sqrt{x} + 5\cos(x-1)$, at the point $(1, 2-\sqrt{5})$

   c) $g(t) = \dfrac{1}{\sqrt{2t+2}} - te^{2t+1} - \tan(4t^2 + \frac{\pi}{4})$, at the point $(0, \frac{1}{\sqrt{2}} - 1)$

   d) $A(\theta) = 3\sin\theta\cos\theta + e^\theta\cos(5\theta)$, at the point $(0, 1)$

   e) $q(s) = \dfrac{e^{s^2+1} - \sin(\sqrt{s})}{s^2+1}$, at the point $(0, e)$

10. Suppose a particle moves along a straight line in such a way that its position (measured as distance from the origin) is given by $s(t) = t^2 - 2\sqrt{t^2+1} + e^t$ at time t. Find the velocity and acceleration of the particle when $t = 1$.

11. If the position (measured as height from the ground) of an object thrown straight up from an initial height of 32 feet is given by $s(t) = -16t^2 + 16t + 32$ at time t, find both the velocity and acceleration at the moment the object hits the ground.

12. The equation $x^2+y^3+y = 1$ implicitly defines a function $y = y(x)$ near the point $(1,0)$. Find the equation of the tangent line to the curve $x^2 + y^3 + y = 1$ at the point $(1,0)$.

13. Let $f(x) = x^3 + x - 2$ and $g(x)$ be its inverse function. Compute $g'(0)$ and $g''(0)$.

14. A ladder 15 ft long is resting against a vertical wall. The bottom of the ladder is sliding away from the wall at a speed of 3 ft per second. When the base of the ladder is 12 ft away from the wall, how fast is the top sliding down the wall?

15. A boat is pulled in to a dock by a rope with one end attached to the front of the boat and the other end passing through a ring attached to the dock at a point 5 ft higher than the front of the boat. The rope is being pulled through the ring at a rate of 0.6 ft/sec. How fast is the boat approaching the dock when 13 ft of rope are out?

16. Water is poured into a conical container at the rate of 10 cm^3/sec. The cone points directly down, and it has a height of 30 cm and a base radius of 10 cm. How fast is the water level rising when the water is 3 cm deep (at its deepest point)?