MAT 176 Solution to HW10

a) $\sum_{n=1}^{\infty} \frac{x^n}{2n}$ $L = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{x^{n+1}}{2(n+1)} \frac{2n}{x^n}\right| = \lim_{n\to\infty} \frac{|x|\, n}{n+1}$

$= |x| < 1 \iff -1 < x < 1$

need this to converge

Endpoints: $x = -1$: $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n} = \frac{1}{2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} = \frac{1}{2} \ln 2 < \infty$

converges (Alt. Harmonic Series)

$x = 1$; $\sum_{n=1}^{\infty} \frac{1}{2n} = \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{n} = +\infty$ Diverges (Harmonic Series)

A: $I = [-1, 1)$ or $-1 \le x < 1$

b) $\sum_{n=1}^{\infty} 3\left(\frac{x}{4}\right)^n = \frac{\frac{3x}{4}}{1 - x/4} = \frac{3x}{4 - x}$ if $\left|\frac{x}{4}\right| < 1 \iff -4 < x < 4$

Geometric Series

ratio

Endpoints: $x = -4$; $\sum_{n=1}^{\infty} 3(-1)^n$ diverges (n^{th} term test)

$x = 4$; $\sum_{n=1}^{\infty} 3$ diverges (n^{th} term test)

A: $I = (-4, 4)$ or $-4 < x < 4$

c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (x-2)^n}{n!}$

Ratio test: $\displaystyle L = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left| \frac{(x-2)^{n+1}}{(n+1)!} \cdot \frac{n!}{(x-2)^n} \right|$

$\displaystyle = \lim_{n\to\infty} \frac{|x-2|}{n+1} = 0 < 1$ <u>for all x</u>

$\Rightarrow \quad I = (-\infty, \infty)$, or $-\infty < x < \infty$

(no endpoints to check!)

d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (x-1)^n}{n^3 \cdot 2^n}$

need this for conv.

Root test $\displaystyle L = \lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \frac{|x-1|}{n^{\frac{3}{n}} \cdot 2} = \frac{|x-1|}{2} < 1$

$\displaystyle \lim_{n\to\infty} n^{3/n} = \lim_{n\to\infty} e^{\frac{3}{n}\ln n} = e^0 = 1$

$\Leftrightarrow |x-1| < 2 \Leftrightarrow -1 < x < 3$

-1 0 1 2 3

<u>Endpoints:</u> $x = -1$: $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(-2)^n}{n^3 \cdot 2^n} = \sum_{n=1}^{\infty} \frac{(-1)^{2n} \cancel{2^n}}{\cancel{2^n} \cdot n^3} = \sum_{n=1}^{\infty} \frac{1}{n^3} < \infty$

converges.

p-series $p=3$.

$x = 3$: $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \cancel{2^n}}{\cancel{2^n} \cdot n^3} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^3} < \infty$

converges.

<u>A:</u> $I = [-1, 3]$ or $-1 \le x \le 3$

e) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n (x+3)^n}{\sqrt{n}}$

Ratio test: $\displaystyle L = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{2^{n+1}|x+3|^{n+1}}{\sqrt{n+1}} \frac{\sqrt{n}}{2^n|x+3|^n}$

$\displaystyle = \lim_{n\to\infty} 2|x+3| \cdot \sqrt{\frac{n}{n+1}} = 2|x+3| < 1$

need this for conv.

$\displaystyle \Rightarrow \quad |x+3| < \frac{1}{2} \quad \Rightarrow \quad -\frac{7}{2} < x < -\frac{5}{2}$

$-\frac{7}{2} \quad -3 \quad -\frac{5}{2}$

Endpoints: $x = -\frac{7}{2} = -3 - \frac{1}{2}$

$$\sum_{n=1}^{\infty} \frac{2^n\left(-\frac{1}{2}\right)^n}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} < \infty$$

Alt. Series test (converges)

$x = -\frac{5}{2} = -3 + 1/2$

$$\sum_{n=1}^{\infty} \frac{2^n \cdot \left(\frac{1}{2}\right)^n}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} = +\infty$$

diverges, p-series $p = \frac{1}{2}$

A: $I = \left[-\frac{7}{2}, -\frac{5}{2}\right)$ or $-\frac{7}{2} \le x < -\frac{5}{2}$.