Solutions to HW 3

a) $$\int \sqrt{16-4t^2}\,dt = \int \sqrt{4(4-t^2)}\,dt = 2\int \sqrt{4-t^2}\,dt$$

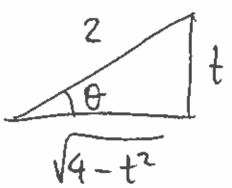
<u>Trig. sub:</u> $t = 2\sin\theta$, $dt = 2\cos\theta\,d\theta$

$$= 2\int \sqrt{4-4\sin^2\theta}\,.\,2\cos\theta\,d\theta$$

$$= 8\int \cos^2\theta\,d\theta = 8\int \frac{1+\cos 2\theta}{2}\,d\theta = 8\left(\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right) + C$$

$$= 4\theta + 2\sin 2\theta + C = \boxed{4\arcsin\left(\frac{t}{2}\right) + t\sqrt{4-t^2} + C}$$

$\theta = \arcsin\left(\frac{t}{2}\right)$



$$\sin\theta = \frac{t}{2} \qquad \cos\theta = \frac{\sqrt{4-t^2}}{2}$$

$$\sin 2\theta = 2\sin\theta\cos\theta = 2\,.\,\frac{t}{2}\,.\,\frac{\sqrt{4-t^2}}{2} = \frac{t\sqrt{4-t^2}}{2}.$$

b) $$\frac{x^2+3x+2}{x(x^2+2)^2} \overset{\text{Partial fractions}}{=} \frac{A}{x} + \frac{Bx+C}{x^2+2} + \frac{Dx+E}{(x^2+2)^2}$$

$$\Rightarrow A(x^2+2)^2 + (Bx+C).x.(x^2+2) + (Dx+E)x = x^2+3x+2$$

$$A(x^4+4x^2+4) + Bx^4 + 2Bx^2 + Cx^3 + 2Cx + Dx^2 + Ex = x^2+3x+2$$

$$(A+B)x^4 + Cx^3 + (4A+2B+D)x^2 + (2C+E)x + 4A = x^2+3x+2$$

$\Longrightarrow$

$$\begin{cases} A+B=0 & \Rightarrow B=-1/2 \\ C=0 & \Rightarrow C=0 \\ 4A+2B+D=1 & \Rightarrow 2-1+D=1 \Rightarrow D=0 \\ 2C+E=3 & \Rightarrow E=3 \\ 4A=2 & \Rightarrow A=1/2 \end{cases}$$

$$\Rightarrow \frac{x^2+3x+2}{x(x^2+2)^2} = \frac{1}{2}\cdot\frac{1}{x} - \frac{1}{2}\cdot\frac{x}{x^2+2} + \frac{3}{(x^2+2)^2}$$

$$\Rightarrow \int \frac{x^2+3x+2}{x(x^2+2)^2}\,dx = \frac{1}{2}\int\frac{1}{x}\,dx - \frac{1}{2}\int\frac{x}{x^2+2}\,dx + 3\int\frac{dx}{(x^2+2)^2}$$

$$\boxed{= \frac{1}{2}\ln|x| - \frac{1}{4}\ln(x^2+2) + 3\left(\frac{\sqrt{2}}{8}\arctan\left(\frac{x}{\sqrt{2}}\right) + \frac{1}{4}\,\frac{x}{x^2+2}\right) + C}$$

b/c:

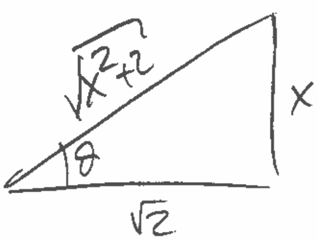
$$\int\frac{dx}{(x^2+2)^2} = \int\frac{\sqrt{2}\sec^2\theta\,d\theta}{(2\tan^2\theta+2)^2} = \frac{\sqrt{2}}{4}\int\frac{\sec^2\theta}{\sec^4\theta}\,d\theta = \frac{\sqrt{2}}{4}\int\cos^2\theta\,d\theta$$

$x=\sqrt{2}\tan\theta$

$dx=\sqrt{2}\sec^2\theta\,d\theta$

$$= \frac{\sqrt{2}}{4}\int\frac{1+\cos 2\theta}{2}\,d\theta = \frac{\sqrt{2}}{4}\left(\frac{\theta}{2}+\frac{\sin 2\theta}{4}\right)+C$$

$$= \frac{\sqrt{2}}{8}\theta + \frac{2\sqrt{2}\sin\theta\cos\theta}{16} + C$$



$$= \frac{\sqrt{2}}{8}\arctan\left(\frac{x}{\sqrt{2}}\right) + \frac{\sqrt{2}}{8}\,\frac{x}{\sqrt{x^2+2}}\cdot\frac{\sqrt{2}}{\sqrt{x^2+2}} + C$$

$$\boxed{= \frac{\sqrt{2}}{8}\arctan\left(\frac{x}{\sqrt{2}}\right) + \frac{1}{4}\,\frac{x}{x^2+2} + C}$$