

a) $\sum_{n=1}^{\infty} \frac{x^n}{2^n}$ $L = \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{x^{n+1}}{2^{n+1}} \cdot \frac{2^n}{x^n} \right| = \lim_{n \rightarrow \infty} \frac{|x|n}{n+1}$

$= |x| < 1 \iff -1 < x < 1$

need this to converge

Endpoints:

$x = -1: \sum_{n=1}^{\infty} \frac{(-1)^n}{2^n} = \frac{1}{2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} = \frac{1}{2} \ln 2 < \infty$
converges (Alt. Harmonic Series)

$x = 1: \sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{n} = +\infty$ Diverges
 (Harmonic Series)

A: $I = [-1, 1)$ or $-1 \leq x < 1$

b) $\sum_{n=1}^{\infty} 3\left(\frac{x}{4}\right)^n = \frac{\frac{3x}{4}}{1 - x/4} = \frac{3x}{4-x}$ $\iff \left| \frac{x}{4} \right| < 1 \iff -4 < x < 4$

Geometric Series

ratio

Endpoint: $x = -4: \sum_{n=1}^{\infty} 3(-1)^n$ diverges (n^{th} term test)

$x = 4: \sum_{n=1}^{\infty} 3$ diverges (n^{th} term test)

A: $I = (-4, 4)$ or $-4 < x < 4$

$$c) \sum_{n=1}^{\infty} \frac{(-1)^n (x-2)^n}{n!}$$

Ratio test: $L = \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{(x-2)^{n+1}}{(n+1)!} \frac{n!}{(x-2)^n} \right|$
 $= \lim_{n \rightarrow \infty} \frac{|x-2|}{n+1} = 0 < 1$ for all x.

$\Rightarrow I = (-\infty, \infty)$, or $-\infty < x < \infty$
 (no endpoints to check!)

$$d) \sum_{n=1}^{\infty} \frac{(-1)^n (x-1)^n}{n^3 \cdot 2^n}$$

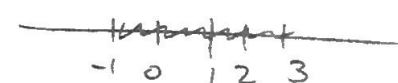
need this for conv.

Root test $L = \lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = \lim_{n \rightarrow \infty} \frac{|x-1|}{n^{\frac{3}{n}} \cdot 2} = \frac{|x-1|}{2} < 1$

$\lim_{n \rightarrow \infty} n^{\frac{3}{n}} = \lim_{n \rightarrow \infty} e^{\frac{3}{n} \ln n} = e^0 = 1$

$\Leftrightarrow |x-1| < 2 \Leftrightarrow -1 < x < 3$

Endpoints: $x = -1$: $\sum_{n=1}^{\infty} \frac{(-1)^n (-2)^n}{n^3 \cdot 2^n} = \sum_{n=1}^{\infty} \frac{(-1)^{2n} 2^n}{2^n \cdot n^3} = \sum_{n=1}^{\infty} \frac{1}{n^3} < \infty$



$x = 3$: $\sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{2^n \cdot n^3} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^3} < \infty$
 Converges. p -series $p=3$.

A: $I = [-1, 3]$ or $-1 \leq x \leq 3$

$$e) \sum_{n=1}^{\infty} \frac{2^n (x+3)^n}{\sqrt{n}}$$

$$\text{Ratio test: } L = \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \frac{2^{n+1} |x+3|^{n+1} \sqrt{n}}{\sqrt{n+1} 2^n |x+3|^n}$$

$$= \lim_{n \rightarrow \infty} 2|x+3| \cdot \sqrt{\frac{n}{n+1}} = 2|x+3| < 1$$

need this for conv.

$$\Rightarrow |x+3| < \frac{1}{2} \Rightarrow -\frac{7}{2} < x < -\frac{5}{2}$$

~~Interval~~

$$-\frac{7}{2} \quad -3 \quad -\frac{5}{2}$$

Endpoints: $x = -\frac{7}{2} = -3 - \frac{1}{2}$

$$\sum_{n=1}^{\infty} \frac{2^n \left(-\frac{1}{2}\right)^n}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} < \infty \quad \text{Alt. Series test (converges)}$$

$$x = -\frac{5}{2} = -3 + \frac{1}{2} \quad \sum_{n=1}^{\infty} \frac{2^n \left(\frac{1}{2}\right)^n}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} = +\infty \quad \text{diverges, } p\text{-series } p = \frac{1}{2}$$

A: $I = \left[-\frac{7}{2}, -\frac{5}{2}\right)$ or $-\frac{7}{2} \leq x < -\frac{5}{2}$