MAT 176

Solution to HW 11

$$\begin{cases} y' + xy = 0 \\ y(0) = 1 \end{cases}$$

$$y(x) = \sum_{n=0}^{+\infty} a_n x^n \Rightarrow y'(x) = \sum_{n=1}^{\infty} n a_n x^{n-1}$$

$$0 = y' + xy = \sum_{n=1}^{\infty} n a_n x^{n-1} + x \cdot \sum_{n=0}^{+\infty} a_n x^n = a_1 + \sum_{n=2}^{+\infty} n a_n x^{n-1} + \sum_{n=0}^{+\infty} a_n x^{n+1}$$

$$= a_1 + \sum_{n=1}^{+\infty} (n+1) a_{n+1} x^n + \sum_{n=1}^{+\infty} a_{n-1} x^n$$

$$= a_1 + \sum_{n=1}^{+\infty} \left[(n+1) a_{n+1} + a_{n-1}\right] x^n$$

$$\Rightarrow a_1 = 0$$

$(n+1) a_{n+1} + a_{n-1} = 0, \ \forall n \geq 1$; i.e., $\boxed{a_{n+1} \overset{\circledast}{=} -\frac{a_{n-1}}{n+1}, \ \forall n \geq 1}$

- $y(0) = 1 \Rightarrow a_0 = 1$
- $a_2 \overset{\circledast}{=} -\frac{a_0}{2} = -\frac{1}{2}$
- $a_3 \overset{\circledast}{=} -\frac{a_1}{3} = 0$
- $a_4 \overset{\circledast}{=} -\frac{a_2}{4} = \frac{1}{4.2}$
- $a_5 \overset{\circledast}{=} -\frac{a_3}{5} = 0$

⋮

so: $a_1 = a_3 = a_5 = a_7 = a_9 = \dots = 0$
(All <u>odd</u> order coeff. vanish)

$$a_{2n} = (-1)^n \frac{1}{(2n)(2n-2)(2n-4)\dots 2.1}$$

$$= (-1)^n \frac{1}{2^n n!}$$

→

$$\boxed{y(x) = \sum_{n=0}^{+\infty} (-1)^n \frac{x^{2n}}{2^n n!}}$$

<u>Note</u>: $y(x) = \sum_{n=0}^{+\infty} \left(-\frac{x^2}{2}\right)^n \cdot \frac{1}{n!} = \underline{\underline{e^{-\frac{x^2}{2}}}}$

Indeed, $y(x) = e^{-x^2/2}$ satisfies $y(0) = 1$ and

$y'(x) = -x e^{-x^2/2}$ so $y' + xy = 0$.